1. Describe each of the following rotations using roll-pitch-yaw angles $(\phi,\ \theta,\ \psi)$, angle-axis $\left(\theta,\ \vec{k}\right)$ and quaternion $\bar{q}$.

   (a) no rotation

   (b) rotation about x by $90°$

   (c) rotation about y by $180°$

   (d) rotation about z by $-90°$

2. Consider the quaternion $\bar{q}_{x,-90°}$ that describes a rotation about the x-axis by $-90°$.

   (a) Show the opposite rotation $\bar{q}_{x,90°}$ is its inverse (conjugate) using the definition.

   (b) Compose $\bar{q}_{x,-90°}$ with $\bar{q}_{x,90°}$ via multiplication $(\otimes)$ on the left and right. Do both yield the identity quaternion?

   (c) Given $\bar{q}_1^0 = \bar{q}_{x,-90°}$ describes the orientation of frame $\{1\}$ relative to frame $\{0\}$, recoordinatize the vector $\vec{v}^1 = [1,\ 1,\ 1]^T$ in frame $\{0\}$, i.e., use the quaternion to find $\vec{v}^0$.

3. How many multiplies and additions are needed for each of the following computations?

   (a) composition of rotations via rotation matrices, $C_1^2 C_0^1$

   (b) composition of rotations via quaternions, $\bar{q}_1^2 \otimes \bar{q}_0^1$

   (c) recoordinatization of a vector via rotation matrix, $C_1^2 \vec{r}^1$

   (d) recoordinatization of a vector via quaternion, $\bar{q}_1^2 \otimes \breve{r}^1 \otimes (\bar{q}_1^2)^{-1}$

4. Consider the time-varying rotation matrix $C_b^a = R_{z,\theta(t)}$ that describes the orientation of frame {b} as it rotates about frame {a}'s $z-$axis by angle $\theta(t)$. Determine the angular velocity as a skew-symmetric matrix (via $\Omega_{ab}^a = \dot{C}_b^a [C_b^a]^T$) and vector $\vec{\omega}_{ab}^a$. Do these answers make sense in light of the type of rotation between the frames?

5. Does the approach of the previous problem extend to quaternions, i.e., if we are given the time-varying quaternion $\bar{q}_b^a(t) = \bar{q}_{z,\theta(t)}$, can we use $\dot{\bar{q}}_b^a(t) = \frac{1}{2}[\breve{\omega}_{ab}^a \otimes]\bar{q}_b^a(t)$, which we derived in class, to find the corresponding angular velocity $\vec{\omega}_{ab}^a$? If so, find it.

6. Consider the time-varying coordinate transformation matrix C_b^n given below that describes the orientation of the body frame as it rotates with respect to the navigation frame.

$$C_b^n = \begin{bmatrix} \cos(t) & \sin(t)\sin(t^2) & \sin(t)\cos(t^2) \\ 0 & \cos(t^2) & -\sin(t^2) \\ -\sin(t) & \cos(t)\sin(t^2) & \cos(t)\cos(t^2) \end{bmatrix}$$

   (a) Determine the analytic form of the time-derivative of C_b^n (i.e., $\dot{C}_b^n = \frac{dC_b^n}{dt}$) via a term-by-term differentiation.

   (b) Develop MATLAB functions that accept "t" (i.e., time) as a numerical input and return C_b^n and $\dot{C}_b^n$, respectively, as numerical outputs.

   (c) Using the C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector $\vec{\omega}_{nb}^n$ at time $t = 0$ sec (**Hint**: you might want to compute Ω_{nb}^n first).

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^n$) has the instantaneous rotation occurred?

   (d) Using the C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector $\vec{\omega}_{nb}^n$ at time $t = 0.5$ sec.

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^n$) has the instantaneous rotation occurred?

   (e) Using C_b^n and $\dot{C}_b^n$ functions from above, compute the angular velocity vector $\vec{\omega}_{nb}^n$ at time $t = 1$ sec.

      i. What is the magnitude (i.e., $\dot{\theta}$, angular speed) of the angular velocity?

      ii. About what unit vector ($\vec{k}_{nb}^n$) has the instantaneous rotation occurred?

   (f) In practice, direct measurement of the angular velocity vector $\vec{\omega}_{nb}^n$ can prove challenging, so a finite-difference approach may be taken given two sequential orientations represented by $C_b^n(t)$ and $C_b^n(t+\Delta t)$ a small time Δt apart. Consider the approximate value of the angular velocity vector $\vec{\omega}_{nb}^n$ derived by using the finite difference

$$\dot{C}_b^n(t) \approx \frac{C_b^n(t+\Delta t) - C_b^n(t)}{\Delta t}$$

   at times $t = 0, 0.5,$ and 1 sec. Compare the "analytic" values for $\dot{\theta}$ and $\vec{k}_{nb}^n$ (found in parts b, c and d) with your approximations from the finite difference using $\Delta t = 0.1$ sec. How large are the errors?

   (g) How small does the sampling time (i.e., Δt) need to be to get a "good" (better than 99.9%) approximation of the angular speed (i.e., $\dot{\theta}$)?