EE 565: Position, Navigation, and Timing

Aided INS

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- Truth value

$$\vec{x}$$

- Measured value

$$\tilde{\vec{x}}$$

- Estimated or computed value

$$\hat{\vec{x}}$$

- Error

$$\delta\vec{x} = \vec{x} - \hat{\vec{x}}$$

Initially the accelerometer and gyroscope measurements, $\tilde{\vec{f}}^{\,b}_{ib}$ and $\tilde{\vec{\omega}}^{\,b}_{ib}$, respectively, will be modeled as

$$\tilde{\vec{f}}^{\,b}_{ib} = \vec{f}^{\,b}_{ib} + \Delta\vec{f}^{\,b}_{ib} = \hat{\vec{f}}^{\,b}_{ib} + \Delta\hat{\vec{f}}^{\,b}_{ib} \qquad (1)$$

$$\tilde{\vec{\omega}}^{\,b}_{ib} = \vec{\omega}^{\,b}_{ib} + \Delta\vec{\omega}^{\,b}_{ib} = \hat{\vec{\omega}}^{\,b}_{ib} + \Delta\hat{\vec{\omega}}^{\,b}_{ib} \qquad (2)$$

where $\vec{f}^{\,b}_{ib}$ and $\vec{\omega}^{\,b}_{ib}$ are the specific force and angular rates, respectively; and $\Delta\vec{f}^{\,b}_{ib}$ and $\Delta\vec{\omega}^{\,b}_{ib}$ represents the errors. In later lectures we will discuss more detailed description of these errors.

Accelerometers

$$\tilde{\vec{f}}\,^b_{ib} = \vec{b}_a + (\mathcal{I} + M_a)\vec{f}\,^b_{ib} + \vec{nl}_a + \vec{w}_a \tag{3}$$

Gyroscopes

$$\tilde{\vec{\omega}}\,^b_{ib} = \vec{b}_g + (\mathcal{I} + M_g)\vec{\omega}\,^b_{ib} + G_g\vec{f}\,^b_{ib} + \vec{w}_g \tag{4}$$

- Position error

$$\delta\vec{r}^{\,\gamma}_{\beta b} = \vec{r}^{\,\gamma}_{\beta b} - \hat{\vec{r}}^{\,\gamma}_{\beta b} \tag{5}$$

- Velocity error

$$\delta\vec{v}^{\,\gamma}_{\beta b} = \vec{v}^{\,\gamma}_{\beta b} - \hat{\vec{v}}^{\,\gamma}_{\beta b} \tag{6}$$

- Specific force errors

$$\delta\vec{f}^{\,b}_{ib} = \vec{f}^{\,b}_{ib} - \hat{\vec{f}}^{\,b}_{ib} \tag{7}$$

$$\Delta_e\vec{f}^{\,b}_{ib} = \Delta\vec{f}^{\,b}_{ib} - \Delta\hat{\vec{f}}^{\,b}_{ib} = -\delta\vec{f}^{\,b}_{ib} \tag{8}$$

- Angular rate errors

$$\delta\vec{\omega}^{\,b}_{ib} = \vec{\omega}^{\,b}_{ib} - \hat{\vec{\omega}}^{\,b}_{ib} \tag{9}$$

$$\Delta_e\vec{\omega}^{\,b}_{ib} = \Delta\vec{\omega}^{\,b}_{ib} - \Delta\hat{\vec{\omega}}^{\,b}_{ib} = -\delta\vec{\omega}^{\,b}_{ib} \tag{10}$$

Recall

$$\begin{pmatrix} \delta\dot{\vec{\psi}}^{e}_{eb} \\ \delta\dot{\vec{v}}^{e}_{eb} \\ \delta\dot{\vec{r}}^{e}_{eb} \end{pmatrix} = \begin{bmatrix} -\Omega^{e}_{ie} & 0_{3\times3} & 0_{3\times3} \\ -[\hat{C}^{e}_{b}\hat{\vec{f}}^{b}_{ib}\times] & -2\Omega^{e}_{ie} & \frac{2g_0(\hat{L}_b)}{r^{e}_{eS}(\hat{L}_b)}\frac{\hat{\vec{r}}^{e}_{eb}}{|\hat{\vec{r}}^{e}_{eb}|^2}(\hat{\vec{r}}^{e}_{eb})^T \\ 0_{3\times3} & \mathcal{I}_{3\times3} & 0_{3\times3} \end{bmatrix} \begin{pmatrix} \delta\vec{\psi}^{e}_{eb} \\ \delta\vec{v}^{e}_{eb} \\ \delta\vec{r}^{e}_{eb} \end{pmatrix} + \begin{bmatrix} 0 & -\hat{C}^{e}_{b} \\ -\hat{C}^{e}_{b} & 0 \\ 0 & 0 \end{bmatrix} \begin{pmatrix} \Delta_e\vec{f}^{b}_{ib} \\ \Delta_e\vec{\omega}^{b}_{ib} \end{pmatrix} \tag{11}$$

In reality there will be error terms in the sensor that can not be calibrated. These terms may be estimated. The error in the estimation of these terms may be expressed as

$$\Delta_e \vec{f}^{\,b}_{ib} = \Delta \vec{f}^{\,b}_{ib} - \Delta \hat{\vec{f}}^{\,b}_{ib} = F_{va}\delta\vec{x}_a + \vec{\varsigma}_a \quad (12)$$

$$\Delta_e \vec{\omega}^{\,b}_{ib} = \Delta \vec{\omega}^{\,b}_{ib} - \Delta \hat{\vec{\omega}}^{\,b}_{ib} = F_{\psi g}\delta\vec{x}_g + \vec{\varsigma}_g \quad (13)$$

These terms represent the difference between what we estimate the errors in the sensors to be (either through calibration or online estimation) and the actual errors in the sensor.

The matrics F_{va} and $F_{\psi g}$, depend on the needed level of complexity in modeling the errors. For example if we only model biases, e.g., $\delta\vec{x}_a = \delta\vec{b}_a$, then $F_{va} = \mathcal{I}$.
If more error terms are modeled, then most likely, we will end up with non-linear equations, and therefore linearization is necessary.

$$\delta\dot{\vec{x}}_a = F_{aa}\delta\vec{x}_a + \vec{w}_a \tag{14}$$

$$\delta\dot{\vec{x}}_g = F_{gg}\delta\vec{x}_g + \vec{w}_g \tag{15}$$

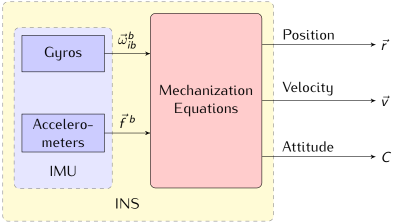
The matrics F_{aa} and F_{gg} are specific to accelerometers and the gyroscopes and there specific configuration within the IMU.

After state augmentation

$$
\begin{pmatrix} \delta\dot{\vec{\psi}}^{\,e}_{eb} \\ \delta\dot{\vec{v}}^{\,e}_{eb} \\ \delta\dot{\vec{r}}^{\,e}_{eb} \\ \delta\dot{\vec{x}}_a \\ \delta\dot{\vec{x}}_g \end{pmatrix} = \begin{bmatrix} -\Omega^e_{ie} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & -\hat{C}^e_b F_{\psi g} \\ -[\hat{C}^e_b \hat{\vec{f}}^{\,b}_{ib}\times] & -2\Omega^e_{ie} & \frac{2g_0(\hat{L}_b)}{r^e_{eS}(\hat{L}_b)} \frac{\hat{\vec{r}}^e_{eb}}{|\hat{\vec{r}}^e_{eb}|^2}(\hat{\vec{r}}^e_{eb})^T & -\hat{C}^e_b F_{va} & 0_{3\times3} \\ 0_{3\times3} & \mathcal{I}_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & F_{aa} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & F_{gg} \end{bmatrix} \begin{pmatrix} \delta\vec{\psi}^{\,e}_{eb} \\ \delta\vec{v}^{\,e}_{eb} \\ \delta\vec{r}^{\,e}_{eb} \\ \delta\vec{x}_a \\ \delta\vec{x}_g \end{pmatrix} +
$$

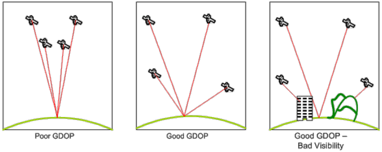
$$
\begin{bmatrix} -\hat{C}^e_b & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & -\hat{C}^e_b & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \mathcal{I}_{3\times3} & 0_{3\times3} \\ 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & 0_{3\times3} & \mathcal{I}_{3\times3} \end{bmatrix} \begin{pmatrix} \vec{\varsigma}_g \\ \vec{\varsigma}_a \\ 0_{3\times1} \\ \vec{w}_a \\ \vec{w}_g \end{pmatrix} \tag{16}
$$

$$
= F(t)\vec{x} + G\vec{w}
$$



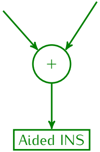
Advantages	Disadvantages
Immune to RF Jaming	Drifts
High data rate	Errors are time dependent
High accuracy in short term	Need Initialization

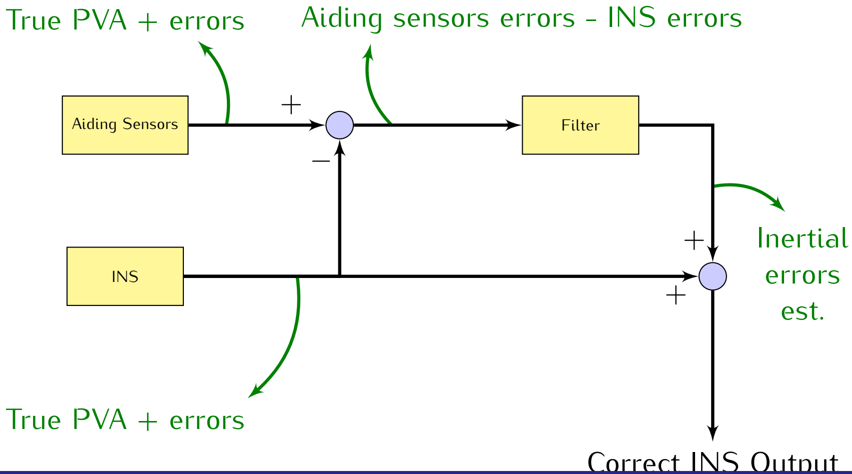
Aiding Sensor (e.g., GPS)



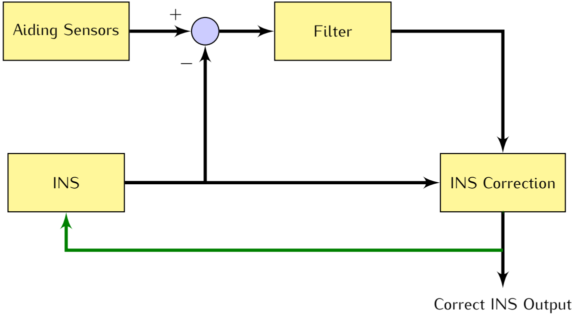
http://nptel.ac.in/courses/105104100/lectureB_11/B_11_3GDOP.htm

Advantages	Disadvantages
Errors time-indep.	Sensitive to RF Interference
No initialization	No attitude information




True PVA + errors
Aiding sensors errors - INS errors
Aiding Sensors
+
−
Filter
INS
+
+
Inertial errors est.
True PVA + errors
Correct INS Output

If error estimates are fedback to correct the INS mechanization, a reset of the state estimates becomes necessary.



Project Ahead

$$\hat{\vec{x}}_{k|k-1} = \mathbf{\Phi}_{k-1}\hat{\vec{x}}_{k-1|k-1} \quad (17)$$

$$\mathrm{P}_{k|k-1} = \mathrm{Q}_{k-1} + \mathbf{\Phi}_{k-1}\mathrm{P}_{k-1|k-1}\mathbf{\Phi}_{k-1}^{T} \quad (18)$$

Update

$$\hat{\vec{x}}_{k|k} = \hat{\vec{x}}_{k|k-1} + \mathrm{K}_k(\vec{z}_k - \mathrm{H}_k\hat{\vec{x}}_{k|k-1}) \quad (19)$$

$$P_{k|k} = (I - K_k H_k)\, P_{k|k-1}\, (I - K_k H_k)^T + K_k R_k K_k^T \quad (20)$$

$$\mathrm{K}_k = \mathrm{P}_{k|k-1}\mathrm{H}_k^T(\mathrm{H}_k\mathrm{P}_{k|k-1}\mathrm{H}_k^T + \mathrm{R}_k)^{-1} \quad (21)$$

Since the errors are being fedback to correct the INS, the state estimate must be reset after each INS correction.

$$\hat{\vec{x}}_{k|k-1} = 0 \tag{22}$$

$$\mathrm{P}_{k|k-1} = \mathrm{Q}_{k-1} + \boldsymbol{\Phi}_{k-1}\mathrm{P}_{k-1|k-1}\boldsymbol{\Phi}_{k-1}^T \tag{23}$$

$$\hat{\vec{x}}_{k|k} = \mathrm{K}_k\vec{z}_k \tag{24}$$

$$P_{k|k} = \left(I - K_k H_k\right) P_{k|k-1} \left(I - K_k H_k\right)^T + K_k R_k K_k^T \tag{25}$$

$$\mathrm{K}_k = \mathrm{P}_{k|k-1}\mathrm{H}_k^T(\mathrm{H}_k\mathrm{P}_{k|k-1}\mathrm{H}_k^T + \mathrm{R}_k)^{-1} \tag{26}$$

$$\Phi_{k-1} \approx \mathrm{I} + \mathrm{F}\Delta t \tag{27}$$

$$\mathrm{Q} = \begin{pmatrix} n_{rg}^2 \mathrm{I}_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & n_{ag}^2 \mathrm{I}_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & n_{bad}^2 \mathrm{I}_{3\times 3} & 0_{3\times 3} \\ 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & 0_{3\times 3} & n_{bgd}^2 \mathrm{I}_{3\times 3} \end{pmatrix} \tag{28}$$

where Δt is the sample time, n_{rg}^2, n_{ag}^2, n_{bad}^2, n_{bgd}^2 are the PSD of the gyro and accel random noise, and accel and gyro bias variation, respectively.

Assuming white noise, small time step, G is constant over the integration period, and the trapezoidal integration

$$Q_{k-1} \approx \frac{1}{2}\left[\boldsymbol{\Phi}_{k-1} G_{k-1} Q(t_{k-1}) G_{k-1}^T \boldsymbol{\Phi}_{k-1}^T + G_{k-1} Q(t_{k-1}) G_{k-1}^T\right] \Delta t \quad (29)$$