Lecture

Navigation Mathematics: Coordinate Frames

EE 565: Position, Navigation and Timing

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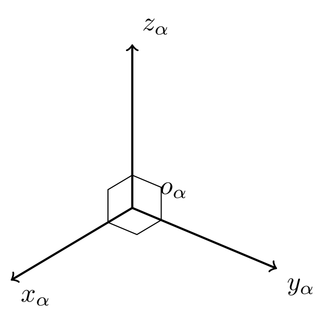
1 Coordinate Frames

Coordinate Frames

Right-hand Cartesian coordinate frame α has

1. origin o_α at which frame is located, and
2. orthonormal vectors $x_\alpha, y_\alpha, z_\alpha$ that serve as axes and indicate positive directions.



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Coordinate Frames

This definition implies

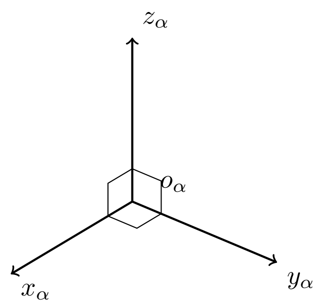
$$x_\alpha \cdot x_\alpha = y_\alpha \cdot y_\alpha = z_\alpha \cdot z_\alpha = 1$$

$$x_\alpha \cdot y_\alpha = y_\alpha \cdot z_\alpha = z_\alpha \cdot x_\alpha = 0$$

$$x_\alpha \times y_\alpha = z_\alpha$$

$$y_\alpha \times z_\alpha = x_\alpha$$

$$z_\alpha \times x_\alpha = y_\alpha$$



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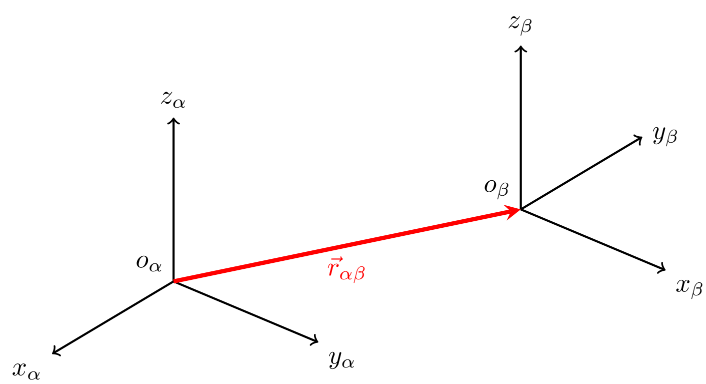
Coordinate Frames

Coordinate frames used as means to describe position and orientation/attitude of one frame with respect to another.



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2 Earth-Centered Inertial (ECI) Frame

Earth-Centered Inertial (ECI) Frame

ECI Frame

- defined as an inertial frame, i.e., it is assumed not to accelerate or rotate with respect to the universe
  - effects of earth's orbit around sun and motion of the galaxy are very small (smaller than can be measured with inertial sensors) and neglected
  - ECI will be attached to earth, but won't spin with earth
- inertial sensors measure "inertial" motion relative to ECI frame
  - Gyroscopes measure rate of change of orientation
  - Accelerometers measure linear acceleration
- referred to as i-frame

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ECI Frame

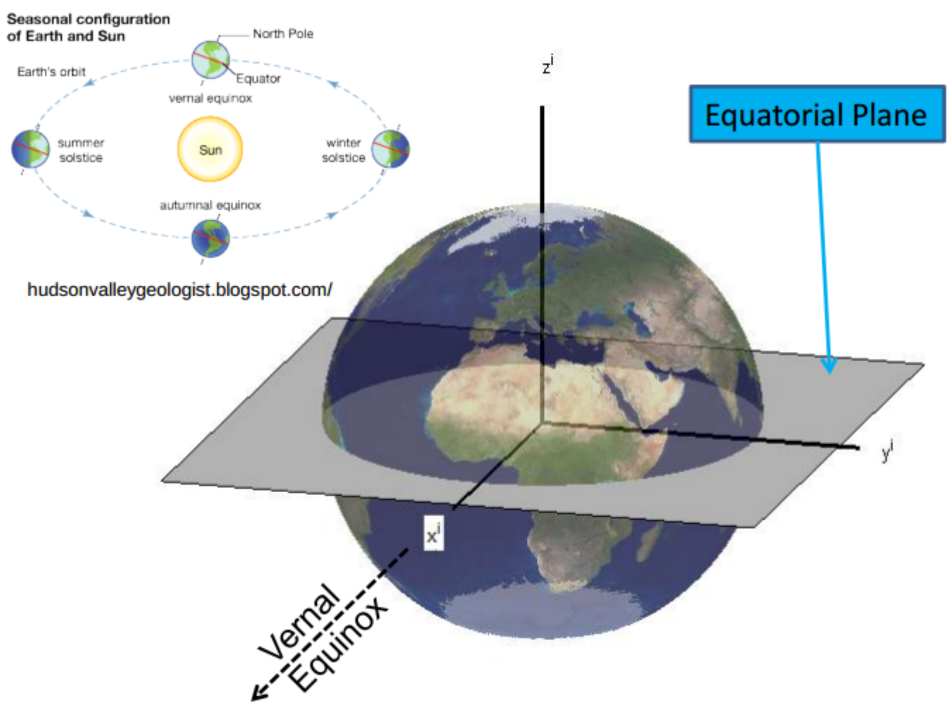
- origin o_i of ECI is located near the center of mass (center of ellipsoidal representation) of the earth
- z_i-axis points along the nominal axis of rotation of the earth
  - true north **not** magnetic north!
  - spin axis moves in circular path with radius of 15 meters, which we'll neglect and use average value

- x_i-axis lies in the equatorial plane and points from the earth to the sun at the vernal (spring) equinox (point in time when sun is in the equatorial plane)
  - defined by the intersection (a line) of the equatorial plane and the earth-sun orbital plane
- y_i-axis chosen to complete right hand coordinate system ($90°$ ahead of x_i in direction of earth's rotation)

The ECI coordinate frame does **not** *rotate with the earth* .6

ECI Frame

- o_i at earth's center
- z_i-axis points along the earth's axis of rotation
- x_i-axis points towards sun at vernal (spring) equinox
- y_i-axis completes a right hand coordinate system



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3 Earth-Centered Earth-Fixed (ECEF) Frame

Earth-Centered Earth-Fixed (ECEF) Frame

ECEF Frame

- **not** an inertial frame
- fixed with respect to the earth, i.e., attached to the earth and spins with earth
- referred to as e-frame .8

ECEF Frame

- origin o_e is located (nearly) at the center of the mass of the earth (co-located with ECI's o_i)
- z_e-axis points along the nominal axis of earth's rotation (same as ECI's z_i)
- x_e-axis lies at the intersection of the equatorial plane and the reference meridian plane (i.e., Greenwich/Prime Meridian)

- tied to concept of latitude and longitude
- x_e points from o_e towards $0°$ longitude and $0°$ latitude (a little west of central Africa)

- y_e-axis is chosen to complete right hand coordinate system

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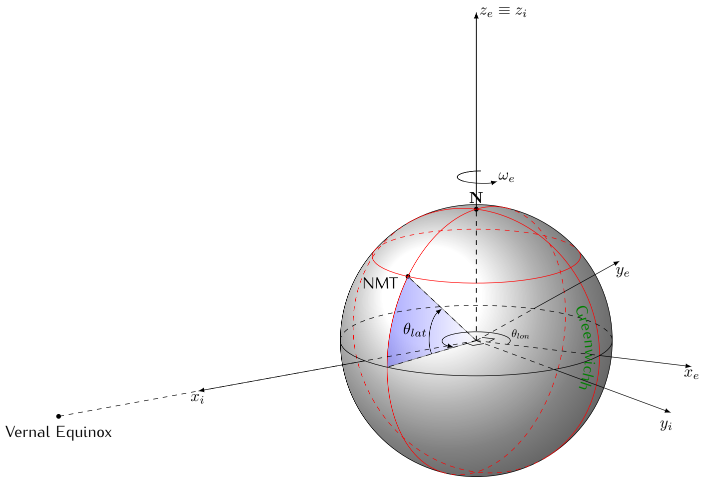
ECEF Frame

- z_e-axis points along axis of earth's rotation
- x_e-axis points towards zero latitude and zero longitude
- y_e-axis completes right hand coordinate system
- NMT's (lat, long) $\approx (34.07°, -106.9°) = (34.07°, 253.1°)$



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4 Local Navigation (Nav) Frame

Local Navigation (Nav) Frame

Nav Frame

- typically **not** fixed with respect to the earth, i.e., free to move, but has specified orientation
- also called geodetic, geographic, locally level, or tangential frame
- referred to as n-frame

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Nav Frame

- origin o_n is located at the center of mass of the body (e.g., air, land or sea vehicle) of interest
- z_n-axis points "down" normal to the earth's surface (approximately towards the center of the earth)
- x_n - y_n axes then constrained to lie in plane locally-level (tangential) to the earth's surface
  - x_n-axis points to the north pole
  - y_n-axis is chosen to complete right hand coordinate system
- frame's configuration is often referred to as the NED frame
  - $x_n \rightarrow$ North, $y_n \rightarrow$ East, and $z_n \rightarrow$ Down

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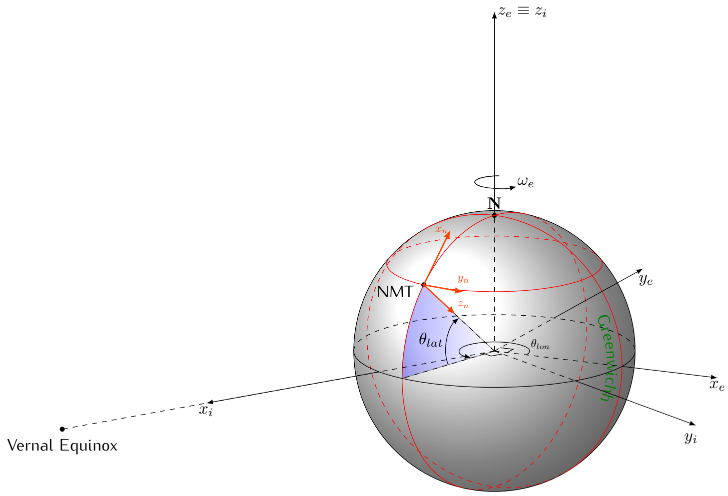
Nav Frame

- o_n on (potentially moving) body
- x_n-axis points north
- y_n-axis points east
- z_n-axis points "down"



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5 Body Frame

Body Frame

Body Frame

- attached to moving body (e.g., land, air or sea vehicle) and moves (position and orientation/attitute) with body
- origin o_b located at the center of mass of the body (co-located with Nav frame's o_n)
- x_b-axis points "forward" *wrt* moving body
- z_b-axis points loosely "down"
  - varies with the roll/pitch of the vehicle
- y_b-axis chosen to complete right hand coordinate system
- referred to as $b-$frame

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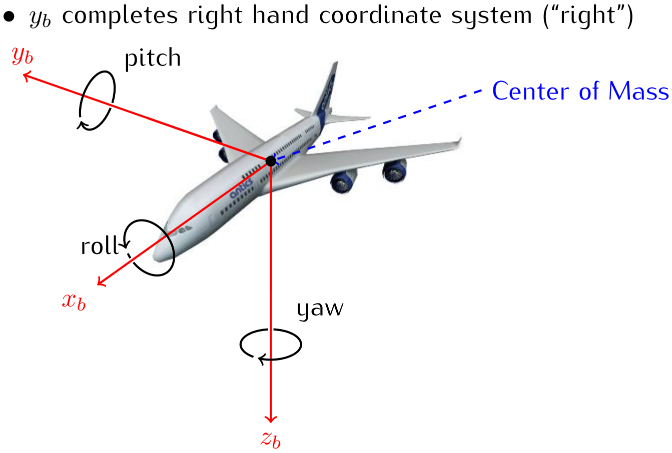
Body Frame

- body frame is fixed with respect to the vehicle
- x_b "forward"
- z_b "down"
- y_b completes right hand coordinate system ("right")



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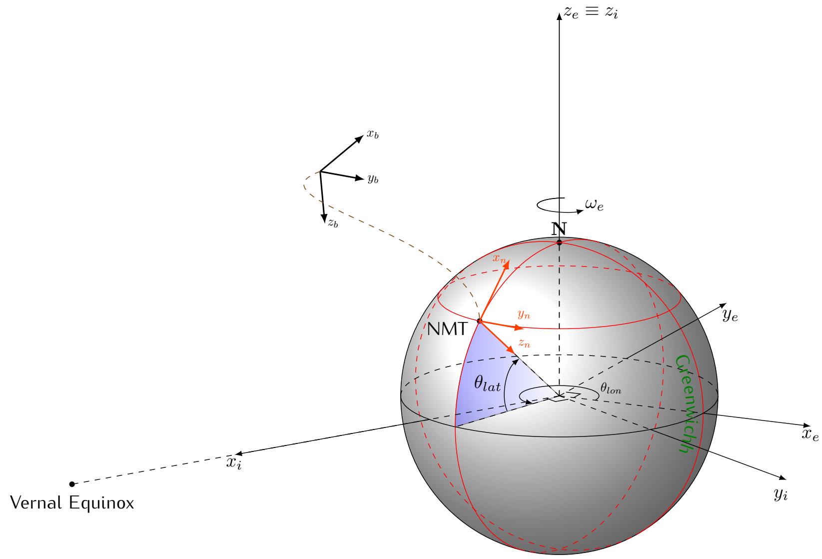
Body Frame



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6 Other Frames

Other Frames

- Wander Azimuth Frame (alternative to the Nav frame)
  - does not always point north ($x-$ and $y-$ axes displaced from north and east by an angle that varies with location on the earth) to avoid numerical stability problems near the poles
- Local Tangential Frame
  - typically, refers to another type of ECEF frame fixed to the Earth's surface (not moving like the n-frame)
  - tangent to the Earth's surface and often aligned with environmental feature such as a building, field, room or road
- Sensor/Instrument Frame
  - attached to body of sensor that may be displaced from a vehicle's center of mass

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The End

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