# EE 565: Position, Navigation and Timing Navigation Mathematics: Rotation Matrices

#### Kevin Wedeward Aly El-Osery

Electrical Engineering Department New Mexico Tech Socorro, New Mexico, USA

In Collaboration with Stephen Bruder Electrical and Computer Engineering Department Embry-Riddle Aeronautical Univesity, Prescott, Arizona, USA

#### January 21, 2020

Kevin Wedewa	ard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020 1 / 21





# 8 Rotation Matrices

## Examples

# **5** Summary

Review	Attitude	Rotation Matrices	Examples	Summa	'y
		0000000			
Kevin Wedeward	l, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020 2 /	21





Review	Attitude 000000	Rotation Matrices 00000000	Examples 00	Su	ımmary D
Kevin Wedewar	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	3 / 21









Review	Attitude 000000	Rotation Matrices 0000000	Examples 00	Su	ımmary Ə
Kevin Wedewar	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	3 / 21





- Coordinate Frames subscript will "name" axes (vectors)
- Earth-Centered Inertial (ECI) Frame - *i*



Review	Attitude 000000	Rotation Matrices 00000000	Examples 00	Sum oo	ımary
Kevin Wedewar	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	3 / 21

Review



 $z_e \equiv z_i$  $\theta_{lat}$ V: Vernal Equinox

Review	Attitude	Rotation Matrices	Examples	Su	ımmary
•	000000	0000000	00		D
Kevin Wedewar	rd. Alu El-Oseru (NMT)	EE 565: Position. Navigation and Timing		lanuaru 21. 2020	3 / 21

- Coordinate Frames subscript will "name" axes (vectors)
- Earth-Centered Inertial (ECI) Frame – *i*
- Earth-Centered Earth-Fixed (ECEF) Frame *e*

Review





- Coordinate Frames subscript will "name" axes (vectors)
- Earth-Centered Inertial (ECI) Frame – *i*
- Earth-Centered Earth-Fixed (ECEF) Frame *e*
- Navigation (Nav) Frame n

Review	Attitude 000000	Rotation Matrices	Examples 00	Si	ummary O
Kevin Wedewa	rd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	3 / 21

Review





view			
Kevin Wedeward	Alu El-Oseru (NMT)	FE 565 <sup>,</sup> Position Navigation and Timing	lanuaru 21 2020 3 / 21

- Coordinate Frames subscript will "name" axes (vectors)
- Earth-Centered Inertial (ECI) Frame – *i*
- Earth-Centered Earth-Fixed (ECEF) Frame *e*
- Navigation (Nav) Frame n
- Body Frame b



• Attitude describes orientation of one coordinate frame with respect to another.

	Attitude ●00000	Rotation Matrices	Examples 00	Si	
Kevin Wedewar	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	4 / 21

# NEW MEXICO TECH

#### Attitude (Orientation)

- Attitude describes orientation of one coordinate frame with respect to another.
- How would one describe orientation of ECEF frame wrt ECI frame at point in time when angular difference is  $\theta$ ?



Review	Attitude	Rotation Matrices	Examples	Si	ummary
0	●00000	00000000	00		O
Kevin Wedeware	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	4 / 21

# NEW MEXICO TECH

#### Attitude (Orientation)

- Attitude describes orientation of one coordinate frame with respect to another.
- How would one describe orientation of ECEF frame wrt ECI frame at point in time when angular difference is  $\theta$ ?



• *e*-frame rotated away from *i*-frame by angle  $\theta$  about  $z_i \equiv z_e$ 

	Attitude			
	000000			
Kevin Wedewa	rd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	4 / 21



• Less obvious, but equally valid, way of describing e-frame wrt i-frame is by giving coordinates of the e-frame's axes in the i-frame.

Review 0	Attitude ⊙●0000	Rotation Matrices	Examples 00	S	ummary
Kevin Wedewa	rd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	5 / 21



- Less obvious, but equally valid, way of describing e-frame wrt i-frame is by giving coordinates of the e-frame's axes in the i-frame.
- Leads to need for further notation:

	Attitude			
	00000			
Kevin Wedeward	I, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	5 / 21



- Less obvious, but equally valid, way of describing e-frame wrt i-frame is by giving coordinates of the e-frame's axes in the i-frame.
- Leads to need for further notation:

• 
$$x_e^i$$
 is  $x_e = x_e^e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  coordinatized (written wrt) the *i*-frame

				ſ
	Attitude			
	00000			
Kevin Wedeward	, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	5 / 21



- Less obvious, but equally valid, way of describing e-frame wrt *i*-frame is by giving coordinates of the e-frame's axes in the i-frame.
- Leads to need for further notation:

•  $x_e^i$  is  $x_e = x_e^e = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$  coordinatized (written wrt) the *i*-frame •  $y_e^i$  is  $y_e = y_e^e = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$  coordinatized (written wrt) the *i*-frame

Review 0	Attitude ⊙●0000	Rotation Matrices	Examples 00	Si	ımmary Ə
Kevin Wedeward,	Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	5 / 21



- Less obvious, but equally valid, way of describing e-frame wrt *i*-frame is by giving coordinates of the e-frame's axes in the i-frame.
- Leads to need for further notation:

eads to need for further notation: •  $x_e^i$  is  $x_e = x_e^e = \begin{bmatrix} 1\\0\\0\\\end{bmatrix}$  coordinatized (written wrt) the *i*-frame •  $y_e^i$  is  $y_e = y_e^e = \begin{bmatrix} 0\\1\\0\\\end{bmatrix}$  coordinatized (written wrt) the *i*-frame •  $z_e^i$  is  $z_e = z_e^e = \begin{bmatrix} 0\\0\\1\\\end{bmatrix}$  coordinatized (written wrt) the *i*-frame Attitude

Kevin Wedeward, Alu El-Oseru (NMT)

00000

EE 565: Position, Navigation and Timing

lanuaru 21, 2020 5/21



•  $x_e^i$ :



	Attitude			
	000000			
Kevin Wedeward	l, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	6 / 21





 Review
 Attitude
 Rotation Matrices
 Examples
 Summary

 o
 ootoooo
 ootooooo
 ootooooo
 ootooooo
 ootooooo

 Kevin Wedeward, Aly El-Osery
 (NMT)
 EE 565: Position, Navigation and Timing
 January 21, 2020
 6 / 21



• y<sub>e</sub><sup>i</sup>:



	Attitude			
	000000			
Kevin Wedeward	, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	7 / 21





$$z_i \equiv z_e$$

$$o_i \equiv o_e$$

$$y_e^i = \begin{bmatrix} y_e \cdot x_i \\ y_e \cdot y_i \\ y_e \cdot z_i \end{bmatrix} = \begin{bmatrix} \|y_e\| \|x_i\| \cos(90^\circ + \theta) \\ \|y_e\| \|y_i\| \cos(\theta) \\ \|y_e\| \|z_i\| \cos(90^\circ) \end{bmatrix} = \begin{bmatrix} -\sin(\theta) \\ \cos(\theta) \\ 0 \end{bmatrix}$$

	Attitude			
	000000			
Kevin Wedewa	rd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020 7 / 2	1



•  $z_e^i$ :



Review	Attitude	Rotation Matrices	Examples	Si	ummary
	000000	00000000			
Kevin Wedeward	I, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	8 / 21





Review 0	Attitude 0000●0	Rotation Matrices	Examples 00	Summary 00	
Kevin Wedeward	l, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020 8 / 21	

Уe



Review	Attitude 00000●	Rotation Matrices	Examples 00	S	ummary O
Kevin Wedeward	l, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	9 / 21



• 
$$C_e^i = \begin{bmatrix} x_e^i \mid y_e^i \mid z_e^i \end{bmatrix} = \begin{bmatrix} \cos(\theta) \mid -\sin(\theta) \mid 0 \\ \sin(\theta) \mid \cos(\theta) \mid 0 \\ 0 \mid 0 \mid 1 \end{bmatrix}$$

Review O	Attitude 00000●	Rotation Matrices	Examples 00	SL	mmary >
Kevin Wedeward,	, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	9 / 21



• 
$$C_e^i = \begin{bmatrix} x_e^i \mid y_e^i \mid z_e^i \end{bmatrix} = \begin{bmatrix} \cos(\theta) \mid -\sin(\theta) \mid 0 \\ \sin(\theta) \mid \cos(\theta) \mid 0 \\ 0 \mid 0 \mid 1 \end{bmatrix}$$

•  $C_e^i$  describes the attitude/orientation of the *e*-frame wrt the *i*-frame

Review	Attitude	Rotation Matrices	Examples	Si	mmaru
	000000				
Kevin Wedeward,	, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	9 / 21



• 
$$C_e^i = \begin{bmatrix} x_e^i \mid y_e^i \mid z_e^i \end{bmatrix} = \begin{bmatrix} \cos(\theta) \mid -\sin(\theta) \mid 0 \\ \sin(\theta) \mid \cos(\theta) \mid 0 \\ 0 \mid 0 \mid 1 \end{bmatrix}$$

- $C_e^i$  describes the attitude/orientation of the *e*-frame wrt the *i*-frame
- $C_e^i$  referred to as a rotation matrix, coordinate transformation matrix, or direct cosine matrix (DCM)

Review	Attitude	Rotation Matrices	Examples 00	Sum 00	ımary
Kevin Wedeward,	Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	9 / 21



• In general, a rotation matrix  $C_2^1$  describes the orientation of frame  $\{2\}$  relative to frame  $\{1\}$ 

**Rotation Matrices** 



		Rotation Matrices		
		0000000		
Kevin Wedeward	I, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	10 / 21



• In general, a rotation matrix  $C_2^1$  describes the orientation of frame  $\{2\}$  relative to frame  $\{1\}$ 



**Rotation Matrices** 

		Rotation Matrices		
		0000000		
Kevin Wedeward	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	10 / 21



• 
$$C_2^1 = \begin{bmatrix} x_2^1, y_2^1, z_2^1 \end{bmatrix} = \begin{bmatrix} x_2 \cdot x_1, y_2 \cdot x_1, z_2 \cdot x_1 \\ x_2 \cdot y_1, y_2 \cdot y_1, z_2 \cdot y_1 \\ x_2 \cdot z_1, y_2 \cdot z_1, z_2 \cdot z_1 \end{bmatrix} = \begin{bmatrix} x_1 \cdot x_2, x_1 \cdot y_2, x_1 \cdot z_2 \\ y_1 \cdot x_2, y_1 \cdot y_2, y_1 \cdot z_2 \\ z_1 \cdot x_2, z_1 \cdot y_2, z_1 \cdot z_2 \end{bmatrix}$$
$$= \begin{bmatrix} (x_1^2)^T \\ (y_1^2)^T \\ (z_1^2)^T \end{bmatrix} = \begin{bmatrix} x_1^2, y_1^2, z_1^2 \end{bmatrix}^T = \begin{bmatrix} C_1^2 \end{bmatrix}^T$$

Review	Attitude	Rotation Matrices	Examples		Summary
O	000000	0000000	00		20
Kevin Wedewar	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	11 / 21



• 
$$C_2^1 = \begin{bmatrix} x_2^1, y_2^1, z_2^1 \end{bmatrix} = \begin{bmatrix} x_2 \cdot x_1, y_2 \cdot x_1, z_2 \cdot x_1 \\ x_2 \cdot y_1, y_2 \cdot y_1, z_2 \cdot y_1 \\ x_2 \cdot z_1, y_2 \cdot z_1, z_2 \cdot z_1 \end{bmatrix} = \begin{bmatrix} x_1 \cdot x_2, x_1 \cdot y_2, x_1 \cdot z_2 \\ y_1 \cdot x_2, y_1 \cdot y_2, y_1 \cdot z_2 \\ z_1 \cdot x_2, z_1 \cdot y_2, z_1 \cdot z_2 \end{bmatrix}$$
$$= \begin{bmatrix} (x_1^2)^T \\ (y_1^2)^T \\ (z_1^2)^T \end{bmatrix} = \begin{bmatrix} x_1^2, y_1^2, z_1^2 \end{bmatrix}^T = \begin{bmatrix} C_1^2 \end{bmatrix}^T$$

• opposite perspective (frame 2 wrt frame 1 given frame 1 wrt frame 2) is as simple as a matrix transpose!

		Rotation Matrices o●oooooo		
Kevin Wedeward	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	11 / 21



• 
$$\begin{bmatrix} C_2^1 \end{bmatrix}^T C_2^1 = C_1^2 C_2^1 = I \implies C_1^2 = \begin{bmatrix} C_2^1 \end{bmatrix}^T = \begin{bmatrix} C_2^1 \end{bmatrix}^{-1}$$

		Rotation Matrices		
	000000	0000000		
Kevin Wedeward	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	12 / 21



# • $[C_2^1]^T C_2^1 = C_1^2 C_2^1 = I \Rightarrow C_1^2 = [C_2^1]^T = [C_2^1]^{-1}$ • $|(C_2^1)^T C_2^1| = |C_2^1||C_2^1| = |I| \Rightarrow |C_2^1| = \pm 1$ (+ for right hand coordinate system)

		Rotation Matrices		
		0000000		
Kevin Wedewa	rd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	12 / 21



# • $\begin{bmatrix} C_2^1 \end{bmatrix}^T C_2^1 = C_1^2 C_2^1 = I \implies C_1^2 = \begin{bmatrix} C_2^1 \end{bmatrix}^T = \begin{bmatrix} C_2^1 \end{bmatrix}^{-1}$

- ②  $|(C_2^1)^T C_2^1| = |C_2^1||C_2^1| = |I| \Rightarrow |C_2^1| = \pm 1$  (+ for right hand coordinate system)
- **(**) columns and rows of  $C_2^1$  are orthogonal

Review	Attitude	Rotation Matrices	Examples	5	Summary
0	000000	oo●ooooo	00		SO
Kevin Wedeward,	, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	12 / 21



# • $\begin{bmatrix} C_2^1 \end{bmatrix}^T C_2^1 = C_1^2 C_2^1 = I \implies C_1^2 = \begin{bmatrix} C_2^1 \end{bmatrix}^T = \begin{bmatrix} C_2^1 \end{bmatrix}^{-1}$

- ◎  $|(C_2^1)^T C_2^1| = |C_2^1||C_2^1| = |I| \Rightarrow |C_2^1| = \pm 1$  (+ for right hand coordinate system)
- **(**) columns and rows of  $C_2^1$  are orthogonal
- magnitude of columns and rows in  $C_2^1$  are 1

Review	Attitude	Rotation Matrices		
		0000000		
Kevin Wedeward	I, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	12 / 21



#### • So far, rotation matrix *C* developed to describe orientation

Review	Attitude	Rotation Matrices	Examples		Summary
Kevin Wedev	vard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	13 / 21



- So far, rotation matrix *C* developed to describe orientation
- C can also perform change of coordinates on vector

		Rotation Matrices		
		0000000		
Kevin Wedewa	ard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	13 / 21



• Consider a point *P* with location described as a vector in coordinate frame {1}



		Rotation Matrices		
		0000000		
Kevin Wedew	vard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	14 / 21



• With  $\vec{P}^{1}$  given, the location of point P can be described in coordinate frame  $\{2\}$  via

$$\vec{P}^{2} = \begin{bmatrix} \vec{P}^{1} \cdot x_{2} \\ \vec{P}^{1} \cdot y_{2} \\ \vec{P}^{1} \cdot z_{2} \end{bmatrix} = \begin{bmatrix} (ux_{1} + vy_{1} + wz_{1}) \cdot x_{2} \\ (ux_{1} + vy_{1} + wz_{1}) \cdot y_{2} \\ (ux_{1} + vy_{1} + wz_{1}) \cdot z_{2} \end{bmatrix}$$
$$= \underbrace{\begin{bmatrix} x_{1} \cdot x_{2} & y_{1} \cdot x_{2} & z_{1} \cdot x_{2} \\ x_{1} \cdot y_{2} & y_{1} \cdot y_{2} & z_{1} \cdot y_{2} \\ x_{1} \cdot z_{2} & y_{1} \cdot z_{2} & z_{1} \cdot z_{2} \end{bmatrix}}_{7} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

		Rotation Matrices		
		00000000		
Kevin Wedew	vard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	15 / 21



$$= \underbrace{ \begin{bmatrix} x_{1} \cdot x_{2} & y_{1} \cdot x_{2} & z_{1} \cdot x_{2} \\ x_{1} \cdot y_{2} & y_{1} \cdot y_{2} & z_{1} \cdot y_{2} \\ x_{1} \cdot z_{2} & y_{1} \cdot z_{2} & z_{1} \cdot z_{2} \end{bmatrix}}_{C_{1}^{2}} \underbrace{ \begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\vec{P}^{1}}$$
$$= C_{1}^{2} \vec{P}^{1}$$

 $\bullet \Rightarrow \vec{P}^{\,2} = C_1^2 \vec{P}^{\,1}$ 

		Rotation Matrices		
		00000000		
Kevin Wedeward	, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	16 / 21



$$= \underbrace{ \begin{bmatrix} x_1 \cdot x_2 & y_1 \cdot x_2 & z_1 \cdot x_2 \\ x_1 \cdot y_2 & y_1 \cdot y_2 & z_1 \cdot y_2 \\ x_1 \cdot z_2 & y_1 \cdot z_2 & z_1 \cdot z_2 \end{bmatrix}}_{C_1^2} \underbrace{ \begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\vec{p}_1}$$
$$= C_1^2 \vec{P}^1$$

- $\bullet \Rightarrow \vec{P}^{\,2} = C_1^2 \vec{P}^{\,1}$
- $C_1^2$  re-coordinatized vector written wrt frame 1 into frame 2 by a matrix-multiplication

		Rotation Matrices		
		00000000		
Kevin Wedewa	nrd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	16 / 21



$$= \underbrace{ \begin{bmatrix} x_1 \cdot x_2 & y_1 \cdot x_2 & z_1 \cdot x_2 \\ x_1 \cdot y_2 & y_1 \cdot y_2 & z_1 \cdot y_2 \\ x_1 \cdot z_2 & y_1 \cdot z_2 & z_1 \cdot z_2 \end{bmatrix}}_{C_1^2} \underbrace{ \begin{bmatrix} u \\ v \\ w \end{bmatrix}}_{\vec{p}_1}$$
$$= C_1^2 \vec{P}^1$$

- $\bullet \Rightarrow \vec{P}^{\,2} = C_1^2 \vec{P}^{\,1}$
- $C_1^2$  re-coordinatized vector written wrt frame 1 into frame 2 by a matrix-multiplication
- superscripts and subscripts help track/denote re-coordinatization

		Rotation Matrices		
		00000000		
Kevin Wedewa	nrd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	16 / 21



Similarly, coordinate transformations can be performed opposite way as well  $\vec{P}^2 = C_1^2 \vec{P}^1$  $\Rightarrow \vec{P}^1 = [C_1^2]^{-1} \vec{P}^2$ 

Revi	ew Attitude	Rotation Matrices	Examples		Summary
	Kevin Wedeward, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	17 / 21



Similarly, coordinate transformations can be performed opposite way as well  $\vec{P}^2 = C_1^2 \vec{P}^1$   $\Rightarrow \vec{P}^1 = [C_1^2]^{-1} \vec{P}^2$  $= [C_1^2]^T \vec{P}^2$ 

		Rotation Matrices		
		0000000		
Kevin Wedewa	nrd, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	17 / 21



Similarly, coordinate transformations can be performed opposite way as well

$$P^{2} = C_{1}^{2}P^{1}$$
  

$$\Rightarrow \vec{P}^{1} = [C_{1}^{2}]^{-1}\vec{P}^{2}$$
  

$$= [C_{1}^{2}]^{T}\vec{P}^{2}$$
  

$$= C_{2}^{1}\vec{P}^{2}$$

D :	A	Detection Martine	<b>F</b> 1		~
		Rotation Matrices			
		0000000			
Kevin Wedeward	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	17 / 21

Example 1



Given 
$$C_b^a = \begin{bmatrix} 0 & -\frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & -\frac{\sqrt{3}}{2} & \frac{1}{2} \\ -1 & 0 & 0 \end{bmatrix}$$
 and frame *a*, sketch frame b.

			Examples		
	000000	0000000	0		
Kevin Wedew	ard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	18 / 21

#### Example 2



Frame 1 has been rotated away from frame 0 by  $30^{\circ}$  about  $z_0$ . Find  $\vec{r}^0$  given  $\vec{r}^1 = [0, 2, 0]^T$ ,  $\cos(30^{\circ}) = \frac{\sqrt{3}}{2}$  and  $\sin(30^{\circ}) = \frac{1}{2}$ .

			Examples		
Kevin Wedewa	ard, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing		January 21, 2020	19 / 21



Rotation matrix can be thought of in three distinct ways:

- It describes the orientation of one coordinate frame wrt another coordinate frame
- It represents a coordinate transformation that relates the coordinates of a point (e.g., P) or vector in two different frames of reference
- It is an operator that takes a vector  $\vec{p}$  and rotates it into a new vector  $C\vec{p}$ , both in the same coordinate frame

				Summary
				•o -
Kevin Wedeward,	Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	20 / 21



				Summary
				oo
Kevin Wedeward	d, Aly El-Osery (NMT)	EE 565: Position, Navigation and Timing	January 21, 2020	21 / 21