

# EE 565: Position, Navigation and Timing

## Navigation Equations: An Overview

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- The fundamental inertial navigation problem:
  - Using inertial sensors (accels & gyros) and an initial position and orientation, determine the vehicle's (i.e., body frame) current position, velocity, and attitude (PVA)

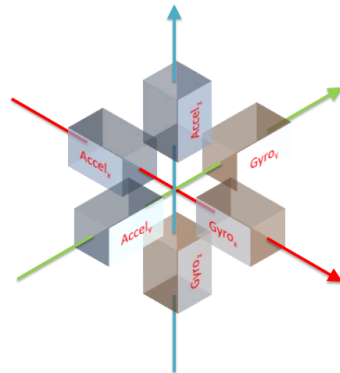
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    - 2 Inertial sensors ( $\vec{\omega}_{ib}^b$  and  $\vec{f}_{ib}^b$ )
    - 3 Have a gravity ( $\vec{g}_b^?$ ) and/or gravitational ( $\vec{\gamma}_{?b}^?$ ) model

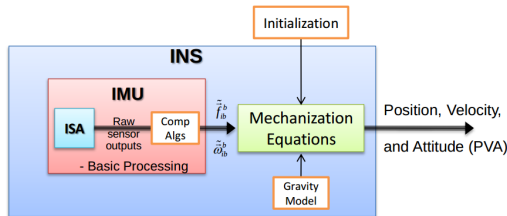
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    - With respect to which frame?

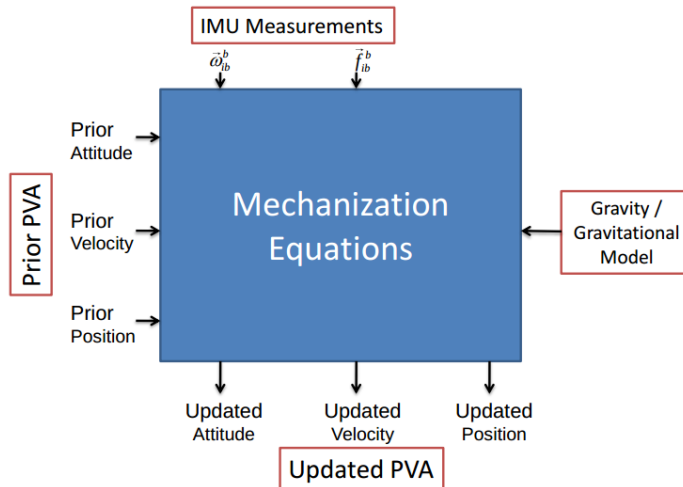
- The process of “integrating” angular velocity & acceleration to determine one’s position, velocity, and attitude (PVA)
- To measure the acceleration and angular velocity vectors we need at least 3-gyros and 3-accelers
  - Typically configured in an orthogonal triad
- The “mechanization” can be performed *wrt*:
  - the ECI frame,
  - the ECEF frame,
  - the Nav frame, or
  - the tangential frame.



- An Inertial Navigation System (INS)
  - ISA — Inertial Sensor Assembly
    - Typically, 3-gyros, 3-accel, and basic electronics
  - IMU — Inertial Measurement Unit
    - ISA + compensation algorithms (i.e., basic processing)
  - INS — Inertial Navigation System
    - IMU + gravity model + “mechanization” algorithm







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- ④ Update the position
  - integrate the result from step 3

# A Four Step Mechanization

