Lecture

Gyro and Accel Noise Characteristics

EE 565: Position, Navigation and Timing

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1 Inertial Sensors Errors

Inertial Sensors — Sensor Models

- Accelerometer model

$$\tilde{\vec{f}}^{\,b}_{ib} = \vec{f}^{\,b}_{ib} + \Delta\vec{f}^{\,b}_{ib} = \vec{b}_a + (\mathcal{I} + M_a)\vec{f}^{\,b}_{ib} + \vec{w}_a \tag{1}$$

- Gyro Model

$$\tilde{\vec{\omega}}^{\,b}_{ib} = \vec{\omega}^{\,b}_{ib} + \Delta\vec{\omega}^{\,b}_{ib} = \vec{b}_g + (\mathcal{I} + M_g)\vec{\omega}^{\,b}_{ib} + G_g\vec{f}^{\,b}_{ib} + \vec{w}_g \tag{2}$$

- Typically, each measures along a signle sense axis requiring three of each to measure the 3-tupple vector
- Bias errors are composite of fixed bias, bias instability, and bias stability

$$b = b_{FB} + b_{BI} + b_{BS}$$

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2 Gyro Noise Characteristics

Gyro Constant Bias ($^\circ/h$)

A constant in the output of a gyro in the absence of rotation, in $^\circ/h$.

Error Growth
Linearly growing error in the angle domain of ϵt.

Model
Random constant.

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Gyro Integrated White Noise

Assuming the rectangular rule is used for integration, a sampling period of T_s and a time span of nT_s.

$$\int_0^t \epsilon(\tau)d\tau = T_s\sum_{i=1}^{n}\epsilon(t_i) \tag{3}$$

since $\mathbb{E}[\epsilon(t_i)] = 0$ and $Cov(\epsilon(t_i), \epsilon(t_j)) = 0$ for all $i \neq j$, $Var[\epsilon(t_i)] = \sigma^2$

$$\mathbb{E}\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s n \mathbb{E}[\epsilon(t_i)] = 0, \forall i \tag{4}$$

$$Var\left[\int_0^t \epsilon(\tau)d\tau\right] = T_s^2 n Var[\epsilon(t_i)] = T_s t \sigma^2, \forall i \tag{5}$$

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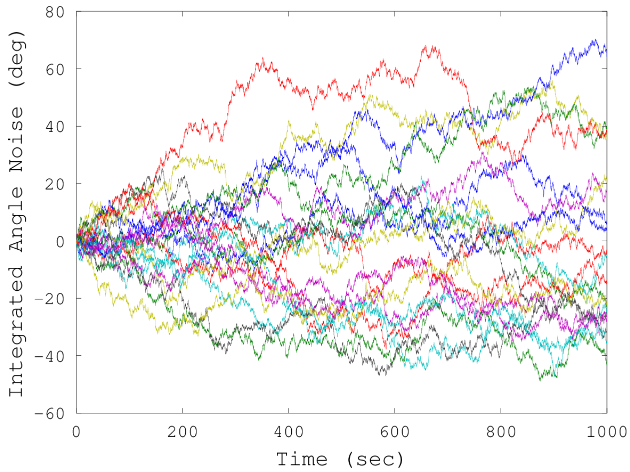
Gyro Integrated White Noise



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Angle Random Walk ($°/\sqrt{h}$)

Integrated noise resulted in zero-mean random walk with standard deviation that grows with time as

$$\sigma_\theta = \sigma\sqrt{T_s t} \tag{6}$$

We define ARW as

$$ARW = \sigma_\theta(1) \qquad (°/\sqrt{h}) \tag{7}$$

In terms of PSD

$$ARW(°/\sqrt{h}) = \frac{1}{60}\sqrt{PSD((°/h)^2/Hz)} \tag{8}$$

Error Growth
ARW times root of the time in hours.

Model
White noise.

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Gyro Bias Instability ($°/h$)

- Due to flicker noise with spectrum $1/F$.
- Results in random variation in the bias.
- Normally more noticeable at low frequencies.
- At high frequencies, white noise is more dominant.

Error Growth
Variance grows over time.

Model
First order Gauss-Markov.

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3 Accel Noise Characteristics

Accel Constant Bias (μg)

A constant deviation in the accelerometer from the true value, in m/s^2.

Error growth
Double integrating a constant bias error of ϵ results in a quadratically growing error in position of $\epsilon t^2/2$.

Model
Random constant. .8

Velocity Random Walk ($m/s/\sqrt{h}$)

Integrating accelerometer output containing white noise results in velocity random walk (VRW) $(m/s/\sqrt{h})$. Similar to development of ARW, if we double integrate white noise we get

$$\iint_0^t \epsilon(\tau) d\tau d\tau = T^2_{s,sensor} \sum_{i=1}^{n} \sum_{j=1}^{i} \epsilon(t_j) \tag{9}$$

Error Growth
Computing the standard deviation results in

$$\sigma_p \approx \sigma t^{(3/2)} \sqrt{\frac{T_s}{3}} \tag{10}$$

Model
White noise. .9

Accel Bias Instability (μg)

Error growth
Grows as $t^{5/2}$.

Model
First order Gauss-Markov. .10

4 Allan Variance

Allan Variance Introduction

It is a time domain analysis techniques designed originally for characterizing noise in clocks. It was first proposed by David Allan in 1966. .11

Allan Variance Computation

1. Divide your N-point data sequence into adjacent windows of size $n = 1, 2, 4, 8, \ldots, M \leq N/2$.
2. For every n generate the sequence

$$y_j(n) = \frac{x_{nj} + x_{nj+1} + \cdots + x_{nj+n-1}}{n}, \quad j = 0, 1, \ldots, \left[\frac{N}{n}\right] - 1 \tag{11}$$

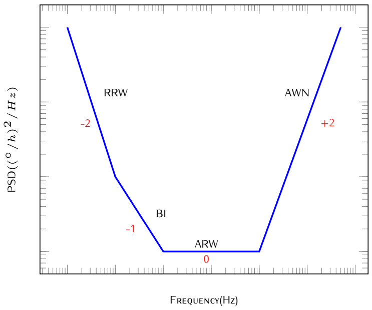
3. Plot log-log of the Allan deviation which is square root of

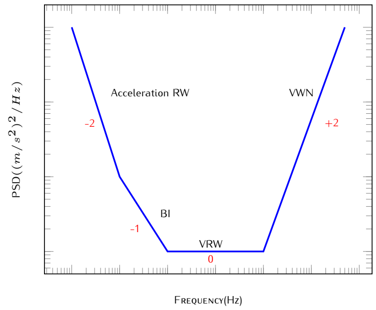
$$\sigma^2_{Allan}(nT_s) = \frac{1}{2(N-1)} \sum_{j=1}^{N-1} (y_j - y_{j-1})^2 \tag{12}$$

versus averaging time $\tau = nT_s$ .12

5 Using PSD and Allan Variance

One-sided PSD - Typical Slopes





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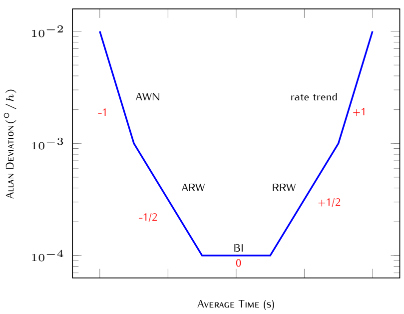
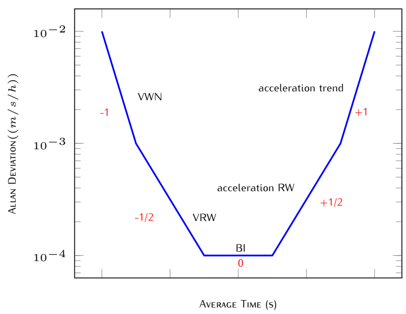
Allan Deviation - Typical Slopes





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Noise Parameters

Noise Type	AV $\sigma^2(\tau)$	PSD (2-sided)
Quantization Noise	$3\frac{\alpha^2}{\tau^2}$	$(2\pi f)^2\alpha^2 T_s$
Angle/Velocity Random Walk	$\frac{\alpha^2}{\tau}$	α^2
Flicker Noise	$\frac{2\alpha^2\ln(2)}{\pi}$	$\frac{\alpha^2}{2\pi f}$
Angular Rate/Accel Random Walk	$\frac{\alpha^2\tau}{3}$	$\frac{\alpha^2}{(2\pi f)^2}$
Ramp Noise	$\frac{\alpha^2\tau^2}{2}$	$\frac{\alpha^2}{(2\pi f)^3}$

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