3.65 An FIR filter is defined by symmetric impulse response, that is $h[0] = h[2]$. Let the input to filter be sum of two cosines of freq. 0.3 rad/samples and 0.6 rad/samples, respectively. Determine the impulse response coeffs so that filter passes only the low-freq. component of input.

$$
\begin{aligned}
H(e^{j\omega}) &= h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} \\
&= h[0] + h[1]e^{-j\omega} + h[0]e^{-j2\omega} \\
&= h[0](1 + e^{-j2\omega}) + h[1]e^{-j\omega} \\
&= e^{-j\omega}\left[h[1] + h[0]e^{j\omega} + h[0]e^{-j\omega}\right] \\
&= e^{-j\omega}\left[h[1] + 2h[0]\cos(\omega)\right]
\end{aligned}
$$

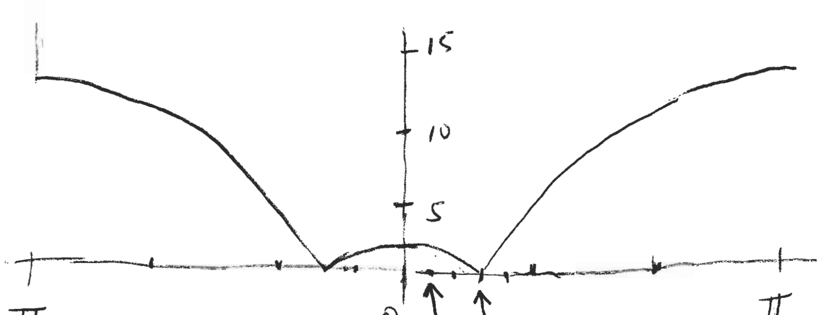
We require

$$|H(e^{j0.3})| = 2h[0]\cos(0.3) + h[1] = 1$$
$$|H(e^{j0.6})| = 2h[0]\cos(0.6) + h[1] = 0$$

$$2h[0](\cos(0.3) - \cos(0.6)) = 1$$
$$h[0] = 3.846$$

$$2(3.846)\cos(0.3) + h[1] = 1$$
$$h[1] = -6.348$$

M3.8 Write a MATLAB program to simulate the filter above (P3.65) and verify its filtering operation



|4.10| Show that the impulse response of the ideal lowpass filter derived in Eq. 4.19 takes the value. $h_r(nT) = \delta[n]$ for all n if $\Omega_c = \Omega_T/2$.

$$h_r(t) = \frac{\sin(\Omega_c t)}{\Omega_T t/2} \qquad h_r(nT) = \frac{\sin(\Omega_c nT)}{\Omega_T nT/2}$$

Since $\Omega_T = 2\pi/T \rightarrow T = 2\pi/\Omega_T$

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$$h_r(nT) = \frac{\sin\left(\frac{2\pi\Omega_c n}{\Omega_T}\right)}{n\pi} \qquad \text{For } \Omega_c = \Omega_T/2 \quad h_r(nT) = \frac{\sin(\pi n)}{\pi n} = \delta$$

[...] Show that the T.F.

$$H_a(s) = \frac{a}{s+a} \qquad a > 0$$

[has] a lowpass magnitude response with $|H_a(j0)| = 1$ and $|H_a(j\infty)| = [\ldots]$ [Deter]mine the 3-dB cutoff freq. Ω_c

$$H_a(j\Omega) = \frac{a}{j\Omega + a}$$

$$|H_a(j\Omega)|^2 = H_a(j\Omega) H_a(-j\Omega) = \frac{a}{j\Omega + a} \cdot \frac{a}{-j\Omega + a} = \frac{a^2}{\Omega^2 + a^2}$$

$$|H_a(j\Omega)|^2 = \frac{a^2}{\Omega_c^2 + a^2} = \frac{1}{2} \qquad \therefore \quad \underline{\underline{\Omega_c = a}}$$

$\boxed{4.19}$ The lowpass T.F. $H_a(s)$ [Eqn. 4.94] and the highpass T.F. $G_a(s)$ [Eqn. 4.95] can be expressed.

$$H_a(s) = \frac{1}{2}\{A_0(s) - A_1(s)\} \qquad G_a(s) = \frac{1}{2}\{A_0(s) + A_1(s)\}$$

where $A_0(s)$, $A_1(s)$ are stable analog allpass functions.

Determine $A_0(s)$, $A_1(s)$

$$\frac{bs}{s^2+bs+\Omega_0^2} = \frac{1}{2}A_0(s) - \frac{1}{2}A_1(s)$$

$$\frac{s^2+\Omega_0^2}{s^2+bs+\Omega_0^2} = \frac{1}{2}A_0(s) + \frac{1}{2}A_1(s)$$

$$\frac{s^2+bs+\Omega_0^2}{s^2+bs+\Omega_0^2} = A_0(s) = 1$$

$$\frac{1}{2}A_1(s) = \frac{1}{2} - \frac{bs}{s^2+bs+\Omega_0^2}$$

$$\frac{1}{\cancel{2}}A_1(s) = \frac{s^2+bs+\Omega_0^2 - 2bs}{\cancel{2}(s^2+bs+\Omega_0^2)}$$

$$A_1(s) = \frac{s^2-bs+\Omega_0^2}{s^2+bs+\Omega_0^2}$$

$|A_0(j\Omega)|^2 = 1 \leftarrow$ All PASS

$$A_1(j\Omega) = \frac{-\Omega^2 - jbw + \Omega_0^2}{-\Omega^2 + jbw + \Omega_0^2}$$

$$|A_1(j\Omega)|^2 = \frac{-\Omega^2 - \cancel{jbw + \Omega_0^2}}{-\Omega^2 + \cancel{jbw + \Omega_0^2}} \cdot \frac{-\Omega^2 \cancel{+ jbw} + \Omega_0^2}{-\Omega^2 \cancel{- jbw} + \Omega_0^2} = 1$$

$\uparrow$

All PASS