HW #5 Due 10/9

6.4 Determine which of the following sequences has the same z-transform

Let us get the z-transforms of all sequences first

$$z\{(0.4)^n \mu[n]\} = \frac{1}{1-0.4z^{-1}} \quad |z| > 0.4 \qquad z\{(-0.6)^n \mu[n]\} = \frac{1}{1+0.6z^{-1}} \quad |z| > 0.6$$

$$z\{(0.4)^n \mu[-n-1]\} = -\frac{1}{1-0.4z^{-1}} \quad |z| < 0.4 \qquad z\{(-0.6)^n \mu[-n-1]\} = -\frac{1}{1+0.6z^{-1}} \quad |z| < 0.6$$

(a) $z\{x_1[n]\} = \frac{1}{1-0.4z^{-1}} + \frac{1}{1+0.6z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.4z^{-1})(1+0.6z^{-1})} \quad |z| > 0.6$

(b) $z\{x_2[n]\} = \frac{1}{1-0.4z^{-1}} + \frac{1}{1+0.6z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.4z^{-1})(1+0.6z^{-1})} \quad 0.4 < |z| < 0.6$

(c) $z\{x_3[n]\} = -\frac{1}{1-0.4z^{-1}} - \frac{1}{1+0.6z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.4z^{-1})(1+0.6z^{-1})} \quad |z| < 0.4$

(d) $z\{x_4[n]\} = \frac{1}{1-0.4z^{-1}} + \frac{1}{1+0.6z^{-1}} = \frac{2+0.2z^{-1}}{(1-0.4z^{-1})(1+0.6z^{-1})}$ THE Z-XFRM DOES NOT CONVERGE FOR $x_4[n]$

6.8 Consider the z-transform

$$G(z) = \frac{(z^2+0.2z+0.1)(z^2-z+0.5)}{(z^2+0.3z-0.18)(z^2-2z+4)}$$

There are four possible nonoverlapping ROCs. Discuss the type of inverse z-transform (left-sided, right-sided, or two-sided sequences) associated with each of the four ROCs. It is not necessary to compute exact inverse transform.

Function has poles at $z_1 = -0.6$, $z_2 = 0.3$, $z_{3,4} = 1 \pm j1.732$

The four ROCs are defined

$R_1: 0<|z|<0.3 \qquad R_2: 0.3<|z|<0.6$

$R_3: 0.6<|z|<2 \qquad R_4: |z|>2$

- Inverse transform of sequence associated with R_1 is left-sided
- Inverse transform of seq. associated with R_2, R_3 is two-sided
- Inverse transform of seq. associated with R_4 is right-sided.

6.14 Determine z-transform of sequence of Prob 3.18 and their ROCs. Show that the ROC includes the unit circle for each transform. Evaluate z transform evaluated on the unit circle for each sequence and show that is is precisely the DTFT as computed in Problem 3.18.

(a) $X_a(z) = Z\{u[n]-u[n-5]\} = \dfrac{1}{1-z^{-1}} - \dfrac{z^{-5}}{1-z^{-1}} = \dfrac{1-z^{-5}}{1-z^{-1}}$

$$
\begin{array}{r|l}
 & 1+z^{-1}+z^{-2}+z^{-3}+z^{-4} \\
\hline
1-z^{-1} & 1-z^{-5} \\
 & -1+z^{-1} \\
 & z^{-1}-z^{-5} \\
 & -z^{-1}+z^{-2} \\
 & z^{-2}-z^{-5} \\
 & -z^{-2}+z^{-3} \\
 & z^{-3}-z^{-5} \\
 & -z^{-3}+z^{-4} \\
 & z^{-4}-z^{-5} \\
 & -z^{-4}+z^{-5} \\
 & 0
\end{array}
$$

All poles are at origin
ROC is entire z plane except point $z=0$.

$X_a(z)\big|_{z=e^{j\omega}} = \dfrac{1-e^{-j5\omega}}{1-e^{-j\omega}}$

$0<\omega\leq 2\pi$

$X_a(j\omega) = (1-e^{-j5\omega})\left[\dfrac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega+2\pi k)\right] = \dfrac{1-e^{-j5\omega}}{1-e^{-j\omega}}$

(using Time-shifting property Table 3.3, 3.4)

(b) $x_b[n] = \alpha^n(u[n] - u[n-8]), \quad |\alpha| < 1$

Let $x[n] = \alpha^n u[n] \quad |\alpha| < 1$

$$X(e^{j\omega}) = \frac{1}{1 - \alpha e^{-j\omega}}$$

Using time-shifting property (Table 3.4)

$$X_b(e^{j\omega}) = (1 - e^{-j8\omega})\, X(e^{j\omega}) = \frac{(1 - e^{-j8\omega})}{1 - \alpha e^{-j\omega}}$$

$$X_b(z) = \frac{1}{1 - \alpha z^{-1}} - \frac{z^{-8}}{1 - \alpha z^{-1}} = \frac{1 - z^{-8}}{1 - \alpha z^{-1}} \quad \text{(Tables 6.1, 6.2)}$$

$|\alpha z^{-1}| < 1 \quad |\alpha| < |z|$. Since $|\alpha| < 1$, then ROC includes unit circle

$$X_b(z)\Big|_{z = e^{j\omega}} = \frac{1 - e^{-j8\omega}}{1 - \alpha e^{-j\omega}}$$

6.22 Show that z-transform of following rational transform

$$H(z) = \frac{1}{1 - 2r\cos\theta\, z^{-1} + r^2 z^{-2}} \qquad |z| > r > 0$$

is given by

$$h[n] = \frac{r^n \sin(n+1)\theta}{\sin\theta}\, u[n]$$

$$H(z) = \frac{1}{(1 - re^{j\theta}z^{-1})(1 - re^{-j\theta}z^{-1})} = \frac{A}{1 - re^{j\theta}z^{-1}} + \frac{B}{1 - re^{-j\theta}z^{-1}}$$

$$A = \frac{1}{1 - re^{-j\theta}z^{-1}}\Big|_{z = re^{j\theta}} = \frac{1}{1 - (re^{-j\theta})(r^{-1}e^{-j\theta})} = \frac{1}{1 - e^{-j2\theta}}$$

$$B = \frac{1}{1 - re^{j\theta}z^{-1}}\Big|_{z = re^{-j\theta}} = \frac{1}{1 - (re^{j\theta})(r^{-1}e^{j\theta})} = \frac{1}{1 - e^{j2\theta}}$$

$$H(z) = \frac{\frac{1}{1-e^{-j2\theta}}}{1-re^{j\theta}z^{-1}} + \frac{\frac{1}{1-e^{j2\theta}}}{1-re^{-j\theta}z^{-1}}$$

$$h[n] = \frac{1}{1-e^{-j2\theta}}(re^{j\theta})^n \mu[n] + \frac{1}{1-e^{j2\theta}}(re^{-j\theta})^n \mu[n]$$

$$= \frac{e^{j\theta}}{e^{j\theta}-e^{-j\theta}} r^n e^{jn\theta}\mu[n] + \frac{e^{-j\theta}}{e^{-j\theta}-e^{j\theta}} r^n e^{-jn\theta}\mu[n]$$

$$= \frac{1}{e^{j\theta}-e^{-j\theta}}\left[r^n e^{j\theta(n+1)} - r^n e^{-j\theta(n+1)}\right]\mu[n]$$

$$= \frac{r^n}{j2\sin\theta}\left[e^{j\theta(n+1)} - e^{-j\theta(n+1)}\right]\mu[n]$$

$$= \frac{r^n}{\sin\theta}\left[\frac{e^{j\theta(n+1)} - e^{-j\theta(n+1)}}{2j}\right]\mu[n]$$

$$= \frac{r^n \sin[\theta(n+1)]}{\sin\theta}\mu[n]$$

$\boxed{6.16}$ (a)(b) Evaluate the linear convolution of P 2.50 using the polynomial multiplication method

(a) $u[n] = x[n] \circledast y[n]$

$x[n] = \{-4 \;\; 5 \;\; 1 \;\; \underset{\uparrow}{-2} \;\; -3 \;\; 0 \;\; 2\}$ $\quad y[n] = \{6 \;\; \underset{\uparrow}{-3} \;\; -1 \;\; 0 \;\; 8 \;\; 7 \;\; -2\}$

$$X(z)Y(z) = (-4z^3 + 5z^2 + z - 2 - 3z^{-1} + 2z^{-3})(6z - 3 - z^{-1} + 8z^{-3} + 7z^{-4} - 2z^{-5})$$

Can use conv in MATLAB

$u[n] = \{-24, 42, -5, -20, \underset{\uparrow}{-45}, 23, 66, -25, -42, -17, 22, 14, -4\}$

(b) $v[n] = x[n] \circledast w[n]$

$$w[n] = \{\underset{\uparrow}{0} \quad 0 \quad 3 \quad 2 \quad 2 \quad -1 \quad 0 \quad -2 \quad 5\}$$

$$X(z)W(z) = (-4z^{3}+5z^{2}+z-2-3z^{-1}+2z^{-3})(3z^{-2}+2z^{-3}+2z^{-4}-z^{-5}-2z^{-7}+5z^{-8})$$

Using conv from MATLAB

$$v[n] = \{-12 \quad \underset{\uparrow}{7} \quad 5 \quad 10 \quad -16 \quad -3 \quad -28 \quad 30 \quad 13 \quad -6 \quad -15 \quad -4 \quad 10\}$$

HW #6 Due 10/16

USE PROGRAM 6_1.m

6.38 Determine the T.F. of the following LTI discrete-time systems. Sketch the pole-zero plot. Is the system BIBO stable?

(a) $$H(z) = \frac{5 + 9.5z^{-1} + 1.4z^{-2} - 24z^{-3}}{1 - 0.1z^{-1} + 0.14z^{-2} + 0.49z^{-3}}$$

Using Program 6_1.m

$$H(z) = \frac{5(1 + 3.1z^{-1} + 4z^{-2})(1 - 1.2z^{-1})}{(1 - 0.8z^{-1} + 0.7z^{-2})(1 + 0.7z^{-1})}$$

zeros:

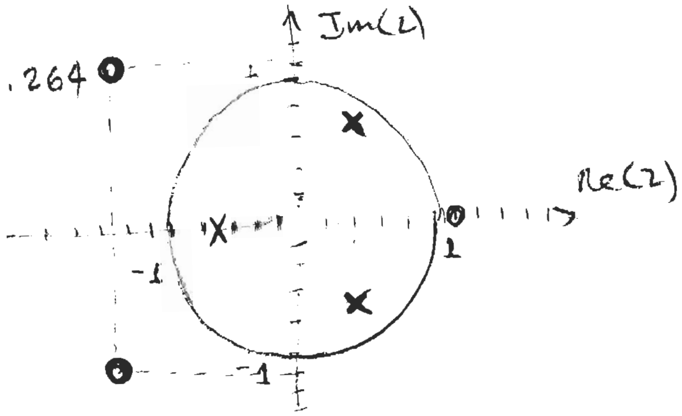
$z_{1,2} = -1.550 \pm j1.264$

$z_3 = 1.2$

poles:

$z_{1,2} = 0.4 \pm j0.73$

$z_3 = -0.7$



All poles inside unit circle, $H(z)$ is BIBO stable

(b) $$H(z) = \frac{5 + 16.5z^{-1} + 14.7z^{-2} - 22.04z^{-3} - 33.6z^{-4}}{0.5z^{-1} - 0.1z^{-2} - 0.3z^{-3} + 0.0936z^{-4}}$$

Using Program 6_1.m

$$H(z) = \frac{5(1 - 1.2z^{-1})(1 + 3.12z^{-1} + 4.05z^{-2})(1 + 1.37z^{-1})}{(1 + 0.6z^{-1})(1 - 0.8z^{-1} + 0.52z^{-2})(1 - 0.3z^{-1})}$$

zeros:

$z_1 = 1.2$

$z_{2,3} = -1.56 \pm j1.27$

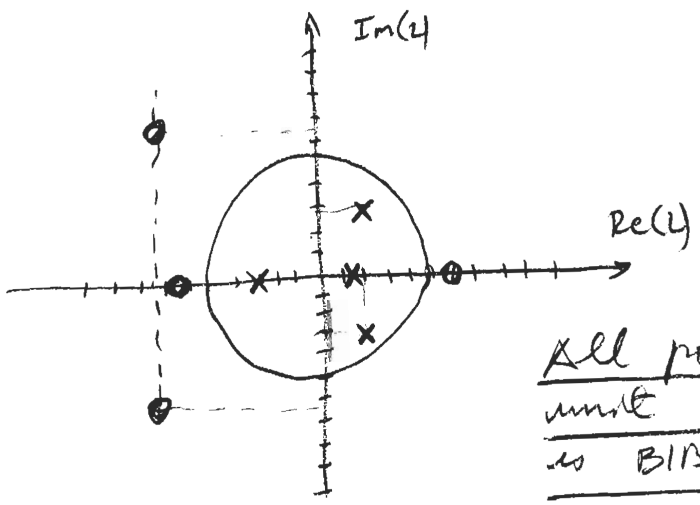
$z_4 = -1.37$

poles:

$z_1 = -0.6$

$z_{2,3} = 0.4 \pm j0.6$

$z_4 = 0.3$



All poles inside unit circle, $H(z)$ is BIBO stable

$\boxed{6.45}$ Determine a closed-form expression for the frequency response $H(e^{j\omega})$ of the LTI system characterized by

$$h[n] = \delta[n] - \alpha\delta[n-R]$$

where $|\alpha| < 1$. What are max and min values of magnitude response. How many peaks & dips occur in range $0 \le \omega < 2\pi$. Locations of peaks & dips? Sketch magnitude & phase responses for $R=6$.

$H(z) = 1 - \alpha z^{-R}$ $\quad H(e^{j\omega}) = 1 - \alpha e^{-j\omega R}$

$H(e^{j\omega}) = 1 - \alpha[\cos(\omega R) - j\sin(\omega R)] = 1 - \alpha\cos(\omega R) - j\alpha\sin(\omega R)$

$|H(e^{j\omega})| = \sqrt{[1 - \alpha\cos(\omega R)]^2 + [\alpha\sin(\omega R)]^2} = \sqrt{1 - 2\alpha\cos(\omega R) + \alpha^2\cos^2(\omega R) + \alpha^2\sin^2(\omega R)}$

$|H(e^{j\omega})| = \sqrt{1 + \alpha^2 - 2\alpha\cos(\omega R)}$

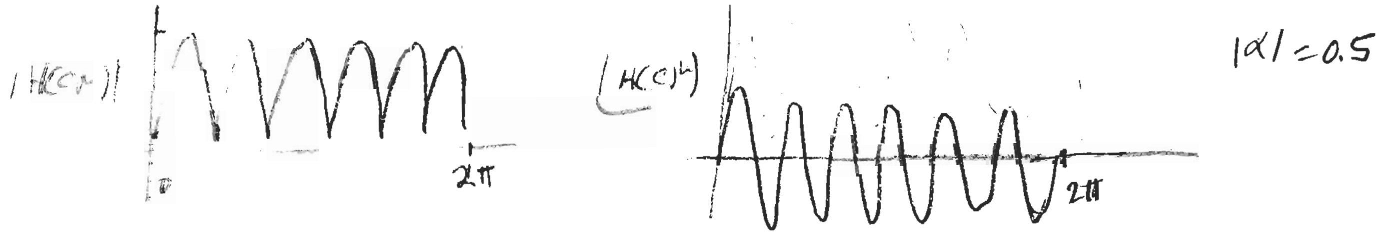
a) $\max(|H(e^{j\omega})|) = \sqrt{1 + \alpha^2 + 2\alpha} = \sqrt{(1+\alpha)^2} = 1 + |\alpha|$

$\min(|H(e^{j\omega})|) = \sqrt{1 + \alpha^2 - 2\alpha} = \sqrt{(1-\alpha)^2} = 1 - |\alpha|$

b) Signal has R peaks/valleys for $0 \le \omega < 2\pi$

c) @ $\omega = \frac{2k\pi}{R}$ $|H(e^{j\omega})|$ is minimum $\quad 0 < k \le R-1$

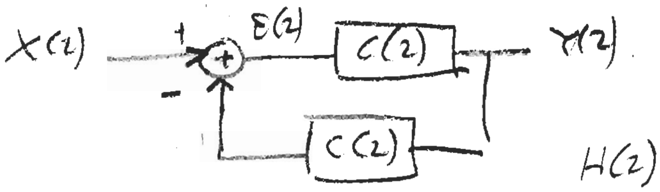
@ $\omega = \frac{(2k+1)\pi}{R}$ $|H(e^{j\omega})|$ is maximum



$\boxed{6.52}$ Let $H(z)$ be a T.F. of a causal, stable LTI discrete-time system. Consider the T.F. $G(z) = H(z)|_{z=F(z)}$. What are the conditions that need to be satisfied by $F(z)$ so that $G(z)$ remains stable.

The transformation $z \to F(z)$ should be such that the unit circle remains inside the ROC after mapping. If the points inside the unit circle after the mapping remain inside the unit circle, $G(z)$ will be causal & stable.

... system. If $G(z) = 2/(1+3z^{-1})$ and $C(z) = K$, determine range of K for which feedback is stable.



$E(z) = X(z) - Y(z)C(z)$

$Y(z) = E(z)G(z)$

$Y(z)(1 + C(z)G(z)) = G(z)X(z)$

$$\frac{Y(z)}{X(z)} = H(z) = \frac{G(z)}{1 + C(z)G(z)}$$

$$H(z) = \frac{\frac{2}{1+3z^{-1}}}{1 + \frac{2K}{1+3z^{-1}}} = \frac{2}{(1+2K) + 3z^{-1}}$$

The only pole is at

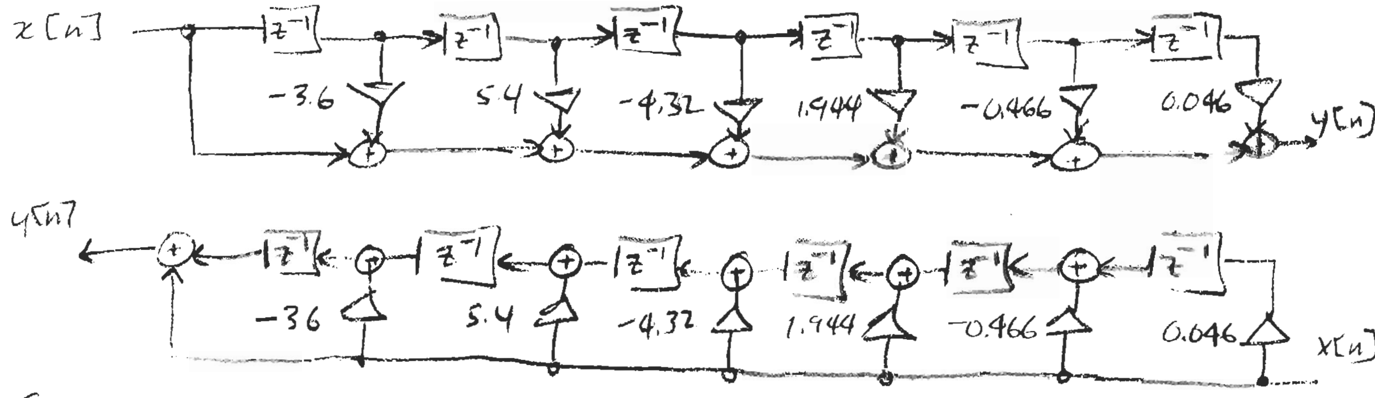
$$z = -\frac{3}{1+2K}$$

System stable if

$$\left|\frac{3}{1+2K}\right| < 1 \qquad K > 1 \text{ or } K < -2$$

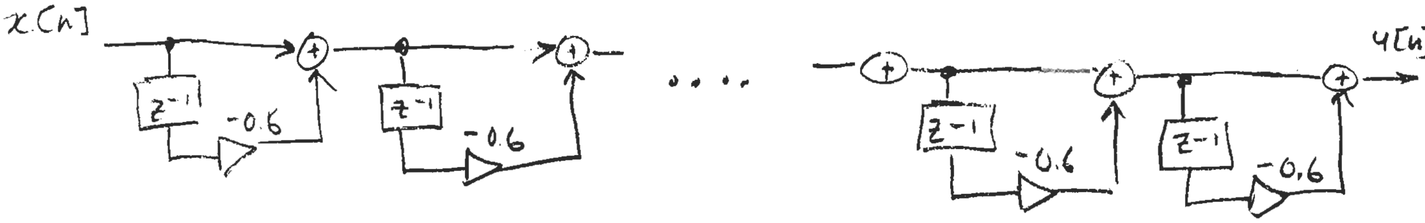
8.10 (a)(b) Realize the FIR T.F $H(z) = (1 - 0.6z^{-1})^6$ in the following forms

(a) Two different direct forms.



(b) Cascade of six first-order sections

$H(z) = (1-0.6z^{-1})(1-0.6z^{-1})(1-0.6z^{-1})(1-0.6z^{-1})(1-0.6z^{-1})(1-0.6z^{-1})$



HW # 7 Due 10/23

$\boxed{8.13}$ (a) Develop a three-branch polyphase realization of $H(z)$ of Problem 8.11 in the form of Fig. 8.7(b), and determine the expressions for $E_0(z)$, $E_1(z)$.
(b) From this realization, develop a canonic two-branch polyphase realization

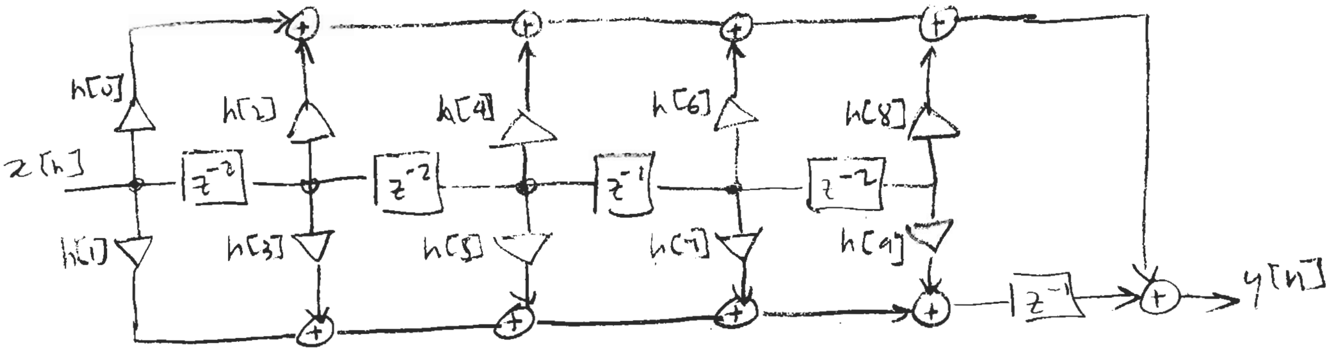
$$H(z) = h[0] + h[1]z^{-1} + h[2]z^{-2} + h[3]z^{-3} + h[4]z^{-4} + h[5]z^{-5} + h[6]z^{-6} + h[7]z^{-7} + h[8]z^{-8} + h[9]z^{-9}$$

a) $H(z) = E_0(z^2) + E_1(z^2)z^{-1}$

$E_0(z) = h[0] + h[2]z^{-1} + h[4]z^{-2} + h[6]z^{-3} + h[8]z^{-4}$

$E_1(z) = h[1] + h[3]z^{-1} + h[5]z^{-2} + h[7]z^{-3} + h[9]z^{-4}$

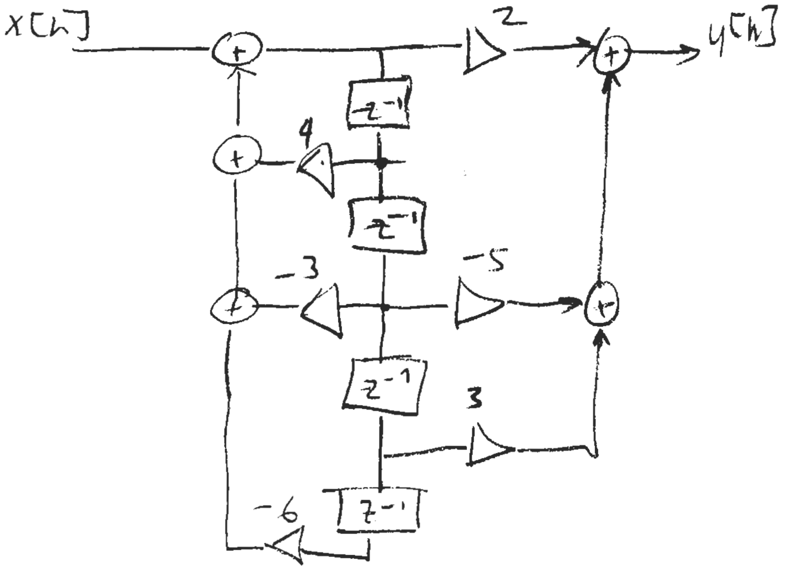
b)

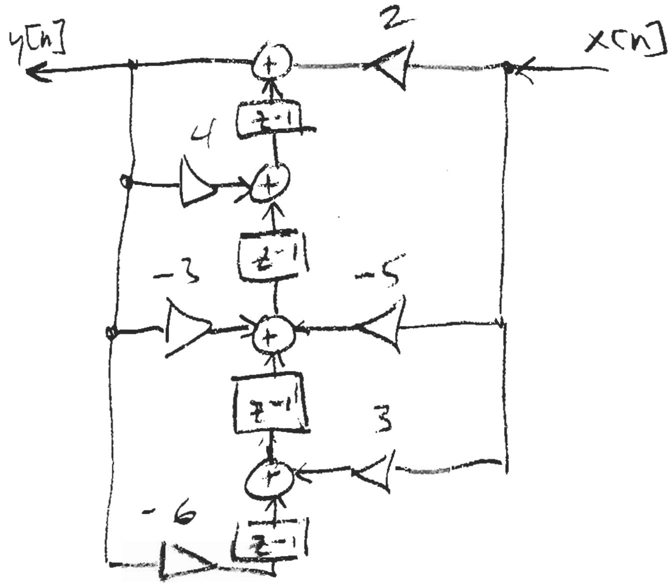


$\boxed{8.20}$ Develop a ~~canonic~~ direct form realization of T.F.

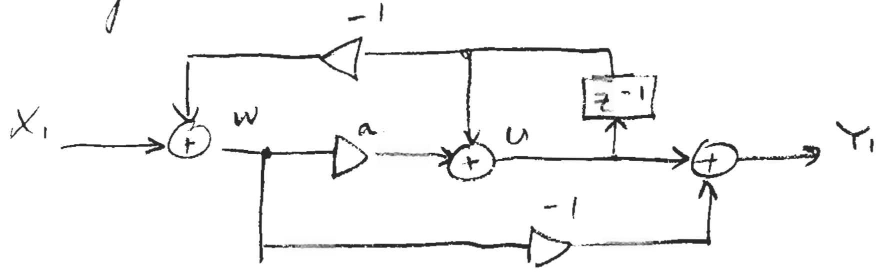
$$H(z) = \frac{2 - 5z^{-2} + 3z^{-3}}{1 - 4z^{-1} + 3z^{-2} + 6z^{-4}}$$

Determine its transpose configuration





$\boxed{8.35}$ Analyze the digital filter structure of Figure P8.15 and show that it is a first-order allpass filter.



$$W = X_1 - z^{-1}U \longrightarrow X_1 = W + \frac{z^{-1}a}{1-z^{-1}}W = \frac{1-z^{-1}+z^{-1}a}{1-z^{-1}}W = \frac{1+(a-1)z^{-1}}{1-z^{-1}}W$$

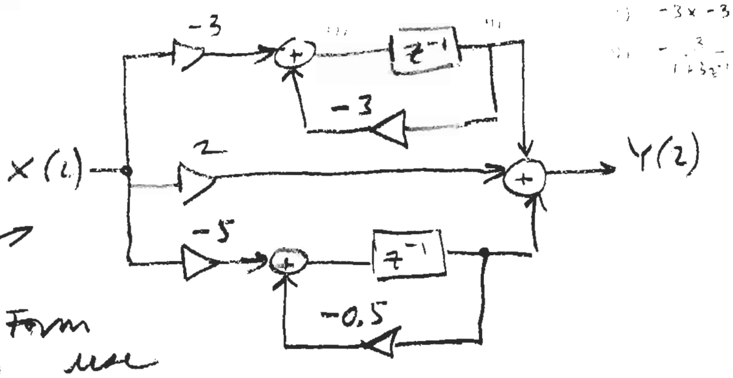
$$U = aW + z^{-1}U \longrightarrow U = \frac{a}{1-z^{-1}}W$$

$$Y = U - W \longrightarrow Y = \frac{a}{1-z^{-1}}W - W = \frac{a-1+z^{-1}}{1-z^{-1}}W$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{(a-1)+z^{-1}}{1+(a-1)z^{-1}}$$

$\boxed{8.26}$ The structure shown in following figure was developed in the course of a realization of the IIR digital TF.

$$H(z) = \frac{2z^2 - 3.2z - 18.6}{z^2 - 2.6z - 1.2}$$



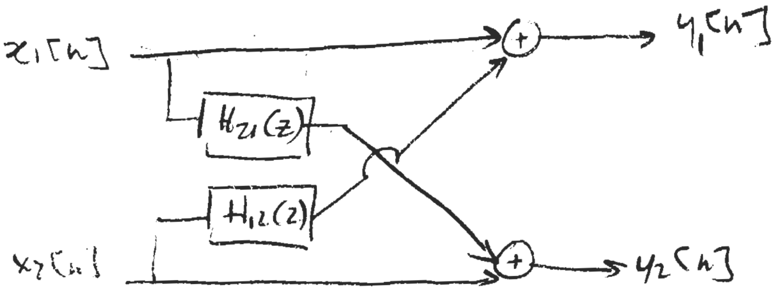
This is clearly a Parallel Form structure, so we need to use partial fraction expansion

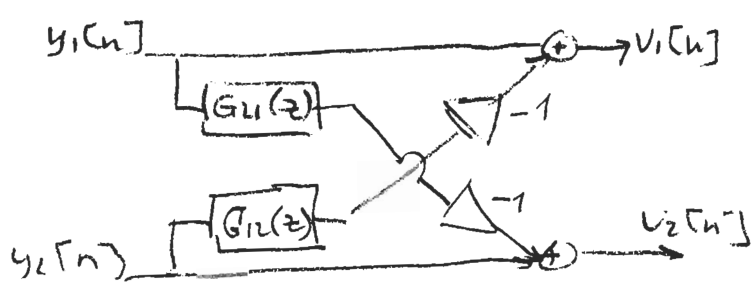
$$H(z) = 2 + \frac{2z-16.2}{(z-3)(z+0.4)} = 2 + \frac{A}{z-3} + \frac{B}{z+0.4} = 2 - \frac{3}{z-3} + \frac{5}{z+0.4}$$

$$= 2 - \frac{3z^{-1}}{1-3z^{-1}} + \frac{5z^{-1}}{1+0.4z^{-1}}$$

- ~~The feedback in upper network should be +3.~~
- The feedforward in lower network should be +5 and feedback should be -0.4.

8.55 Signals generated by multiple sources or multiple sensors, called multichannel signals, are usually transmitted through independent channels in close proximity with each other. As a result, each component of the multichannel often gets corrupted by signals from adjacent channels during transmission, resulting in cross-talk. Separation of multichannel signal at the receiver is thus of practical interest. A model of cross-talk is depicted below, and corresponding system for channel separation is also shown. Determine two possible sets of conditions for perfect channel separation.





$$\begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix} = \begin{bmatrix} 1 & H_{12}(z) \\ H_{21}(z) & 1 \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} V_1(z) \\ V_2(z) \end{bmatrix} = \begin{bmatrix} 1 & -G_{12}(z) \\ -G_{21}(z) & 1 \end{bmatrix} \begin{bmatrix} Y_1(z) \\ Y_2(z) \end{bmatrix}$$

$$\begin{bmatrix} Y_1(z) \\ V_2(z) \end{bmatrix} = \begin{bmatrix} 1 & -G_{12}(z) \\ -G_{21}(z) & 1 \end{bmatrix} \begin{bmatrix} 1 & H_{12}(z) \\ H_{21}(z) & 1 \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \end{bmatrix}$$

$$= \begin{bmatrix} 1 - H_{21}(z)G_{12}(z) & H_{12}(z) - G_{12}(z) \\ -G_{21}(z) + H_{21}(z) & -G_{21}(z)H_{12}(z) + 1 \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \end{bmatrix}$$

The cross-talk can be eliminated if off diagonal elements are zero.

$H_{12}(z) = G_{12}(z)$ and $H_{21}(z) = G_{21}(z)$

$$\begin{bmatrix} V_1(z) \\ V_2(z) \end{bmatrix} = \begin{bmatrix} 1 - H_{21}(z)G_{12}(z) & 0 \\ 0 & 1 - H_{12}(z)G_{21}(z) \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \end{bmatrix}$$

$G_{12}(z) = H_{21}(z)^{-1}$ & $G_{21}(z) = H_{12}(z)^{-1}$

$$\begin{bmatrix} V_1(z) \\ V_2(z) \end{bmatrix} = \begin{bmatrix} 0 & G_{12}(z) - H_{12}(z) \\ -G_{21}(z) + H_{21}(z) & 0 \end{bmatrix} \begin{bmatrix} X_1(z) \\ X_2(z) \end{bmatrix}$$