By KCL at node v_2, we get

$$7(v_2 - v_1) + 3v_2 + i = 0 \qquad (2.17)$$

But i, the current through a voltage source, can be anything—we cannot use Ohm's law for a voltage source! Can we express i in terms of v_1, v_2, and v_3 other than what we get from Equation (2.17)? The answer is yes.

By KCL at node v_3,

$$i = 2(v_3 - v_1) + 5v_3 \qquad (2.18)$$

and substituting this into Equation (2.17) yields

$$7(v_2 - v_1) + 3v_2 + 2(v_3 - v_1) + 5v_3 = 0 \qquad (2.19)$$

from which

$$-9v_1 + 10v_2 + 7v_3 = 0 \qquad (2.20)$$

and this is the second of the three equations needed to solve for v_1, v_2, and v_3.

So far we have applied KCL at all three nonreference nodes, and we only have two equations [Equations (2.16) and (2.20)] to show for it. Where is the third equation?

It was the voltage source that required us to combine two equations [Equations (2.17) and (2.18)] into a single equation [Equation (2.20)] in order to get an expression that does not include i. Yet, it is the voltage source that gives us the third needed equation. Specifically, by KVL,

$$v_3 - v_2 = 3 \qquad (2.21)$$

Solving Equations (2.16), (2.20), and (2.21), we get $\Delta = 72$, $\Delta_1 = -36$, $\Delta_2 = -108$, and $\Delta_3 = 108$. Therefore,

$$v_1 = \frac{\Delta_1}{\Delta} = \frac{-36}{72} = -0.5 \text{ V} \quad v_2 = \frac{\Delta_2}{\Delta} = \frac{-108}{72} = 1.5 \text{ V} \quad v_3 = \frac{\Delta_3}{\Delta} = \frac{108}{72} = 1.5 \text{ V}$$

Now that nodal analysis is complete, we can determine i if we so desire. By KCL at node v_3,

$$i = 2(v_3 - v_1) + 5v_3 = 2(1.5 + 0.5) + 5(1.5) = 11.5 \text{ A}$$

Alternatively, by KCL at node v_2,

$$i = 7(v_1 - v_2) - 3v_2 = 7(-0.5 + 1.5) - 3(-1.5) = 11.5 \text{ A}$$

DRILL EXERCISE 2.5

For the circuit shown in Fig. DE2.5, use nodal analysis to find the node voltages v_1, v_2, and v_3. Use these results to determine i_1 and i_2.
Answer: -4 V; 6 V; 8 V; 4 A; 0 A

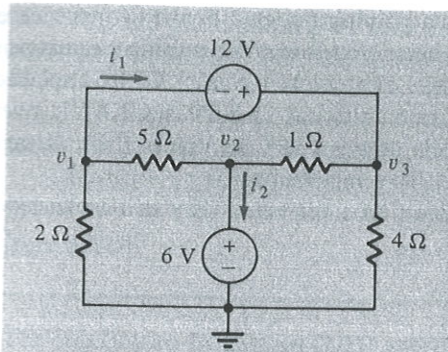

Fig. DE2.5

Supernodes

For the circuit given in Fig. 2.10, because there is a voltage source connected between the two nonreference nodes labeled v_2 and v_3, we designated a current i through the voltage source. Having applied KCL to the node labeled v_2, we obtained an equation [Equation (2.17)] with i in it. Having applied KCL to the node labeled v_3, we got another equation [Equation (2.18)] with i in it. We then combined the two equations to get an expression [Equation (2.19)] that does not contain i. But note that Equation (2.19) is the expression that is obtained by applying KCL to the region, shown in Fig. 2.11, which surrounds the 3-V source and the nodes to which it is connected. This region is referred to as a **supernode**.

When employing nodal analysis on a circuit with a voltage source (either independent or dependent) that is connected between two nonreference nodes, either a current through the voltage source can be designated or a region (supernode) can

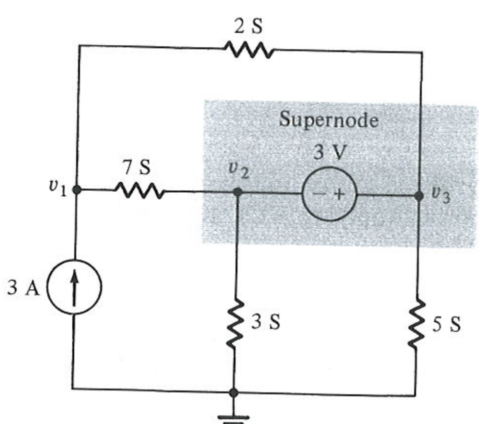

Fig. 2.11 Supernode corresponding to voltage source.

4. A set of mesh equations can be expressed as a single matrix equation of the form $\mathbf{Ri} = \mathbf{v}$.

5. To use loop analysis (on planar or nonplanar circuits) apply KVL around each loop formed from one nonbranch edge and branches, express the resulting equations in terms of the loop currents, and solve.

6. A set of loop equations can be expressed as a single matrix equation of the form $\mathbf{Ri} = \mathbf{v}$.

PROBLEMS FOR CHAPTER 2

2.1 For the circuit shown in Fig. P2.1, select node a to be the reference node. (a) Use nodal analysis to find the nonreference node voltages. (b) Find i_1, i_2, i_3, and i_4.

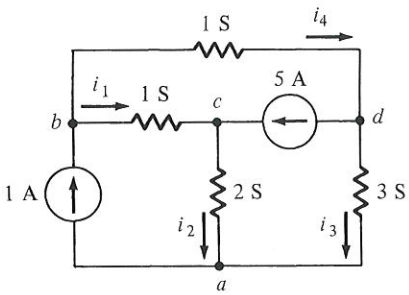

Fig. P2.1

2.2 Repeat Problem 2.1 using node b as the reference node.

2.3 Repeat Problem 2.1 using node c as the reference node.

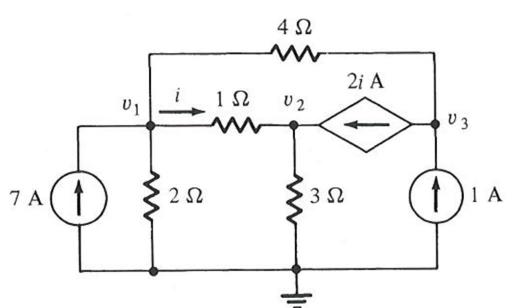

Fig. P2.5

2.4 Repeat Problem 2.1 using node d as the reference node.

2.5 Find the node voltages v_1, v_2, and v_3 for the circuit shown in Fig. P2.5.

2.6 Find the node voltages v_1, v_2, and v_3 for the circuit shown in Fig. P2.6.

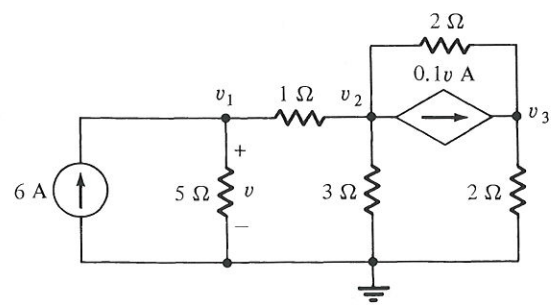

Fig. P2.6

2.7 For the circuit shown in Fig. P2.7, find (a) the node voltages v_1, v_2, v_3, and v_4, (b) i_s, and (c) the conductance $G_{eq} = i_s/v_s$ seen by the voltage source.

2.8 Repeat Problem 2.7 for the circuit shown in Fig. P2.8.

2.9 Find the node voltages v_1, v_2, v_3, and v_4 for the circuit shown in Fig. P2.9.

2.10 Find the node voltages v_1, v_2, v_3, and v_4 for the circuit shown in Fig. P2.10.

2.11 For the circuit shown in Fig. P2.11, find the node voltages v_1, v_2, and v_3.