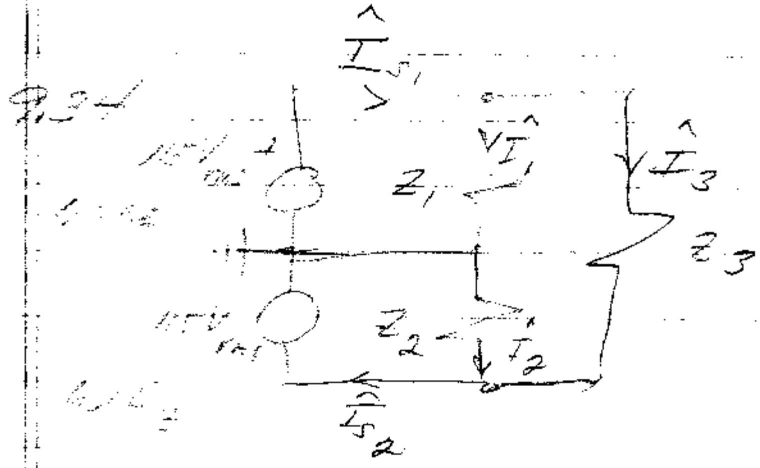
EE 212 Homework 1

$P = VI\cos\theta$



for Z_1 $\begin{cases} P = 500W \\ pf = 0.8 \text{ lagging} \end{cases}$

for Z_2 $\begin{cases} P = 1000W \\ pf = 0.9 \text{ lagging} \end{cases}$

for Z_3 $\begin{cases} P = 1500W \\ pf = 0.95 \text{ leading} \end{cases}$

$|\hat{I}_1| = \frac{500}{115 \times 0.8} = 5.435$; $\theta = \cos^{-1} 0.8 = 36.87°$

$\therefore \hat{I}_1 = 5.435\, e^{-j36.87°}$

$\hat{I}_2 = \frac{1000}{115 \times 0.9}\, e^{-j\cos^{-1}(0.9)} = 9.66\, e^{-j25.84°}$

$\hat{I}_3 = \frac{1500}{230 \times 0.95}\, e^{+j\cos^{-1}(0.95)} = 6.865\, e^{j18.19°}$

$\hat{I}_{S_1} = \hat{I}_1 + \hat{I}_3 = 4.348 - j3.261 + 6.522 + j2.143$

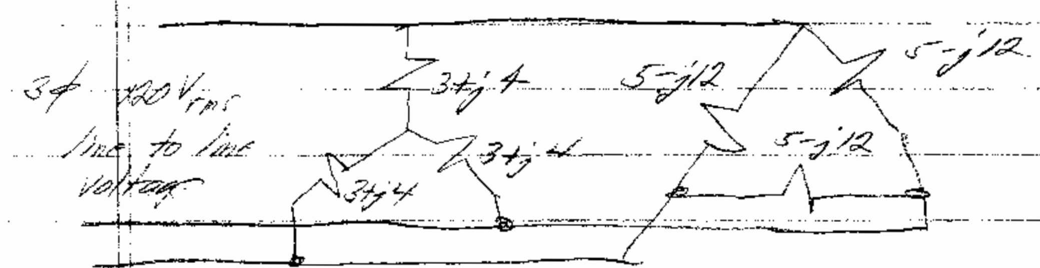
$\hat{I}_{S_1} = \boxed{10.87 - j1.118} = 10.92\, e^{-j5.87°}$

$\hat{I}_{S_2} = \hat{I}_2 + \hat{I}_3 = 8.694 - j4.21 + 6.522 + j2.143 = 15.22 - j2.07$

$\hat{I}_{S_2} = 15.36\, e^{-j7.75°}$

EE 212 Homework 2

9.44

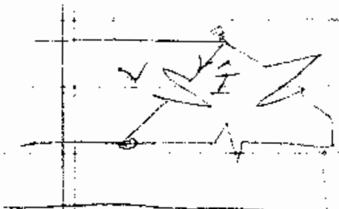


convert Y load to delta $Z_\Delta = 3Z_Y = 9+j12$

this gives a Δ connected load with each impedance $= \dfrac{(9+j12)(5-j12)}{14} = \dfrac{(45+144)+j(-48)}{14}$

$\therefore\ Z_{\Delta\,total} = 13.5 - j3.429 = 13.93\,e^{-j14.25^\circ}$

$pf = \cos(14.25) = 0.97$



$$\hat{I}_{rms} = \frac{120}{13.93\,e^{-j14.25}} = 8.615\,e^{j14.25}$$

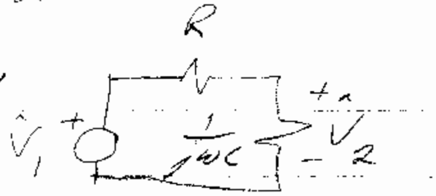
$$P_{/phase} = |\hat{I}_{rms}|^2 \times 13.5 = 1001.9 \text{ Watts}$$

$$\text{total } P = 3\,P_{/phase} = \boxed{3005.8\text{ W}}$$

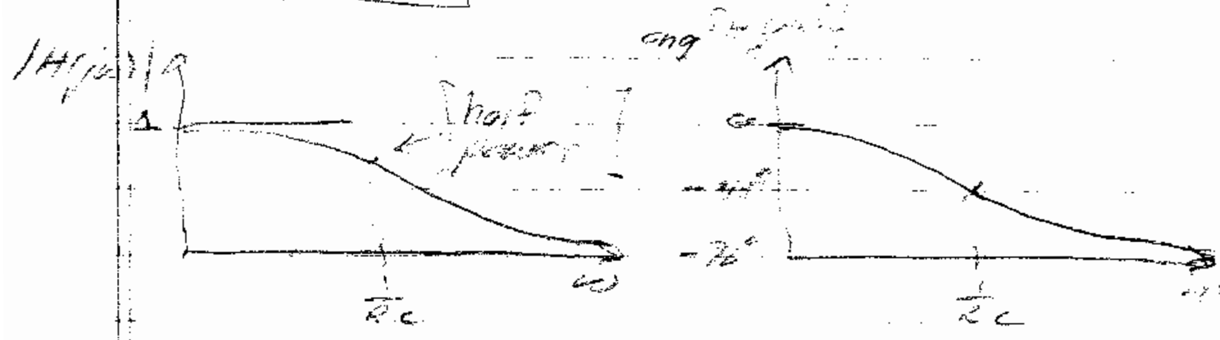
or $P_{/phase} = V_{rms} I_{rms} \cos\theta = 120 \times 8.615 \cos(14.25^\circ) = 1001.99$

EE 212 Homework 3

10.1

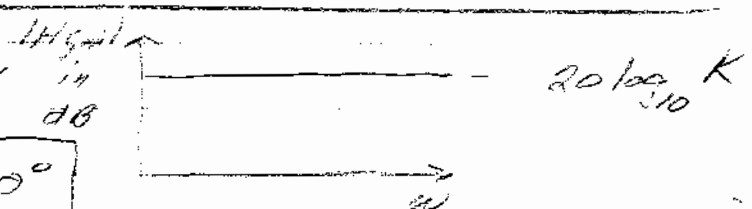


$$H(j\omega) = \frac{\hat{V}_2}{\hat{V}_1} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$



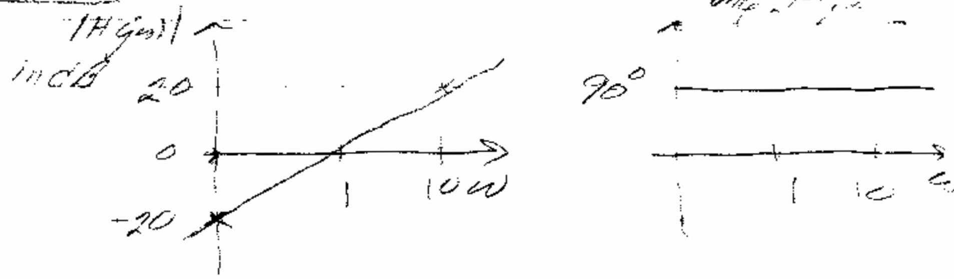
(Low pass network)

10.11 a) $H(j\omega) = K$

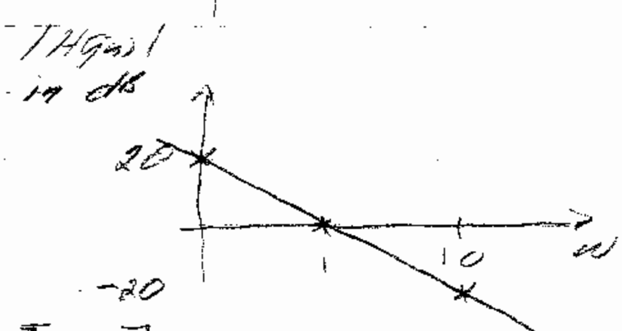


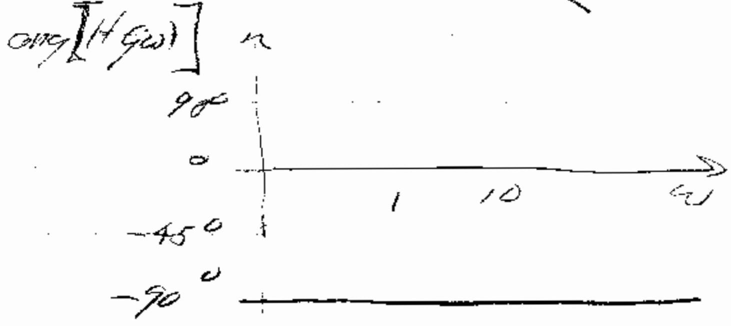
$\longrightarrow \boxed{\text{ang } H(j\omega) = 0^\circ}$

b) $H(j\omega) = j\omega$



c) $H(j\omega) = \frac{1}{j\omega}$

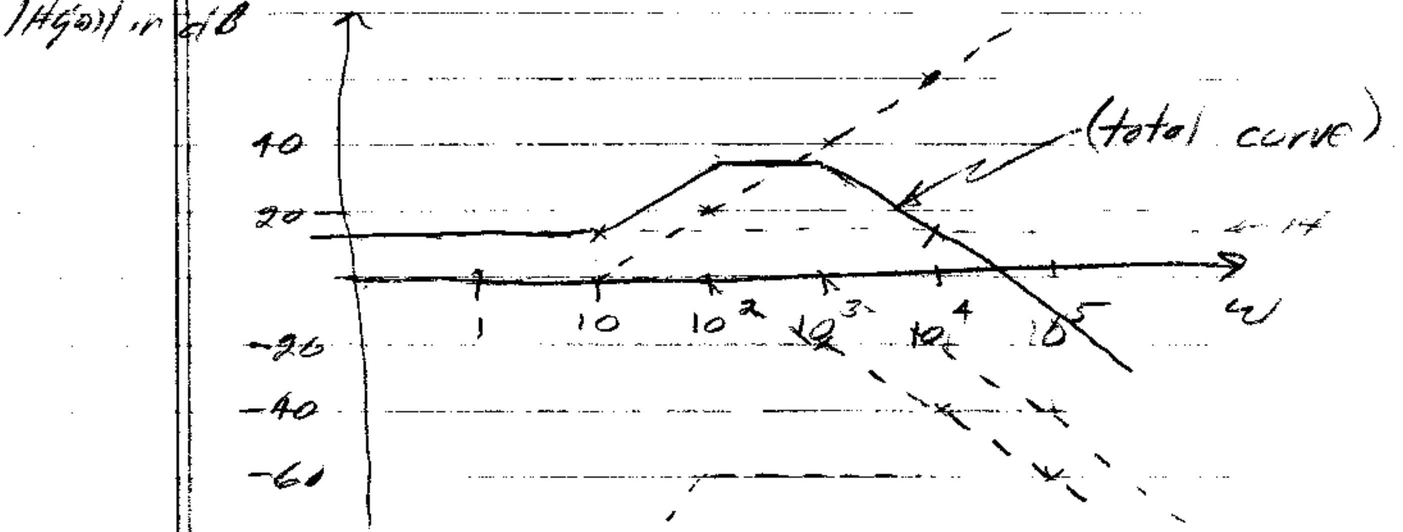


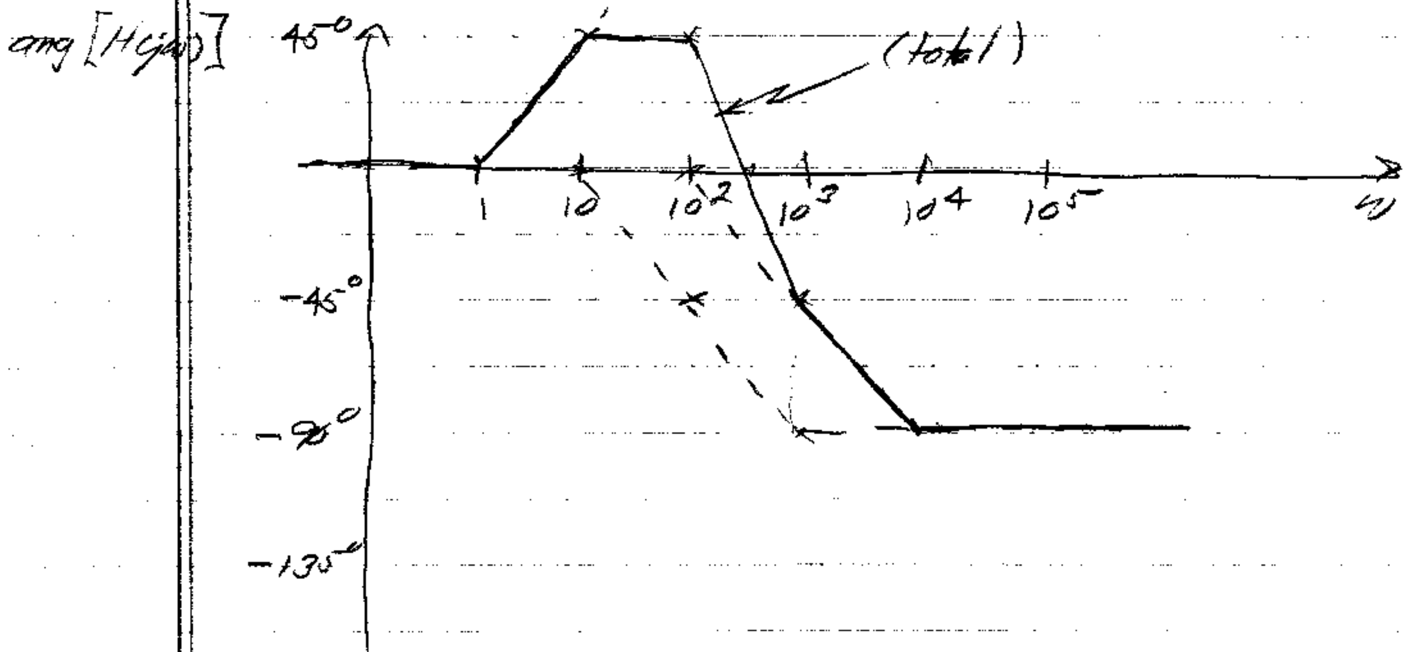


EE 212 Homework 4

Special

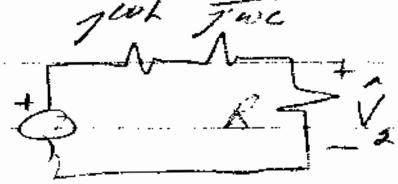
$$H(j\omega) = \frac{5\left(1+j\frac{\omega}{10}\right)}{\left(1+j\frac{\omega}{100}\right)\left(1+j\frac{\omega}{10^3}\right)} \quad ; \; 20\log_{10} 5 \cong 14$$





EE 212 Homework 5

10.12

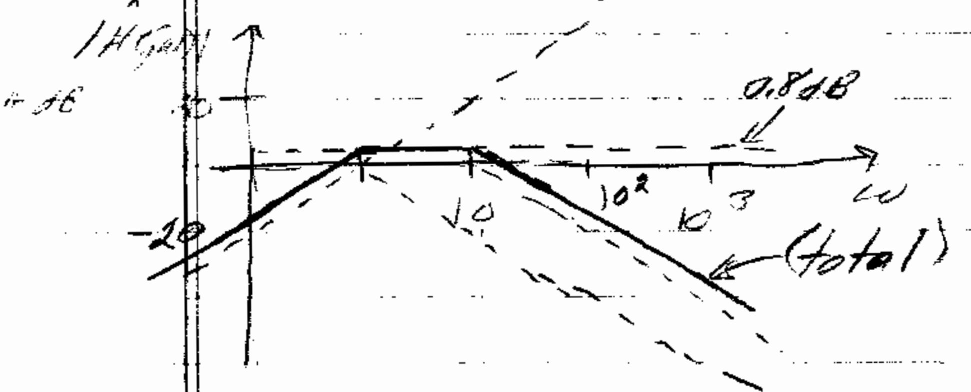


$$\hat{H}(j\omega) = \frac{R}{R + j\omega L + \frac{1}{j\omega C}}$$

$$\hat{H}(j\omega) = \frac{j\omega RC}{1 + j\omega RC + (j\omega)^2 LC} = \frac{j\omega \frac{R}{L}}{(j\omega)^2 + j\omega \frac{R}{L} + \frac{1}{LC}}$$

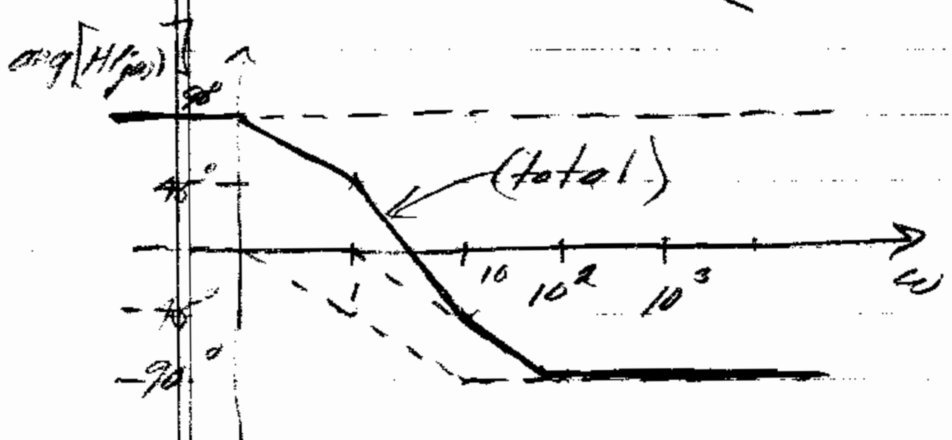
for $R = 11$, $L = 1$, $C = \frac{1}{10}$ $\quad \hat{H}(j\omega) = \frac{j\omega 11}{(j\omega)^2 + j\omega 11 + 10}$

or $\hat{H}(j\omega) = \frac{11 j\omega}{(j\omega + 10)(j\omega + 1)} = \frac{11}{10} \cdot \frac{j\omega}{(1 + j\frac{\omega}{10})(1 + j\omega)}$



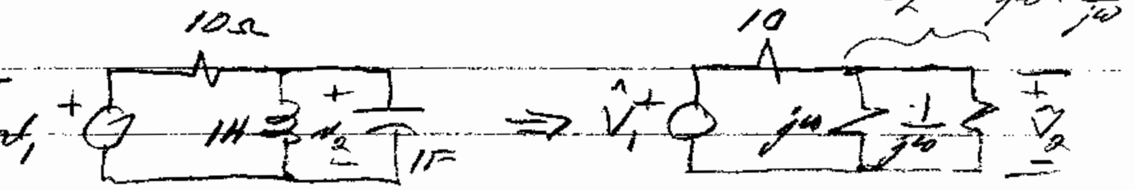
(Bandpass)

$\boxed{20\log_{10}\left(\frac{11}{10}\right) = 0.827}$



EE 212 Homework 6

10.15

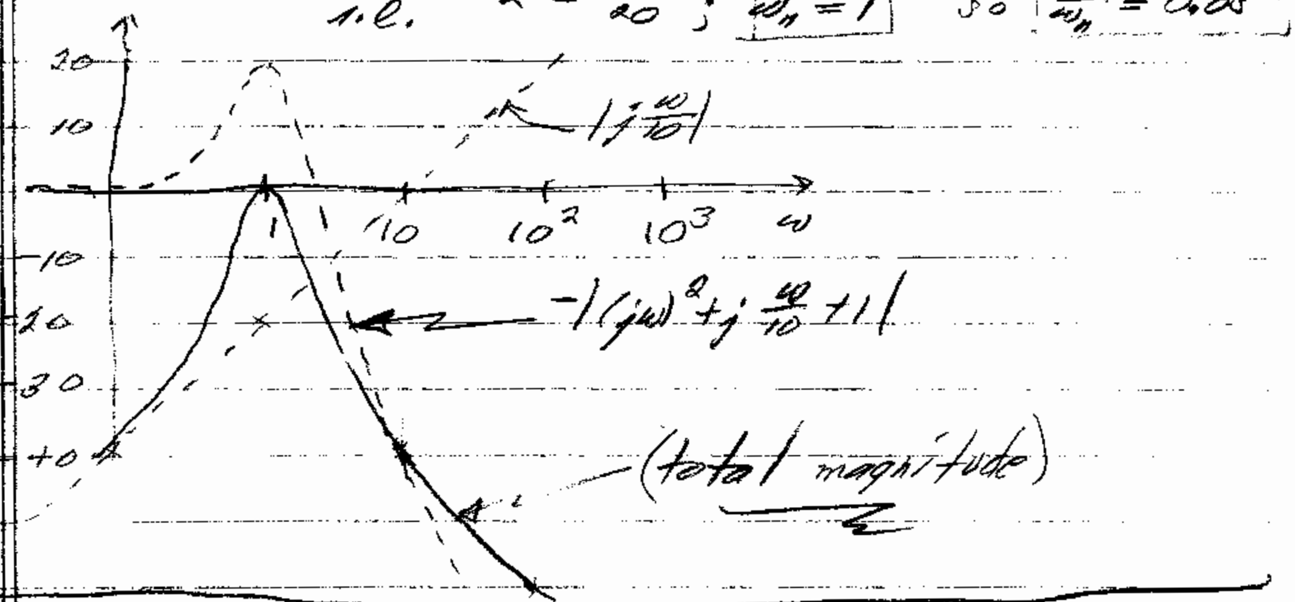


$$\hat{H}(j\omega) = \frac{\hat{V}_2}{\hat{V}_1} = \frac{Z_{eq}}{10 + Z_{eq}} = \frac{j\omega}{10(j\omega)^2 + j\omega + 10} = \frac{j\omega}{(j\omega - s_1)(j\omega - s_2)} = \frac{j\frac{\omega}{10}}{(j\omega)^2 + j\frac{\omega}{10} + 1}$$

$$s_{1,2} = \frac{-1 \pm \sqrt{1-400}}{20} = -\frac{1}{20} \pm j\sqrt{1-\left(\tfrac{1}{20}\right)^2}$$

i.e. $\alpha = \frac{1}{20}$; $\boxed{\omega_n = 1}$ so $\boxed{\frac{\alpha}{\omega_n} = 0.05}$

$|\hat{H}(j\omega)|$ in dB



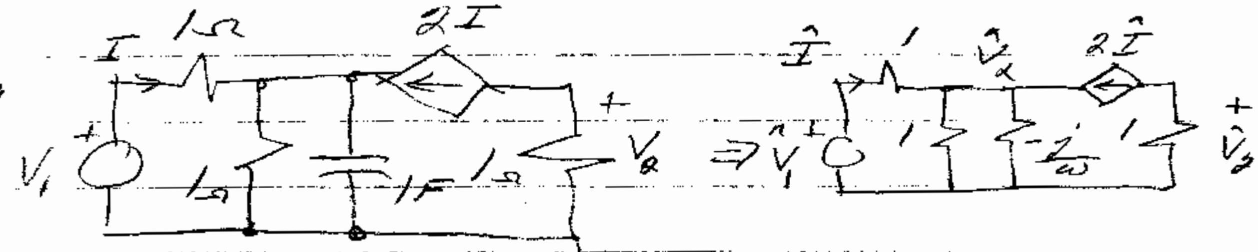
Special (scope probe)

$$\frac{\hat{V}_2}{\hat{V}_1} = \frac{\dfrac{\frac{R_2}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}}}{\dfrac{\frac{R_2}{j\omega C_2}}{R_2 + \frac{1}{j\omega C_2}} + \dfrac{\frac{R_1}{j\omega C_1}}{R_1 + \frac{1}{j\omega C_1}}} = \frac{\dfrac{R_2}{1 + j\omega R_2 C_2}}{\dfrac{R_2}{1 + j\omega R_2 C_2} + \dfrac{R_1}{1 + j\omega R_1 C_1}}$$

if $R_1 C_1 = R_2 C_2$ this gives: $\boxed{\frac{\hat{V}_2}{\hat{V}_1} = \frac{R_2}{R_1 + R_2}}$ ←

EE 212 Homework 7

12.9



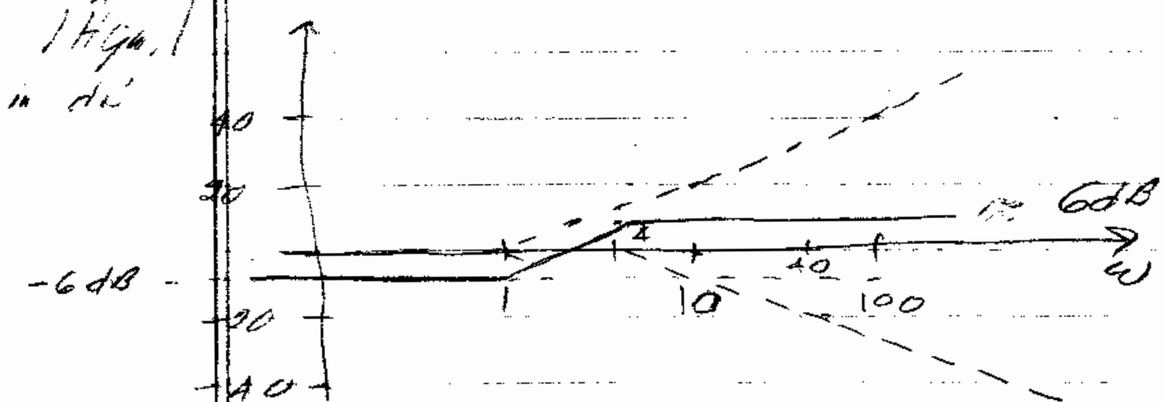
$\hat V_x - \hat V_1 + \hat V_x + j\omega \hat V_x - 2\hat I = 0$ but $\hat I = \hat V_1 - \hat V_x$

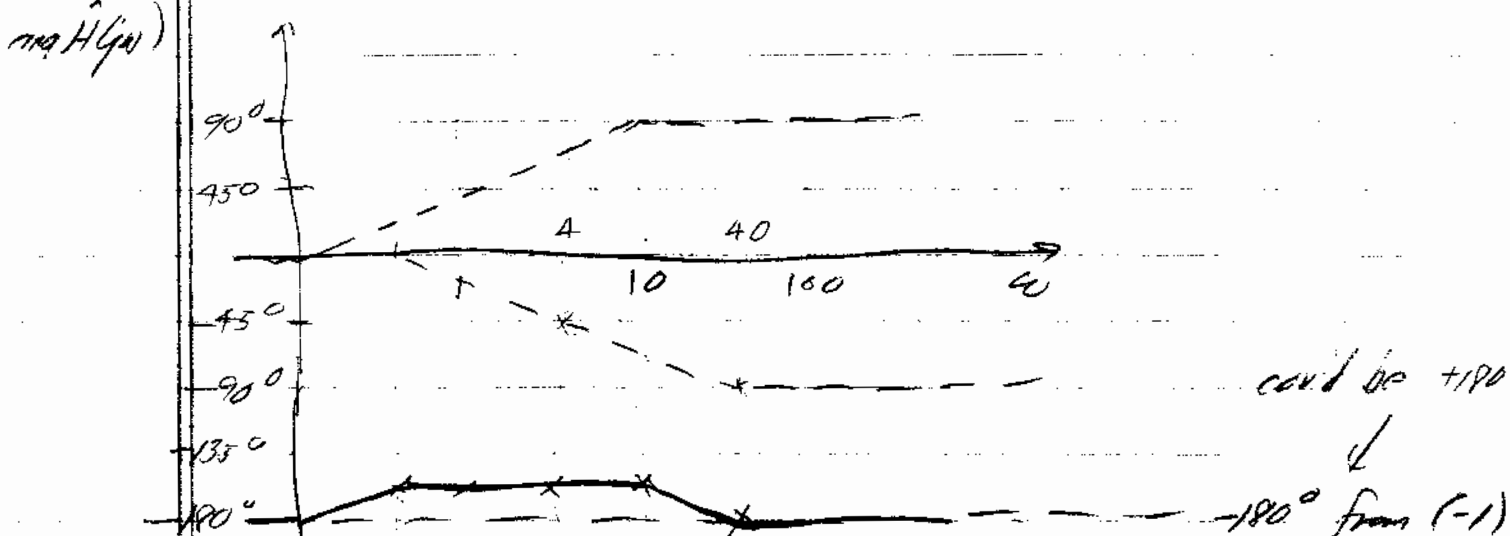
so $2\hat V_x - \hat V_1 + j\omega \hat V_x - 2\hat V_1 + 2\hat V_x = 0 \Rightarrow \hat V_x (4 + j\omega) = 3\hat V_1$

$\hat V_2 = -2\hat I = -2\hat V_1 + 2\hat V_x = -2\hat V_1 + 2\,\frac{3\hat V_1}{4+j\omega} = \frac{(-8-2j\omega+6)\hat V}{4+j\omega}$

or $\frac{\hat V_2}{\hat V_1} = -\frac{(2+j2\omega)}{4+j\omega} = \left(-\frac{1}{2}\right)\frac{1+j\omega}{1+j\frac{\omega}{4}}$

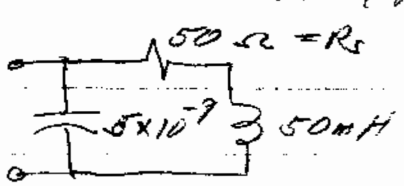
$20\log_{10}(\frac{1}{2}) = -6$; @ $\omega = 4$ $20\log_{10}|4| = 12$ dB





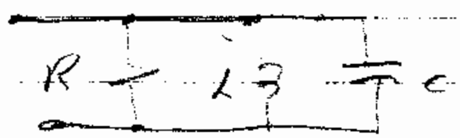
EE 212 Homework 8

10.33



$$Y = j\omega C + \frac{1}{R_s + j\omega L} = \frac{(j\omega)^2 LC + j\omega R_s C + 1}{R_s + j\omega L}$$

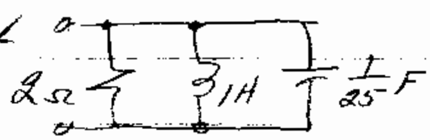
If $R_s << \omega L$ we can replace this circuit with



where $\boxed{R = \frac{L}{R_s C} = 200\,k\Omega}$

$$\text{so } Q = R\sqrt{\frac{C}{L}} = 2\times10^{5}\sqrt{\frac{5\times10^{-9}}{5\times10^{-2}}} = \boxed{63.24}$$

10.34



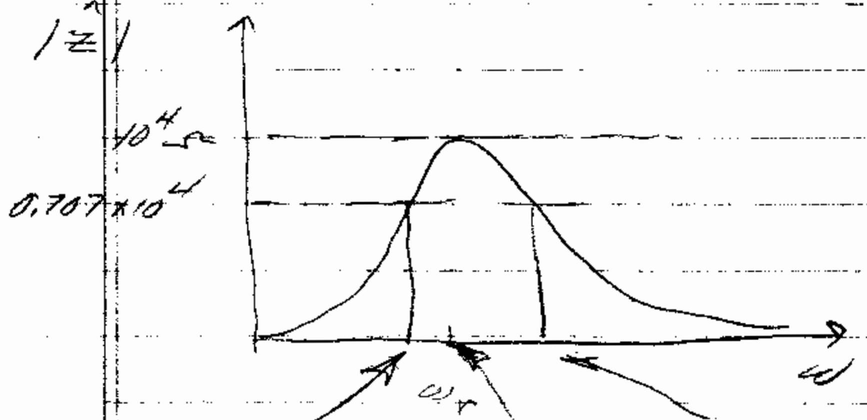
$K_m = 5\times10^{3}$

$K_f = 10^{5}$

New R, L, C
$$\left\{\begin{array}{l} R_{new} = 10^{4} \\ L_{new} = 5\times10^{-2} \\ C_{new} = \frac{1}{125}\times10^{-8} = 8\times10^{-11} \end{array}\right\}$$

$$\therefore \boxed{\omega_r = \frac{1}{\sqrt{LC}} = 5\times10^{5}} \;;\; Q = R\sqrt{\frac{C}{L}} = 10^{4}\sqrt{\frac{8\times10^{-11}}{5\times10^{-2}}} = \boxed{0.4}$$

$$\omega_{1,2} = \pm\frac{\omega_r}{2Q} + \omega_r\sqrt{\left(\frac{1}{2Q}\right)^2 + 1} = \pm 6.25\times10^{5} + 8.0\times10^{5} = \left\{\begin{array}{l} 14.25\times10^{5} \\ 1.75\times10^{5} \end{array}\right\}$$



$\omega_1 = 1.75\times10^{5}$ $\omega_r = 5\times10^{5}$ $\omega_2 = 14.25\times10^{5}$

Homework 9

11.1 a) $f(t)=2e^{-8t}-e^{-2t}$; $\boxed{F(s)=\frac{2}{s+8}-\frac{1}{s+2}}$

b) $f(t)=6+2e^{-6t}-12e^{-t}$; $\boxed{F(s)=\frac{6}{s}+\frac{2}{s+6}-\frac{12}{s+1}}$

c) $f(t)=(2+3t)e^{-2t}$; $\boxed{F(s)=\frac{2}{s+2}+\frac{3}{(s+2)^2}}$

d) $f(t)=(\cos 4t-\sin 4t)e^{-3t}$

$$\boxed{F(s)=\frac{s+3}{(s+3)^2+16}-\frac{4}{(s+3)^2+16}}$$

EE 212 Homework 10

11.3 a) $\mathcal{L}[\sin(\beta t-\phi)\,u(t)] = \mathcal{L}[\sin\beta t\cos\phi - \cos\beta t\sin\phi]$

$$= \frac{\beta\cos\phi}{s^2+\beta^2} - \frac{s\sin\phi}{s^2+\beta^2}$$

b) $\mathcal{L}[\cos(\beta t-\phi)] = \mathcal{L}[\cos\phi\cos\beta t + \sin\phi\sin\beta t] = \frac{s\cos\phi}{s^2+\beta^2} + \frac{\beta\sin\phi}{s^2+\beta^2}$

c) $\mathcal{L}[e^{-\alpha t}\sin(\beta t-\phi)] = \frac{\beta\cos\phi}{(s+\alpha)^2+\beta^2} - \frac{(s+\alpha)\sin\phi}{(s+\alpha)^2+\beta^2}$

d) $\mathcal{L}[e^{-\alpha t}\cos(\beta t-\phi)] = \frac{(s+\alpha)\cos\phi}{(s+\alpha)^2+\beta^2} + \frac{\beta\sin\phi}{(s+\alpha)^2+\beta^2}$

EE 212 Homework 11

11.11 a) $F(s) = \dfrac{6}{s(s+1)(s+3)} = \dfrac{2}{s} + \dfrac{-3}{s+1} + \dfrac{1}{s+3}$

$\therefore f(t) = (2 - 3e^{-t} + e^{-3t})\, u(t)$

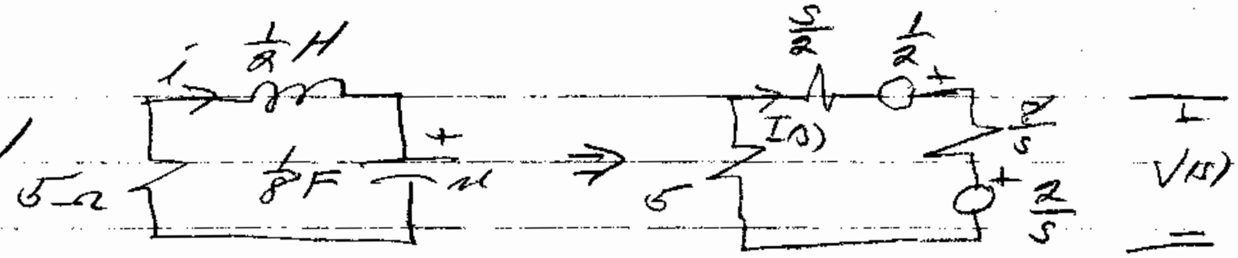
b) $F(s) = \dfrac{60(s+4)}{s(s+2)(s+12)} = \dfrac{\frac{240}{24}}{s} + \dfrac{\frac{120}{-20}}{s+2} + \dfrac{\frac{-480}{120}}{s+12}$

$F(s) = \dfrac{10}{s} - \dfrac{6}{s+2} - \dfrac{4}{s+12}$

$\therefore f(t) = (10 - e^{-2t} - 4e^{-12t})\, u(t)$

EE 212 Homework 12

14.21



$v(0) = 2V,\ i(0) = 1A$

KVL $\quad I\left(5 + \frac{s}{2} + \frac{8}{s}\right) = \frac{1}{2} - \frac{2}{s}$

$$I(s) = \frac{\frac{s-4}{2s}}{\frac{10s+s^2+16}{2s}} = \frac{s-4}{s^2+10s+16} = \frac{s-4}{(s+2)(s+8)}$$

$$I(s) = \frac{-1}{s+2} + \frac{2}{s+8} \qquad \therefore\ i(t) = \left[-1e^{-2t} + 2e^{-8t}\right]u(t)$$

$$V(s) = \frac{8}{s}I(s) + \frac{2}{s} = \frac{8(s-4)}{s(s+2)(s+8)} + \frac{2}{s}$$

$$V(s) = \cancel{\frac{\frac{-32}{16}}{s} + \frac{2}{s}} + \frac{\frac{8(-6)}{-2(6)}}{s+2} + \frac{\frac{8(-12)}{-8(-6)}}{s+8} = \frac{4}{s+2} - \frac{2}{s+8}$$

$$\therefore\ v(t) = \left[4e^{-2t} - 2e^{-8t}\right]u(t)$$

EE 212 Homework 13

11.14 b) $F(s) = \dfrac{(s+2)(s+3)}{s(s+1)^2} = \dfrac{1}{s+1}\left\{\dfrac{(s+2)(s+3)}{s(s+1)}\right\}$

$$\begin{array}{r} 1 \\ s^2+s \,\overline{)\, s^2+5s+6} \\ \underline{s^2+s} \\ 4s+6 \end{array}$$

$\therefore F(s) = \dfrac{1}{s+1}\left\{1+\dfrac{4s+6}{s(s+1)}\right\}$

$$F(s) = \frac{1}{s+1}\left\{1+\frac{6}{s}+\frac{-2}{s+1}\right\} = \frac{1}{s+1} + \frac{6}{s(s+1)} - \frac{2}{(s+1)^2}$$

$$F(s) = \frac{1}{s+1} + \frac{6}{s} + \frac{-6}{s+1} - \frac{2}{(s+1)^2} = \frac{6}{s} - \frac{5}{s+1} - \frac{2}{(s+1)^2}$$

$$f(t) = \{6 - 5e^{-t} - 2te^{-t}\}u(t) \longleftarrow$$

EE 212 Homework 14

11.14c) $F(s)=\frac{4(1-e^{-2s})}{s^2(s+2)} = \left\{\frac{K_1}{s}+\frac{2}{s^2}+\frac{1}{s+2}\right\}(1-e^{-2s})$

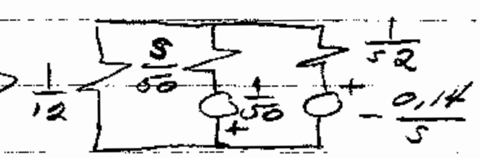
$\frac{d}{ds}\{F(s)s^2\} \Rightarrow K_1 = \frac{-4}{(s+2)^2}\Big|_{s=0} = -1$

so $F(s)=\left\{\frac{-1}{s}+\frac{2}{s^2}+\frac{1}{s+2}\right\}(1-e^{-2s})$

$$\boxed{f(t)=\{-1+2t+e^{-2t}\}u(t)-\{-1+2(t-2)+e^{-2(t-2)}\}u(t-2)}$$

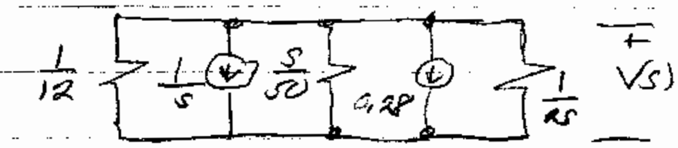
11.22

⇒



$\left\{\begin{array}{l} v(0) = -0.14V \\ i(0) = 1A \end{array}\right\}$ converting to current sources

⇓



$V\left(12+\frac{50}{s}+2s\right) = -\frac{1}{s} - 0.28$

$V = -\frac{1+0.28s}{2s^2+12s+50} = -\frac{0.5+0.14s}{s^2+6s+25} = -\frac{0.5+0.14s}{(s+3)^2+16}$

$V(s) = -\frac{0.14(s+3)-0.42+0.5}{(s+3)^2+16} = -\frac{0.14(s+3)+4(0.02)}{(s+3)^2+16}$

$$\boxed{v(t) = -\{0.14e^{-3t}\cos 4t + 0.02e^{-3t}\sin 4t\}u(t)} \leftarrow$$

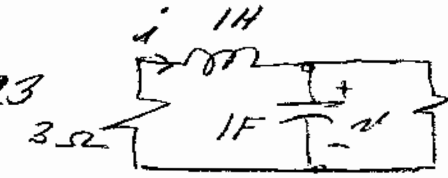
$I(s) = \frac{V(s)+\frac{1}{50}}{\frac{s}{50}} = \frac{50V+1}{s} = \frac{-\frac{25+7s}{s^2+6s+25}+1}{s}$

$I(s) = \frac{\cancel{-25}-7s+s^2+6s\cancel{+25}}{s(s^2+6s+25)} = \frac{s-1}{(s+3)^2+16} = \frac{(s+3)-4}{(s+3)^2+16}$

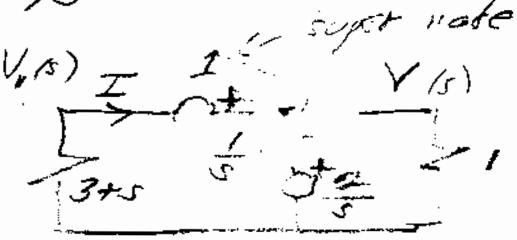
$$\boxed{i(t) = \{e^{-3t}\cos 4t - e^{-3t}\sin 4t\}u(t)} \leftarrow$$

EE 212 Homework 15

11.23



⇒



$v(0) = 2V$, $i(0) = 1A$

$\boxed{V_1 = V - 1}$

@ supernode $\frac{V_1}{s+3} + \frac{V - \frac{2}{s}}{\frac{1}{s}} + \frac{V}{1} = 0$; $V - V_1 = 1$

$\therefore \frac{V-1}{s+3} + \frac{V - \frac{2}{s}}{\frac{1}{s}} + V = 0 = \frac{V-1}{s+3} + Vs - 2 + V = 0$

or $V - 1 + (Vs - 2 + V)(s+3) = 0 = V - 1 + Vs^2 + 3Vs - 2s - 6 + Vs + 3V$

$$V(s^2 + 4s + 4) = 2s + 7$$

$$V = \frac{2s+7}{s^2+4s+4} = \frac{2s+7}{(s+2)^2} = \frac{2}{s+2} + \frac{3}{(s+2)^2}$$

$$v(t) = \{2e^{-2t} + 3te^{-2t}\}u(t)$$

$$I = \frac{V - \frac{2}{s}}{\frac{1}{s}} + V = Vs - 2 + V = V(s+1) - 2$$

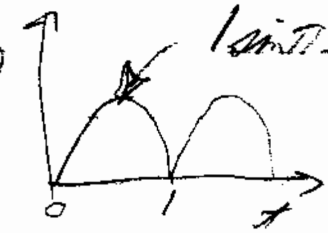
$$V(s+1) = \frac{2s^2 + 7s + 2s + 7}{s^2+4s+4} = \frac{2s^2+9s+7}{s^2+4s+4} = 2 + \frac{s-1}{s^2+4s+4}$$

$$\left\{ \begin{array}{r} 2 \\ s^2+4s+4 \,\overline{)\, 2s^2+9s+7} \\ \underline{2s^2+8s+8} \\ s-1 \end{array} \right\}$$

$\therefore I(s) = \frac{s-1}{(s+2)^2} = \frac{1}{s+2} + \frac{-3}{(s+2)^2}$

and $i(t) = \{e^{-2t} - 3te^{-2t}\}u(t)$

11.5



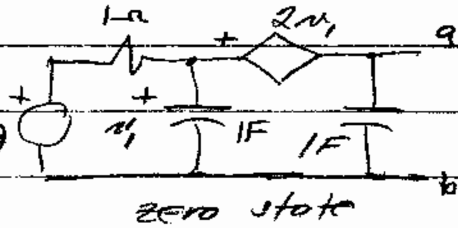
$\mathcal{L}\{\sin \pi t\,[u(t) - u(t-1)]\} = F_1(s)$

but $\sin \pi(t-1) = -\sin \pi t$

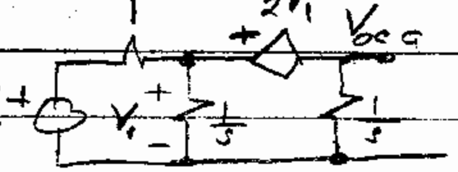
so $F_1(s) = \mathcal{L}\{\sin \pi t\, u(t) + \sin \pi(t-1)\, u(t-1)\}$; $\mathcal{L}\{f(t)\} = \left\{\frac{\pi}{s^2+\pi^2} + \frac{\pi e^{-s}}{s^2+\pi^2}\right\}\frac{1}{1-e^{-s}}$

EE 212 Homework 16

11.49

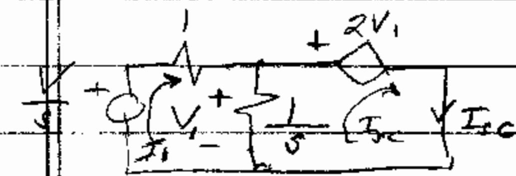


$\Rightarrow$



$V_1 - \frac{V}{s} + sV_1 + V_{oc}s = 0$; $V_1 - V_{oc} = 2V_1$ or $V_1 = -V_{oc}$

$\therefore -V_{oc} - \frac{V}{s} - \cancel{sV_{oc}} + \cancel{sV_{oc}} = 0 \Rightarrow \boxed{V_{oc} = -\frac{V}{s} = V_{Th}}$



$-\frac{V}{s} + I_1 + (I_1 - I_{sc})\frac{1}{s} = 0$

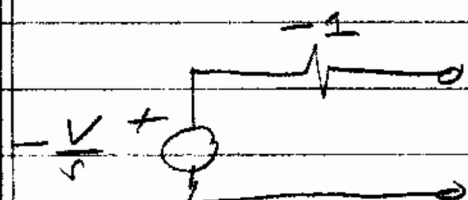
$(I_{sc} - I_1)\frac{1}{s} + 2V_1 = 0$ but $V_1 = (I_1 - I_{sc})\frac{1}{s}$

so $I_1(1 + \frac{1}{s}) - I_{sc}\frac{1}{s} = \frac{V}{s}$ $\quad\left.\right\}$ $I_1(s+1) - I_{sc} = V$

$(I_{sc} - I_1)\frac{1}{s} + \frac{2}{s}(I_1 - I_{sc}) = 0$ $\quad$ $I_1 - I_{sc} = 0$

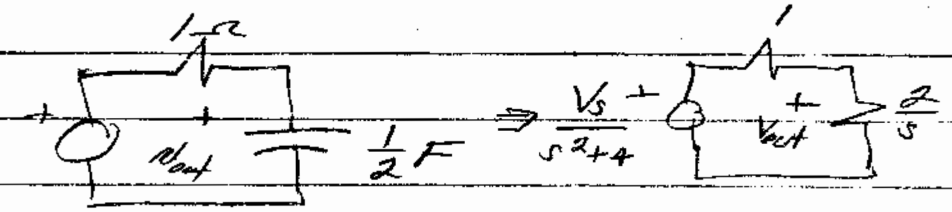
$\therefore +I_{sc}(s+1) - I_{sc} = V \Rightarrow I_{sc} = \frac{V}{s}$

$\therefore Z_{Th} = \frac{V_{oc}}{I_{sc}} = \frac{-\frac{V}{s}}{\frac{V}{s}} = -1$



EE 212 Homework 17

Special



$$\therefore V_{out}(s) = \frac{Vs}{s^2+4}\cdot\frac{\frac{2}{s}}{1+\frac{2}{s}} = 2V\left\{\frac{s}{s^2+4}\cdot\frac{1}{s+2}\right\}$$

$$V_{out}(s) = 2V\left\{\frac{As+B}{s^2+4}+\frac{C}{s+2}\right\} = 2V\left\{\frac{As+B}{s^2+4}+\frac{-\frac{1}{4}}{s+2}\right\}$$

so $\quad s = (As+B)(s+2) - \frac{1}{4}(s^2+4)$

s^2 terms: $\quad 0 = A - \frac{1}{4} \quad \therefore A = \frac{1}{4}$

s terms: $\quad 1 = B + 2A \quad \therefore B = 1 - 2A = \frac{1}{2}$

constants: $\quad 0 = 2B - 1 = 0$

$$V_{out}(s) = 2V\left\{\frac{\frac{1}{4}s+\frac{1}{2}}{s^2+4} - \frac{\frac{1}{4}}{s+2}\right\}$$

$$\boxed{\therefore v_{out}(t) = V\left\{\frac{1}{2}\cos 2t + \frac{1}{2}\sin 2t - \frac{1}{2}e^{-2t}\right\}u(t)}$$

EE 212 Homework 18

11.(25 a) $h(t)=e^{-2t}u(t)$, $x(t)=e^{-t}u(t)$

$y(t)=\int_0^{\infty} e^{-2\tau}\overset{1}{u(\tau)}e^{-(t-\tau)}u(t-\tau)\,d\tau=\int_0^t e^{-t}e^{-\tau}\,d\tau\,u(t)$

$y(t)=e^{-t}(-1)e^{-\tau}\Big|_0^t u(t)=\left[e^{-t}-e^{-2t}\right]u(t)$

b) $h(t)=e^{-2t}u(t)$, $x(t)=2u(t)-2u(t-1)$

$y(t)=\int_0^{\infty} e^{-2\tau}\overset{1}{u(\tau)}\left[2u(t-\tau)-2u(t-\tau-1)\right]d\tau$

$y(t)=\int_0^t 2e^{-2\tau}\,d\tau\,u(t)-2\int_0^{t-1} e^{-2\tau}\,d\tau\,u(t-1)$

$y(t)=-e^{-2\tau}\Big|_0^t u(t)+e^{-2\tau}\Big|_0^{t-1} u(t-1)$

$y(t)=\left[1-e^{-2t}\right]u(t)+\left[e^{-2(t-1)}-1\right]u(t-1)$

EE 212 Homework 19

14.51 $H(s)=\frac{1}{s+2}$

a) $x(t)=u(t)$ $\therefore$ $X(s)=\frac{1}{s}$ and $Y(s)=\frac{1}{s(s+2)}=\frac{\frac{1}{2}}{s}+\frac{-\frac{1}{2}}{s+2}$

$$y(t)=\left[\tfrac{1}{2}-\tfrac{1}{2}e^{-2t}\right]u(t)$$

b) $x(t)=e^{-t}u(t)$; $X(s)=\frac{1}{s+1}$; $Y(s)=\frac{1}{(s+1)(s+2)}=\frac{1}{s+1}+\frac{-1}{s+2}$

$$y(t)=\left[e^{-t}-e^{-2t}\right]u(t)$$

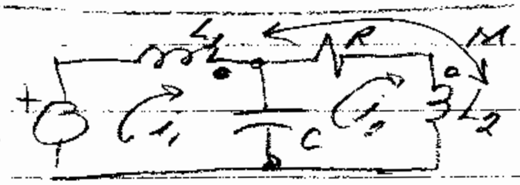
c) $x(t)=(1-e^{-t})u(t)$; $X(s)=\frac{1}{s}-\frac{1}{s+1}$; $Y(s)=\frac{1}{s(s+2)}-\frac{1}{(s+1)(s+2)}$

from a) & b) $y(t)=\left[\tfrac{1}{2}+\tfrac{1}{2}e^{-2t}-e^{-t}\right]u(t)$

d) $x(t)=e^{-2t}u(t)$; $X(s)=\frac{1}{s+2}$; $Y(s)=\frac{1}{(s+2)^2}$

$$y(t)=te^{-2t}u(t)$$

Special



$10\cos 2t$

a) $10\cos 2t = L_1\frac{di_1}{dt} - M\frac{di_2}{dt} + \frac{1}{C}\int_0^t (i_1-i_2)\,dt$

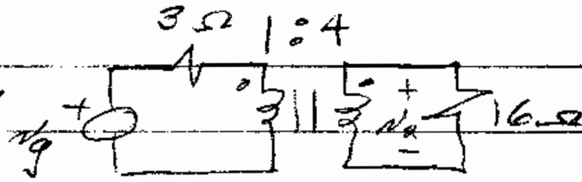
$$0=\frac{1}{C}\int_0^t (i_2-i_1)\,dt + i_2R + L_2\frac{di_2}{dt} - M\frac{di_1}{dt}$$

b) $10 = j\omega L_1\hat{I}_1 - j\omega M\hat{I}_2 + \frac{1}{j\omega C}(\hat{I}_1-\hat{I}_2)$

$$0=\frac{1}{j\omega C}(\hat{I}_2-\hat{I}_1)+\hat{I}_2R+j\omega L_2\hat{I}_2-j\omega M\hat{I}_1$$

EE 212 Homework 20

12.17

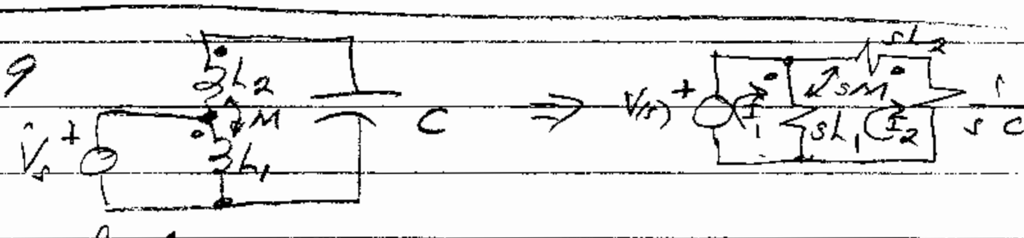


a) $Z = 3 + 1 = \boxed{4\ \Omega}$

b) $v_2 = \frac{1}{4} v_g \times 4 = v_g$; $\boxed{\frac{v_2}{v_g} = 1}$

c) $R_{\text{looking into transformer}} = 3 = \frac{1}{16} R_x \quad \therefore \boxed{R_x = 48\ \Omega}$

12.29



$k = 1$

$$\begin{cases} V(s) = sL_1(I_1 - I_2) - sMI_2 \\ 0 = sL_1(I_2 - I_1) + sMI_2 + sL_2 I_2 + sM(I_2 - I_1) + I_2 \frac{1}{sC} \end{cases}$$

$$V = sL_1 I_1 - (sL_1 + sM) I_2$$

$$0 = I_1(-sL_1 - sM) + I_2\left(sL_1 + sM + sL_2 + \frac{1}{sC} + sM\right)$$

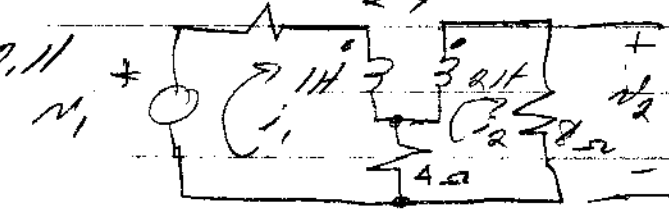
$$I_1 = \frac{\begin{vmatrix} V & -sL_1 - sM \\ 0 & sL_1 + 2sM + sL_2 + \frac{1}{sC} \end{vmatrix}}{\begin{vmatrix} sL_1 & -sL_1 - sM \\ -sL_1 - sM & sL_1 + 2sM + sL_2 + \frac{1}{sC} \end{vmatrix}} = \frac{V\left(sL_1 + 2sM + sL_2 + \frac{1}{sC}\right)}{\cancel{s^2L_1^2} + \cancel{2s^2L_1M} + s^2L_1L_2 + L_1/C - \cancel{s^2L_1^2} - \cancel{2s^2L_1M} - s^2M^2}$$

$$Y_{in} = \frac{I_1}{V} = \frac{sL_1 + 2sM + sL_2 + \frac{1}{sC}}{\underbrace{s^2L_1L_2 - s^2M^2}_{=0 \text{ for } k=1} + L_1/C} = \frac{s^2(L_1 + L_2 + 2M)C + 1}{sL_1}$$

$$Y_{in} = s\left(1 + \underset{\substack{\uparrow \\ N^2}}{\frac{L_2}{L_1}} + 2\sqrt{\frac{L_2}{L_1}}\right)C + \frac{1}{sL_1} = \boxed{s(1+N)^2 C + \frac{1}{sL_1}}$$

EE 212 Homework 21

12.11



$$H(s) = \frac{V_2(s)}{V_1(s)}$$

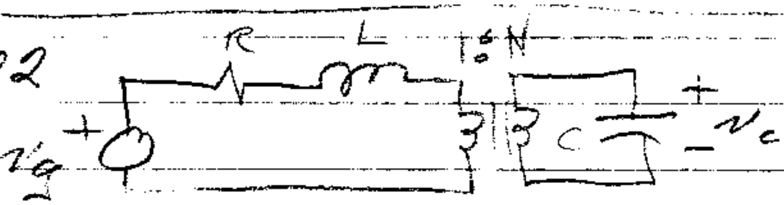
$$V_2(s) = 8I_2(s)$$

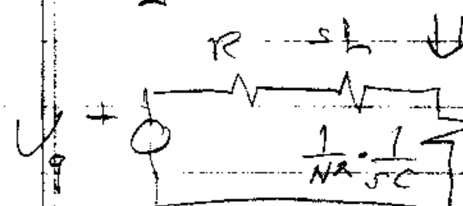
$$\left.\begin{aligned} V_1 &= 3I_1 + sI_1 - sI_2 + 4(I_1 - I_2) \\ 0 &= 4(I_2 - I_1) + 2sI_2 - sI_1 + 8I_2 \end{aligned}\right\} \quad \begin{aligned} V_1 &= I_1(7+s) - I_2(s+4) \\ 0 &= -I_1(s+4) + I_2(12+2s) \end{aligned}$$

$$I_2 = \frac{\begin{vmatrix} 7+s & V_1 \\ -s-4 & 0 \end{vmatrix}}{\begin{vmatrix} 7+s & -s-4 \\ -s-4 & 12+2s \end{vmatrix}} = \frac{V_1(s+4)}{(7+s)(12+2s) - (-s-4)^2} = \frac{V_1(s+4)}{84 + 26s + 2s^2 - s^2 - 8s - 16}$$

$$I_2 = \frac{V_1(s+4)}{s^2+18s+68} \quad \text{so} \quad \boxed{H(s) = \frac{8(s+4)}{s^2+18s+68}}$$

12.22





← RLC series circuit with equivalent capacitance $= N^2C$

a) $\omega_0 = \frac{1}{\sqrt{LCN^2}} = \boxed{\frac{1}{N\sqrt{LC}}}$

b) $Q = \omega_0 \frac{L}{R} = \frac{L}{N\sqrt{LC}\,R} = \boxed{\sqrt{\frac{L}{C}} \cdot \frac{1}{NR}}$

c) $BW = \frac{\omega_0}{Q} = \frac{\frac{1}{N\sqrt{LC}}}{\sqrt{\frac{L}{C}} \cdot \frac{1}{NR}} = \frac{R}{\sqrt{LC}}\sqrt{\frac{C}{L}} = \boxed{\frac{R}{L}}$

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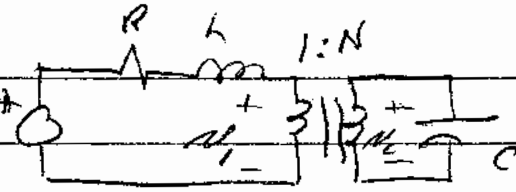
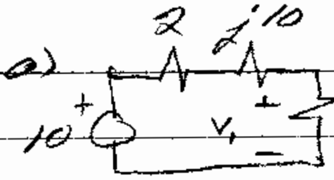
12.25



a)



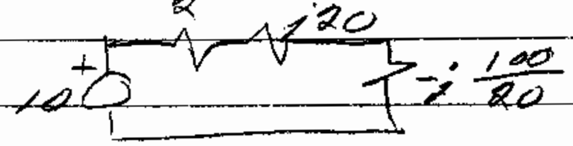
$\frac{100}{2j5} = -j10$

$\omega = 5$

$R=2,\ L=2,\ C=2,\ N=\frac{1}{10}$

$\hat{V}_1 = \frac{-j10 \times 10}{2} = -j50 \quad \therefore \hat{V}_c = -j5\ ;\ \boxed{v_c(t) = 5\cos(5t - 90°)}$

b) $v_g = 10\cos 10t$



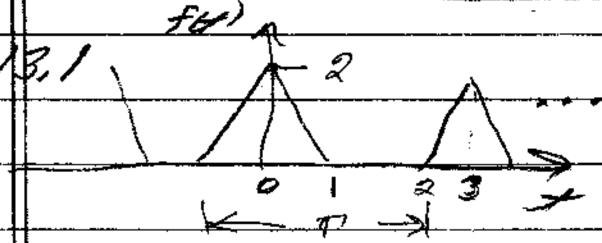
$-j\frac{100}{20} = -j5$

$\hat{V}_1 = \frac{-j5 \times 10}{2+j15}\ ;\ \hat{V}_c = \frac{-j5}{2+j15} = \frac{5e^{-j90°}}{15.13e^{j82.4°}} = 0.33e^{-j172.4°}$

$\boxed{v_c(t) = 0.33\cos(10t - 172.4°)}$

EE 212 Homework 23

13.1



Even function ∴ $\boxed{b_n = 0}$

$\boxed{a_0 = \frac{2}{3}}$; $T = 3$

a)

$$a_n = \frac{4}{T}\int_0^1 \left(-2t+2\right)\cos\frac{2\pi t}{3}\,dt$$

$$a_n = -\frac{8}{3}\int_0^1 t\cos\frac{n2\pi t}{3}\,dt + \frac{8}{3}\int_0^1 \cos\frac{2\pi t n}{3}\,dt \qquad \text{let } u = \frac{2\pi n t}{3},\; du = \frac{2\pi n}{3}\,dt$$

$$a_n = -\frac{8}{3}\int_0^{\frac{2\pi n}{3}} \left(\frac{3}{2\pi n}\right)^2 u\cos u\,du + \frac{\cancel{8}^{\,4}}{\cancel{3}}\int_0^{\frac{2\pi n}{3}} \left(\frac{\cancel{3}}{2\pi n}\right)\cos u\,du$$

$$a_n = -\frac{8}{3}\left(\frac{3}{2\pi n}\right)^2\Big\{\cos u + u\sin u\Big\}_0^{\frac{2\pi n}{3}} + \frac{4}{\pi n}\sin u\,\Big|_0^{\frac{2\pi n}{3}}$$

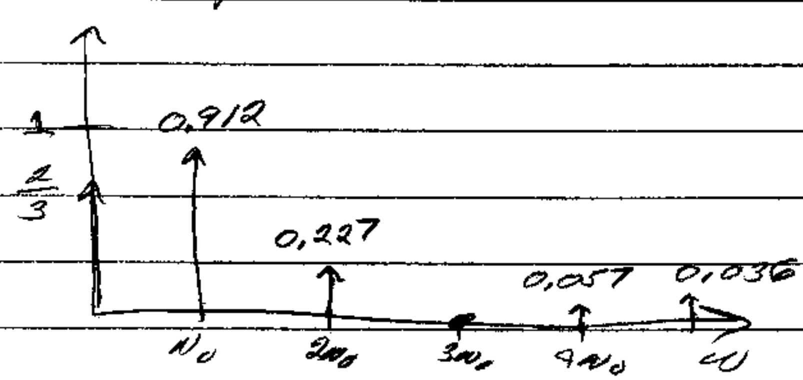
$$a_n = -\frac{8}{3}\left(\frac{3}{2\pi n}\right)^2\Big\{\cos\frac{2\pi n}{3} - 1 - \frac{2\pi n}{3}\sin\frac{2\pi n}{3}\Big\} + \frac{4}{n\pi}\sin\frac{2\pi n}{3}$$

$$a_n = -\frac{6}{(n\pi)^2}\Big\{\cos\frac{2\pi n}{3} - 1\Big\} - \cancel{\frac{2\pi n}{3}\left(\frac{6}{(n\pi)^2}\right)\sin\frac{2\pi n}{3} + \frac{4}{n\pi}\sin\frac{2\pi n}{3}}$$

$$a_n = \frac{6}{(n\pi)^2}\Big\{1 - \cos\frac{2\pi n}{3}\Big\} \longleftarrow$$

harmonic magnitude

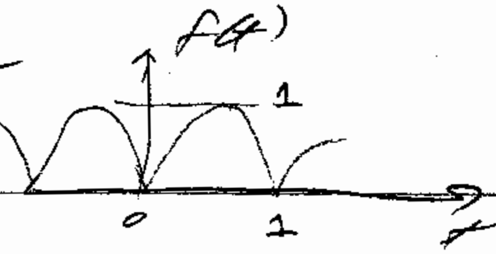
b)



$$\boxed{\omega_0 = \frac{2\pi}{3}}$$

EE212 Homework ~~25~~ 24

$f(t) = |\sin \pi t|$



(6) even so $b_n = 0$

$$a_0 = \frac{1}{1}\int_0^1 \sin\pi t\, dt = \frac{-1}{\pi}\cos\pi t\Big|_0^1 = -\frac{1}{\pi}[-1-1] = \frac{2}{\pi}$$

$$a_n = \frac{2}{1}\int_0^1 \sin\pi t \cos n\frac{2\pi t}{1}\,dt = \int_0^1[\sin(\pi+n2\pi)t\,dt + \sin(\pi - n2\pi)t\,dt]$$

$$\cancel{= \int_0^1[\sin\pi t\cos n2\pi t + \cos\pi t \sin n2\pi t]\,dt + \int_0^1[\sin\pi t\cos n2\pi t - \cos\pi t\sin n2\pi t]\,dt}$$

$$\cancel{= 2\int_0^1 \sin\pi t\cos n2\pi t\,dt = \int_0^1[\sin(\pi+n2\pi)t + \sin(\pi-n2\pi)t]\,dt}$$

$$= \left[\frac{-\cos(\pi+n2\pi)t}{\pi+n2\pi} - \frac{\cos(\pi-n2\pi)t}{\pi-n2\pi}\right]_0^1$$

$$= \frac{-\cos(\pi+n2\pi)+1}{\pi+n2\pi} + \frac{-\cos(\pi-n2\pi)+1}{\pi-n2\pi}$$

$$= \frac{[1-\cos\pi\cos n2\pi + \sin\pi\sin n2\pi][\pi-n2\pi] + [1-\cos\pi\cos n2\pi - \sin\pi\sin n2\pi][\pi+n2\pi]}{\pi^2 - n^2 4\pi^2}$$

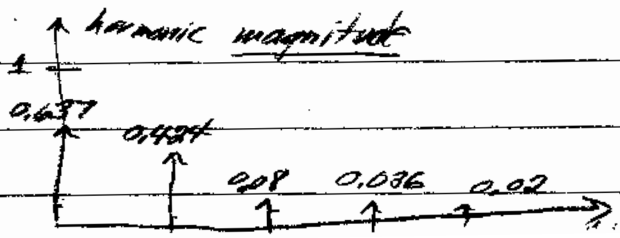
2

$$= \frac{\pi - n2\pi + \pi + n2\pi + \overbrace{\cos n2\pi}^{2}\,[\pi - n2\pi + \pi + n2\pi]}{\pi^2(1-4n^2)}$$

$$= \frac{4}{\pi(1-4n^2)}$$

$$f(t) = \frac{2}{\pi} - 0.424\cos 2\pi t - 0.085\cos 4\pi t - 0.036\cos 6\pi t - 0.02\cos 8\pi t \ldots$$

a)



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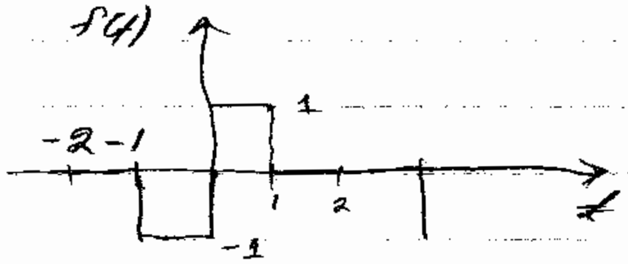
13.4



$T = 4$
odd function
$\therefore \; a_n = 0$
also $a_0 = 0$

$$b_n = \frac{4}{T}\int_0^1 \sin n \frac{2\pi}{4} t \, dt = \frac{-4}{T}\cdot\frac{2}{n\pi}\cos n\frac{\pi}{2}t\Big|_0^1$$

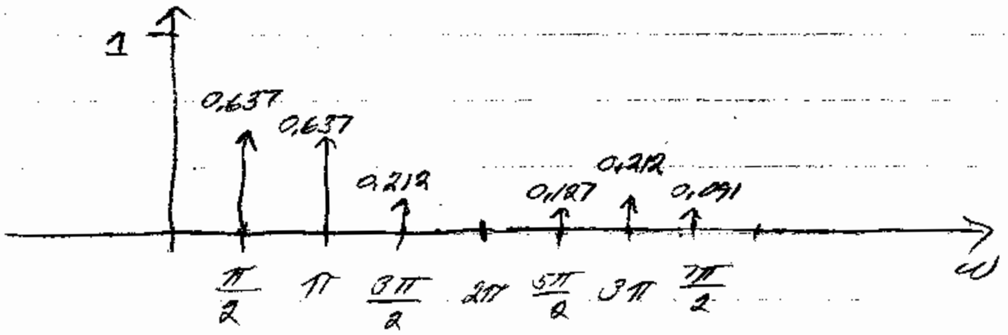
$$b_n = -\frac{8}{4n\pi}\left[\cos n\frac{\pi}{2} - 1\right] = \frac{2}{n\pi}\left[1-\cos n\frac{\pi}{2}\right]$$

$$f(t) = \sum_{n=1}^{\infty} \frac{2}{n\pi}\left[1-\cos n\frac{\pi}{2}\right]\sin n\frac{\pi}{2}t$$

$$f(t) = 0.637\sin\frac{\pi}{2}t + 0.637\sin\pi t + 0.212\sin\frac{3\pi}{2}t + 0.127\sin\frac{5\pi}{2}t + 0.212\sin 3\pi t + 0.091\sin\frac{7\pi}{2}t + \cdots$$

~~harmonic magnitude~~

b)



EE 212 Homework 26

13.22 $f(t)$ is $\boxed{\text{odd}}$ + periodic

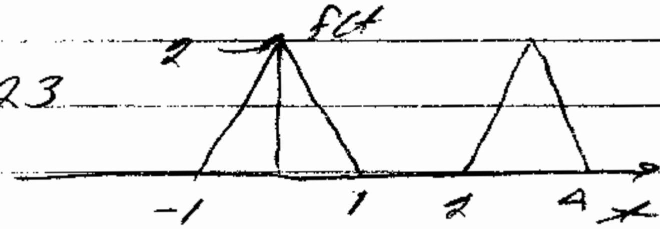
in general $\hat{C}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) e^{-jn\omega_0 t}\,dt$

$$\hat{C}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\left[\cos n\omega_0 t - j\sin n\omega_0 t\right]dt$$ but $f(t)$ is "odd"

$$\therefore \hat{C}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \underbrace{-f(t)\, j\sin n\omega_0 t}_{\text{"even"}}\,]\,dt = \frac{-2j}{T}\int_{0}^{\frac{T}{2}} f(t)\sin(n\omega_0 t)\,dt$$

EE 212 Homework 27

13.23



$T = 3$

$$\hat{C}_n = \frac{1}{3}\int_{-1}^{1} f(t)\, e^{-jn\frac{2\pi}{3}t}\, dt = \frac{2}{3}\int_{0}^{1}(2-2t)\cos\left(\frac{2\pi n t}{3}\right) dt$$

let $u = \frac{2\pi n t}{3}$; $du = \frac{2\pi n}{3}\,dt$

$$\hat{C}_n = \frac{2\cdot 2}{3}\times\frac{3}{2\pi n}\sin\left(\frac{2\pi n t}{3}\right)\Big|_0^1 - \frac{4}{3}\int_0^{\frac{2\pi n}{3}}\left(\frac{3}{2\pi n}\right)^2 u\cos u\, du$$

$$\hat{C}_n = \frac{2}{\pi n}\sin\left(\frac{2\pi n}{3}\right) - \frac{3}{(n\pi)^2}\left\{\cos u + u\sin u\right\}_0^{\frac{2\pi n}{3}}$$

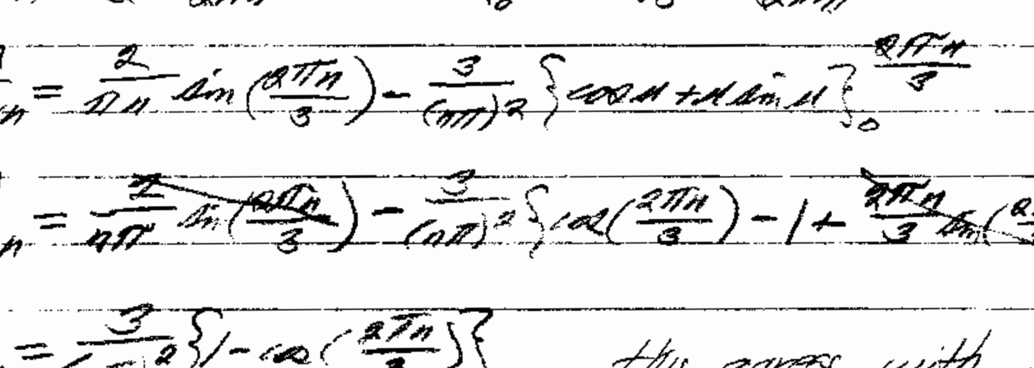
$$\hat{C}_n = \cancel{\frac{2}{n\pi}\sin\left(\frac{2\pi n}{3}\right)} - \frac{3}{(n\pi)^2}\left\{\cos\left(\frac{2\pi n}{3}\right) - 1 + \cancel{\frac{2\pi n}{3}\sin\left(\frac{2\pi n}{3}\right)}\right\}$$

$$\hat{C}_n = \frac{3}{(n\pi)^2}\left\{1-\cos\left(\frac{2\pi n}{3}\right)\right\}$$

this agrees with solution we obtained for problem 13.1

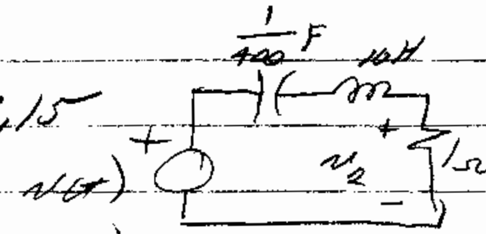
$$C_0 = \lim_{n\to 0}\hat{C}_n = \lim_{n\to 0}\frac{3\left(\frac{2\pi}{3}\right)\sin\left(\frac{2\pi n}{3}\right)}{2\pi^2 n} = \lim_{n\to 0}\frac{\cancel{2\pi}\left(\frac{2\pi}{3}\right)\cos\left(\frac{2\pi n}{3}\right)}{\cancel{2\pi}} = \frac{2}{3}$$

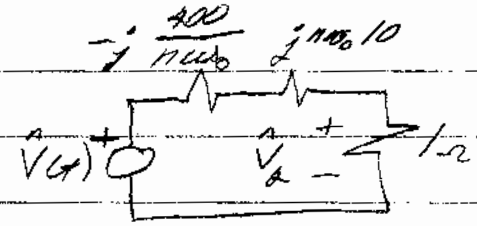
harmonic magnitude or "frequency spectrum"



EE 212 Homework 28

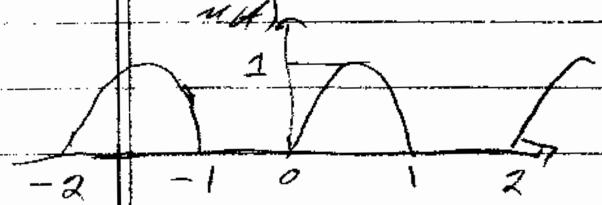
13.15





$T = 2$

$\omega_0 = \pi$



from page 602

$$v(t) = \frac{1}{\pi} + \frac{1}{2}\sin \pi t + \sum_{n=2}^{\infty}\left(\frac{1+\cos(n)\pi}{\pi(1-n^2)}\right)\cos n\pi t$$

or $v(t) = 0.32 + 0.5\sin \pi t - 0.21\cos 2\pi t - 0.04\cos 4\pi t \cdots$

$$\therefore \hat{V}_{2n} = \hat{V}_{1n}\frac{1}{1+j\left(n\pi 10 - \frac{400}{n\pi}\right)} = \hat{V}_{1n}\frac{n\pi}{n\pi + j(10n^2\pi^2 - 400)} \leftarrow$$

or $$\hat{V}_{2n} = \hat{V}_{1n}\frac{n\pi\, e^{-j\tan^{-1}\left(\frac{10n^2\pi^2-400}{n\pi}\right)}}{\sqrt{(n\pi)^2 + (10n^2\pi^2 - 400)^2}}$$ this gives:

$$|\hat{V}_{2n}| = |\hat{V}_{1n}|\frac{1}{\sqrt{1+\left(10n\pi - \frac{400}{n\pi}\right)^2}}$$

n	$\lvert\hat{V}_{2n}\rvert$
0	0
1	5.213×10^{-3}
2	-0.162
3	0
4	-4.5×10^{-4}
5	0
6	-1.1×10^{-4}

We see that 2^{nd} harmonic is dominant!

$$\therefore \hat{V}_2 \approx -0.162\, e^{-j\tan^{-1}\left(\frac{40\pi^2-400}{2\pi}\right)}$$

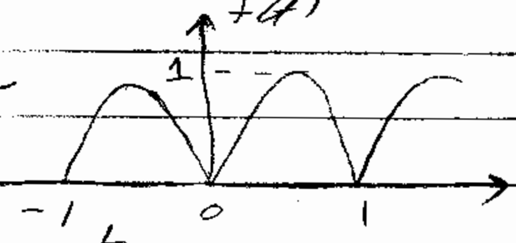
or $\hat{V}_2 \approx -0.162\, e^{j39.7^\circ}$

and $v_2(t) = -0.162\cos(2\pi t + 39.7^\circ) \leftarrow$

or $v_2(t) = 0.162\cos(2\pi t - 140.3^\circ) \leftarrow$

EE 212 Homework 29

13.25



$T = 1$

$\omega_0 = \frac{2\pi}{T} = 2\pi$

even function

$$\hat{C}_n = \frac{1}{T}\int_{-\frac{1}{2}}^{\frac{1}{2}} |\sin \pi t|\, e^{-jn2\pi t}\, dt = \int_{-\frac{1}{2}}^{\frac{1}{2}} |\sin \pi t| \left[\cos(2\pi n t) - j \sin(2\pi n t)\right] dt$$

$$\hat{C}_n = 2\int_0^{\frac{1}{2}} \sin \pi t \cos(2\pi n t)\, dt = \int_0^{\frac{1}{2}} \left[\sin(\pi + 2\pi n)t + \sin(\pi - 2\pi n)t\right] dt$$

$$\hat{C}_n = \left\{-\frac{\cos(\pi+2\pi n)t}{\pi+2\pi n} - \frac{\cos(\pi-2\pi n)t}{\pi-2\pi n}\right\}_0^{\frac{1}{2}}$$

$$\hat{C}_n = \frac{-\cos(\pi+2\pi n)\frac{1}{2} + 1}{\pi+2\pi n} + \frac{-\cos(\pi-2\pi n)\frac{1}{2} + 1}{\pi - 2\pi n}$$

$$\hat{C}_n = \frac{(2\pi n - \pi)\cos(\pi+2\pi n)\frac{1}{2} - (\pi+2\pi n)\cos(\pi-2\pi n)\frac{1}{2} + \pi \cancel{-2\pi n} + \pi \cancel{+2\pi n}}{\pi^2 - 4\pi^2 n^2}$$

$$\hat{C}_n = \frac{(2\pi n - \pi)\left[\underbrace{\cos\frac{\pi}{2}}_{0}\cos n\pi - \underbrace{\sin\frac{\pi}{2}}_{1}\sin n\pi\right] - (\pi + 2\pi n)\left[\underbrace{\cos\frac{\pi}{2}}_{0}\cos n\pi + \underbrace{\sin\frac{\pi}{2}}_{1}\sin n\pi\right] + 2\pi}{\pi^2 - 4\pi^2 n^2}$$

$$\hat{C}_n = \frac{-4\pi n \underbrace{\sin n\pi}_{0} + 2\pi}{\pi^2 - 4\pi^2 n^2} = \frac{2}{\pi - 4\pi n^2} = \frac{2}{\pi(1-4n^2)}$$

$$\boxed{f(t) = \sum_{n=-\infty}^{\infty} \frac{2}{\pi(1-4n^2)} e^{j2\pi n t}}$$

$$f(t) = 0.637 - 0.312 e^{j2\pi t} - 0.212 e^{-j2\pi t}$$
$$- 0.0424 e^{j4\pi t} - 0.0424 e^{-j4\pi t} \ldots$$

EE 212 Homework 30

14.6 $\mathcal{F}[f(t-a)] = \int_{-\infty}^{\infty} f(t-a) e^{-j\omega t} dt = \int_{-\infty}^{\infty} f(x) e^{-j\omega(x+a)} dx$

$x = t-a$

$= e^{-j\omega a} F(j\omega)$ ←

14.9
$F(j\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$ but $f(t)$ is even

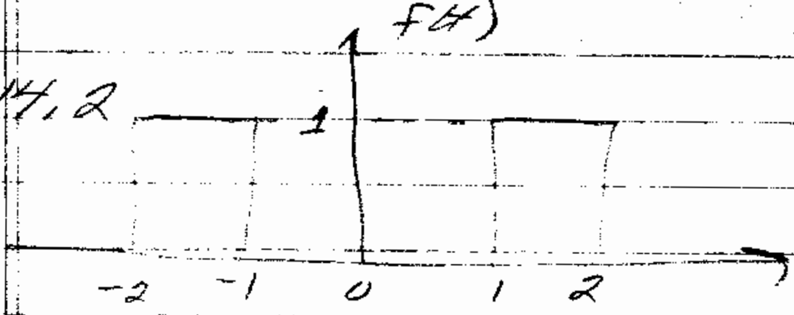
$\therefore F(j\omega) = \int_{-\infty}^{\infty} f(t) \cos\omega t \, dt = 2\int_0^{\infty} f(t) \cos\omega t \, dt$ ←

14.13 $\mathcal{F}[e^{-\alpha t} f(t)] = \int_{-\infty}^{\infty} e^{-\alpha t} f(t) e^{-j\omega t} dt$

$= \int_{-\infty}^{\infty} f(t) e^{-(\alpha + j\omega)t} dt = F(j\omega + \alpha)$ ←

EE 812 Homework 31

14.2



even function

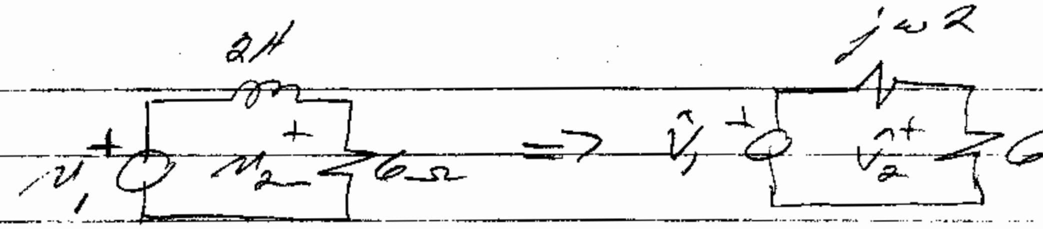
$$\mathcal{F}[f(t)] = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt = 2\int_0^{\infty} f(t) \cos\omega t \, dt$$

$$= 2\int_1^2 \cos\omega t \, dt = 2\frac{1}{\omega}\sin\omega t \Big|_1^2$$

$$\mathcal{F}[f(t)] = \frac{2}{\omega}[\sin 2\omega - \sin\omega]$$

EE 212 Homework 32

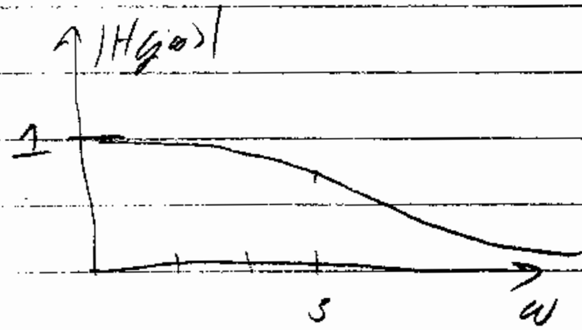
14.19



a) $\frac{\hat{V}_2}{\hat{V}_1} = H(j\omega) = \boxed{\frac{6}{6+j\omega 2}} = \boxed{\frac{3}{3+j\omega}}$

b) $H(j\omega) = \frac{3}{j\omega+3}$ $\therefore$ $\boxed{h(t) = 3e^{-3t}u(t)}$

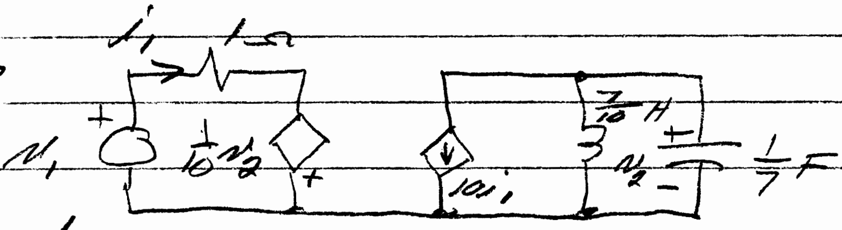
c)

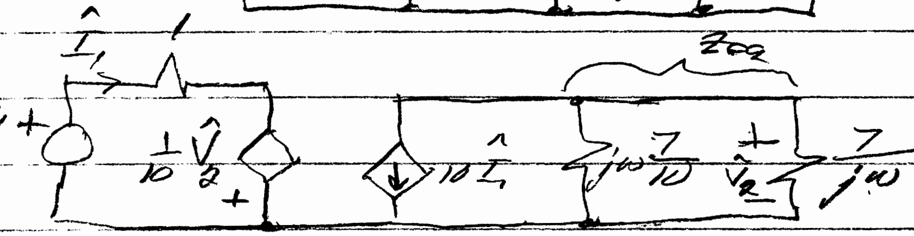


d) low pass

EE 212 Homework 33

14.23





$$-\hat{V}_2 = 10\hat{I}_1 Z_{eq} = 10\hat{I}_1 \frac{\frac{49}{10}}{j\omega\frac{7}{10}+\frac{7}{j\omega}} = \hat{I}_1 \frac{49j\omega}{7+\frac{7}{10}(j\omega)^2} = \hat{I}_1 \frac{490j\omega}{70+7(j\omega)^2}$$

or $-\hat{V}_2 = \hat{I}_1 \dfrac{70j\omega}{10+(j\omega)^2}$; but $\hat{I}_1 = \hat{V}_1 + \frac{1}{10}\hat{V}_2$

so $-\hat{V}_2 = (\hat{V}_1 + \frac{1}{10}\hat{V}_2)\dfrac{490j\omega}{70+7(j\omega)^2}$

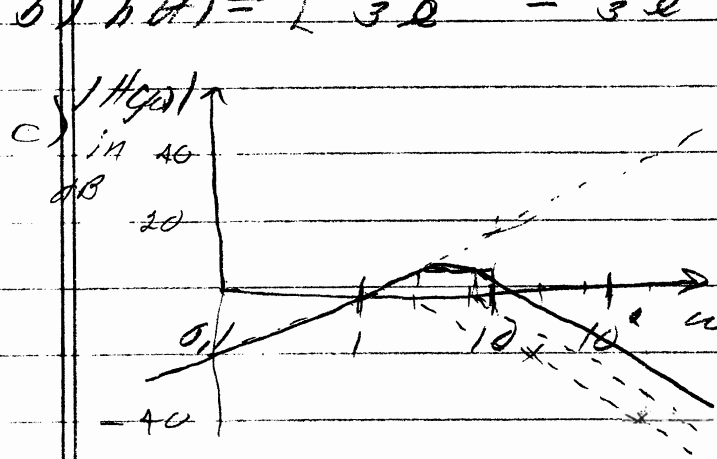
or $\hat{V}_2\left(-1 - \dfrac{49j\omega}{70+7(j\omega)^2}\right) = \hat{V}_1 \dfrac{490j\omega}{70+7(j\omega)^2}$

$$H(j\omega) = \frac{\hat{V}_2}{\hat{V}_1} = \frac{490j\omega}{-70-7(j\omega)^2-49j\omega} = \frac{-70j\omega}{(j\omega)^2+7j\omega+10}$$

$$\boxed{H(j\omega) = \frac{-70j\omega}{(j\omega+2)(j\omega+5)} = \frac{\frac{140}{3}}{j\omega+2} + \frac{\frac{350}{3}}{j\omega+5}}$$

b) $h(t) = \left[\frac{140}{3}e^{-2t} - \frac{350}{3}e^{-5t}\right]u(t)$

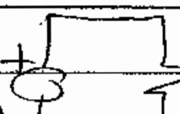
c)



$$H(j\omega) = \frac{-10j\omega}{(1+j\frac{\omega}{2})(1+j\frac{\omega}{5})}$$

d) Bandpass filter

EE 212 Homework 34

14.17  $te^{-at}u(t)$ 1Ω

~~I used table of~~ integrals

a) $W_{1\Omega} = \int_0^\infty t^2 e^{-2at}\,dt = \left[e^{-2at}\left(\frac{t^2}{-2a} - \frac{2t}{4a^2} - \frac{2}{8a^3}\right)\right]_0^\infty$

or $\boxed{W_{1\Omega} = \frac{1}{4a^3}}$

2nd method $W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |F(j\omega)|^2\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{(j\omega+a)^2}\right|^2 d\omega$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left|\frac{1}{a^2-\omega^2+j2\omega a}\right|^2 d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{(a^2-\omega^2)^2+4\omega^2a^2}\,d\omega$$

$$W_{1\Omega} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{a^4+\omega^4+2\omega^2a^2}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{d\omega}{(a^2+\omega^2)^2}$$

$$W_{1\Omega} = \frac{1}{2\pi}\left\{\frac{\omega}{2a^2(a^2+\omega^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{\omega}{a}\right)\right\}_{-\infty}^{\infty} = \frac{1}{2\pi}\left\{\frac{1}{2a^3}\left[\frac{\pi}{2}+\frac{\pi}{2}\right]\right\}$$

or $\boxed{W_{1\Omega} = \frac{1}{4a^3}}$

---

b) $$W_{\omega=-a\to a} = \frac{1}{2\pi}\int_{-a}^{a}\frac{d\omega}{(a^2+\omega^2)^2} = \frac{1}{2\pi}\left\{\frac{\omega}{2a^2(a^2+\omega^2)} + \frac{1}{2a^3}\tan^{-1}\left(\frac{\omega}{a}\right)\right\}_{-a}^{a}$$

$$= \frac{1}{2\pi}\left\{\frac{a}{4a^4} + \frac{1}{2a^3}\cdot\frac{\pi}{4} + \frac{a}{4a^4} - \frac{1}{2a^3}\left(-\frac{\pi}{4}\right)\right\}$$

$$= \frac{1}{2\pi}\left\{\frac{1}{2a^3} + \frac{\pi}{4a^3}\right\} = \frac{1}{4a^3}\left\{\frac{1}{\pi} + \frac{1}{2}\right\}$$

% in this band $= \left\{\frac{1}{\pi} + \frac{1}{2}\right\} \times 100$ $\boxed{= 81.83\ \%}$