EE 333 — Homework 1

1.2 $\bar{A} = 5\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z$, $\bar{B} = 2\bar{a}_y + \bar{a}_z$, $\bar{C} = -6\bar{a}_z$

a) $\bar{D} = \bar{A} + \bar{B} + \bar{C} = 5\bar{a}_x + 5\bar{a}_y - \bar{a}_z$

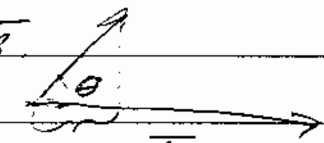
b) $|\bar{A}| = \sqrt{25+9+16} = 7.07$

$|\bar{D}| = \sqrt{25+25+1} = 7.14$

c) $2\bar{A} - \bar{C} = \bar{E} = 10\bar{a}_x + 6\bar{a}_y + 14\bar{a}_z$

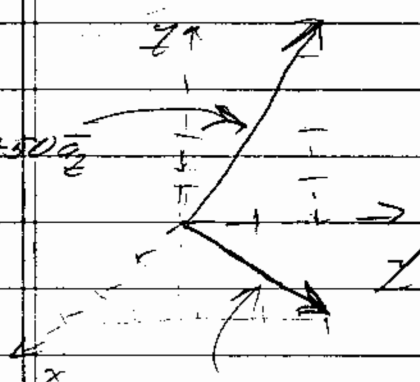
$|E| = \sqrt{100+36+196} = 18.22$

1-8 $\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta$



projection of $\bar{B}$ onto $\bar{A}$ $\longrightarrow$ $B\cos\theta$ $\bar{A}$

$\bar{G} = 20\bar{a}_y + 50\bar{a}_z$



$\bar{F} = 30\bar{a}_x + 40\bar{a}_y$

$F = 50$

$\bar{F}\cdot\bar{G} = 800 = FG\cos\theta$

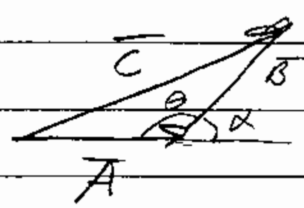
but $G = \sqrt{400+2500} = 53.85$

$\therefore F\cos\theta = \dfrac{800}{53.85} = \boxed{14.85}$

$\cos\theta = \dfrac{800}{FG} = 0.297$

$\boxed{\text{so } \theta = 72.72^\circ}$

1-11



$\bar{A} + \bar{B} = \bar{C}$

$\bar{C}\cdot\bar{C} = (\bar{A}+\bar{B})\cdot(\bar{A}+\bar{B}) = A^2 + B^2 + 2AB\cos\alpha$

but $\alpha = 180^\circ - \theta$ so $\cos\alpha = \cos(\pi - \theta)$

$= \cos\pi\cos\theta + \sin\theta\sin\pi$

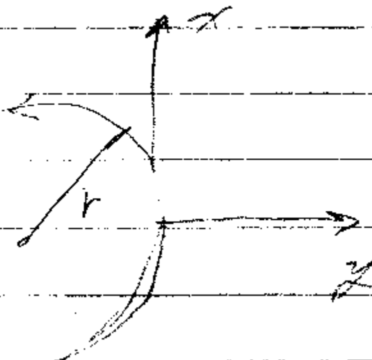
$= -\cos\theta$

$\therefore \bar{C}\cdot\bar{C} = C^2 = \boxed{A^2 + B^2 - 2AB\cos\theta}$

EE 333 Homework 2

1-25 $q = -e$, $\bar{v} = 10^5 \bar{a}_x$, $\bar{B} = 10^{-4}\bar{a}_z$

$$\bar{F} = q(\bar{E} + \bar{v}\times\bar{B})$$



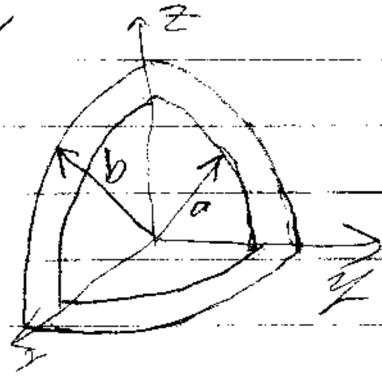
Constant force normal to velocity produces a circular trajectory with r

$$r = \frac{mv^2}{F}$$

$$\therefore r = \frac{9.1\times10^{-31}\times10^{10}}{1.6\times10^{-19}\times10^{5}\times10^{-4}} = \boxed{5.69\times10^{-3}\ m}$$

for no deflection $\bar{E} = -\bar{v}\times\bar{B} = -10^5\bar{a}_x\times10^{-4}\bar{a}_z = \boxed{10\bar{a}_y}$

1-28



$$\oint \epsilon_0 \bar{E}\cdot d\bar{S} = \int \rho_v\, dV$$

$\bar{E} = E_r(r)\,\bar{a}_r$ from symmetry

for $\boxed{r<a}$ $\rho_v = 0$

$\therefore 4\pi r^2 \epsilon_0 E_r = 0$ or $\boxed{E_r = 0}$

for $\boxed{a<r<b}$ $4\pi r^2\epsilon_0 E_r = \rho_v \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=a}^{r} r^2\sin\theta\, d\theta\, d\phi\, dr = \left(\frac{r^3}{3}\right)_a^r 4\pi\rho_v$

$$\therefore \boxed{E_r = \frac{\rho_v}{3\epsilon_0 r^2}(r^3 - a^3)}$$

for $r>b$ $4\pi r^2\epsilon_0 E_r = \left(\frac{r^3}{3}\right)_a^b 4\pi\rho_v$

$$\text{or } E_r = \frac{\rho_v}{3\epsilon_0 r^2}(b^3 - a^3)$$

EE 333 Homework 3

1-29 for $r<a$ $\quad \rho_v = \rho_0(1+kr)$

q_{total} = total charge $= \int_{r=0}^{a}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0(1+kr)\, r^2 \sin\theta \, d\theta\, d\phi\, dr$

$q_{total} = \frac{4}{3}\pi a^3 \rho_0 + k\rho_0 \frac{a^4}{4} 4\pi = 0$ when $\frac{1}{3} = -k\frac{a}{4}$ or $\boxed{k = -\frac{4}{3a}}$

for $r>a$ in this case Gauss's law gives:

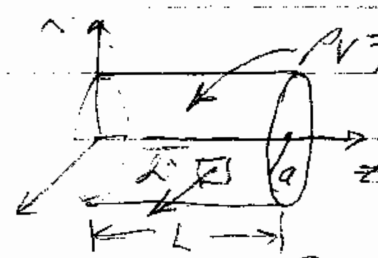
$$\boxed{\epsilon_0 E_r 4\pi r^2 = 0 \quad \therefore \quad E_r = 0}$$

for $r<a$ $\quad \epsilon_0 E_r 4\pi r^2 = \int_{r=0}^{r}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0\left(1-\frac{4r}{3a}\right) r^2 \sin\theta \, d\theta\, d\phi\, dr$

$$= 4\pi\rho_0\left[\frac{r^3}{3} - \frac{4r^4}{4\cdot 3a}\right]$$

so $E_r = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{r^2}{3a}\right]$

1-31 $\quad \rho_v = \rho_0\left(\frac{\rho}{a}\right)^2$



$Q = \int_{z=0}^{L}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \rho_0\left(\frac{\rho}{a}\right)^2 d\rho\, \rho\, d\phi\, dz$

$Q = L\, 2\pi \frac{\rho_0}{a^2}\cdot\frac{a^4}{4} = \boxed{\rho_0 L \frac{\pi a^2}{2}}$

Gauss's law $\rho > a$ $\quad \int_{\phi=0}^{2\pi}\int_{z=0}^{L} \epsilon_0 E_\rho \, dz\, \rho\, d\phi = Q = \epsilon_0 E_\rho L \rho 2\pi$

$$\therefore \quad E_\rho = \frac{Q}{2\pi\epsilon_0\rho L} = \frac{\rho_0 \pi a^2 L}{4\pi\epsilon_0\rho L} = \boxed{\frac{\rho_0 a^2}{4\epsilon_0\rho}}$$

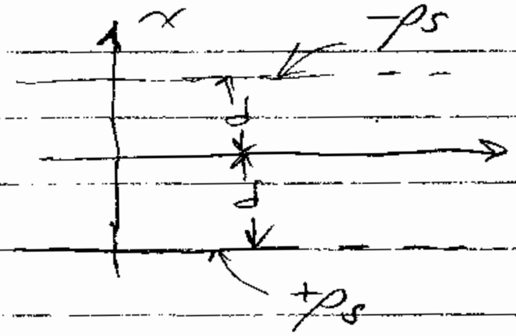
or $\boxed{E_\rho = \frac{\rho_l}{2\pi\epsilon_0\rho}}$

for $\rho<a$ $\quad \int_{\phi=0}^{2\pi}\int_{z=0}^{L} \epsilon_0 E_\rho \, dz\, \rho\, d\phi = \int_{z=0}^{L}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \rho_0\left(\frac{\rho}{a}\right)^2 d\rho\, \rho\, d\phi\, dz$

or $\epsilon_0 E_\rho L \rho 2\pi = L\rho_0 2\pi \frac{\rho^4}{4a^2}$

$$\therefore \quad \boxed{E_\rho = \frac{\rho_0\rho^3}{4\epsilon_0 a^2}}$$

1-32



a) between the charged sheets both sheets produce a constant E_x in the positive x direction

$$\therefore \quad -d<x<d \quad \boxed{E_x = \frac{\rho_s}{\epsilon_0}}$$

for $x>d$ or $x<-d$ fields cancel so $\boxed{E_x = 0}$

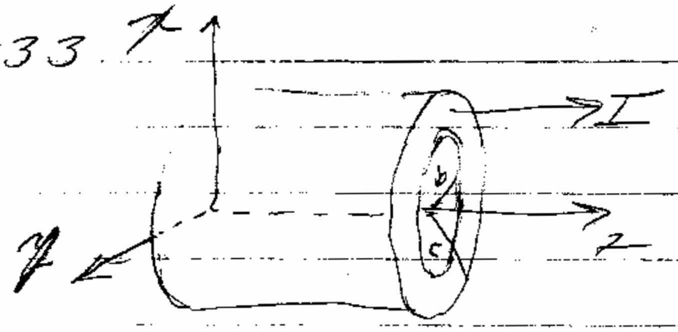
b) both sheets positively charged

$$x>d \quad E_x = \frac{\rho_s}{\epsilon_0}$$

$$x<-d \quad E_x = -\frac{\rho_s}{\epsilon_0}$$

$$-d<x<d \quad E_x = 0$$

1.33



a) $\overline{J} = \dfrac{I\,\overline{a}_z}{\pi c^2 - \pi b^2}$

(assuming uniform current density $J = \dfrac{I}{\text{cross section area}}$)

b) $\oint \dfrac{\overline{B}}{\mu_o}\cdot \overline{dl} = I$ for $\rho > c$

$$\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_o}\rho\, d\phi = I \quad \text{or} \quad \boxed{B_\phi = \frac{\mu_o I}{2\pi\rho} \text{ for } \rho > c}$$

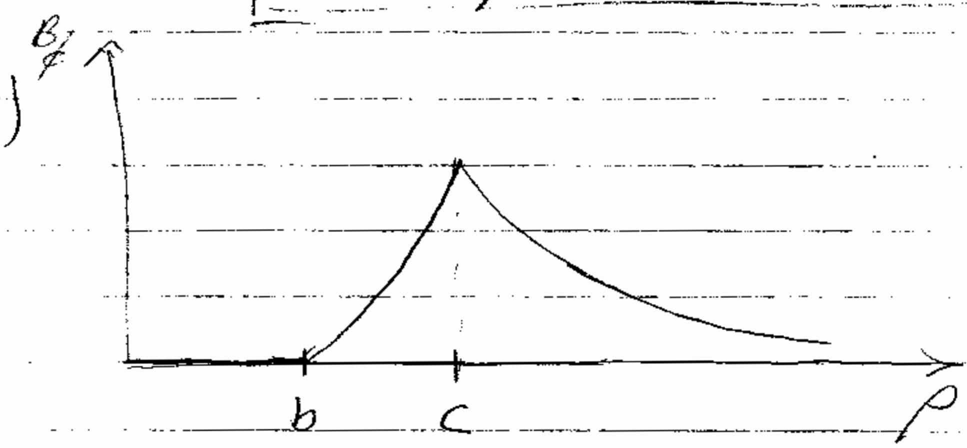
$$\boxed{\text{for } \rho \le b} \quad 2\pi\rho\frac{B_\phi}{\mu_o} = 0 \quad \boxed{\text{or } B_\phi = 0}$$

$$\text{for } a < \rho < b \quad 2\pi\rho\frac{B_\phi}{\mu_o} = \int_{\rho=b}^{\rho}\int_{\phi=0}^{2\pi} \frac{I\,\overline{a}_z}{\pi c^2 - \pi b^2}\cdot \rho\, d\phi\, d\rho$$

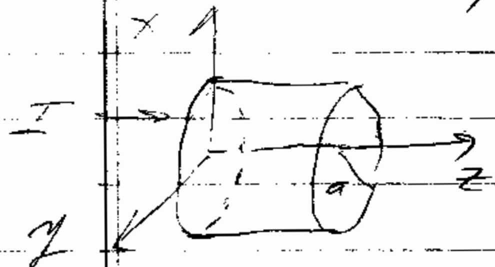
$$\text{or } 2\pi\rho\frac{B_\phi}{\mu_o} = \frac{\cancel{2}\cancel{\pi} I}{\cancel{\pi}(c^2-b^2)}\cdot \frac{\rho^2}{\cancel{2}}\Bigg|_b^\rho = I\,\frac{\rho^2-b^2}{c^2-b^2}$$

$$\text{so } \boxed{B_\phi = \frac{\mu_o I}{2\pi\rho}\left[\frac{\rho^2-b^2}{c^2-b^2}\right] \quad b < \rho < c}$$

c)



1-35 from example 1-15



$$B_\phi = \frac{\mu_0 I}{2\pi\rho} \quad \text{for } \rho > a$$

$$B_\phi = \frac{\mu_0 I \rho}{2\pi a^2} \quad \text{for } \rho < a$$

Using our solution to problem 1-33 and superposition we have

$$\boxed{B_\phi = \frac{\mu_0 I \rho}{2\pi a^2} \quad \text{for } \rho < a}$$

$$\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho} \quad \text{for } a < \rho < b}$$

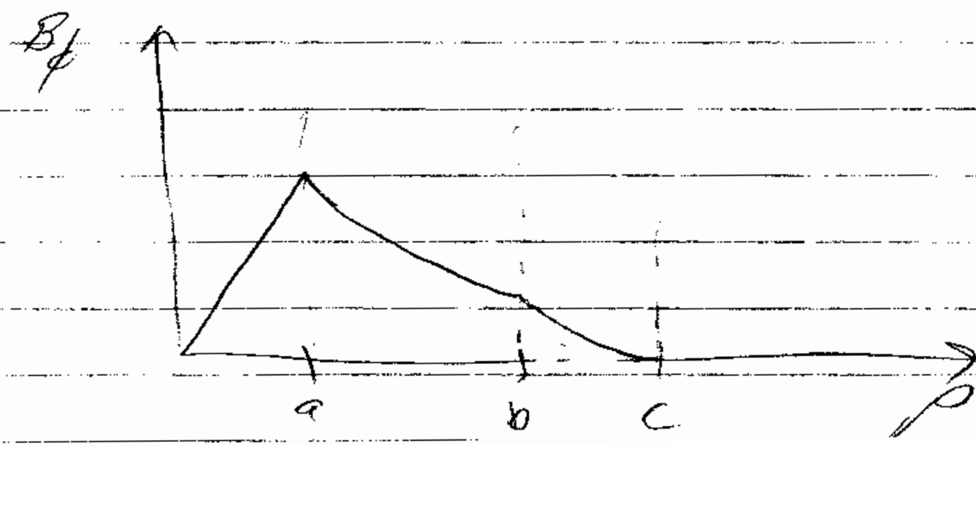
$$B_\phi = \frac{\mu_0 I}{2\pi\rho} - \frac{\mu_0 I}{2\pi\rho}\left[\frac{\rho^2 - b^2}{c^2 - b^2}\right] = \frac{\mu_0 I}{2\pi\rho}\left[\frac{c^2 - b^2 - \rho^2 + b^2}{c^2 - b^2}\right]$$

$$\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}\left[\frac{c^2 - \rho^2}{c^2 - b^2}\right] \quad b < \rho < c}$$

Same as answer for 1-34

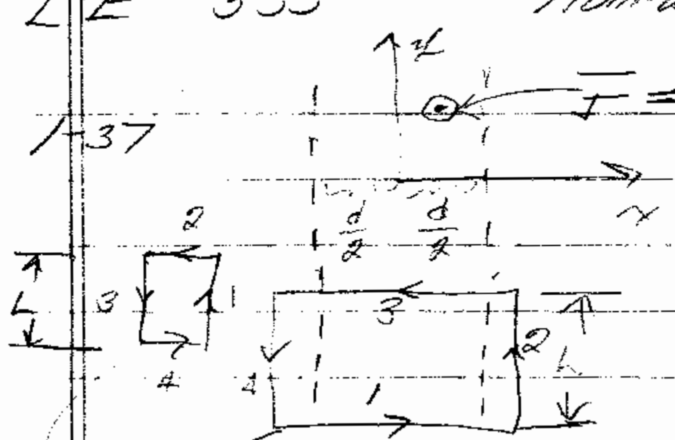
$$\boxed{B_\phi = 0 \quad \text{for } \rho > c}$$

(no current enclosed)



EE 333 Homework 5

1-37

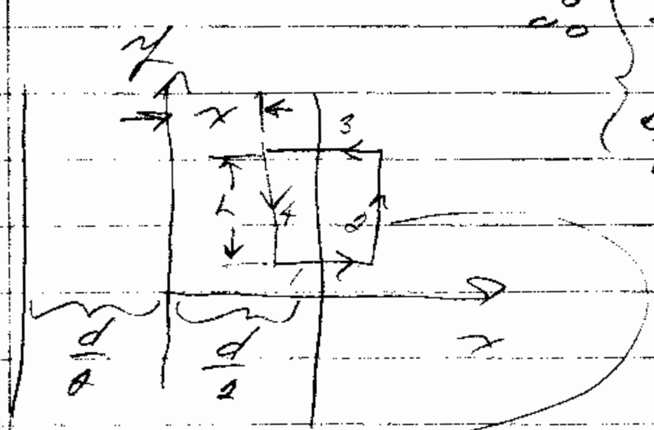


From sheet current problem we know that $\bar{B}$ is only in the $\pm y$ direction

for this contour $\frac{B_{y_1}}{\mu_0} L - \frac{B_{y_3}}{\mu_0} L = 0 \therefore \left\{ B_y = \text{constant outside of slab} \right.$

$$\frac{B_{y_2}}{\mu_0} \cancel{L} + \frac{B_{y_2}}{\mu_0} \cancel{L} = J_z \cancel{L} d$$

$$\therefore \begin{cases} B_y = \dfrac{\mu_0 J_z d}{2} & \text{for } x > d/2 \\ B_y = \dfrac{-\mu_0 J_z d}{2} & \text{for } x < d \end{cases}$$



$$\frac{\cancel{\mu_0} J_z d}{\cancel{\mu_0}\, 2} \cancel{L} - \frac{B_{y_4}}{\mu_0} \cancel{L} = J_z \cancel{L} \left(\frac{d}{2} - x\right)$$

$$\text{so } \boxed{B_{y_4}} = \frac{\mu_0 J_z d}{2} - \mu_0 J_z \left(\frac{d}{2} - x\right) = \boxed{\mu_0 J_z x \quad |x| < \frac{d}{2}}$$

2-17 $\bar{E} = \frac{\rho_0}{\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{3a}\right) \bar{a}_r$

$$\nabla \circ \bar{E} = \frac{1}{r^2} \frac{d}{dr}\left[\frac{\rho_0}{\epsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{3a}\right)\right] = \frac{\rho_0}{r^2 \epsilon_0}\left[r^2 - \frac{4}{3}\frac{r^3}{a}\right]$$

$$\boxed{\text{or } \nabla \circ \bar{E} = \frac{\rho_v}{\epsilon_0}}$$

EE 333 Homework 5 (page 2)

2-41 $\hat{E}_x^+ = 1885\, e^{-j\beta_0 z}$ $f = 10^8$

a) amplitude = 1885 V/m
traveling in +z direction
vector direction $\bar{a}_x$ (x polarization)

b) $E_x = 1885 \cos(\omega t - \beta_0 z) = 1885 \cos(2\pi \times 10^8 t - \beta_0 z)$

where $\beta_0 = \frac{\omega}{c} = \frac{2\pi \times 10^8}{3 \times 10^8} = 2.094$ ←

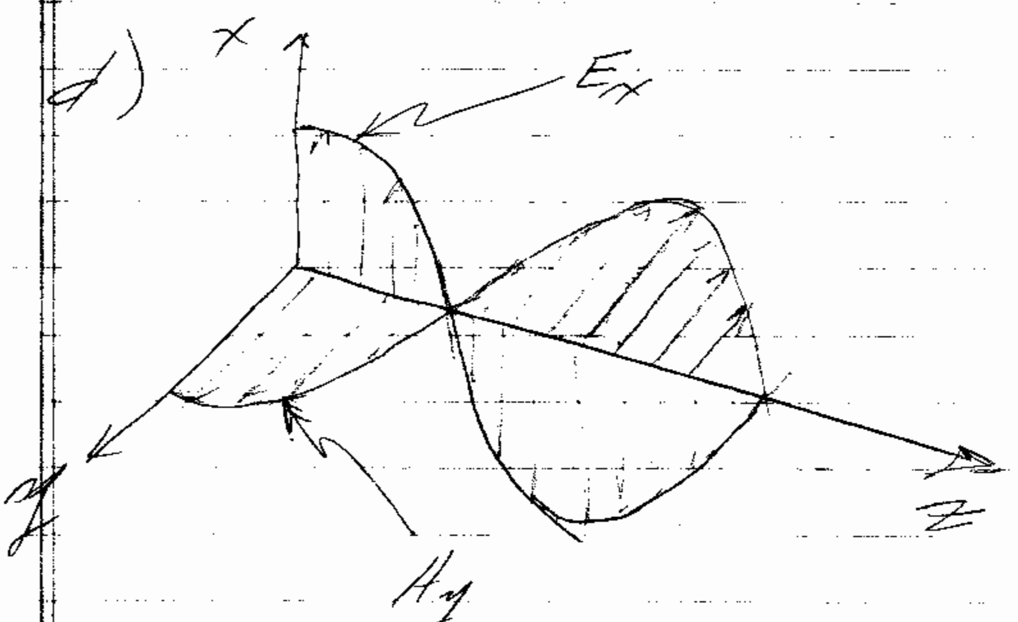
$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^8} = 3\,m$ ←

c) $\hat{B}_y^+ = \frac{1885}{c} e^{-j\beta_0 z} = \boxed{2\pi \times 10^{-6} e^{-j\beta_0 z}}$ ←

$\hat{H}_y^+ = \frac{1885}{377} e^{-j\beta_0 z} = \boxed{5\, e^{-j\beta_0 z}}$ ←

so $\boxed{H_y^+ = 5 \cos(2\pi \times 10^8 t - \beta_0 z)}$ ←

d)

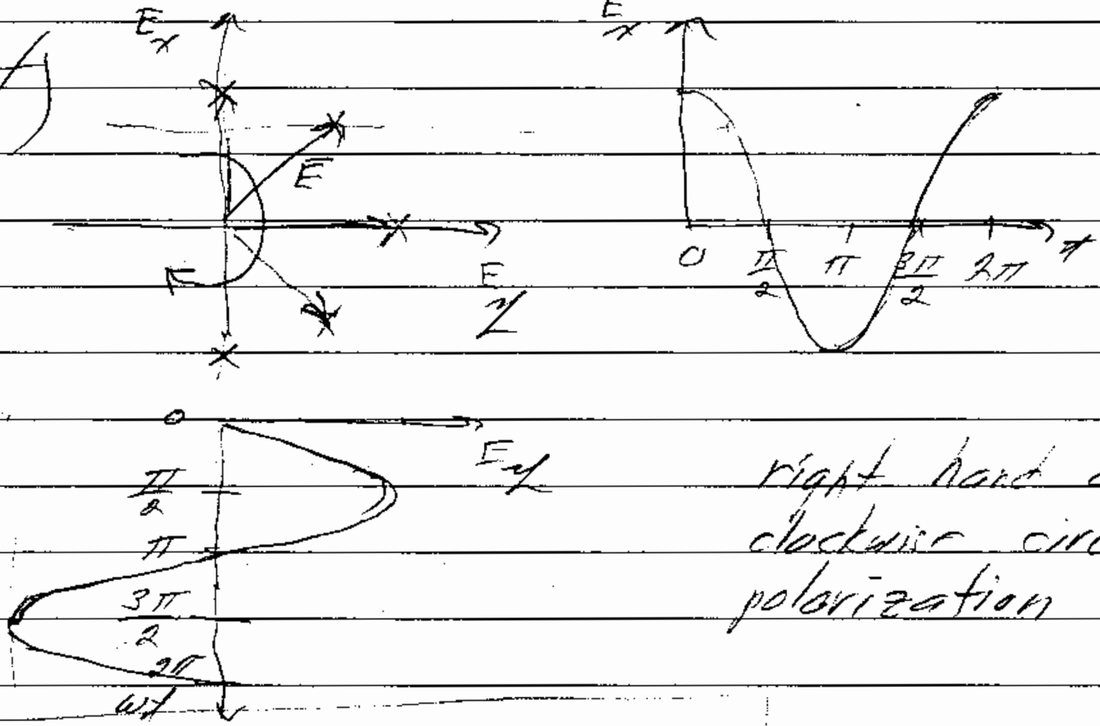


EE 333   Homework 6

2-46 $\hat{\bar{E}}(z) = 500\, e^{-j\beta_0 z}(\bar{a}_x - j\bar{a}_y)$

$\bar{E} = 500\cos(\omega t - \beta_0 z)\bar{a}_x + 500\sin(\omega t - \beta_0 z)\bar{a}_y$

(plot at $z=0$)



right hand or clockwise circular polarization

$$\hat{\bar{H}}_z = \frac{500}{\eta_0} e^{-j\beta_0 z}\bar{a}_y + j\frac{500}{\eta_0} e^{-j\beta_0 z}\bar{a}_x$$

3.1 $n = 10^{29}$; $\sigma = 5.8\times10^{7}$ mho/m

a) $\mu_e = \frac{e\tau_c}{m}$; $\sigma = \frac{ne^2}{m}\tau_c$

$\therefore \mu_e = \sigma\left(\frac{1}{ne}\right) = \frac{5.8\times10^{7}}{10^{29}\times1.6\times10^{-19}} = \boxed{3.625\times10^{-3}}$

b) charge density $= -10^{29}\frac{\text{elec}}{m^3}\times\frac{1.6\times10^{-19}C}{\text{elec}}\times\frac{1m^3}{10^9 mm^3} = \boxed{-16\frac{C}{mm^3}}$

c) $E = 1\bar{a}_x$; $v_d = -\mu_e E = \boxed{-3.625\times10^{-3}\ m/sec}$

$\bar{J} = \sigma\bar{E} = 5.8\times10^{7}\bar{a}_x$ A/m^2

$\bar{E}$

$\bar{J}$

$\bar{v}_d$

EE 333 Homework 7

3.3 $\boxed{He}$ 10^{25} atoms/m^3 ; $\chi_e = 1.5\times10^{-4}$; $\bar{E} = 10^3$ V/m

a) $\bar{P} = \epsilon_0 \chi_e \bar{E} = \frac{10^{-9}}{36\pi}\times 1.5\times10^{-4}\times10^{3} = \boxed{1.33\times10^{-12}}$

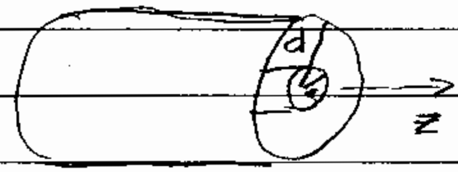
2 electrons/atom

b) $\rho_v^+ = 2\times10^{25}\times1.6\times10^{-19} = \boxed{3.2\times10^{6}\ C/m^3}$

$\bar{P} = \rho_v^+ \, \overline{dl}_{ave}$ $\therefore$ $dl_{ave} = \frac{1.33\times10^{-12}}{3.2\times10^{6}} = \boxed{0.416\times10^{-18}\ m}$

c) $\epsilon_r = 1+\chi_e = \boxed{1.00015}$

3-12



$\oint \bar{D}\cdot d\bar{s} = \rho_l L$ charge/m

$\int_{z=0}^{L}\int_{\phi=0}^{2\pi} D_\rho \rho \, d\phi \, dz = \rho_l L$

or $2\pi\rho \cancel{L} D_\rho = \rho_l \cancel{L}$ $\therefore$ $D_\rho = \frac{\rho_l}{2\pi\rho}$; $\boxed{E_\rho = \frac{\rho_l}{2\pi\epsilon\rho}}$

a) E_ρ is constant if $\boxed{\epsilon = \frac{K}{\rho}}$

want $\epsilon = \epsilon_1$ @ $\rho = d$ $\therefore$ $\boxed{K = d\epsilon_1}$

and $\boxed{E_\rho = \frac{\rho_l}{2\pi\epsilon_1 d}}$

b) $D_\rho = \epsilon_0 E_\rho + P_\rho$ so $P_\rho = D_\rho - \epsilon_0 E_\rho = \frac{\rho_l}{2\pi\rho} - \frac{\epsilon_0\rho_l}{2\pi\epsilon_1 d}$

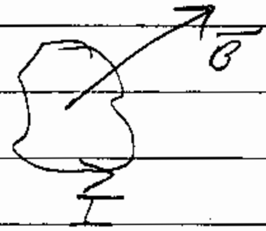
$$\boxed{P_\rho = \frac{\rho_l}{2\pi}\left(\frac{1}{\rho} - \frac{\epsilon_0}{\epsilon_1 d}\right)}$$

$$\rho_p = -\nabla\cdot\bar{P} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho_l\epsilon_0\rho}{2\pi\epsilon_1 d}\right) = \boxed{\frac{\rho_l\epsilon_0}{2\pi\epsilon_1 d\rho}}$$

c) Use layers that approximate $\frac{\epsilon_1 d}{\rho}$ variation

EE 333 Homework 7 (solutions - pg 2)

3.16

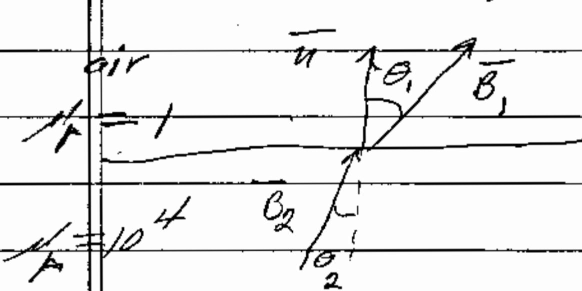


$$\overline{F} = \oint dq\, \overline{u} \times \overline{B}$$

$$F = \oint I\, \overline{dl} \times \overline{B} = -I \oint \overline{B} \times \overline{dl}$$

$$\overline{F} = -I\, \overline{B} \times \oint \overline{dl} \equiv 0$$

3.25 $\quad H_{t_1} = H_{t_2} \qquad B_{n_1} = B_{n_2}$

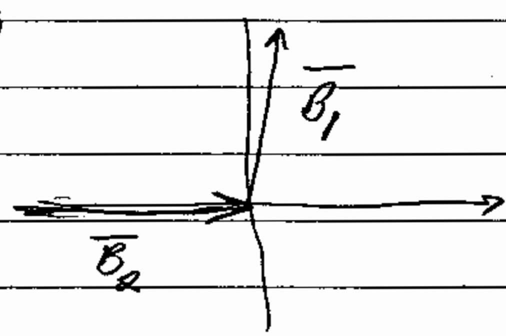


$$\tan\theta_2 = \frac{\mu_2}{\mu_1}\tan\theta_1 \quad (3.76)$$

$$\text{or } \theta_1 = \tan^{-1}\left[10^{-4}\tan\theta_2\right]$$

θ_2	θ_1
0	0
$45°$	5.7×10^{-3} °
$89°$	$0.328°$
$89.9°$	$3.27°$
$89.9675°$	$10°$ ✓

①

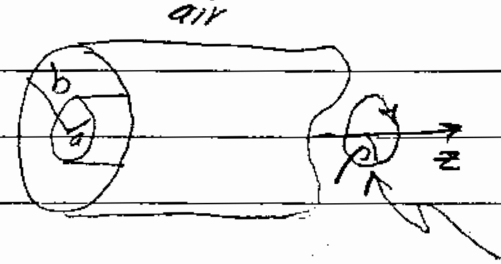


②

EE 333 Homework 8

29, 33

3-28



I carried in conductor of radius "a" ($\mu = \mu_0$)

for $a < \rho < b$ $\mu = \mu$

$\oint \bar{H} \cdot d\bar{l} = \int \bar{J} \cdot d\bar{s}$ Integrate around circular contour

a) for $\rho < a$ $H_\phi 2\pi\rho = \frac{I}{\pi a^2}\pi\rho^2$ or $H_\phi = \frac{I\rho}{2\pi a^2}$

$B_\phi = \frac{\mu_0 I \rho}{2\pi a^2}$

for $a < \rho < b$ $H_\phi = \frac{I}{2\pi\rho}$

$B_\phi = \frac{\mu I}{2\pi\rho}$

for $\rho > b$ $H_\phi = \frac{I}{2\pi\rho}$

$B_\phi = \frac{\mu_0 I}{2\pi\rho}$

b) $\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_\phi\left[\frac{\mu_r I}{2\pi\rho} - \frac{I}{2\pi\rho}\right] = \frac{I}{2\pi\rho}\left[\mu_r - 1\right]\bar{a}_\phi$

H_ϕ is continuous across all boundaries ($H_{tangential}$)

B_ϕ is discontinuous

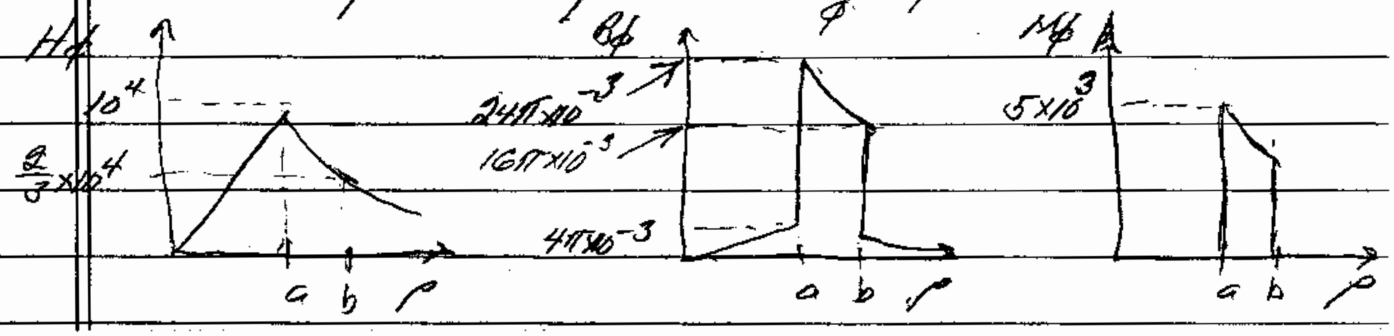
@ $\rho = a$ $H_\phi = \frac{628}{2\pi \times 10^{-2}} = 10^4$; $B_\phi = 24\pi \times 10^{-3}$ ($\rho = a^+$)

$B_\phi = 4\pi \times 10^{-3}$ ($\rho = a^-$)

$\rho = b$ $H_\phi = \frac{628}{2\pi \times 1.5 \times 10^{-2}} = \frac{2}{3} \times 10^4$, $B_\phi = 4\pi \times 10^{-7} \times 6 \times \frac{2}{3} \times 10^4 = 16\pi \times 10^{-3}$ ($\rho = b^-$)

$B_\phi = \frac{8}{3}\pi \times 10^{-3}$ ($\rho = b^+$)

$M = 0$ for $\rho < a$ and $\rho > b$ $M_\phi = \frac{50}{\rho}$ $a < \rho < b$



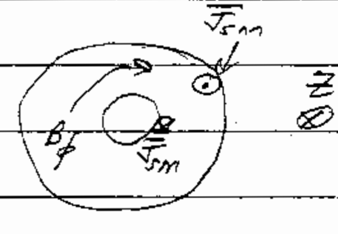
EE 333 Homework 8 (page 2)

3.28 continued: $\bar{J}_m = \nabla \times \bar{M}$

$$\therefore \bar{J}_m = \bar{a}_\rho\left(-\frac{\partial M_\phi}{\partial z}\right) + \bar{a}_z\left(\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho M_\phi)\right) = 0$$

$$\bar{J}_{SM}(\rho = a) = \bar{a}_\rho \times \bar{M} = 5 \times 10^3 \bar{a}_z$$

$$\bar{J}_{SM}(\rho = b) = -\bar{a}_\rho \times \bar{M} = -\frac{10}{3} \times 10^3 \bar{a}_z$$



3.29

Region 1 $\epsilon = \epsilon_0$ $\quad \bar{n} = \bar{a}_z$

Region 2 $\epsilon = 4\epsilon_0$

$$\bar{E}_1 = -15\bar{a}_x + 20\bar{a}_y + 30\bar{a}_z$$

$$\left\{\begin{array}{l} E_{tang_1} = E_{tang_2} \\ D_{norm_1} = D_{norm_2} \end{array}\right\}$$

a) $\boxed{\bar{E}_2 = -15\bar{a}_x + 20\bar{a}_y + \frac{15}{2}\bar{a}_z}$ $\longleftrightarrow$ $\left\{\begin{array}{l} D_{z_1} = 30\epsilon_0 = D_{z_2} \\ E_{z_2} = \frac{D_{z_2}}{4\epsilon_0} = \frac{30}{4} \end{array}\right.$

$$\bar{D}_1 = \epsilon_0 \bar{E}_1$$
$$\bar{D}_2 = 4\epsilon_0 \bar{E}_2$$

b) $\bar{n}\cdot\bar{E}_1 = |\bar{E}_1|\cos\theta_1 = 30 \quad \therefore \theta_1 = \cos^{-1}\left\{\frac{30}{\sqrt{15^2+20^2+30^2}}\right\} = \boxed{39.8^\circ}$

$$\theta_2 = \cos^{-1}\left\{\frac{\frac{15}{2}}{\sqrt{15^2+20^2+7.5^2}}\right\} = \boxed{73.3^\circ}$$

c:
$$\left.\begin{array}{l} \tan\theta_2 = \frac{\epsilon_2}{\epsilon_1}\tan\theta_1 \\ 3.333 = 4 \times 0.833 \\ 3.333 = 3.333 \quad ✓ \end{array}\right\}$$

3-33 $\alpha = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{\frac{1}{2}}$

$$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}+1\right]^{\frac{1}{2}}$$

$$\eta = \sqrt{\frac{\mu}{\epsilon - j\frac{\sigma}{\omega}}}$$

for $\sigma = 0$, $\mu = \mu_0$, $\epsilon = \epsilon_0$ these give:

$$\left.\begin{aligned} \alpha &= 0 \\ \beta &= \omega\sqrt{\mu_0\epsilon_0} \\ \eta &= \sqrt{\frac{\mu_0}{\epsilon_0}} \end{aligned}\right\} \leftarrow$$

EE 333 Homework 9 (12 total)

3.38 $\hat{E}_m^+ = 1$ @ $z=0$; $\left\{ \text{for sea water } \begin{array}{l} \epsilon_r = 81 \\ \sigma = 4 \end{array} \right.$

(6) $\boxed{f = 10^4}$

$\frac{\sigma}{\omega\epsilon} = \frac{4 \times 36\pi}{10^{-9} \times 2\pi \times 10^4} = 7.2 \times 10^6 >> 1$ ∴ good conductor ✓

so $\alpha = \beta = \sqrt{\frac{\omega\sigma\mu}{2}} = \sqrt{\frac{2\pi \times 10^4 \times 4 \times 4\pi \times 10^{-7}}{2}} = \boxed{0.397}$ ←

for $e^{-\alpha z} = 0.05$; $\boxed{z_{5\%} = -\frac{\ln 0.05}{\alpha} = 7.5\text{ m}}$ ✓ ←

for $f = 10^3$ $\frac{\sigma}{\omega\epsilon} = 7.2 \times 10^7$ (good conductor)

$\alpha = \beta = 4\pi \times 10^{-2} \boxed{= 0.1257}$ ← ✓

$z_{5\%} = -\frac{\ln 0.05}{0.1257} \boxed{= 23.8\text{ m}}$ ✓ ←

RF communication in sea water is only possible at very low frequencies. ✓

6 3.37 $\hat{E}_m^+ = 200\text{ V/m}$; $f = 10^8\text{ Hz}$, $\mu = \mu_0$, $\epsilon = 6\epsilon_0$, $\sigma = 10^{-2}$

$\gamma = 0.761 + j5.187 = \alpha + j\beta$

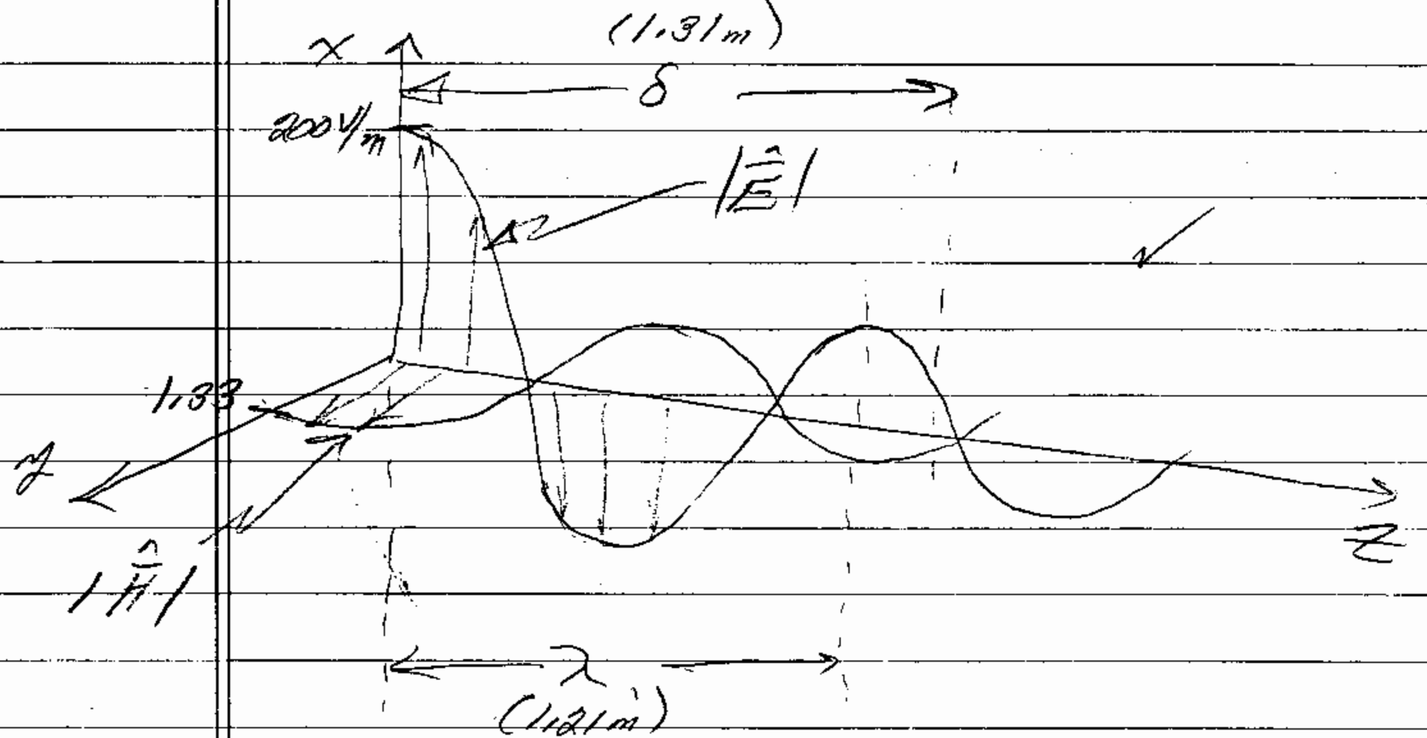
a) $\boxed{\lambda = \frac{2\pi}{\beta} = 1.21\text{ m}}$ ✓ $\boxed{\delta = \frac{1}{\alpha} = 1.31\text{ m}}$ ✓

$u_{ph} = \frac{\omega}{\beta} = \frac{2\pi \times 10^8}{5.187} \boxed{= 1.21 \times 10^8\text{ m/sec}}$ ✓ ←

b) $\hat{\eta} = \sqrt{\frac{\mu}{\epsilon - j\frac{\sigma}{\omega}}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{6 \times 10^{-9}}{36\pi} - j\frac{10^{-2}}{2\pi \times 10^8}}} = \boxed{1.506 \times 10^2 \angle 8.34^\circ}$ ✓ ←

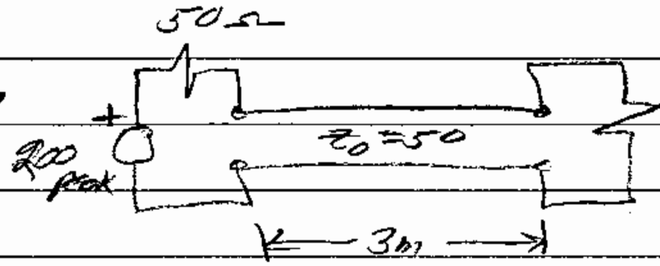
→ so $\boxed{\hat{H}_y^+ = \frac{200}{\hat{\eta}} = 1.33 e^{-j8.34^\circ}}$ ✓

EE 333 Homework 9 (page 2)



EE 333 Homework 10

10.3



$f = 50\,MHz$

$\epsilon_r = 2.25$

$v_{ph} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8\, m/sec$; $\lambda = \frac{2\times10^8}{5\times10^7} = 4\,m$

so line length $= \frac{3}{4}\lambda$

a) $\boxed{\Gamma(l) = \frac{Z_L - Z_0}{Z_L + Z_0} = 0}$ $\therefore$ $\Gamma(z)$ is zero at all points on the line

$\boxed{Z(z)} = Z_0 \frac{1+\Gamma(z)}{1-\Gamma(z)} = \boxed{Z_0}$ everywhere

b) $\hat{V}(z) = \hat{V}_m^+ e^{-j\beta z}$; $\hat{I}(z) = \frac{\hat{V}_m^+}{Z_0} e^{-j\beta z}$

these become $\hat{V}_m^+$ and $\hat{V}_m^+/Z_0$ at $z=0$

c) $\hat{I}_{in} = \hat{I}_m^+ = \frac{\hat{V}_m^+}{Z_0} = \frac{200}{100} = \boxed{2A}$; $\hat{V}_{in} = \hat{V}_m^+ = \boxed{100}$

$$\left.\begin{aligned}\therefore\ \hat{V}(z) &= 100 e^{-j\beta z} = 100 e^{-j\frac{2\pi}{\lambda}z} \\ \hat{I}(z) &= 2 e^{-j\beta z} = 2 e^{-j\frac{2\pi}{\lambda}z}\end{aligned}\right\}$$

$\boxed{P_{in_{ave}} = \frac{1}{2}\, 100\times2 = 100\ Watts}$

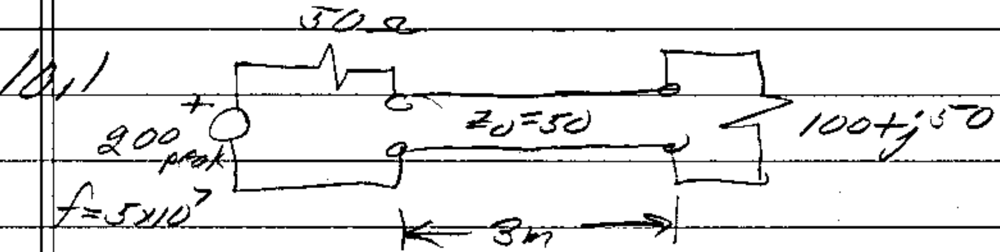
$$\left.\begin{aligned}\hat{V}_{load} &= 100 e^{-j\frac{2\pi}{\lambda}\left(\frac{3\lambda}{4}\right)} = 100 e^{-j\frac{3\pi}{2}} = j100 \\ \text{so } \hat{I}_{load} &= j2\end{aligned}\right\}$$

and $P_{load} = \frac{1}{2}Re\{j100(-j2)\} = 100\,Watts$

or $P_{load} = \frac{1}{2}|\hat{I}|^2 50 = \frac{1}{2}\frac{|\hat{V}|^2}{50} = 100W$

EE 333 Homework 11.

10.1 $\boxed{\epsilon_r = 2.25}$



$$j\beta = j\omega\sqrt{\mu\epsilon} = j\omega\frac{\sqrt{2.25}}{3\times10^8} = j2\pi\times5\times10^7\frac{1.5}{3\times10^8} = \boxed{j\frac{1}{2}\pi}$$

$\boxed{v_p = \frac{3\times10^8}{\sqrt{\epsilon_r}} = 2\times10^8\ m/sec}$; $\boxed{\lambda = \frac{2\pi}{\beta} = 4\ m}$ $\boxed{\text{line length} = \frac{3}{4}\lambda}$

$$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j50}{150 + j50} = 0.4 + j0.2 = \boxed{0.4472\,e^{j26.565^\circ}}$$

$$\Gamma_{in} = \Gamma_{load}\,e^{-j2\beta\left(\frac{3\lambda}{4}\right)} = (0.4 + j0.2)e^{-j3\pi} = -(0.4 + j0.2) = \boxed{-0.4472\,e^{j26.56^\circ}}$$

∴ approximately 45% voltage reflection

$$Z_{in} = 50\frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 50\frac{0.6 - j0.2}{1.4 + 0.2} = 20 - j10$$

$$\hat{I}_{in} = \frac{200}{50 + Z_{in}} = \frac{200}{70 - j10} = 2.828\,e^{j8.13^\circ}$$

$$P_{in} = \frac{1}{2}|\hat{I}_{in}|^2 R_{in} = \frac{1}{2}(2.828)^2(20) = \boxed{79.98\ Watts}$$

$$\hat{I} = \hat{I}_m^+ e^{-j\beta z}\left[1 - \Gamma(z)\right] \quad ∴ \hat{I}_m^+ = \frac{\hat{I}_{in}}{1 - \Gamma(0)}$$

$$\text{so } \hat{I}_{load} = \hat{I}_m^+ e^{-j\frac{2\pi}{\lambda}\cdot\frac{3\lambda}{4}}\left[1 - \Gamma_{load}\right] = \hat{I}_{in}\,e^{-j\frac{3\pi}{2}}\frac{1 - \Gamma_{load}}{1 - \Gamma_{in}}$$

$$\text{or } \hat{I}_{load} = 2.828\,e^{j8.13^\circ}\cdot j\,\frac{0.6 - j0.2}{1.4 + j0.2} = 1.2647\,e^{j71.56^\circ}$$

$$∴ P_{load} = \frac{1}{2}(1.2647)^2\times100 = 79.97\ Watts$$

10.2 $P_{ave} = \frac{1}{2} Re\left\{ \hat{V}_m^+ e^{-j\beta z}\left[1+\Gamma\right] \frac{\hat{V}_m^{+*}}{Z_0} e^{+j\beta z}\left[1-\Gamma^*\right]\right\}$

$$P_{ave} = \frac{1}{2} Re\left\{ \frac{|\hat{V}_m^+|^2}{Z_0}\left[\underbrace{1-\Gamma^*+\Gamma}_{-\Gamma_r+j\Gamma_i+\Gamma_r+j\Gamma_i}-|\Gamma|^2\right]\right\}$$

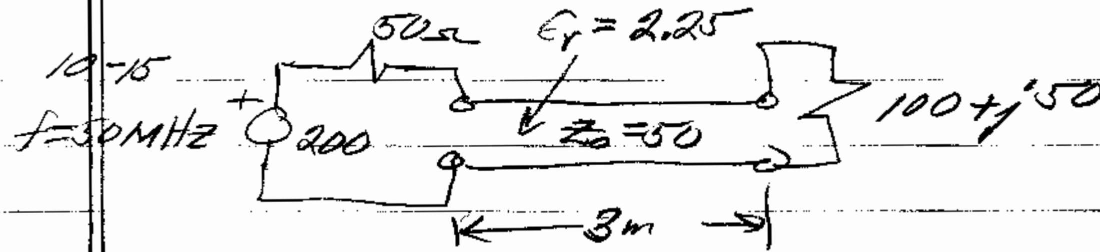
so $P_{ave} = \frac{|\hat{V}_m^+|^2}{2Z_0}\left[1-|\Gamma|^2\right]$

$$\Gamma = \frac{\hat{V}_m^-}{\hat{V}_m^+} e^{-j2\beta z} \quad \therefore |\Gamma| = \frac{|\hat{V}_m^-|}{|\hat{V}_m^+|}$$

so $P_{ave} = \frac{|\hat{V}_m^+|^2}{2Z_0} - \frac{|\hat{V}_m^-|^2}{2Z_0} = P_{ave}^+ - P_{ave}^-$

EE 333 Homework 12 (page 1)

10-15



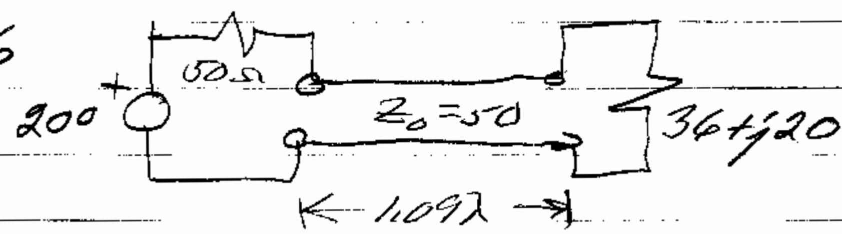
$$\lambda = \frac{3\times10^8}{5\times10^7 \times \sqrt{2.25}} = 4m \quad \therefore \text{ line length} = \frac{3}{4}\lambda$$

$Z_{L_{norm}} = 2 + j1$; from chart: $\Gamma_L = 0.46\, e^{j26^\circ}$

[chart scales $0.214 + 0.25 = 0.464\lambda$]

from chart: $\left\{ \begin{array}{l} \Gamma_{in} = 0.46\, e^{j154^\circ} \\ Z_{in_{norm}} = 0.4 - j0.195 \text{ or } Z_{in} = 20 - j9.75 \end{array} \right\}$

10-16



$Z_{L_{norm}} = 0.72 + j0.4$

from chart: $\Gamma_L = 0.29\, e^{j112^\circ}$

chart rotation $= 0.094 + 0.09 = 0.184\lambda$

from chart: $\Gamma_{in} = 0.29\, e^{j48^\circ}$ and $Z_{in_{norm}} = 1.32 + j0.58$

or $\boxed{Z_{in} = 66 + j29}$

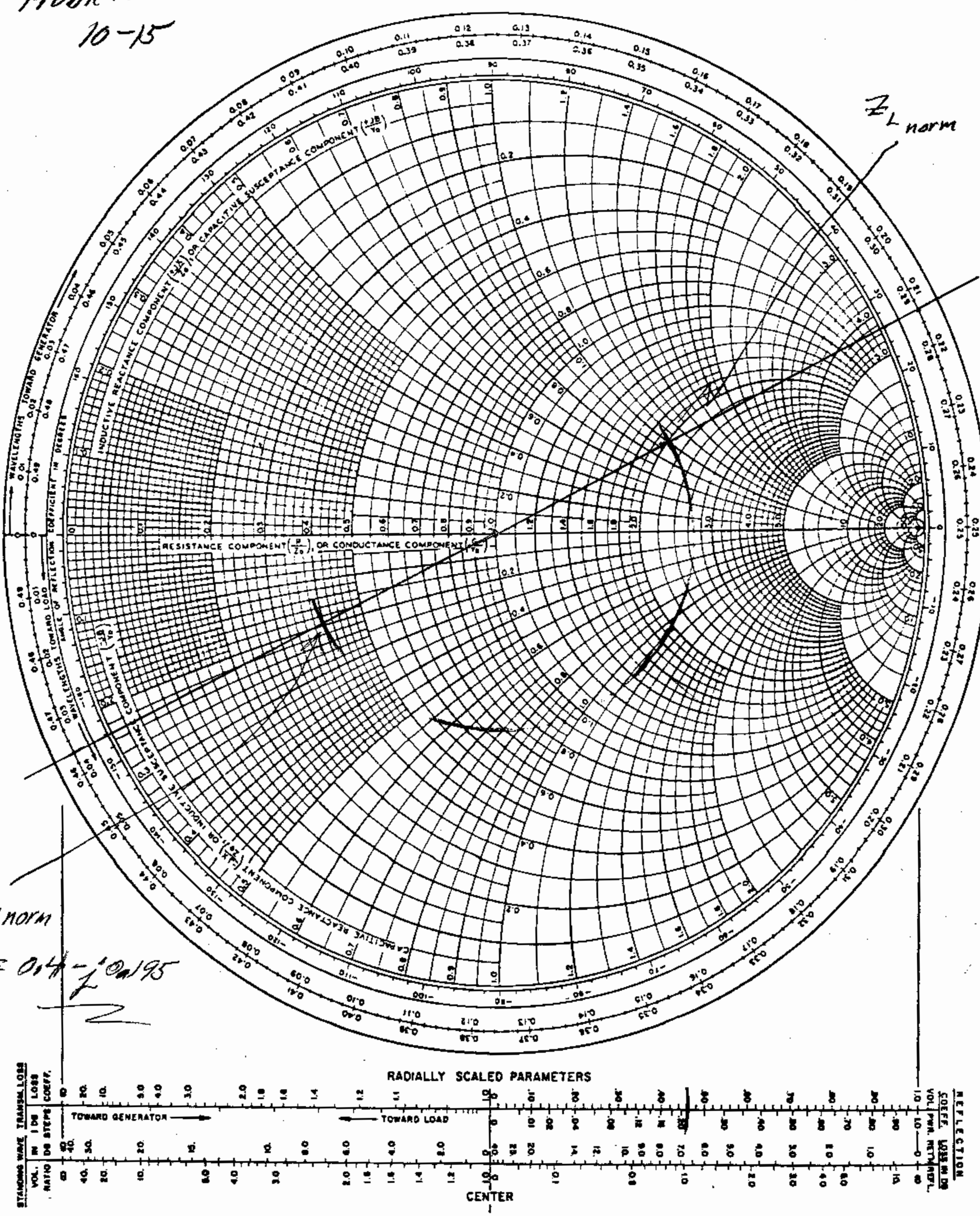
10-21 $Z_L = 36 + j20$; $Z_0 = 50$ $\therefore$ $Z_{L_{norm}} = 0.72 + j0.4$

from chart $Y_{L_{norm}} = 1.07 - j0.59$

$$\frac{1}{Z_{L_{norm}}} = \frac{1}{0.72 + j0.4} = \frac{1}{0.8237\, e^{j29.05}} = 1.214\, e^{-j29.05^\circ} = 1.06 - j0.59$$

Problem 10-15

IMPEDANCE OR ADMITTANCE COORDINATES

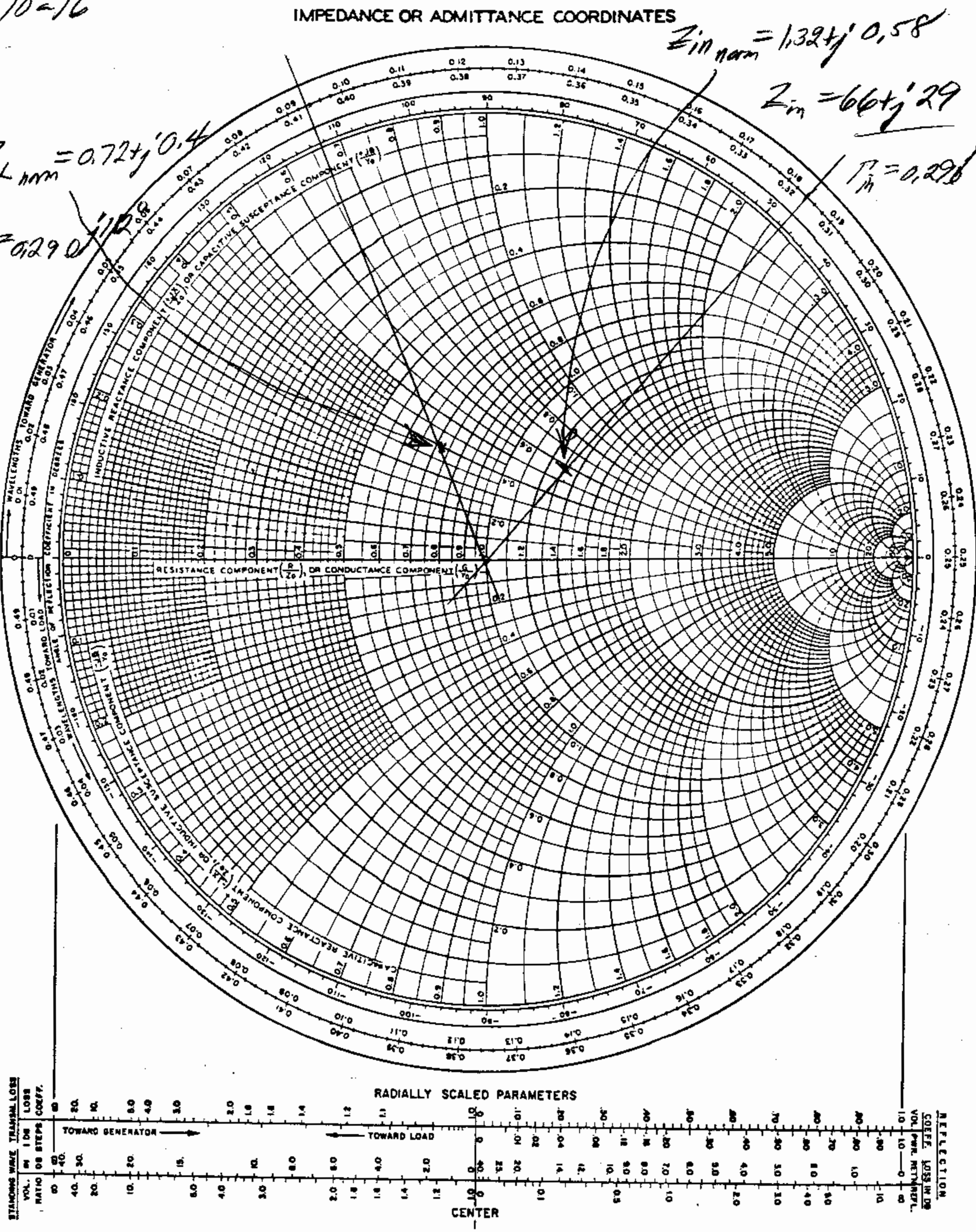


$Z_{L\,norm}$

$Z_{in\,norm} = 0.4 - j0.195$

Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

10-16



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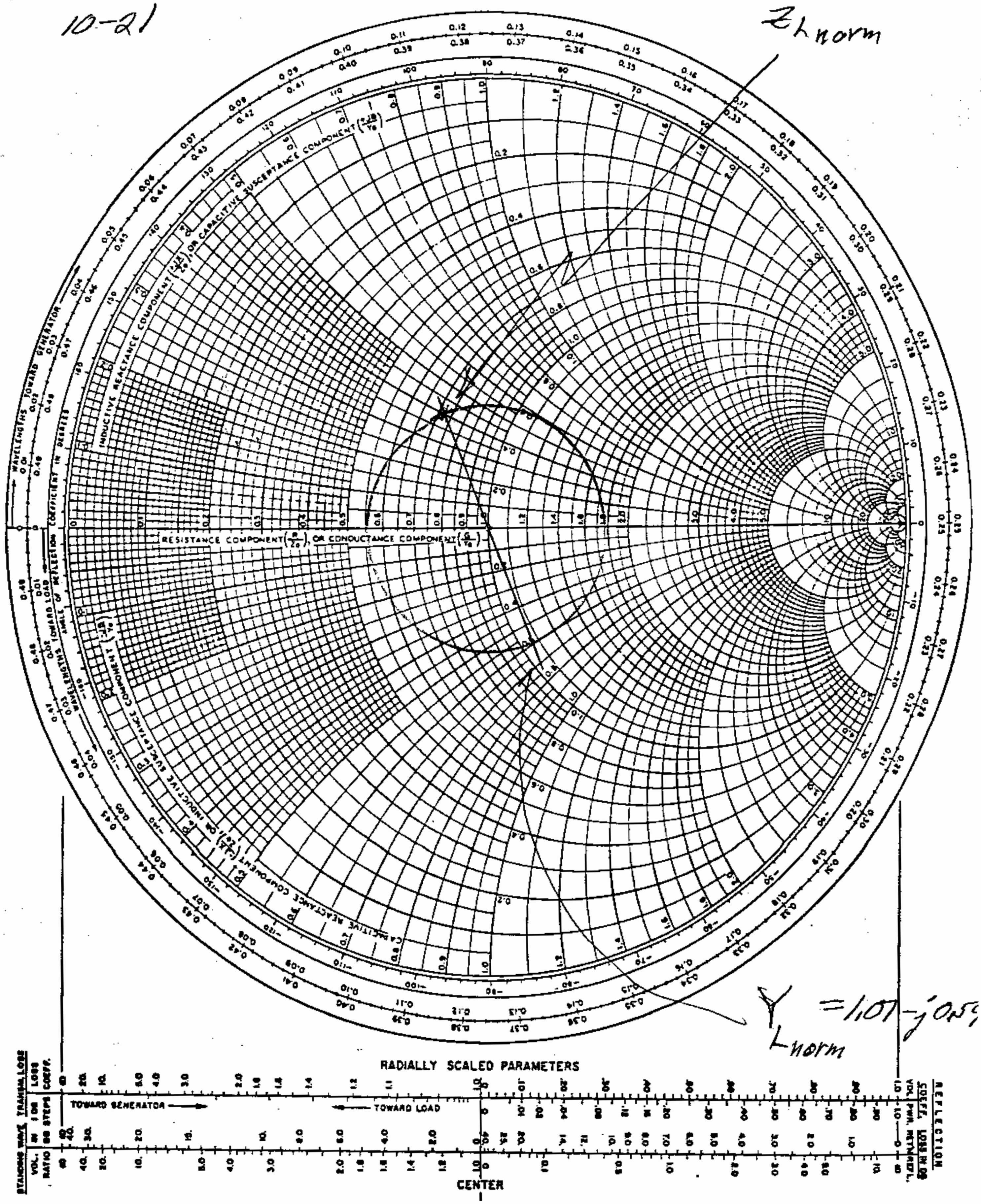
Problem 10-21

IMPEDANCE OR ADMITTANCE COORDINATES



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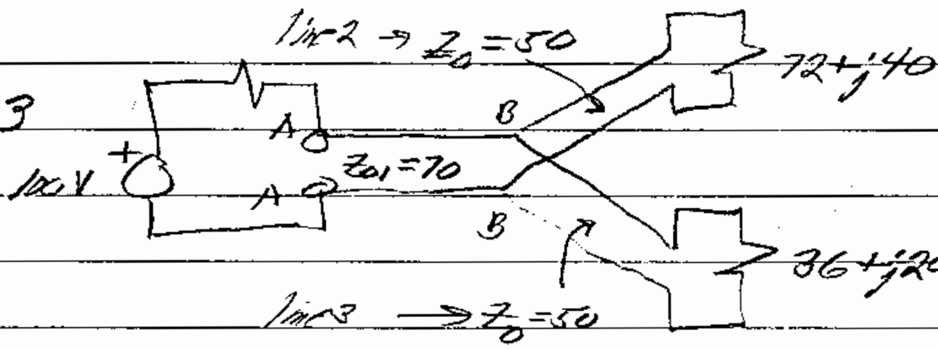
EE 333 Homework 12 (page 2)

10-23



from example 10-6 $P_{ave_{\text{into } A\text{-}A}} = 8.5$ Watts

$Y_{in_2 @ BB} = G_2 + jB_2$; $Y_{in_3 @ BB} = G_3 + jB_3$

$P_{ave} = \frac{1}{2}\,\mathrm{Re}\{\hat{V}\hat{I}^*\} = \frac{1}{2}\,\mathrm{Re}\{\hat{V}\hat{V}^* Y^*\} = \frac{1}{2}|\hat{V}|^2 G$

$\therefore \boxed{\dfrac{P_{ave_2}}{P_{ave_3}} = \dfrac{G_2}{G_3}}$

$\frac{1}{2}|\hat{V}_{BB}|^2 (G_2 + G_3) = 8.5 = \frac{1}{2}|\hat{V}|^2 G_2 \left(1 + \frac{G_3}{G_2}\right)$

$= P_{ave_2}\left(1 + \frac{G_3}{G_2}\right)$

but from example: $G_2 = 0.0366$ + $G_3 = 0.012$

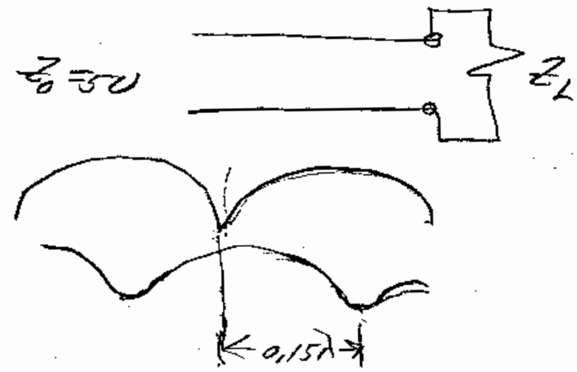
so $P_{ave_2} = \dfrac{8.5}{1 + \frac{0.012}{0.0366}} = \boxed{6.4 \text{ Watts}}$

$P_{ave_3} = 8.5 - 6.4 = \boxed{2.1 \text{ Watts}}$

lossless lines

EE333 Homework 13 (Solution)

10-27

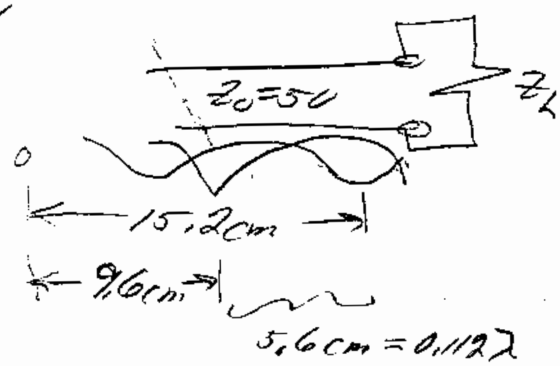


SWR = 4.0

from chart $Z_{L_{norm}} = 0.67 + j1.14$

$\therefore \boxed{Z_L = 33.5 + j57}$

10.28



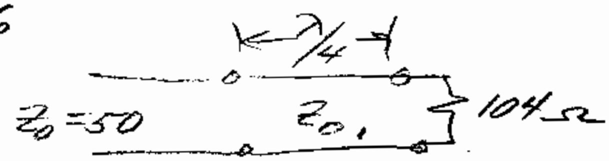
$f = 600\,MHz$; SWR = 3.5

$\lambda = \frac{3 \times 10^8}{6 \times 10^8} = 0.5\,m$

from chart $Z_{L_{norm}} = 0.475 + j0.73$

$\boxed{\text{or } Z_L = 23.75 + j36.5}$

10.36

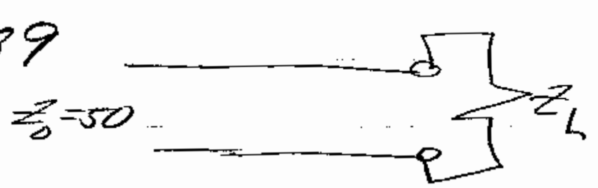


$f = 300\,MHz$, $\epsilon_r = 2.26$

$\boxed{Z_{01} = \sqrt{50 \times 104} = 72.1\,\Omega}$

$\lambda = \frac{3 \times 10^8}{\sqrt{2.26}\; 5 \times 10^8} = \frac{0.6}{1.5} = 0.4\,m \quad \therefore \boxed{\lambda/4 = 0.1\,m}$

10.39



$f = 500\,MHz$; $\lambda = 60\,cm$

SWR = 3.5

(see chart)

place stub $(0.25 - 0.171)\lambda = 0.08\lambda = 4.8\,cm$ either direction from V_{min}

a) for stub toward the load $Y_{norm_{stub}} = -j1.3$; stub length = 0.355 − 0.25 = 0.105

or stub $\boxed{\text{length} = 6.3\,cm}$

b) SWR from generator to stub = 1

SWR from stub to load = 3.5

c) for stub toward the generator $Y_{norm_{stub}} = +j1.3$; stub length = $(0.145 + 0.25)\lambda = \boxed{23.7\,cm}$

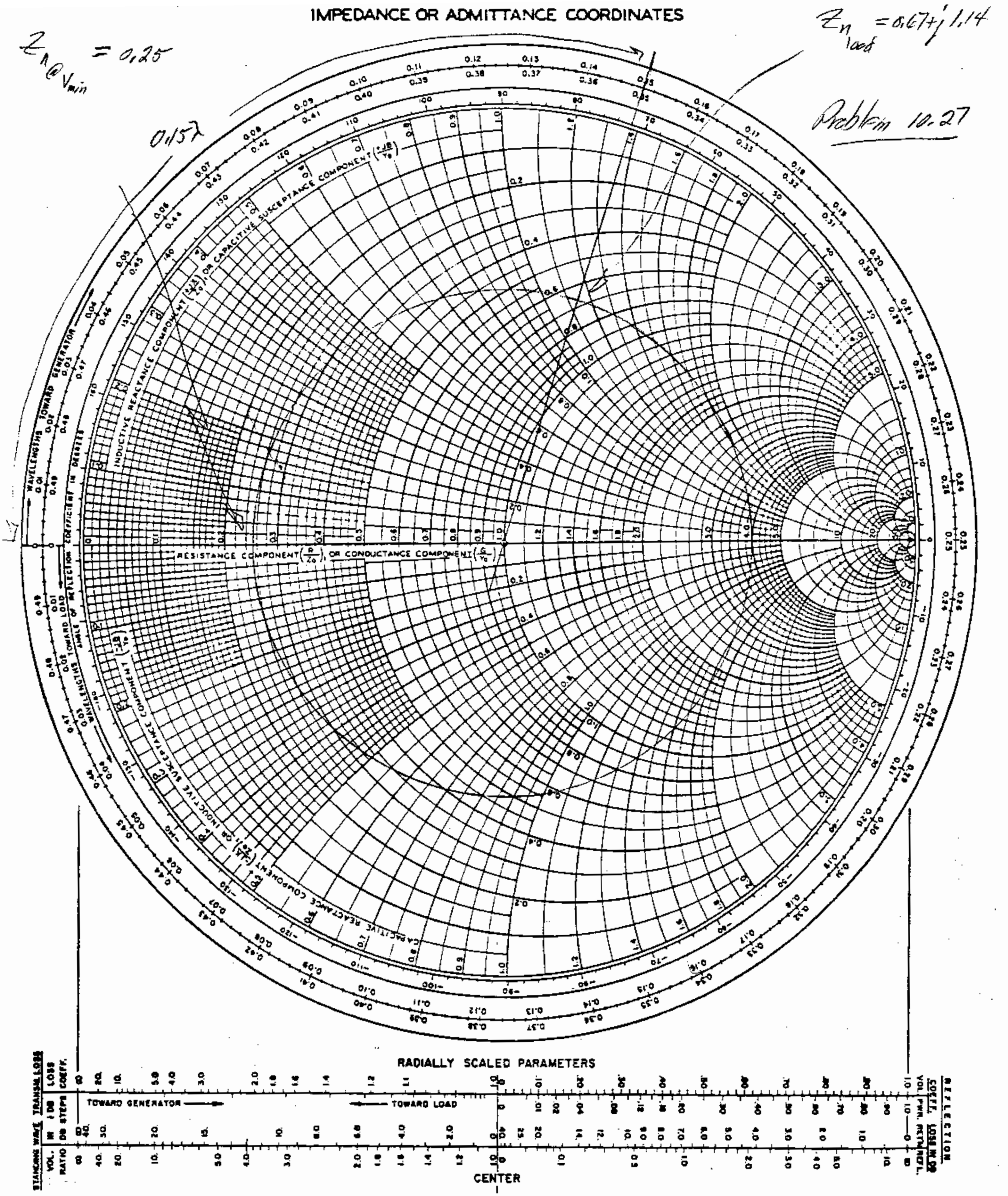


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

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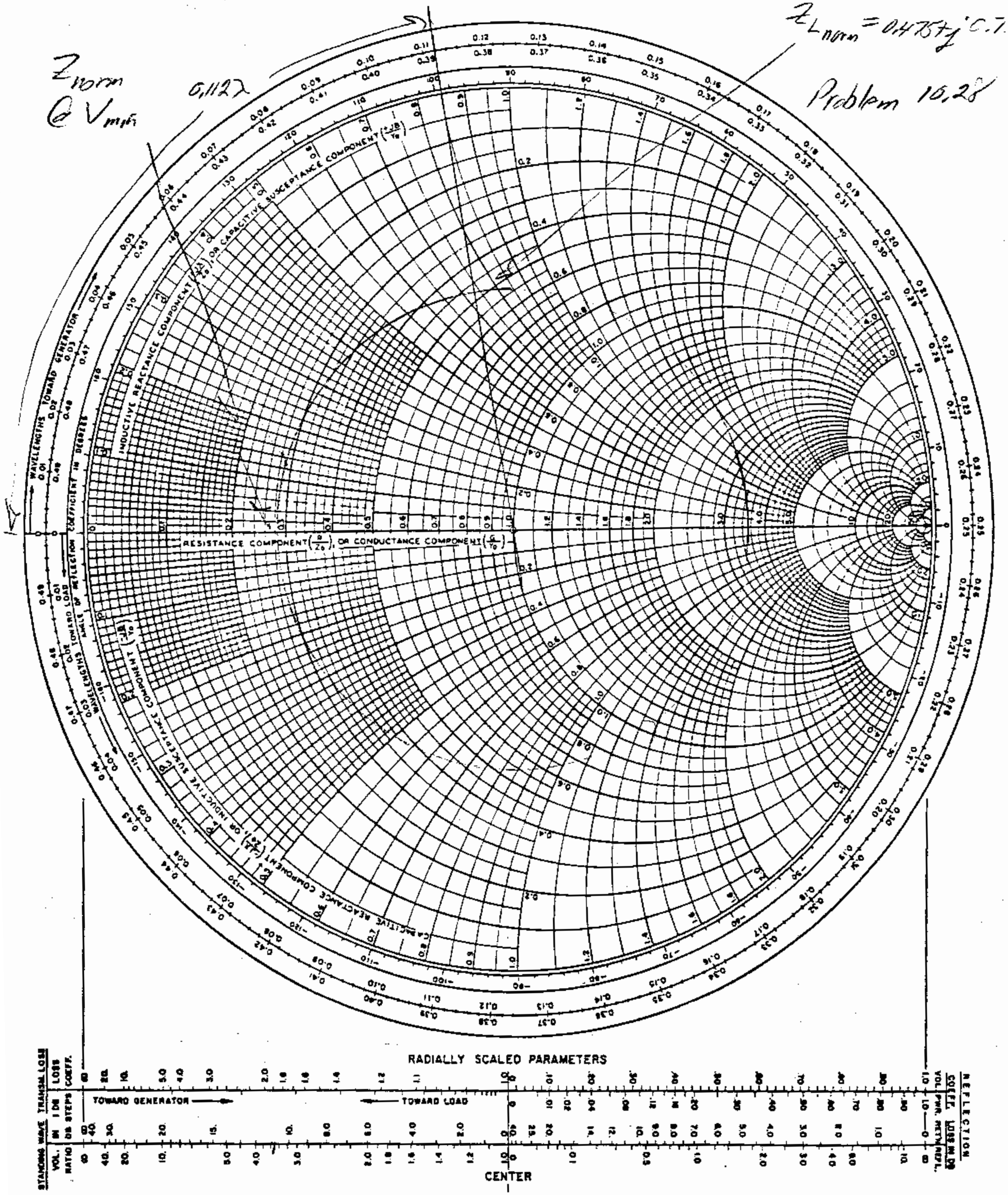

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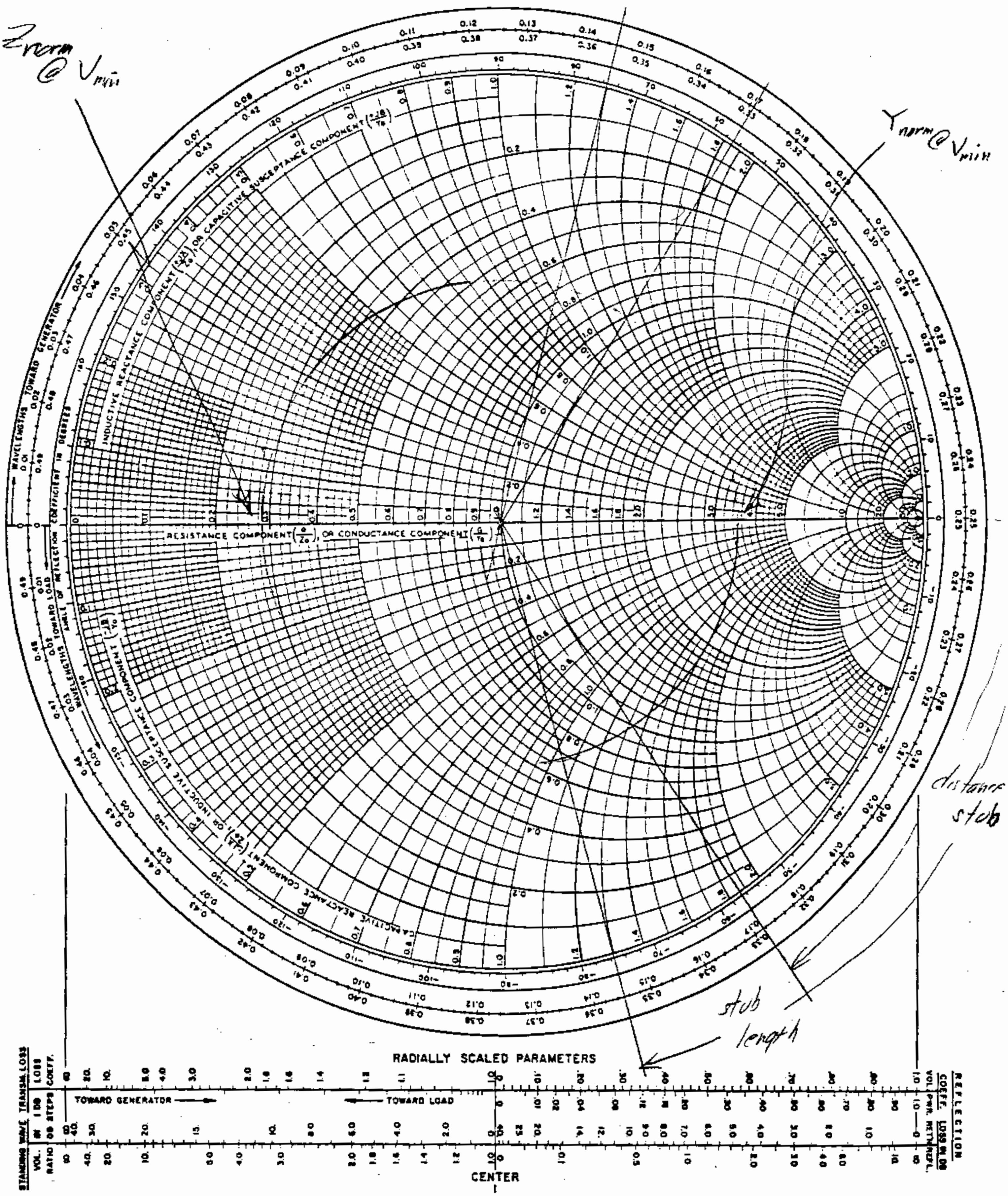
IMPEDANCE OR ADMITTANCE COORDINATES

Problem 10.39



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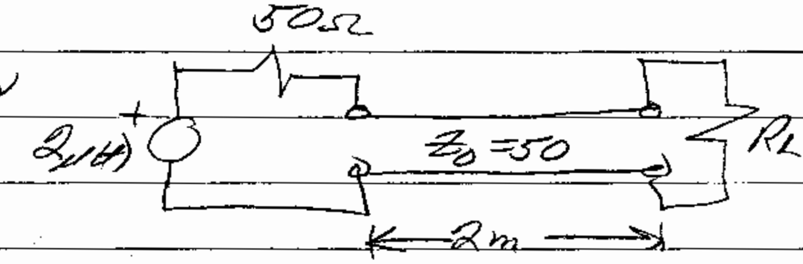
EE 333 Homework 14

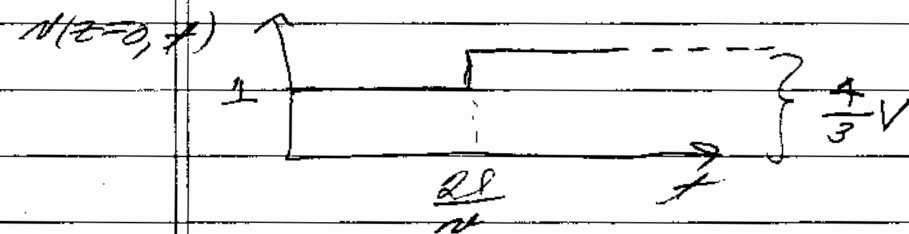
9-38



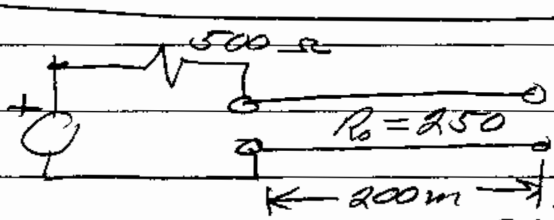
a) $R_L = 0$, $\Gamma_L = -1$

$\left\{ \text{where } \frac{\ell}{u} = \frac{2}{3\times10^8} = \frac{2}{3}\times10^{-8}\,s \right\}$

b) $R_L = 100$ $\therefore$ $\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}$



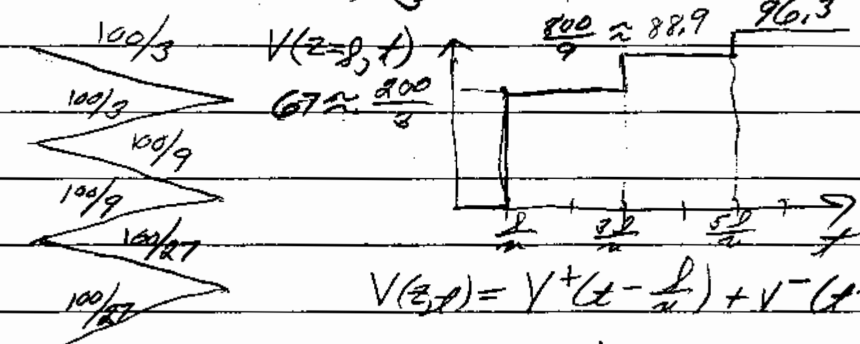
9-39



$\left\{ \begin{array}{l} V^+(z=0,t) = 100\left(\frac{1}{3}\right) V \\ I^+(z=0,t) = \frac{V^+}{250} = \frac{2}{15} A \end{array} \right\}$

$\Gamma(0) = \frac{1}{3}$ $\Gamma(\ell) = 1$

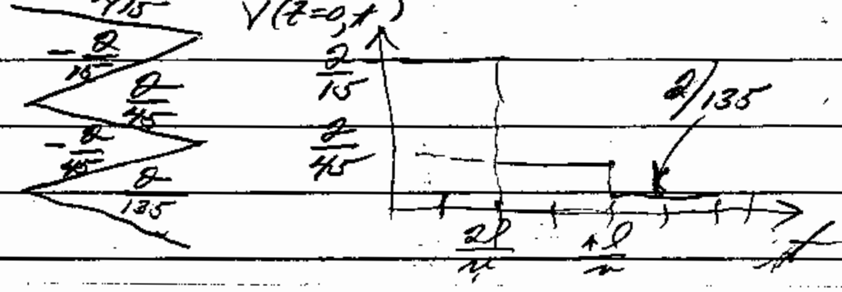
a)



$\boxed{V_{final} = 100}$

$V(z,t) = V^+\left(t - \frac{\ell}{u}\right) + V^-\left(t + \frac{\ell}{u}\right)$

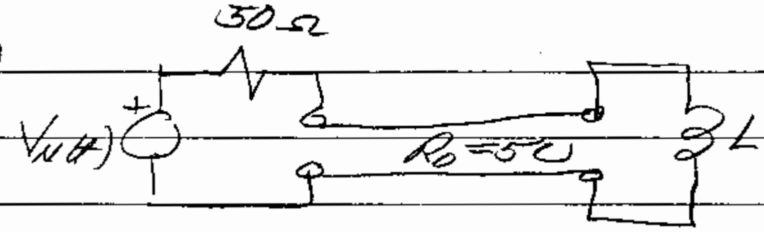
b) $I(z,t) = \frac{1}{R_0}\{V^+ - V^-\} = \frac{V^+}{R_0}\{1 - \Gamma\}$



$\boxed{I_{final} = 0}$

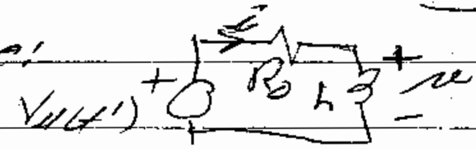
EE 333 Homework 15

9-40



@ load $V^+(t-\frac{l}{u}) = \frac{V}{2}u(t-\frac{l}{u}) = \frac{V}{2}u(t')$; $\boxed{t' = t - \frac{l}{u}}$

so at load we have:



$V = iR_0 + L\frac{di'}{dt'}$

$i_{homo} = Ae^{st'}$; $s = -\frac{R_0}{L}$ so $i_h = Ae^{-\frac{R_0 t'}{L}}$

$i_{particular} = K = \frac{V}{R_0}$ ∴ $i = Ae^{-\frac{R_0 t'}{L}} + \frac{V}{R_0}$

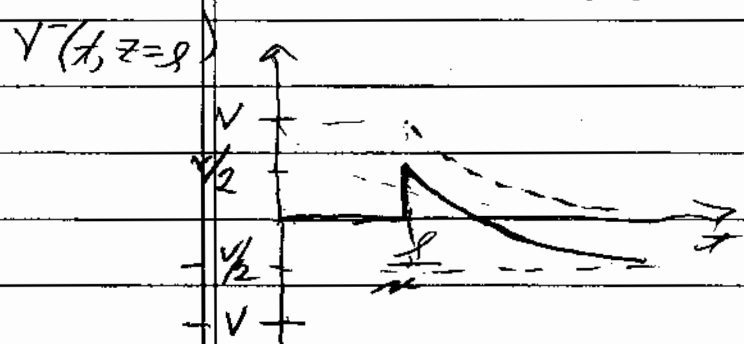
Assuming no initial energy storage [i.e. $i(0)=0$]

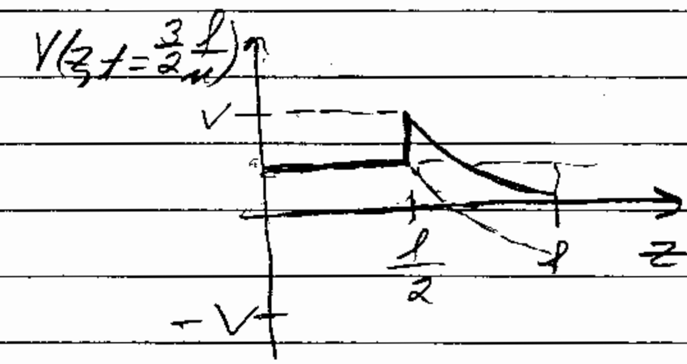
$$i = \frac{V}{R_0}\left(1 - e^{-\frac{R_0 t'}{L}}\right) = \left[V^+(t-\frac{l}{u}) - V^-(t+\frac{l}{u})\right]\frac{1}{R_0}$$

giving $V^-(t+\frac{l}{u}) = V^+(t-\frac{l}{u}) - V\left(1 - e^{-\frac{R_0(t-\frac{l}{u})}{L}}\right)$

or $V^-(t+\frac{l}{u}) = \frac{V}{2}u(t-\frac{l}{u}) - V\left(1 - e^{-\frac{R_0(t-\frac{l}{u})}{L}}\right)u(t-\frac{l}{u})$

$$V^-(t+\frac{l}{u}) = \left[-\frac{V}{2} + Ve^{-\frac{R_0(t-\frac{l}{u})}{L}}\right]u(t-\frac{l}{u})$$





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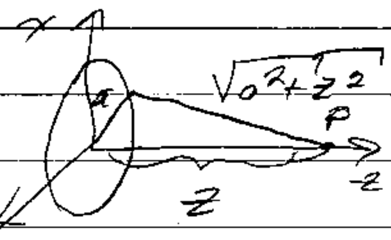
4-12



a) $\Phi_p = \int_{\phi=0}^{2\pi} \frac{a\rho_l\, d\phi}{4\pi\epsilon(a^2+z^2)^{1/2}}$

$\Phi_p = \frac{a\rho_l}{2\epsilon(a^2+z^2)^{1/2}}$

$\bar{E}_{\text{on z axis}} = -\nabla\Phi = -\frac{\partial\Phi}{\partial z}\bar{a}_z = \frac{a\rho_l\, 2z}{4\epsilon(a^2+z^2)^{3/2}}\bar{a}_z$

b) $\Phi(z) = -\int_{\infty}^{z} \frac{a\rho_l z\, \bar{a}_z \cdot dz\, \bar{a}_z}{2\epsilon(a^2+z^2)^{3/2}} = -\int_{\infty}^{a^2+z^2} \frac{a\rho_l\, du}{4\epsilon u^{3/2}}$

let $u = a^2+z^2$
$du = 2z\,dz$

$\Phi(z) = + \frac{a\rho_l\, 2}{4\epsilon u^{1/2}}\Big|_{\infty}^{a^2+z^2}$

so $\Phi(z) = \frac{a\rho_l}{2\epsilon(a^2+z^2)^{1/2}}$

4-16 for coax $C/m = \frac{2\pi\epsilon}{\ln(\frac{b}{a})} = 96.8\times10^{-12}$

$\boxed{\epsilon_r = 2.26}$

so $\ln(\frac{b}{a}) = \frac{2\pi\times10^{-9}\times2.26}{36\pi\times96.8\times10^{-12}} = 1.297$

$\therefore \frac{b}{a} = 3.6585$, so $\boxed{b = 0.713\text{ in or } 1.81\text{ cm}}$

4.29 $U_e = \iiint \bar{D}\cdot\bar{E}\, dv = \frac{1}{2}\int_{\phi=0}^{\pi}\int_{\rho=a}^{b}\int_{z=0}^{l} \frac{\epsilon_1 V^2 \rho\, d\phi\, dz\, d\rho}{\rho^2[\ln\frac{b}{a}]^2}$

[using (4-71)]

$+ \frac{1}{2}\int_{\phi=\pi}^{2\pi}\int_{\rho=a}^{b}\int_{z=0}^{l} \frac{\epsilon_2 V^2 \rho\, d\phi\, dz\, d\rho}{\rho^2[\ln\frac{b}{a}]^2}$

$U_e = \frac{\pi V^2 l}{2(\ln\frac{b}{a})^2}\ln(\frac{b}{a})(\epsilon_1+\epsilon_2) = \frac{1}{2}CV^2$

$\therefore C = \frac{\pi l}{\ln(\frac{b}{a})}(\epsilon_1+\epsilon_2)$

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4-61 $30 \to 500\,pF$ in 0 to $260°$

$V = 5{,}000\,V$

$$\boxed{T = \frac{\partial U_e}{\partial \theta} = \frac{\Delta U_e}{\Delta \theta}}$$

and $U_e = \frac{1}{2} C V^2$.

so $\Delta U_e = \frac{1}{2} V^2 (30 - 500) \times 10^{-12} = -V^2 \frac{470}{2} \times 10^{-12}$

$$T = -\frac{25 \times 10^{6} \times 470 \times 10^{-12}}{2 \times 260 \times \pi/180} = 1.294 \times 10^{-3}\ N\text{-}m$$