EE 335 Homework 1

1-2 $\bar{A} = 5\bar{a}_x + 3\bar{a}_y + 4\bar{a}_z$, $\bar{B} = 2\bar{a}_y + \bar{a}_z$, $\bar{C} = -6\bar{a}_z$

a) $\bar{A} + \bar{B} + \bar{C} = 5\bar{a}_x + 5\bar{a}_y - \bar{a}_z$ ←

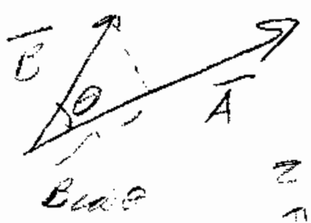
b) $|\bar{A}| = \sqrt{5^2 + 3^2 + 4^2} = \sqrt{50} = 7.07$ ←

$|\bar{D}| = \sqrt{51} = 7.14$ ←

c) $2\bar{A} - \bar{C} = \bar{E} = 10\bar{a}_x + 6\bar{a}_y + 14\bar{a}_z$ ←

$|\bar{E}| = \sqrt{332} = 18.22$ ←

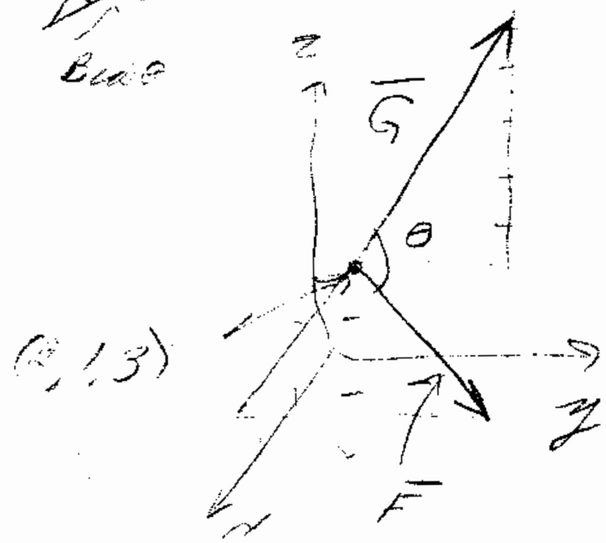
1-8



a) $\bar{A} \cdot \bar{B} = AB\cos\theta = |\bar{A}| \times$ projection of $\bar{B}$ on $\bar{A}$

$\bar{F} = 30\bar{a}_x + 40\bar{a}_y$; $\bar{G} = 20\bar{a}_y + 50\bar{a}_z$

b)



$\bar{F} \cdot \bar{G} = 800 = FG\cos\theta$

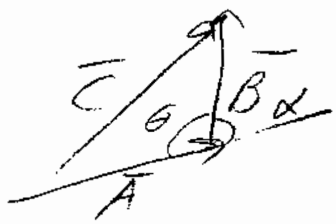
$= F\cos\theta \times \sqrt{2900}$

or $\boxed{F\cos\theta = \frac{800}{\sqrt{2900}} = 14.86}$

$\cos\theta = \frac{800}{FG} = 0.297$

∴ $\boxed{\theta = 72.72^\circ}$

1-11



$\bar{C} = \bar{A} + \bar{B}$

$\bar{C} \cdot \bar{C} = A^2 + B^2 + 2AB\cos\alpha = C^2$

but $\alpha = 180^\circ - \theta$

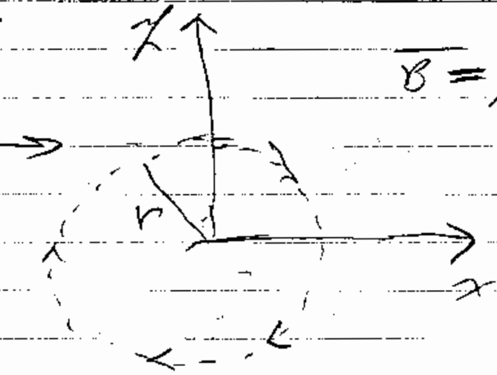
$\cos(180^\circ - \theta) = -\cos\theta$

∴ $\boxed{C^2 = A^2 + B^2 - 2AB\cos\theta}$ ←

EE 333 Homework 2

1-25

$\bar{v} = 10^5 \bar{a}_x$



$\bar{B} = 10^{-4} a_z$

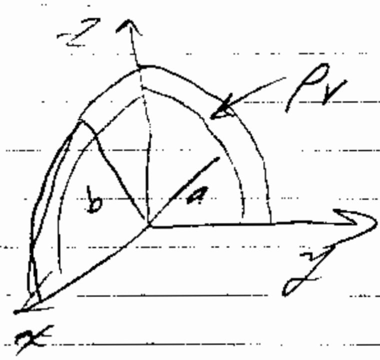
$\left[F_{initial} = -q\bar{v} \times \bar{B} = 10q\, \bar{a}_y \right]$

force is always normal to $\bar{v}$ and constant so path is a circle ($|\bar{v}|$ = constant)

$$r = ? = \frac{mv^2}{F} = \frac{9.1\times10^{-31} \times 10^{10}}{1.6\times10^{-19}\times10} = \boxed{5.688\times10^{-3}\ m}$$

for no deflection $\bar{E} = -\bar{v}\times\bar{B} = -10^5 \bar{a}_x \times 10^{-4} a_z = \boxed{10\,\bar{a}_y}$

1-28



for $r < a$ charge enclosed $= 0$ and for any spherical charge distribution $\bar{E} = E(r)\,\bar{a}_r$

$\therefore\ 4\pi r^2 \epsilon_0 E_r = 0$ for $r < a$

$\boxed{\text{so } E_r = 0 \text{ for } r < a}$

for $a < r < b$ we have: $4\pi r^2 \epsilon_0 E_r = \int_{r=a}^{r}\int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \rho_v r^2 \sin\theta\, d\phi\, d\theta\, dr$

or $\cancel{4\pi} r^2 \epsilon_0 E_r = \cancel{4\pi}\rho_v \left\{\frac{r^3}{3} - \frac{a^3}{3}\right\}$

and $\boxed{E_r = \frac{\rho_v}{\epsilon_0}\left\{\frac{r}{3} - \frac{a^3}{3r^2}\right\}}$

for $r > b$ $\cancel{4\pi} r^2 \epsilon_0 E_r = \cancel{4\pi}\rho_v \left\{\frac{b^3}{3} - \frac{a^3}{3}\right\}$

or $\boxed{E_r = \frac{\rho_v}{3\epsilon_0 r^2}\left\{b^3 - a^3\right\}}$

EE 333 Homework 3

1-29 for $r<a$ $\quad \rho_v = \rho_0(1+kr)$

$$q_{total} = \text{total charge} = \int_{r=0}^{a}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0(1+kr)\, r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$q_{total} = \frac{4}{3}\pi a^3 \rho_0 + k\rho_0 \frac{a^4}{4} 4\pi = 0 \quad \text{when } \frac{1}{3} = -k\frac{a}{4} \text{ or } \boxed{k = -\frac{4}{3a}}$$

for $r>a$ in this case Gauss's Law gives:

$$\boxed{\epsilon_0 E_r 4\pi r^2 = 0 \quad \therefore \quad E_r = 0}$$

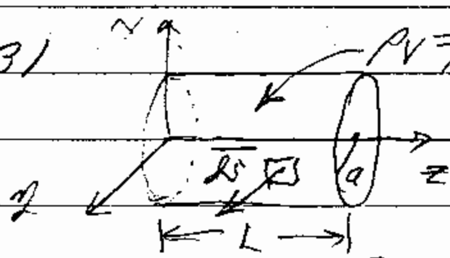
for $r<a$

$$\epsilon_0 E_r 4\pi r^2 = \int_{r=0}^{r}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0\left(1-\frac{4r}{3a}\right) r^2 \sin\theta \, d\theta \, d\phi \, dr$$

$$= 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{4r^4}{4\cdot 3a}\right]$$

$$\text{so } E_r = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{r^2}{3a}\right]$$

1-31 $\quad \rho_v = \rho_0 \left(\frac{\rho}{a}\right)^2$



$$Q = \int_{z=0}^{L}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \rho_0 \left(\frac{\rho}{a}\right)^2 d\rho \, \rho \, d\phi \, dz$$

$$Q = L\, 2\pi \frac{\rho_0}{a^2}\cdot\frac{a^4}{4} = \boxed{\rho_0 L \frac{\pi a^2}{2}}$$

Gauss's Law $\rho > a$

$$\int_{\phi=0}^{2\pi}\int_{z=0}^{L} \epsilon_0 E_\rho \, dz \, \rho \, d\phi = Q = \epsilon_0 E_\rho L \rho 2\pi$$

$$\therefore \quad E_\rho = \frac{Q}{2\pi\epsilon_0\rho L} = \frac{\rho_0 \pi a^2 L}{4\pi\epsilon_0\rho L} = \boxed{\frac{\rho_0 a^2}{4\epsilon_0\rho}}$$

$$\text{or } \boxed{E_\rho = \frac{\rho_l}{2\pi\epsilon_0\rho}}$$

for $\rho < a$

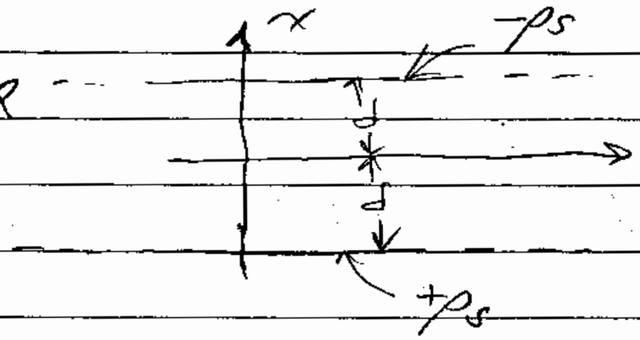
$$\int_{\phi=0}^{2\pi}\int_{z=0}^{L} \epsilon_0 E_\rho \, dz \, \rho \, d\phi = \int_{z=0}^{L}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \rho_0\left(\frac{\rho}{a}\right)^2 d\rho \, \rho \, d\phi \, dz$$

$$\text{or } \epsilon_0 E_\rho L \rho 2\pi = L\rho_0 2\pi \frac{\rho^4}{4a^2}$$

$$\therefore \quad \boxed{E_\rho = \frac{\rho_0 \rho^3}{4\epsilon_0 a^2}}$$

EE 333 Homework 4 (page 1)

1-32



a) between the charged sheets both sheets produce a constant E_x in the positive x direction

$$\therefore \quad -d < x < d \quad \boxed{E_x = \frac{\rho_s}{\epsilon_0}}$$

for $x > d$ or $x < -d$ fields cancel so $\boxed{E_x = 0}$

b) both sheets positively charged

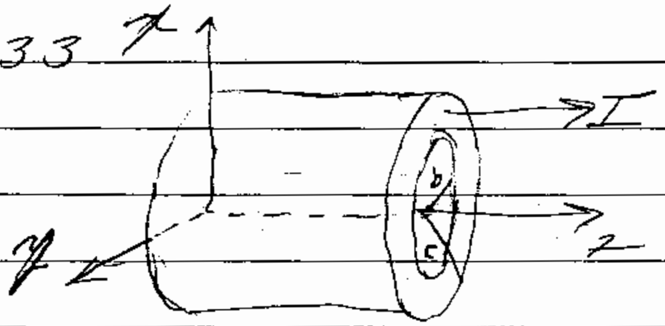
$$x > d \quad E_x = \frac{\rho_s}{\epsilon_0}$$

$$x < -d \quad E_x = -\frac{\rho_s}{\epsilon_0}$$

$$-d < x < d \quad E_x = 0 \, .$$

EE 333 Homework 4 (page 2)

1-33



a) $\bar{J} = \dfrac{I\,\bar{a}_z}{\pi c^2 - \pi b^2}$

(assuming uniform current density $J = \dfrac{I}{\text{cross section area}}$)

b) $\oint \frac{\bar{B}}{\mu_0} \cdot \overline{dl} = I$ for $\rho > c$

$\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_0} \rho\, d\phi = I$ or $\boxed{B_\phi = \dfrac{\mu_0 I}{2\pi\rho} \text{ for } \rho > c}$

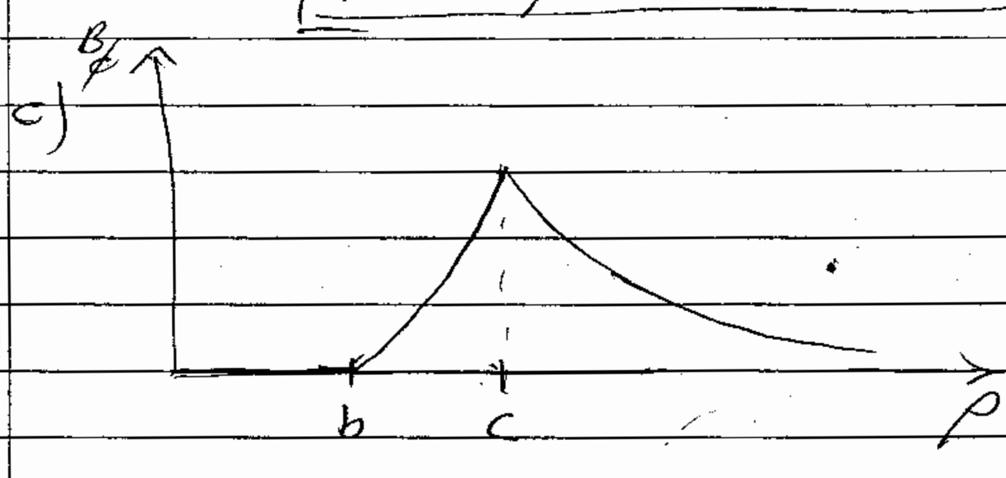
$\boxed{\text{for } \rho < b}$ $2\pi\rho \frac{B_\phi}{\mu_0} = 0$ $\boxed{\text{or } B_\phi = 0}$

for $a < \rho < b$ $2\pi\rho \frac{B_\phi}{\mu_0} = \int_{\rho=b}^{\rho} \int_{\phi=0}^{2\pi} \frac{I\,\bar{a}_z}{\pi c^2 - \pi b^2} \cdot \rho\, d\phi\, d\rho$

or $2\pi\rho \frac{B_\phi}{\mu_0} = \frac{2\pi I}{\pi(c^2-b^2)} \cdot \frac{\rho^2}{2}\Big|_b^\rho = I \frac{\rho^2 - b^2}{c^2-b^2}$

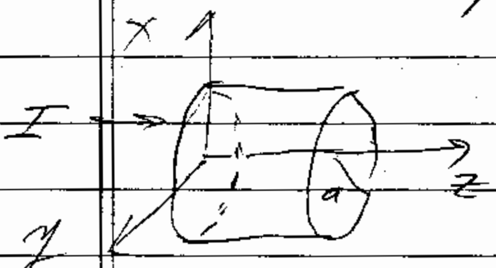
so $\boxed{B_\phi = \dfrac{\mu_0 I}{2\pi\rho}\left[\dfrac{\rho^2-b^2}{c^2-b^2}\right] \quad b < \rho < c}$

c)



EE 333 Homework 4 (page 3)

1-35 from example 1-15



$$B_\phi = \frac{\mu_0 I}{2\pi\rho} \quad \text{for } \rho > a$$

$$B_\phi = \frac{\mu_0 I \rho}{2\pi a^2} \quad \text{for } \rho < a$$

Using our solution to problem 1-33 and superposition we have

$$\boxed{B_\phi = \frac{\mu_0 I \rho}{2\pi a^2} \quad \text{for } \rho < a}$$

$$\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho} \quad \text{for } a < \rho < b}$$

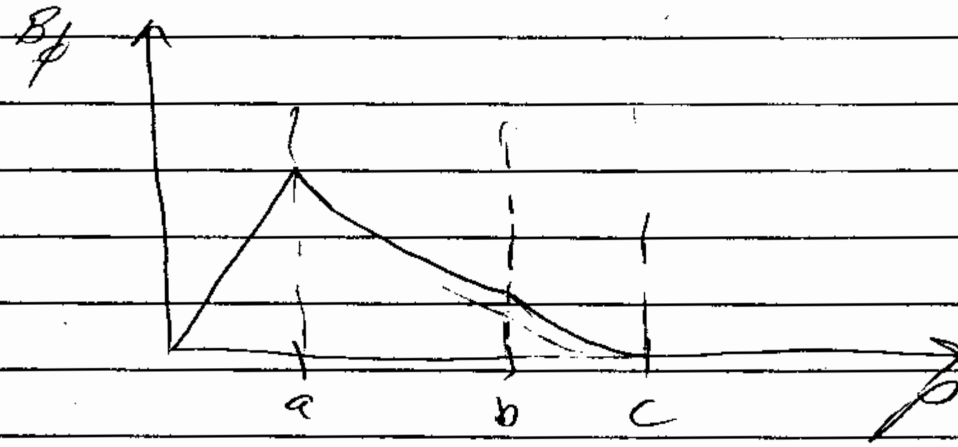
$$B_\phi = \frac{\mu_0 I}{2\pi\rho} - \frac{\mu_0 I}{2\pi\rho}\left[\frac{\rho^2 - b^2}{c^2 - b^2}\right] = \frac{\mu_0 I}{2\pi\rho}\left[\frac{c^2 - b^2 - \rho^2 + b^2}{c^2 - b^2}\right]$$

$$\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}\left[\frac{c^2 - \rho^2}{c^2 - b^2}\right] \quad b < \rho < c}$$

Same a. answer f. 1-34

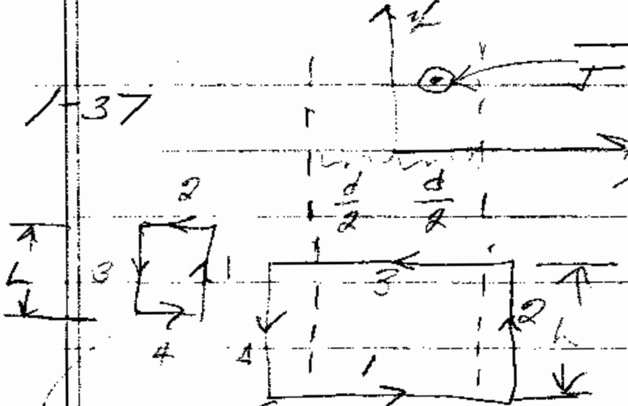
$$\boxed{B_\phi = 0 \quad \text{for } \rho > c}$$

(no current enclosed)



EE 333 Homework 5

1-37

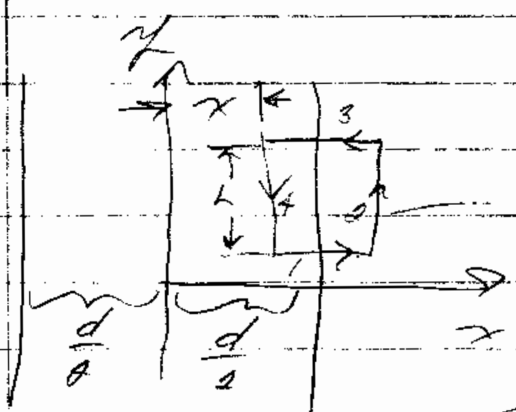


From direct current problem we know that $\bar{B}$ is only in the $\pm y$ direction

for this contour $\frac{B_{y1}}{\mu_0} L - \frac{B_{y3}}{\mu_0} L = 0$ $\therefore$ $\left\{ B_y = \text{constant} \text{ outside of slab} \right.$

$\frac{B_{y2}}{\mu_0} \cancel{L} + \frac{B_{y2}}{\mu_0} \cancel{L} = J_z \cancel{L} d$

$\therefore \left\{ \begin{array}{l} B_y = \frac{\mu_0 J_z d}{2} \text{ for } x > d \\ B_y = \frac{-\mu_0 J_z d}{2} \text{ for } x < d \end{array} \right.$



$\frac{\cancel{\mu_0} J_z d}{\cancel{\mu_0} 2} \cancel{L} - \frac{B_{y4}}{\mu_0} \cancel{L} = J_z L\left(\frac{d}{2} - x\right)$

so $\boxed{B_{y4}} = \frac{\mu_0 J_z d}{2} - \mu_0 J_z \left(\frac{d}{2} - x\right) = \boxed{\mu_0 J_z x \quad |x| < \frac{d}{2}}$

2-17 $\bar{E} = \frac{\rho_0}{\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{3a}\right)\bar{a}_r$

$\nabla \cdot \bar{E} = \frac{1}{r^2}\frac{d}{dr}\left[\frac{\rho_0}{\epsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{3a}\right)\right] = \frac{\rho_0}{r^2 \epsilon_0}\left[r^2 - \frac{4r^3}{3a}\right]$

$\boxed{\text{or } \nabla \cdot \bar{E} = \frac{\rho_v}{\epsilon_0}}$

2-41 $\hat{E}_x^+ = 1885\, e^{-j\beta_0 z}$ $f = 10^8$

a) amplitude = 1885 V/m

traveling in +z direction

vector direction $\bar{a}_x$ (x polarized)

b) $E_x = 1885 \cos(\omega t - \beta_0 z) = 1885 \cos(2\pi \times 10^8 t - \beta_0 z)$

where $\beta_0 = \frac{\omega}{c} = \frac{2\pi \times 10^8}{3 \times 10^8} = \boxed{2.094}$

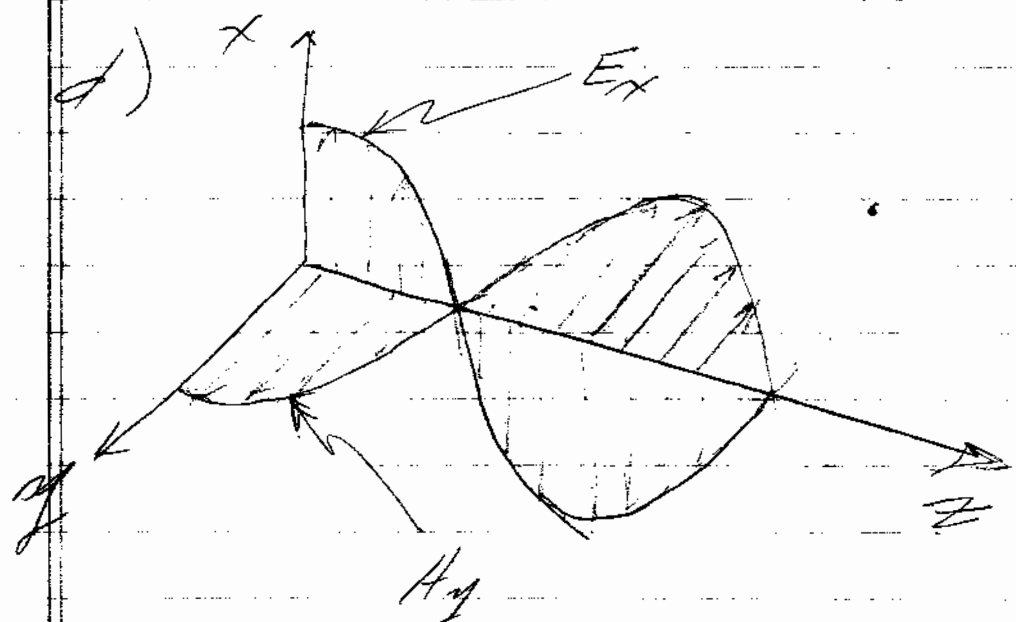
$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{1 \times 10^8} = \boxed{3\ m}$

c) $\hat{B}_y^+ = \frac{1885}{c} e^{-j\beta_0 z} = \boxed{2\pi \times 10^{-6} e^{-j\beta_0 z}}$

$\hat{H}_y^+ = \frac{1885}{377} e^{-j\beta_0 z} = \boxed{5 e^{-j\beta_0 z}}$

so $\boxed{H_y^+ = 5 \cos(2\pi \times 10^8 t - \beta_0 z)}$

d)

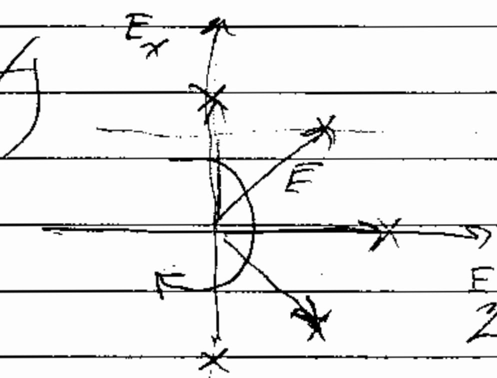


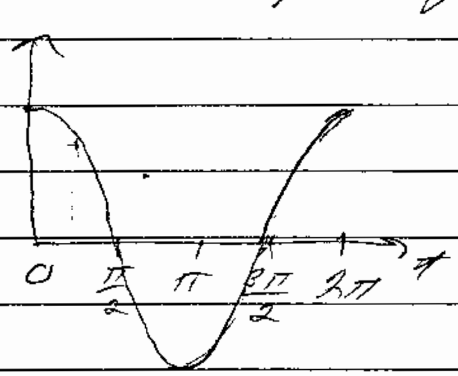
EE 333 Homework 6

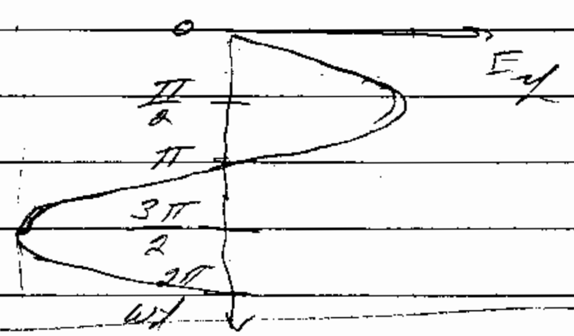
2-46 $\hat{\bar{E}}(z) = 500\, e^{-j\beta_0 z}(\bar{a}_x - j\bar{a}_y)$

$\bar{E} = 500\cos(\omega t - \beta_0 z)\bar{a}_x + 500\sin(\omega t - \beta_0 z)\bar{a}_y$

(plot at $z=0$)







right hand or clockwise circular polarization

$$\boxed{\hat{H}_z = \frac{500}{\eta_0} e^{-j\beta_0 z}\bar{a}_y + j\frac{500}{\eta_0} e^{-j\beta_0 z}\bar{a}_x}$$

3.1 $n = 10^{29}$; $\sigma = 5.8\times10^{7}$ mho/m

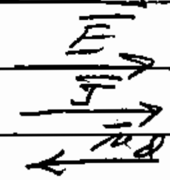
a) $\mu_e = \frac{e\tau_c}{m}$; $\sigma = \frac{ne^2}{m}\tau_c$.

$\therefore \mu_e = \sigma\left(\frac{1}{ne}\right) = \frac{5.8\times10^{7}}{10^{29}\times1.6\times10^{-19}} = \boxed{3.625\times10^{-3}}$

b) charge density $= -10^{29}\frac{\text{elec}}{\text{m}^3}\times\frac{1.6\times10^{-19}\,\text{C}}{\text{elec}}\times\frac{1\,\text{m}^3}{10^9\,\text{mm}^3} = \boxed{-16\,\frac{\text{C}}{\text{mm}^3}}$

c) $E = 1a_x$; $v_d = -\mu_e E = \boxed{-3.625\times10^{-3}\ \text{m/sec}}$

$\bar{J} = \sigma\bar{E} = 5.8\times10^{7}\bar{a}_x\ \text{A/m}^2$



Homework 6 (alternate solution)

2-46 $\hat{\bar{E}}_z = 500 e^{-j\beta_0 z}(\bar{a}_x - j\bar{a}_y)$

$\nabla \times \hat{\bar{E}} = -j\omega \hat{\bar{B}}$

$$\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ \hat{E}_x & \hat{E}_y & \hat{E}_z \end{vmatrix} = -j\omega(\hat{B}_x\bar{a}_x + \hat{B}_y\bar{a}_y + \hat{B}_z\bar{a}_z)$$

or $\frac{\partial \hat{E}_y}{\partial z} = j\omega \hat{B}_x$

$\frac{\partial \hat{E}_x}{\partial z} = -j\omega \hat{B}_y$

$\hat{B}_z = 0$

$\therefore$ for above $\hat{\bar{E}}$ field we have:

$\hat{B}_x = \frac{1}{j\omega}\frac{\partial \hat{E}_y}{\partial z} = \frac{1}{j\omega}(-j)(-j\beta_0)500 e^{-j\beta_0 z}$

$\hat{B}_x = j\frac{\omega\sqrt{\mu_0\varepsilon_0}}{\omega} 500 e^{-j\beta_0 z}$

or $\hat{H}_x = j\frac{500}{\eta_0} e^{-j\beta_0 z}$.

similarly $\hat{B}_y = \frac{-1}{j\omega} 500(-j\beta_0) e^{-j\beta_0 z}$

$B_y = \frac{500}{c} e^{-j\beta_0 z}$

and $\hat{H}_y = \frac{500}{\eta_0} e^{-j\beta_0 z}$

EE 333 Homework 7

3.3 $\boxed{He}$ 10^{25} atoms/m^3 ; $\chi_e = 1.5\times10^{-4}$; $E = 10^3$ V/m

a) $\bar{P} = \epsilon_0 \chi_e \bar{E} = \frac{10^{-9}}{36\pi}\times 1.5\times10^{-4}\times10^{3} = \boxed{1.33\times10^{-12}}$

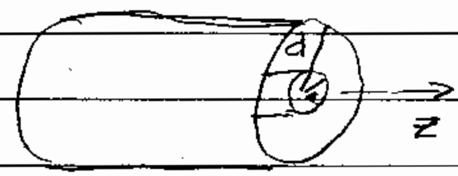
2 electrons/atom

b) $\rho_v^+ = 2\times10^{25}\times 1.6\times10^{-19} = \boxed{3.2\times10^{6}\ C/m^3}$

$\bar{P} = \rho_v^+ \, \bar{dl}_{ave}$ ∴ $dl_{ave} = \frac{1.33\times10^{-12}}{3.2\times10^{6}} = \boxed{0.416\times10^{-18}\ m}$

c) $\epsilon_r = 1+\chi_e = \boxed{1.00015}$

3-12



$\oint \bar{D}\cdot d\bar{s} = \rho_l L$ charge/m

$$\int_{z=0}^{L}\int_{\phi=0}^{2\pi} D_\rho\, \rho\, d\phi\, dz = \rho_l L$$

or $2\pi\rho L D_\rho = \rho_l L$ ∴ $D_\rho = \frac{\rho_l}{2\pi\rho}$; $\boxed{E_\rho = \frac{\rho_l}{2\pi\epsilon\rho}}$

a) E_r is constant if $\boxed{\epsilon = \frac{K}{\rho}}$

want $\epsilon = \epsilon_1$ @ $\rho = d$ ∴ $\boxed{K = d\epsilon_1}$

and $\boxed{E_\rho = \frac{\rho_l}{2\pi\epsilon_1 d}}$

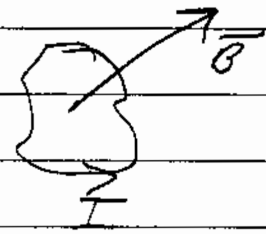
b) $D_\rho = \epsilon_0 E_\rho + P_\rho$ so $P_\rho = D_\rho - \epsilon_0 E_\rho = \frac{\rho_l}{2\pi\rho} - \frac{\epsilon_0\rho_l}{2\pi\epsilon_1 d}$

$$\boxed{P_\rho = \frac{\rho_l}{2\pi}\left(\frac{1}{\rho} - \frac{\epsilon_0}{\epsilon_1 d}\right)}$$

$$\rho_p = -\nabla\cdot\bar{P} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho_l\epsilon_0\rho}{2\pi\epsilon_1 d}\right) = \boxed{\frac{\rho_l\epsilon_0}{2\pi\epsilon_1 d\rho}}$$

c) Use layers that approximate $\frac{\epsilon_1 d}{\rho}$ variation

EE 333 Homework 7 (solutions - page 2)

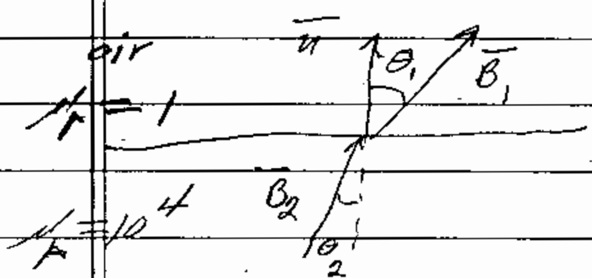
3.16 

$\bar{F} = \oint dq\,\bar{u} \times \bar{B}$

$F = \oint I\,\overline{d\ell} \times \bar{B} = -I \oint \bar{B} \times \overline{d\ell}$

$\bar{F} = -I\,\bar{B} \times \oint \overline{d\ell} \equiv 0$

3.25 $H_{t_1} = H_{t_2}$ $B_{n_1} = B_{n_2}$

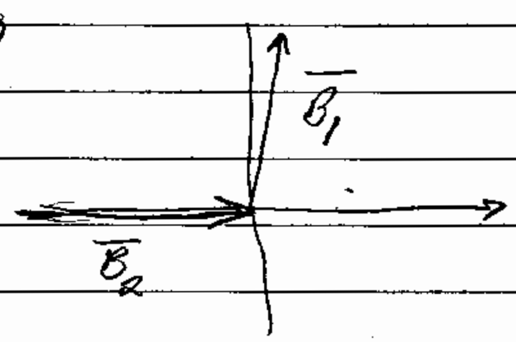


$\tan\theta_2 = \frac{\mu_2}{\mu_1}\tan\theta_1$ (3.76)

or $\theta_1 = \tan^{-1}\left[10^{-4}\tan\theta_2\right]$

θ_2	θ_1
0	0
45°	5.7×10^{-3} °
89°	0.328°
89.9°	3.27°
89.9675°	10° ✓

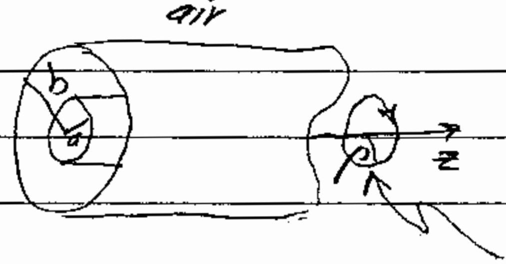
①



②

EE 333 Homework 8

3-28



I carried in conductor of radius "a" ($\mu=\mu_0$)
for $a<\rho<b$ $\mu=\mu$

$\oint \bar{H}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{s}$ Integrate around circular contour

a) for $\rho<a$ $H_\phi 2\pi\rho = \frac{I}{\pi a^2}\pi\rho^2$ or $H_\phi = \frac{I\rho}{2\pi a^2}$

$B_\phi = \frac{\mu_0 I \rho}{2\pi a^2}$

for $a<\rho<b$

$H_\phi = \frac{I}{2\pi\rho}$

$B_\phi = \frac{\mu I}{2\pi\rho}$

for $\rho>b$

$H_\phi = \frac{I}{2\pi\rho}$

$B_\phi = \frac{\mu_0 I}{2\pi\rho}$

b) $\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \bar{a}_\phi\left[\frac{\mu_r I}{2\pi\rho} - \frac{I}{2\pi\rho}\right] = \frac{I}{2\pi\rho}\left[\mu_r - 1\right]\bar{a}_\phi$

H_ϕ is continuous across all boundaries ($H_{tangential}$

B_ϕ is discontinuous

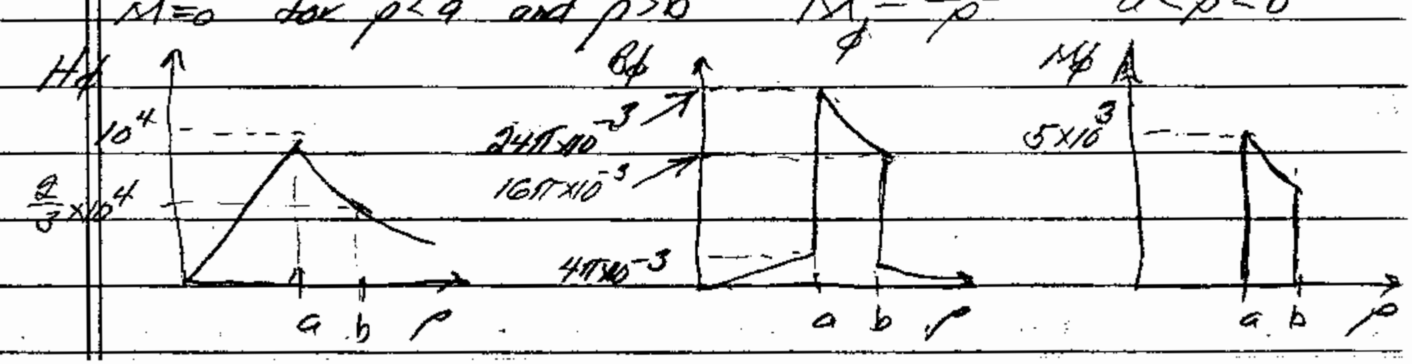
$B_\phi = 24\pi\times10^{-3}$ ($\rho=a^+$)

@ $\rho=a$ $H_\phi = \frac{628}{2\pi\times10^{-2}} = 10^4$; $B_\phi = 4\pi\times10^{-3}$ ($\rho=a^-$)

$\rho=b$ $H_\phi = \frac{628}{2\pi\times1.5\times10^{-2}} = \frac{2}{3}\times10^4$; $B_\phi = 4\pi\times10^{-7}\times6\times\frac{2}{3}\times10^4 = 16\pi\times10^{-3}$ ($\rho=b^-$)

$B_\phi = \frac{8}{3}\pi\times10^{-3}$ ($\rho=b^+$)

$M=0$ for $\rho<a$ and $\rho>b$ $M_\phi = \frac{50}{\rho}$ $a<\rho<b$

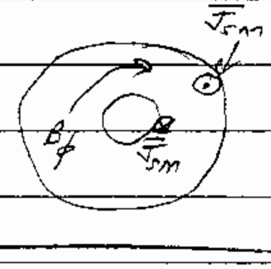


3.28 continued: $\bar{J}_m = \nabla \times \bar{M}$

$$\therefore \bar{J}_m = \bar{a}_\rho\left(-\frac{\partial M_\phi}{\partial z}\right) + \bar{a}_z\left(\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho M_\phi)\right) = 0$$

$$\bar{J}_{sM}(\rho=a) = \bar{a}_\rho \times \bar{M} = 5\times 10^{3}\,\bar{a}_z$$

$$\bar{J}_{sM}(\rho=b) = -\bar{a}_\rho \times \bar{M} = -\frac{10}{3}\times 10^{3}\,\bar{a}_z$$



3.29

Region 1 $\epsilon = \epsilon_0$ $\quad \bar{n} = \bar{a}_z$

Region 2 $\epsilon = 4\epsilon_0$

$$\bar{E}_1 = -15\bar{a}_x + 20\bar{a}_y + 30\bar{a}_z$$

$$\left\{\begin{array}{l} E_{tang_1} = E_{tang_2} \\ D_{norm_1} = D_{norm_2} \end{array}\right\}$$

a) $\boxed{\bar{E}_2 = -15\bar{a}_x + 20\bar{a}_y + \frac{15}{2}\bar{a}_z} \longleftrightarrow \left\{\begin{array}{l} D_{z_1} = 30\epsilon_0 = D_{z_2} \\ E_{z_2} = \frac{D_{z_2}}{4\epsilon_0} = \frac{30}{4} \end{array}\right.$

$$\bar{D}_1 = \epsilon_0 \bar{E}_1$$

$$\bar{D}_2 = 4\epsilon_0 \bar{E}_2$$

b) $\bar{n}\cdot\bar{E}_1 = |\bar{E}_1|\cos\theta_1 = 30 \quad \therefore \quad \theta_1 = \cos^{-1}\left\{\frac{30}{\sqrt{15^2+20^2+30^2}}\right\} = 39.8$

$$\theta_2 = \cos^{-1}\left\{\frac{\frac{15}{2}}{\sqrt{15^2+20^2+7.5^2}}\right\} = \boxed{73.3^\circ}$$

ck: $\tan\theta_2 = \frac{\epsilon_2}{\epsilon_1}\tan\theta_1$;

$$3.333 = 4\times 0.833$$

$$3.333 = 3.333$$

3-38 $\hat{E}_m^+ = 1$ @ $z = 0$ $\left\{ \begin{matrix} \text{sea water} \\ \epsilon_r = 81, \; \sigma = 4 \end{matrix} \right\}$

$\boxed{f = 10^4}$

$$\frac{\sigma}{\omega\epsilon} = \frac{2 \cdot 4 \times 36\pi}{8\pi \times 10^4 \times 10^{-9}} = 72 \times 10^5 >> 1$$ ∴ good conductor

so $\alpha = \beta = \sqrt{\dfrac{\omega\sigma\mu}{2}} = \sqrt{\dfrac{2\pi \times 10^4 \times 4 \times 4\pi \times 10^{-7}}{2}} = 0.397$

for $e^{-\alpha z} = 0.05$ (5%) ; $z = -\dfrac{\ln 0.05}{\alpha} \boxed{= 7.5\text{ m}}$

if $f = 10^3$; $\dfrac{\sigma}{\omega\epsilon} = 7.2 \times 10^7$ (good conductor)

$$\alpha = \beta \doteq 4\pi \times 10^{-2} \boxed{= 0.1257}$$

$$z_{5\%} = -\frac{\ln 0.05}{0.1257} \boxed{= 23.8\text{ m}}$$

Thus demonstrating the rf communication in sea water is only possible at very low frequencies.

EE 333 Homework 9

3-37 $\hat{E}_m^+ = 200\ V/m$; $f = 10^8\ Hz$, $\mu = \mu_0$

$\epsilon = 6\epsilon_0$, $\sigma = 10^{-2}\ \mho/m$

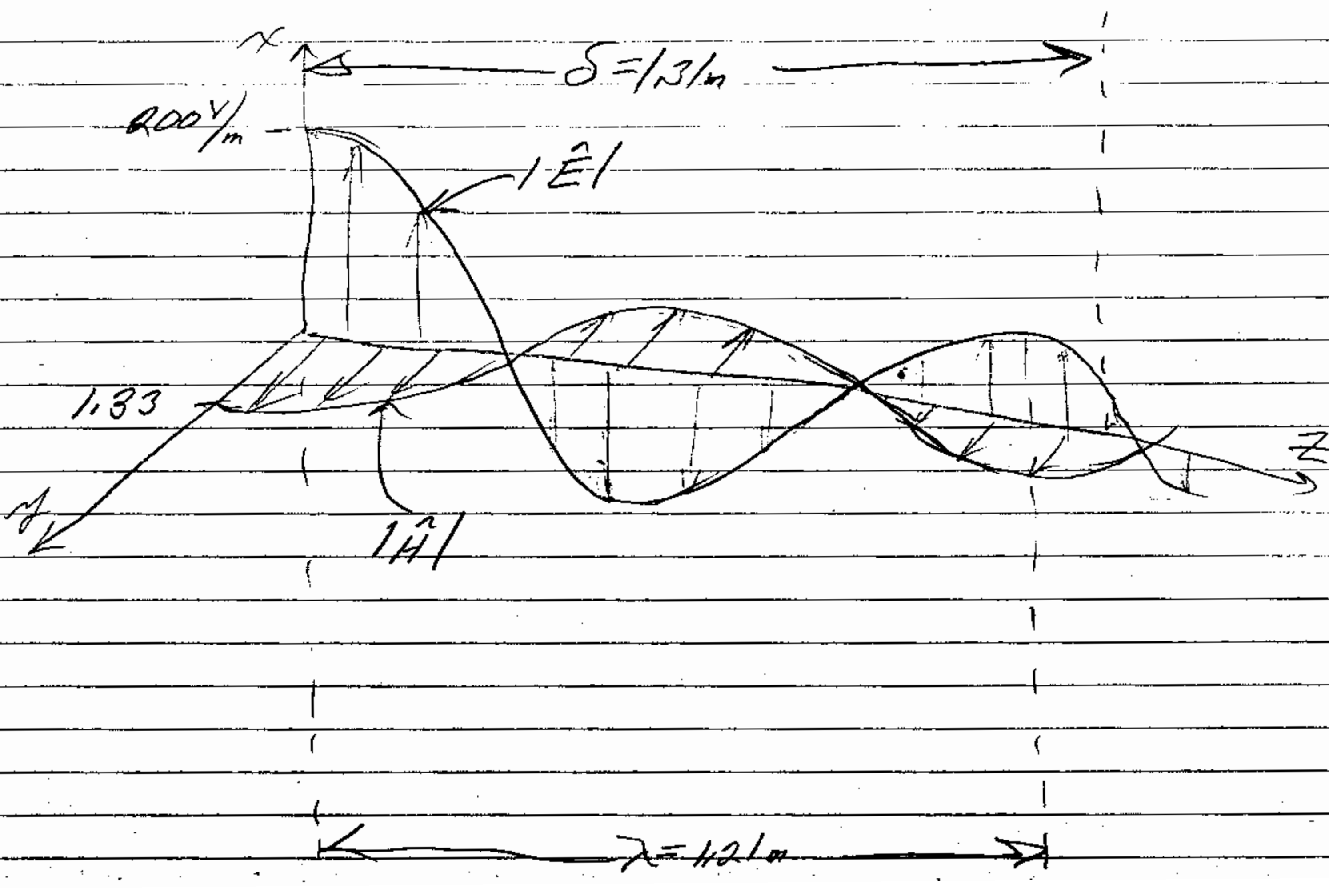
$\gamma = 0.761 + j5.187$

a) $\boxed{\lambda = \frac{2\pi}{\beta} = 1.21\,m}$ $\boxed{\delta = \frac{1}{\alpha} = 1.31\,m}$

$v_{ph} = \frac{\omega}{\beta} = \frac{2\pi \times 10^8}{5.187} = \boxed{1.21 \times 10^8\ m/sec}$

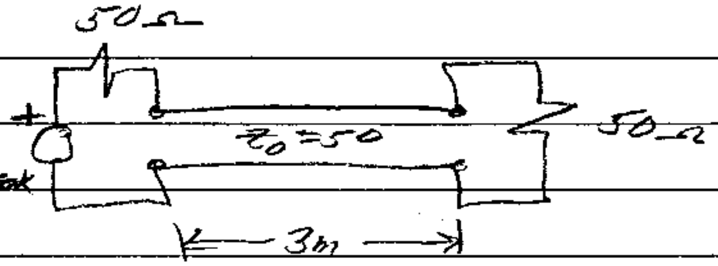
b) $\hat{\eta} = \sqrt{\frac{\mu}{\epsilon - j\frac{\sigma}{\omega}}} = \sqrt{\frac{4\pi \times 10^{-7}}{\frac{6 \times 10^{-9}}{36\pi} - j\frac{10^{-2}}{2\pi \times 10^8}}} = \boxed{1.506 \times 10^2\, e^{j8.34°}}$

$\therefore \hat{H}_y^+ = \frac{200}{\hat{\eta}} = \boxed{1.33\, e^{-j8.34°}}$



EE 333 Homework 9 (page 2)

10.3



$f = 50\,MHz$

$\epsilon_r = 2.25$

$v_{ph} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8\,m/sec$; $\lambda = \frac{2\times10^8}{5\times10^7} = 4\,m$

so line length $= \frac{3}{4}\lambda$

a) $\boxed{\Gamma(\ell) = \frac{Z_L - Z_0}{Z_L + Z_0} = 0}$ $\therefore$ $\Gamma(z)$ is zero at all points on the line

$\boxed{Z(z)} = Z_0 \frac{1+\Gamma(z)}{1-\Gamma(z)} = \boxed{Z_0}$ everywhere

b) $\hat{V}(z) = \hat{V}_m^+ e^{-j\beta z}$; $\hat{I}(z) = \frac{\hat{V}_m^+}{Z_0} e^{-j\beta z}$

these become $\hat{V}_m^+$ and $\hat{V}_m^+/Z_0$ at $z=0$

c) $\hat{I}_{in} = \hat{I}_m^+ = \frac{\hat{V}_m^+}{Z_0} = \frac{200}{100} = \boxed{2A}$; $\hat{V}_{in} = \hat{V}_m^+ = \boxed{100}$

$\therefore$ $\left.\begin{aligned}\hat{V}(z) &= 100 e^{-j\beta z} = 100 e^{-j\frac{2\pi}{\lambda}z} \\ \hat{I}(z) &= 2 e^{-j\beta z} = 2 e^{-j\frac{2\pi}{\lambda}z}\end{aligned}\right\}$

$\boxed{P_{in_{ave}} = \frac{1}{2}\, 100\times2 = 100\text{ Watts}}$

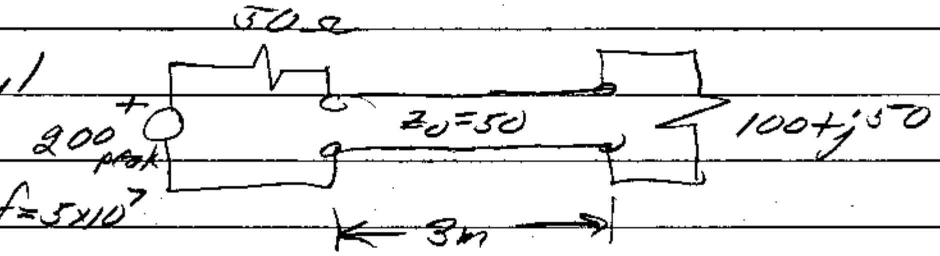
$\left.\begin{aligned}\hat{V}_{load} &= 100 e^{-j\frac{2\pi}{\lambda}\left(\frac{3\lambda}{4}\right)} = 100 e^{-j\frac{3\pi}{2}} = j100 \\ \text{so } \hat{I}_{load} &= j2\end{aligned}\right\}$

and $P_{load} = \frac{1}{2}\mathrm{Re}\{j100(-j2)\} = 100\text{ Watts}$

or $P_{load} = \frac{1}{2}|\hat{I}|^2 50 = \frac{1}{2}\frac{|\hat{V}|^2}{50} = 100W$

EE 333 Homework 10

10.1



$\boxed{\epsilon_r = 2.25}$

$j\beta = j\omega\sqrt{\mu\epsilon} = j\omega\frac{\sqrt{2.25}}{3\times10^8} = j2\pi\times5\times10^7\frac{1.5}{3\times10^8} = \boxed{j\frac{1}{2}\pi}$

$\boxed{v_p = \frac{3\times10^8}{\sqrt{\epsilon_r}} = 2\times10^8\, m/sec}$; $\boxed{\lambda = \frac{2\pi}{\beta} = 4\,m}$ $\boxed{\text{line length} = \frac{3}{4}\lambda}$

$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50+j50}{150+j50} = 0.4+j0.2 = \boxed{0.4472\, e^{j26.565^\circ}}$

$\Gamma_{in} = \Gamma_{load}\, e^{-j2\beta\left(\frac{3\lambda}{4}\right)} = (0.4+j0.2)e^{-j3\pi} = -(0.4+j0.2) = \boxed{-0.4472\, e^{j26.56^\circ}}$

∴ approximately 45% voltage reflection

$Z_{in} = 50\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50\frac{0.6-j0.2}{1.4+0.2} = 20-j10$

$\hat{I}_{in} = \frac{200}{50+Z_{in}} = \frac{200}{70-j10} = 2.828\, e^{j8.13^\circ}$

$P_{in_{ave}} = \frac{1}{2}|\hat{I}_{in}|^2 R_{in} = \frac{1}{2}(2.828)^2(20) = \boxed{79.98\text{ Watts}}$

$\hat{I} = \hat{I}_m^+ e^{-j\beta z}[1-\Gamma(z)]$ ∴ $\hat{I}_m^+ = \frac{\hat{I}_{in}}{1-\Gamma(0)}$

so $\hat{I}_{load} = \hat{I}_m^+ e^{-j\frac{2\pi}{\lambda}\cdot\frac{3\lambda}{4}}[1-\Gamma_{load}] = \hat{I}_{in}\, e^{-j\frac{3\pi}{2}}\frac{1-\Gamma_{load}}{1-\Gamma_{in}}$

or $\hat{I}_{load} = 2.828\, e^{j8.13^\circ}\cdot j\,\frac{0.6-j0.2}{1.4+j0.2} = 1.2647\, e^{j71.56^\circ}$

∴ $P_{load} = \frac{1}{2}(1.2647)^2\times100 = 79.97$ Watts

EE 333 Homework 10 (page 2)

10.2 $P_{ave} = \frac{1}{2} Re\left\{ \hat{V}_m^+ e^{-j\beta z}[1+\Gamma] \frac{\hat{V}_m^{+*}}{Z_0} e^{+j\beta z}[1-\Gamma^*] \right\}$

$P_{ave} = \frac{1}{2} Re\left\{ \frac{|\hat{V}_m^+|^2}{Z_0} [1-\Gamma^* + \Gamma - |\Gamma|^2] \right\}$

$-\Gamma_r + j\Gamma_i + \Gamma_r + j\Gamma_i$

so $P_{ave} = \frac{|\hat{V}_m^+|^2}{2Z_0}[1-|\Gamma|^2]$

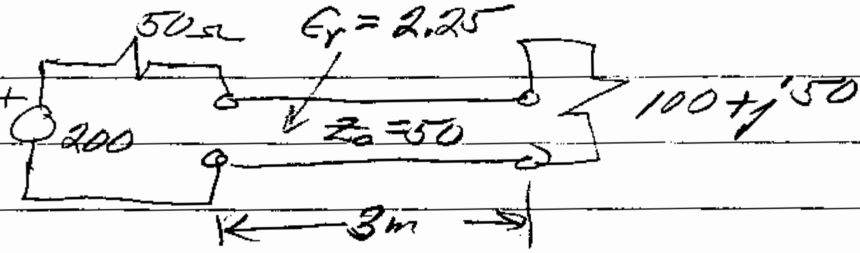
$\Gamma = \frac{\hat{V}_m^-}{\hat{V}_m^+} e^{-j2\beta z}$ so $|\Gamma| = \frac{|\hat{V}_m^-|}{|\hat{V}_m^+|}$

so $P_{ave} = \frac{|\hat{V}_m^+|^2}{2Z_0} - \frac{|\hat{V}_m^-|^2}{2Z_0} = P_{ave}^+ - P_{ave}^-$

EE 333 Homework 12 (page 1)

10-15

f = 50MHz



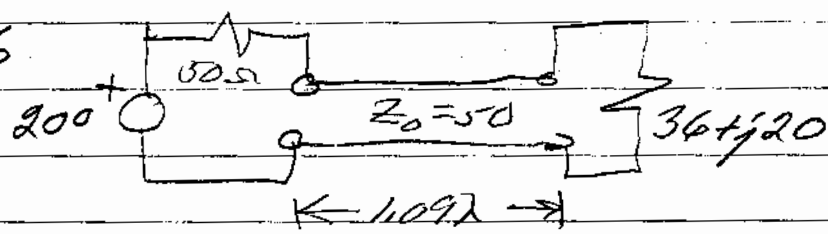
$$\lambda = \frac{3 \times 10^8}{5 \times 10^7 \times \sqrt{2.25}} = 4m \quad \therefore \text{ line length} = \frac{3}{4}\lambda$$

$Z_{L_{norm}} = 2 + j1$; from chart: $\Gamma_L = 0.46 e^{j26^\circ}$ ←

[chart scales $0.214 + 0.25 = 0.464\lambda$]

from chart: $\left\{ \begin{array}{l} \Gamma_{in} = 0.46 e^{-j154^\circ} \\ Z_{in_{norm}} = 0.4 - j0.195 \text{ or } Z_{in} = 20 - j9.75 \end{array} \right\}$ ←

10-16



$Z_{L_{norm}} = 0.72 + j0.4$

from chart: $\Gamma_L = 0.29 e^{j112^\circ}$

chart rotation $= 0.094 + 0.09 = 0.184\lambda$

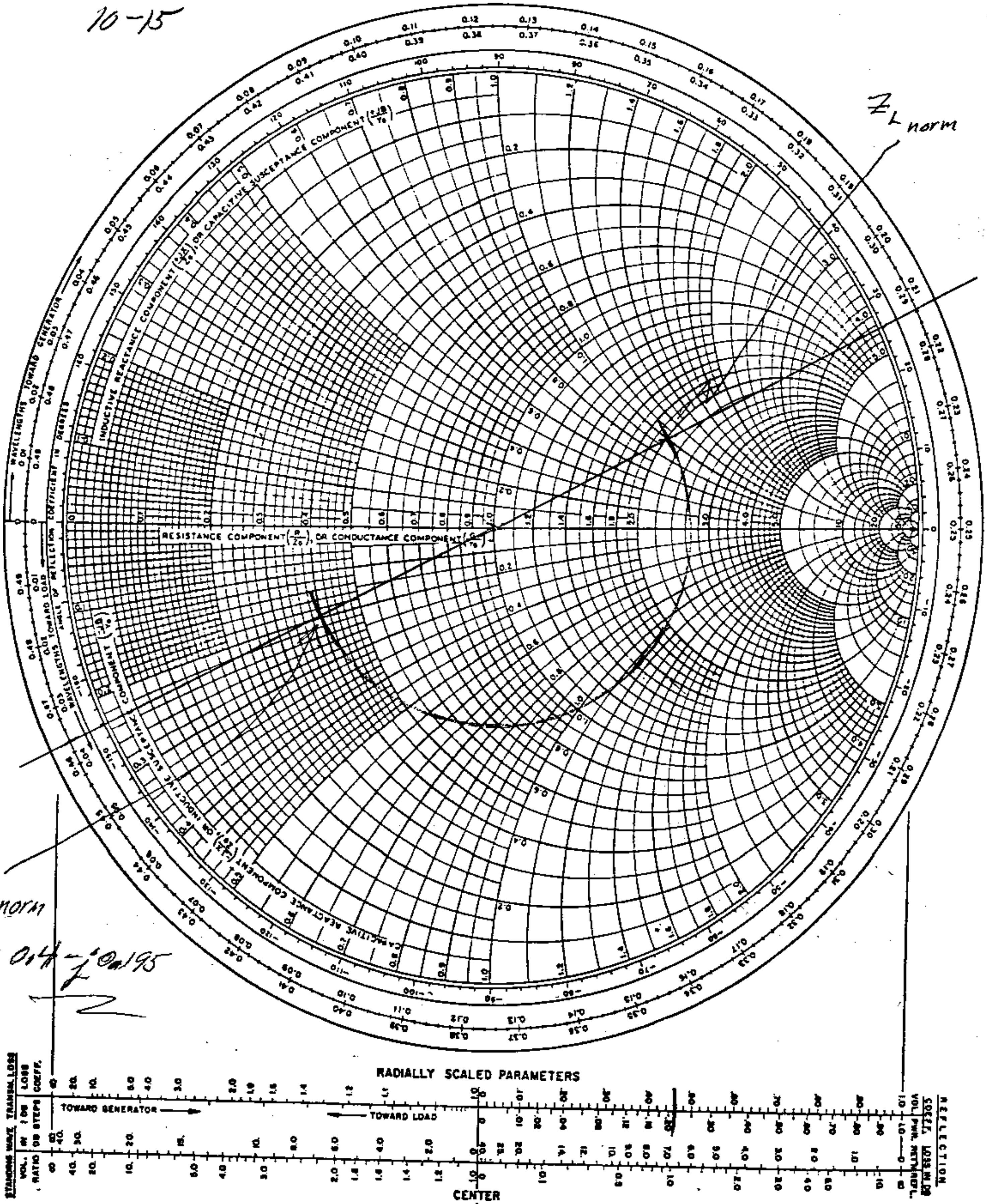
from chart: $\Gamma_{in} = 0.29 e^{j48^\circ}$ and $Z_{in_{norm}} = 1.32 + j0.58$

or $\boxed{Z_{in} = 66 + j29}$ ←

Problem 10-15

IMPEDANCE OR ADMITTANCE COORDINATES

Z_L norm



Z_{in} norm $= 0.4 - j0.195$

Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

10-16

IMPEDANCE OR ADMITTANCE COORDINATES

$Z_{in\,norm} = 1.32 + j0.58$

$Z_{in} = 66 + j29$

$Z_{L\,norm} = 0.72 + j0.4$

$\Gamma_{in} = 0.29$

$\Gamma_L = 0.29 \angle 112.8$

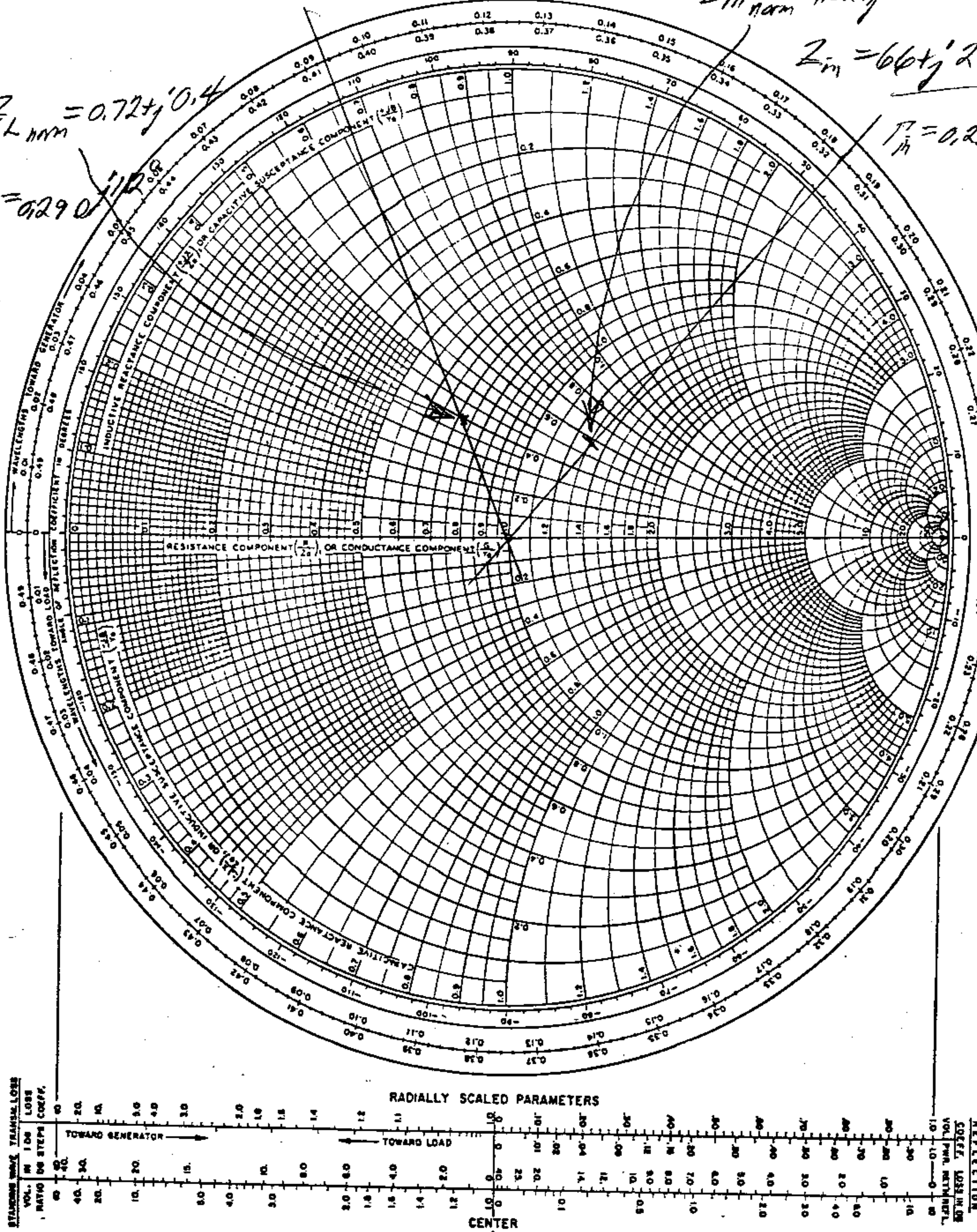


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 333 Homework 13

10-21 $Z_L = 36 + j20$; $Z_0 = 50$

$\therefore Z_{L_{norm}} = 0.72 + j0.4$

from chart (next page)

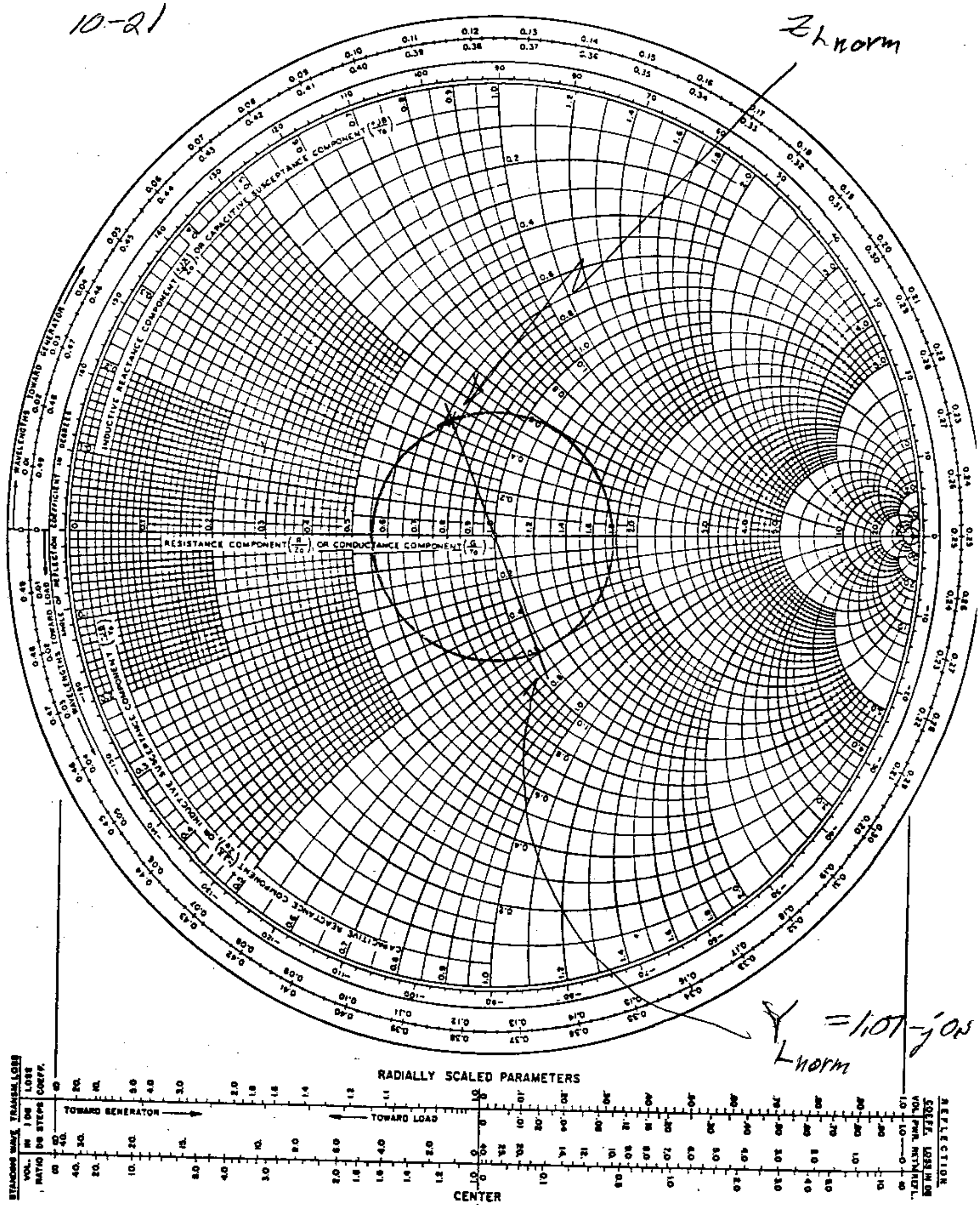
$$\boxed{Y_{L_{norm}} = 1.07 - j0.59}$$

$$\frac{1}{Z_{L_{norm}}} = \frac{1}{0.72 + j0.4} = \frac{1}{0.8237 e^{j29.05}} = 1.214 e^{-j29.05^\circ}$$

$$\boxed{\text{or } \frac{1}{Z_{L_{norm}}} = 1.06 - j0.59}$$

Problem 10-21

IMPEDANCE OR ADMITTANCE COORDINATES



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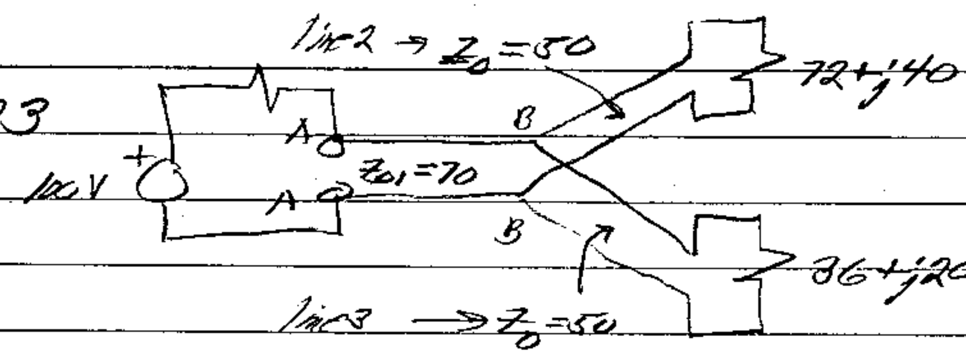
EE 333 Homework 13 (page 2)

10-23



from example 10-6 $P_{ave_{\text{into } A\text{-}A}} = 8.5$ Watts

$Y_{in_2 @ BB} = G_2 + jB_2$; $Y_{in_3 @ BB} = G_3 + jB_3$

$P_{ave} = \frac{1}{2}\mathrm{Re}\{\hat{V}\hat{I}^*\} = \frac{1}{2}\mathrm{Re}\{\hat{V}\hat{V}^* Y^*\} = \frac{1}{2}|\hat{V}|^2 G$

$\therefore \boxed{\dfrac{P_{ave_2}}{P_{ave_3}} = \dfrac{G_2}{G_3}}$

$\frac{1}{2}|\hat{V}_{BB}|^2 (G_2 + G_3) = 8.5 = \frac{1}{2}|\hat{V}|^2 G_2 \left(1 + \frac{G_3}{G_2}\right)$

$= P_{ave_2}\left(1 + \frac{G_3}{G_2}\right)$

but from example: $G_2 = 0.0366$ & $G_3 = 0.012$

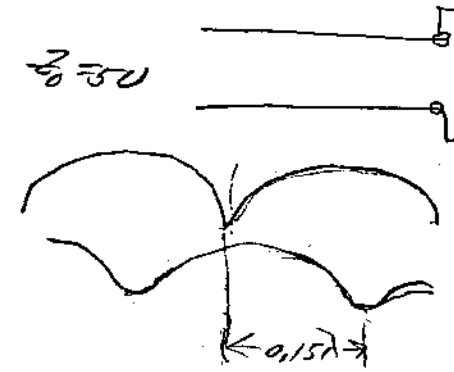
so $P_{ave_2} = \dfrac{8.5}{1 + \frac{0.012}{0.0366}} = \boxed{6.4 \text{ Watts}}$

$P_{ave_3} = 8.5 - 6.4 = \boxed{2.1 \text{ Watts}}$

lossless lines

EE333 Homework 13 (Solution)

10-27

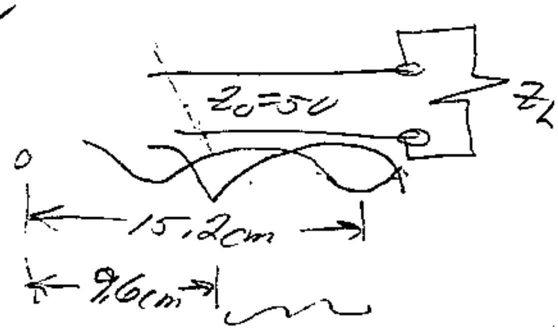


SWR = 4.0

from chart $Z_{L_{norm}} = 0.67 + j1.14$

$\therefore \boxed{Z_L = 33.5 + j57}$

10.28



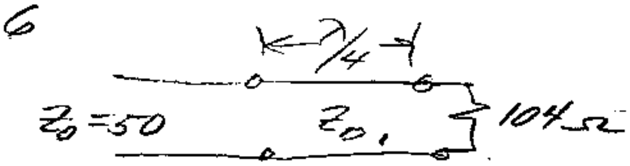
$f = 600\,MHz$; $SWR = 3.5$

$\lambda = \frac{3\times10^8}{6\times10^8} = 0.5\,m$

from chart $Z_{L_{norm}} = 0.475 + j0.73$

$\boxed{\text{or } Z_L = 23.75 + j36.5}$

10.36

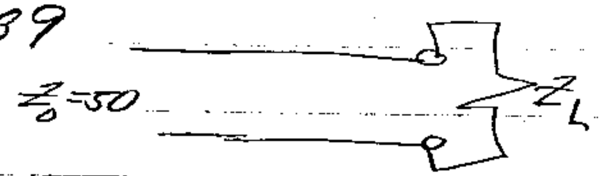


$f = 300\,MHz$, $\epsilon_r = 2.26$

$\boxed{Z_{01} = \sqrt{50\times104} = 72.1\,\Omega}$

$\lambda = \frac{3\times10^8}{\sqrt{2.26}\;5\times10^8} = \frac{0.6}{1.5} = 0.4\,m \quad \therefore \boxed{\lambda/4 = 0.1\,m}$

10.39



$f = 500\,MHz$; $\lambda = 60\,cm$

$SWR = 3.5$

(see chart)

place stub $(0.25-0.171)\lambda = 0.08\lambda = 4.8\,cm$ either direction from V_{min}

a) for stub toward the load $Y_{norm_{stub}} = -j1.3$; stub length $= 0.355-0.25 = 0.1$ or stub $\boxed{\text{length} = 6.3\,cm}$

SWR from generator to stub = 1

b) SWR from stub to load = 3.5

c) for stub toward the generator $Y_{norm_{stub}} = +j1.3$; stub length $= (0.145+0.25)\lambda = 23.7\,cm$

IMPEDANCE OR ADMITTANCE COORDINATES

$Z_{n\,@V_{min}} = 0.25$

$Z_{n\,load} = 0.67 + j1.14$

Problem 10.27

0.157λ

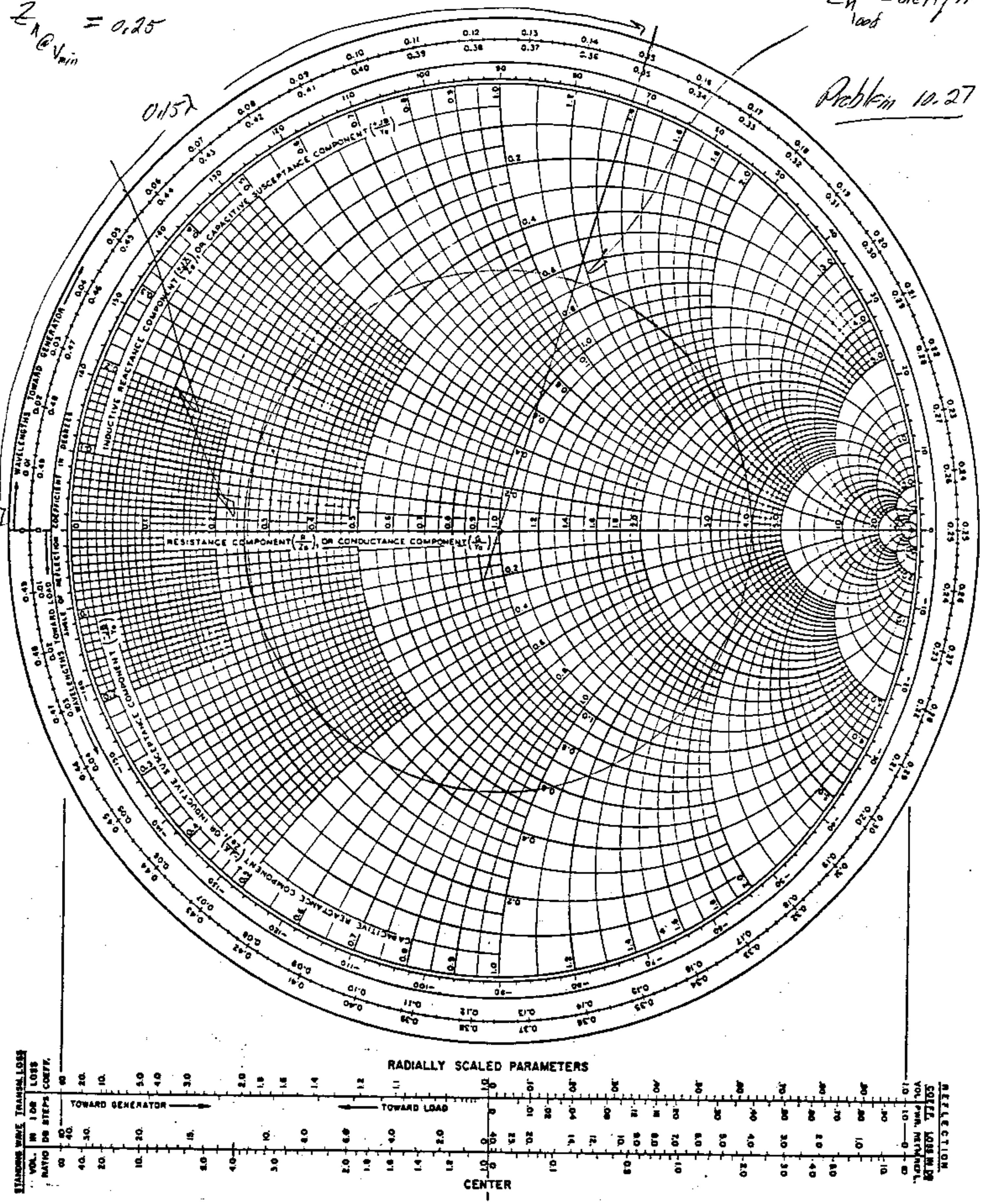


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

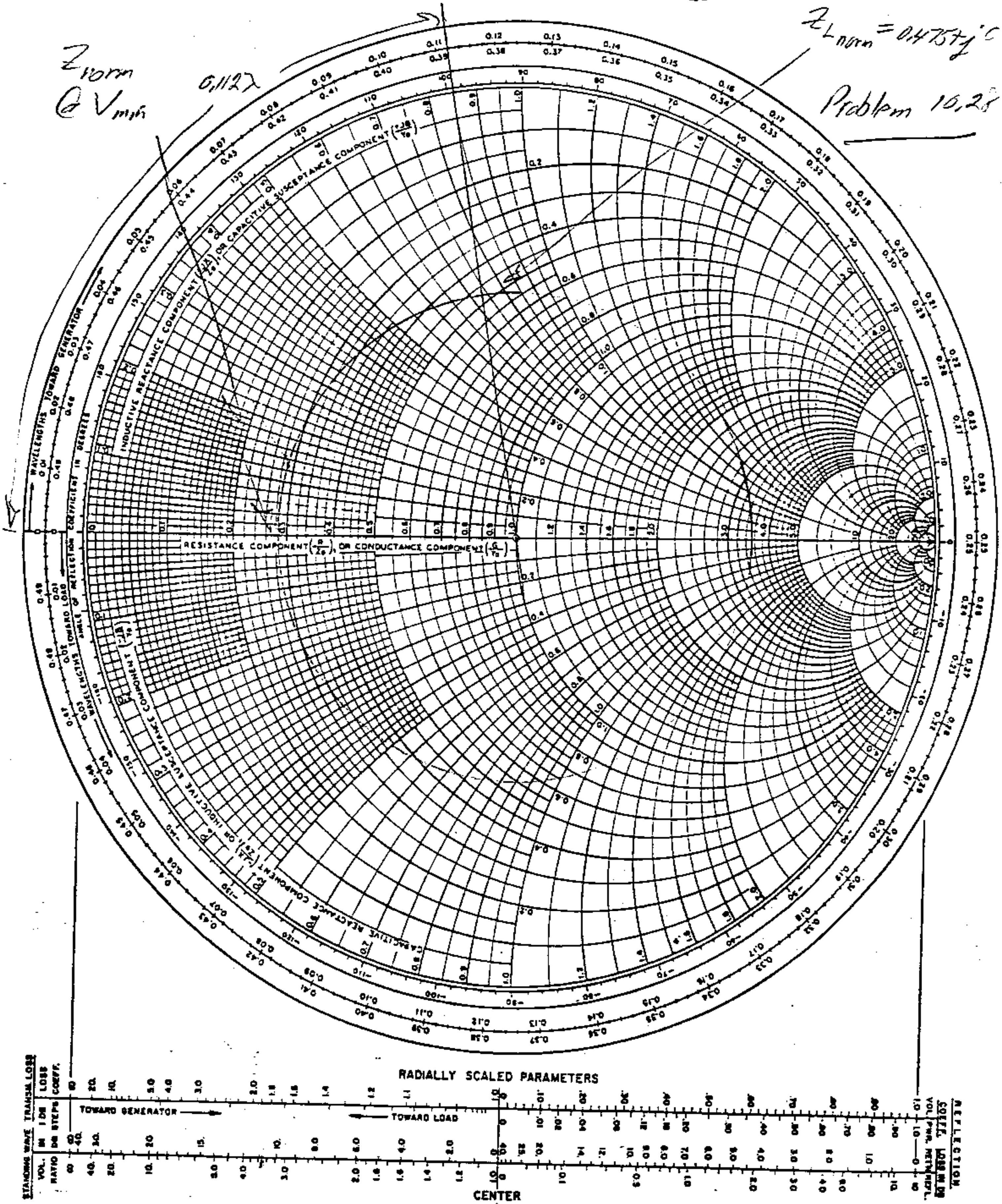
IMPEDANCE OR ADMITTANCE COORDINATES

$Z_{L\,norm} = 0.475+j$

Problem 10.28

Z_{norm} @ V_{min}

0.112λ



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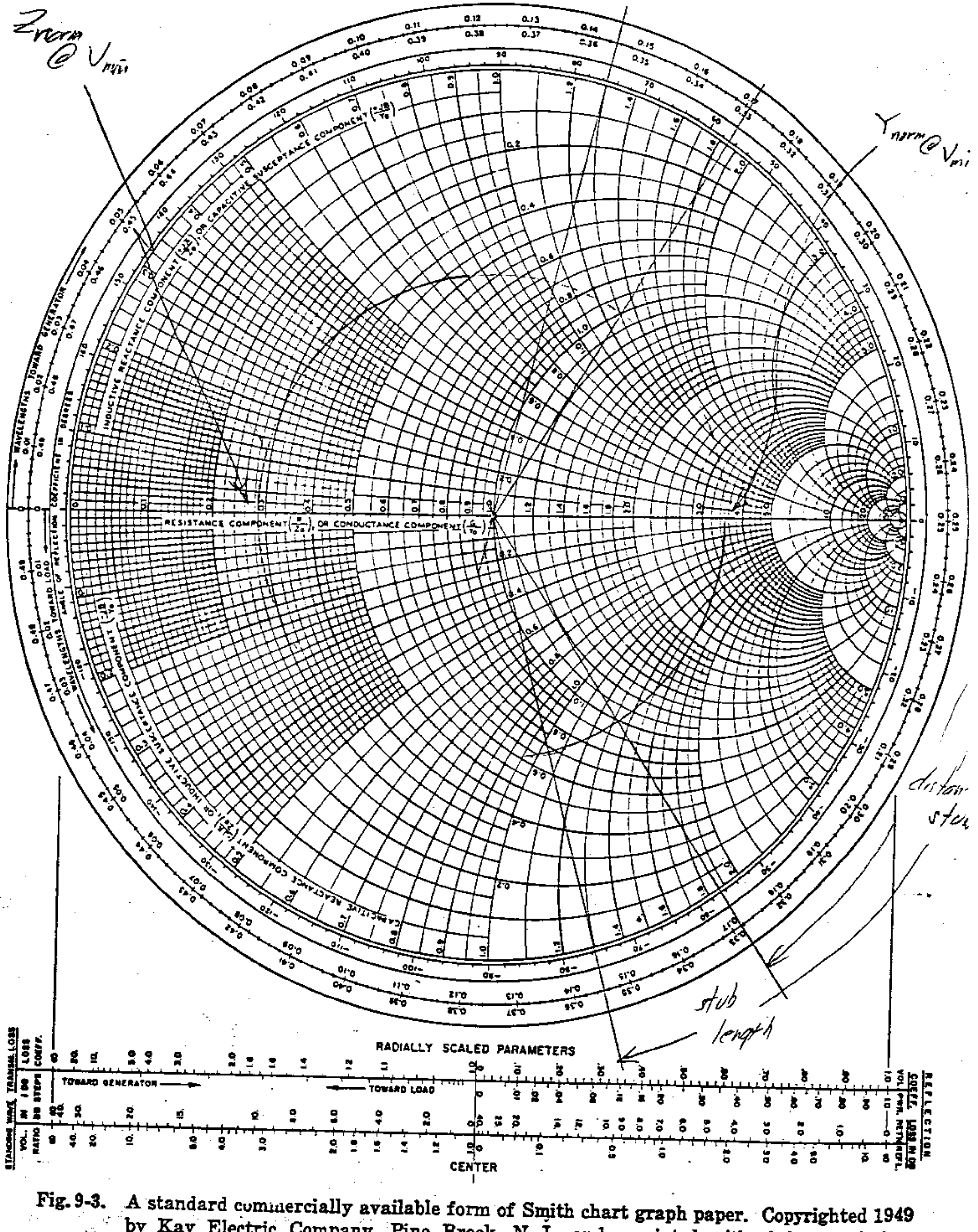
Problem 10.39



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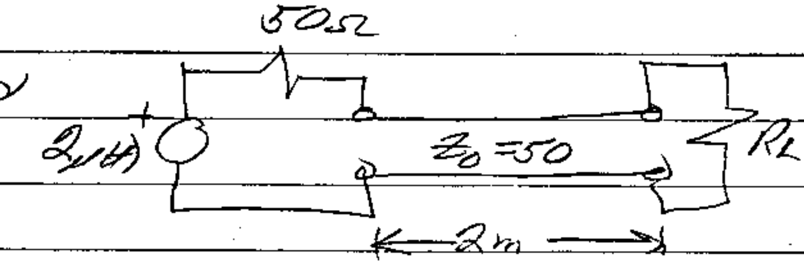
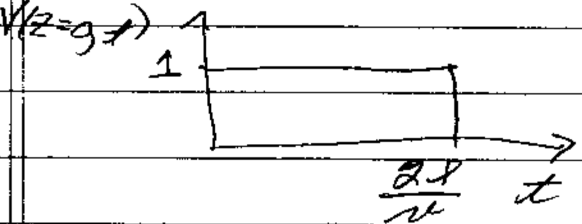
EE 333 Homework 14

9-38

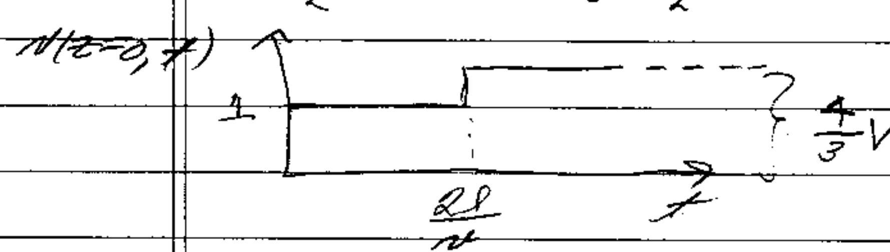


a) $R_L = 0$, $\Gamma_L = -1$

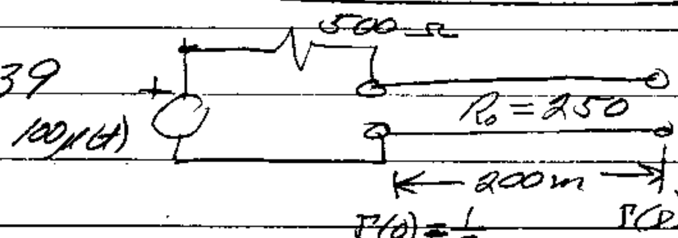


$\left\{ \text{where } \frac{l}{u} = \frac{2}{3\times10^{8}} = \frac{2}{3}\times10^{-8}\,s \right\}$

b) $R_L=100$ $\therefore$ $\Gamma_L = \frac{100-50}{100+50} = \frac{1}{3}$

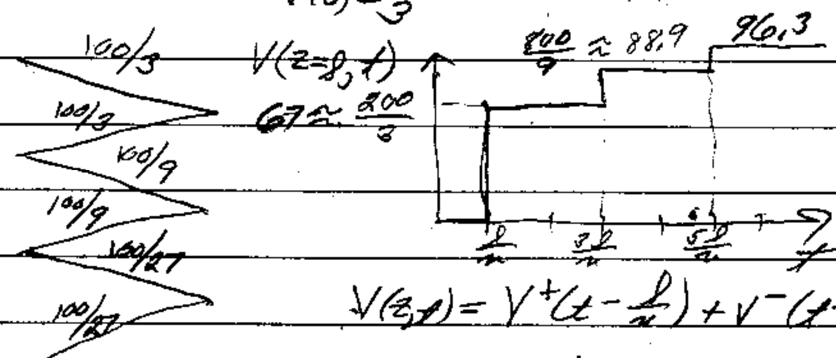


9-39



$\left\{ \begin{aligned} V^+(z=0,t) &= 100\left(\tfrac{1}{3}\right) V \\ I^+(z=0,t) &= \frac{V^+}{250} = \frac{2}{15} A \end{aligned} \right.$

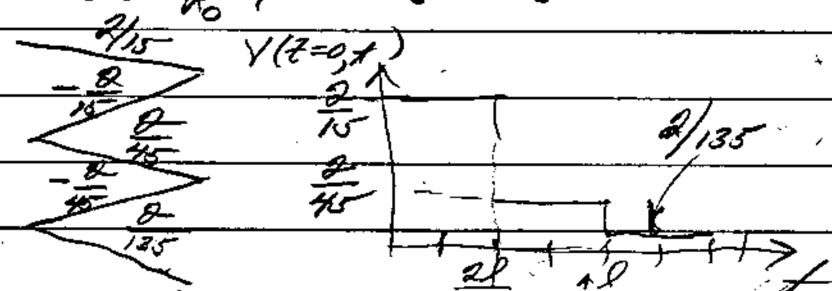
a)



$\boxed{V_{final} = 100}$

$V(z,t) = V^+\left(t-\frac{l}{v}\right) + V^-\left(t+\frac{l}{v}\right)$

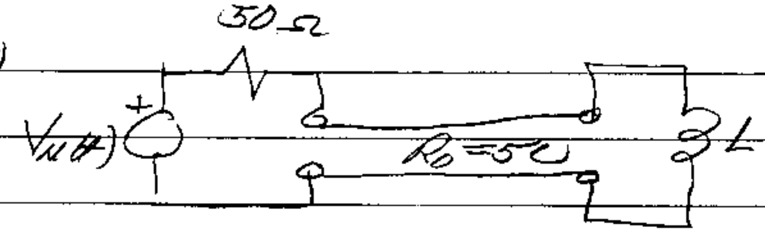
b) $I(z,t) = \frac{1}{R_0}\{V^+ - V^-\} = \frac{V^+}{R_0}\{1-\Gamma\}$



$\boxed{I_{final} = 0}$

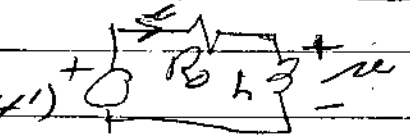
EE 333 Homework 14 (page 2)

9-40



@ load $V^+(t-\frac{l}{u}) = \frac{V}{2}u(t-\frac{l}{u}) = \frac{V}{2}u(t')$; $t' = t - \frac{l}{u}$

so at load we have:



$V = iR_0 + L\frac{di'}{dt'}$

$i_{homo} = Ae^{st'}$; $s = -\frac{R_0}{L}$ so $i_h = Ae^{-\frac{R_0 t'}{L}}$

$i_{particular} = K = \frac{V}{R_0}$ ∴ $i' = Ae^{-\frac{R_0 t'}{L}} + \frac{V}{R_0}$

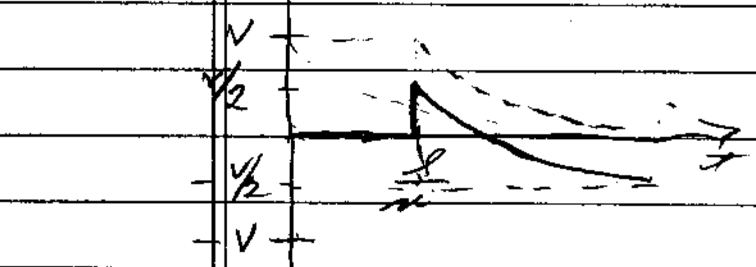
Assuming no initial energy storage [i.e. $i(0)=0$]

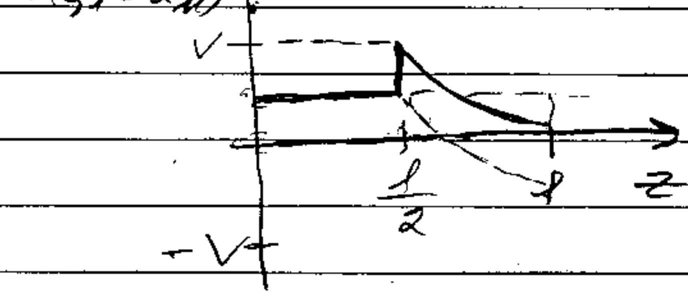
$$i = \frac{V}{R_0}(1-e^{-\frac{R_0 t'}{L}}) = \left[V^+(t-\frac{l}{u}) - V^-(t+\frac{l}{u})\right]\frac{1}{R_0}$$

giving $V^-(t+\frac{l}{u}) = V^+(t-\frac{l}{u}) - V(1-e^{-\frac{R_0(t-\frac{l}{u})}{L}})$

or $V^-(t+\frac{l}{u}) = \frac{V}{2}u(t-\frac{l}{u}) - V(1-e^{-\frac{R_0(t-\frac{l}{u})}{L}})u(t-\frac{l}{u})$

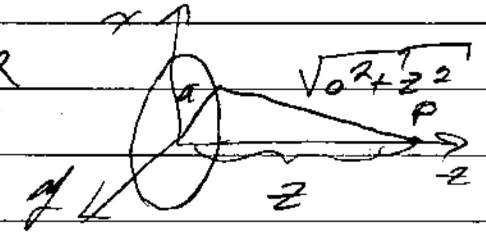
$$V^-(t+\frac{l}{u}) = \left[-\frac{V}{2} + Ve^{-\frac{R_0(t-\frac{l}{u})}{L}}\right]u(t-\frac{l}{u})$$





EE 333 Homework 15

4-12



a) $\Phi_P = \int_{\phi=0}^{2\pi} \frac{a\rho_\ell\, d\phi}{4\pi\epsilon (a^2+z^2)^{1/2}}$

$\Phi_P = \frac{a\rho_\ell}{2\epsilon (a^2+z^2)^{1/2}}$ ←

$\bar{E}_{\text{on } z \text{ axis}} = -\nabla\Phi = -\frac{\partial\Phi}{\partial z}\bar{a}_z = \frac{a\rho_\ell\, 2z}{4\epsilon(a^2+z^2)^{3/2}}\bar{a}_z$ ←

b) $\Phi(z) = -\int_{\infty}^{z} \frac{a\rho_\ell\, z\, \bar{a}_z \cdot dz\, \bar{a}_z}{2\epsilon(a^2+z^2)^{3/2}} = -\int_{\infty}^{a^2+z^2} \frac{a\rho_\ell\, du}{4\epsilon u^{3/2}}$

let $u = a^2+z^2$
$du = 2z\,dz$ $\quad \Phi(z) = + \left.\frac{a\rho_\ell\, 2}{4\epsilon u^{1/2}}\right|_{\infty}^{a^2+z^2}$

so $\Phi(z) = \frac{a\rho_\ell}{2\epsilon(a^2+z^2)^{1/2}}$ ←

4-16 for coax $C/m = \frac{2\pi\epsilon}{\ln(\frac{b}{a})} = 96.8\times10^{-12}$

$\boxed{\epsilon_r = 2.26}$ so $\ln(\frac{b}{a}) = \frac{2\pi\times10^{-9}\times2.26}{36\pi\times96.8\times10^{-12}} = 1.297$

$\therefore \frac{b}{a} = 3.6585$; so $\boxed{b = 0.713\text{ in or } 1.81\text{ cm}}$

4.29 $U_e = \iiint \bar{D}\cdot\bar{E}\, dv = \frac{1}{2}\int_{\phi=0}^{\pi}\int_{\rho=a}^{b}\int_{z=0}^{\ell} \frac{\epsilon_1 V^2 \rho\, d\phi\, dz\, d\rho}{\rho^2[\ln\frac{b}{a}]^2}$

[using (4-71)] $\quad + \frac{1}{2}\int_{\phi=\pi}^{2\pi}\int_{\rho=a}^{b}\int_{z=0}^{\ell} \frac{\epsilon_2 V^2 \rho\, d\phi\, dz\, d\rho}{\rho^2[\ln\frac{b}{a}]^2}$

$U_e = \frac{\pi V^2 \ell}{2(\ln\frac{b}{a})^2}\ln(\frac{b}{a})(\epsilon_1+\epsilon_2) = \frac{1}{2}CV^2$

$\therefore C = \frac{\pi\ell}{\ln(\frac{b}{a})}(\epsilon_1+\epsilon_2)$ ←

EE 333 Homework 16

4-61 $30 \to 500\,pF$ in 0 to 260°

$V = 5{,}000\,V$

$$\boxed{T = \frac{\partial U_e}{\partial \theta} = \frac{\Delta U_e}{\Delta \theta}}$$

and $U_e = \frac{1}{2}CV^2$.

$$\text{so } \Delta U_e = \frac{1}{2}V^2(30-500)\times 10^{-12} = -V^2\,\frac{470}{2}\times 10^{-12}$$

$$T = -\frac{25\times 10^{6}\times 470\times 10^{-12}}{2\times 260\times \pi/180} = 1.294\times 10^{-3}\ N\text{-}m$$