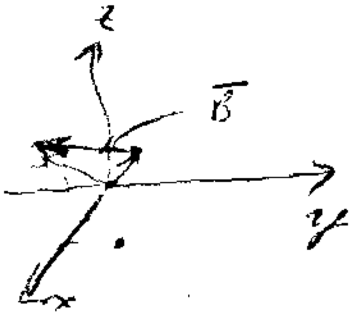
EE 333 Assignment 1 (page 1)



1-2 $\bar{A} = \bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$

a) $\boxed{\bar{B} = -2\bar{a}_x - 3\bar{a}_y - \bar{a}_z}$

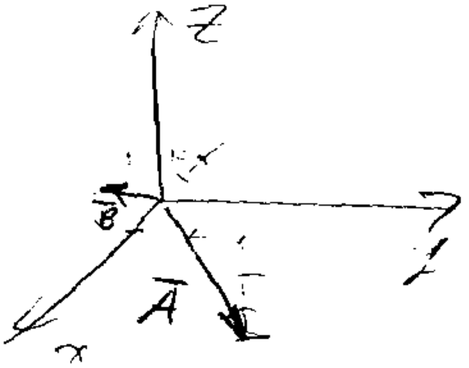
b) $|\bar{B}\cdot\bar{A}| = |-2-6+3| = 5$

projection of $\bar{B}$ on $\bar{A}$ $= \frac{|\bar{B}\cdot\bar{A}|}{|\bar{A}|} = \frac{5}{\sqrt{1+4+9}} = \boxed{\frac{5}{\sqrt{14}}}$

c) $\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta = -5$

$\therefore \cos\theta = \frac{-5}{\sqrt{14}\cdot\sqrt{14}} = -0.357$ (2nd or 3rd quadrant)

$\boxed{\theta = \pm 110.92^\circ}$



d) unit vector $= \frac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|} = \frac{\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 1 & 2 & -3 \\ -2 & -3 & -1 \end{vmatrix}}{|\bar{A}\times\bar{B}|} = \frac{\bar{a}_x(-11) - \bar{a}_y(-7) + \bar{a}_z(1)}{\sqrt{121+49+1}}$

$\boxed{\text{unit vector} = -0.841\bar{a}_x + 0.535\bar{a}_y + 0.076\bar{a}_z}$

1-15 $F = \frac{Q_1 Q_2}{4\pi\epsilon_0 r^2} = \frac{1.6^2\times10^{-38}\times 9\times10^{9}}{10^{-20}} = \boxed{23.04\times10^{-9}\,N}$

1-24 $\bar{B} = B_0(\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)$; $\bar{v} = v_0(3\bar{a}_x - \bar{a}_y + 2\bar{a}_z)$

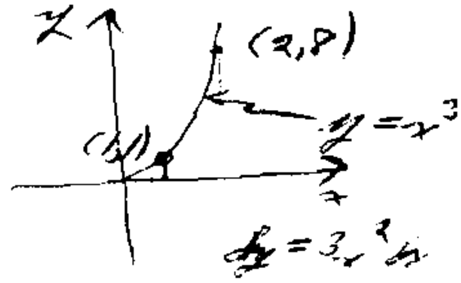
$\bar{F} = q(\bar{E} + \bar{v}\times\bar{B})$ for $\bar{F} = 0$

$$\boxed{\frac{-\bar{E}}{v_0 B_0} = + \begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 3 & -1 & 2 \\ 1 & 2 & -4 \end{vmatrix} = -\bar{a}_y(-12-2) + \bar{a}_z(6+1) = 14\bar{a}_y + 7\bar{a}_z}$$

$$\therefore \boxed{\bar{E} = v_0 B_0 \{-14\bar{a}_y - 7\bar{a}_z\}}$$

1-27 $\bar{E} = (5xy - 6x^2)\bar{a}_x + (2y - 4x)\bar{a}_y$

$$\text{Work} = Q\int_{x=1}^{2} \bar{E}\cdot d\bar{l} = Q\int_{x=1}^{2} \bar{E}\cdot(dx\,\bar{a}_x + 3x^2 dx\,\bar{a}_y)$$



$$\text{Work} = Q\int_{x=1}^{2} \{(5xy - 6x^2)dx + (2y - 4x)3x^2 dx\}$$

$$= Q\int_{x=1}^{2} (5x^4 - 6x^2 + 6x^5 - 12x^3)\,dx$$

$$= Q\left\{\cancel{5}\frac{x^5}{\cancel{5}} - \cancel{6}^{2}\frac{x^3}{\cancel{3}} + \cancel{6}\frac{x^6}{\cancel{6}} - \cancel{12}^{3}\frac{x^4}{\cancel{4}}\right\}_1^2$$

$$= Q\{32 - 1 - 2(8-1) + 64 - 1 - 3(16-1)\} = Q\{31 - 14 + 63 - 45\}$$

$$\boxed{\text{Work} = Q\{35\} = 35\times10^{-6}\ \text{Joules}}$$

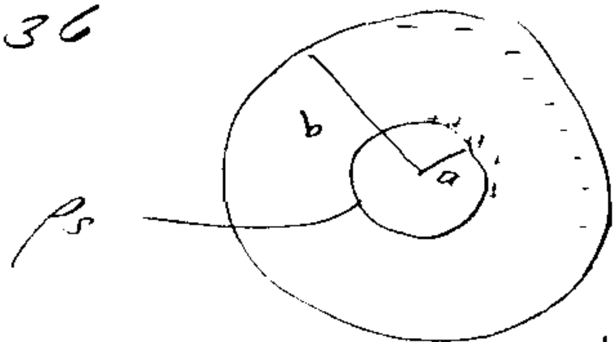
1-40 $\text{emf} = -\frac{d}{dt}\int\bar{B}\cdot d\bar{s} = -\frac{d}{dt}(0.02\sin 10^3 t) = -20\cos 10^3 t$

I (clockwise current) $= \frac{\text{emf}}{5} = 4\cos 10^3 t$

opposite of direction of integration

EE 333 Assignment 2

1-36



a) for $a<r<b$

$4\pi r^2 \epsilon_0 E_r = \rho_s 4\pi a^2$

$$\boxed{\text{or } E_r = \frac{\rho_s a^2}{\epsilon_0 r^2}}$$

$$\boxed{\text{for } r>b \quad E_r = 0}$$

b) $\bar{J}_D = \frac{\partial(\epsilon_0 \bar{E})}{\partial t}$ $\quad \rho_s(t) = 2\times10^{-9}\cos 10^5 t$ C/m²

$\therefore \bar{J}_D = \frac{\partial}{\partial t}\left(\frac{a^2}{r^2} 2\times10^{-9}\cos 10^5 t\right)\bar{a}_r = \boxed{-\frac{2\times10^{-4}a^2}{r^2}\sin 10^5 t\,\bar{a}_r}$

1-37 $\rho_v = \rho_0\left(1-\frac{r^2}{a^2}\right)$ $\quad r<a$

$\rho_v = 0$ $\quad r>a$

for $0<r<a$ $\quad \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\epsilon_0 E_r r^2 \sin\theta\, d\phi\, d\theta = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\int_{r=0}^{r}\rho_0\left(1-\frac{r^2}{a^2}\right)r^2\sin\theta\, dr\, d\phi\, d\theta$

or $\epsilon_0 E_r 4\pi r^2 = 4\pi\rho_0\int_0^r\left(1-\frac{r^2}{a^2}\right)r^2 dr = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5a^2}\right]$

so $\boxed{E_r = \frac{\rho_0}{\epsilon_0}\left\{\frac{r}{3}-\frac{r^3}{5a^2}\right\}}$ $\quad 0<r<a$

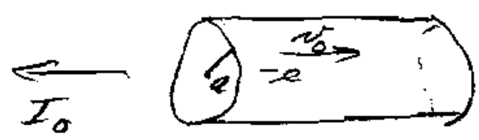
for $r>a$ [$Q_{enclosed}$ = above rhs integral integrated from $r=0$ to $r=a$]

$\therefore \epsilon_0 4\pi r^2 E_r = 4\pi\rho_0\left\{\frac{a^3}{3}-\frac{a^3}{5}\right\} = 4\pi\rho_0\frac{2a^3}{15}$

so $\boxed{E_r = \frac{2\rho_0 a^3}{15\epsilon_0 r^2}}$ $\quad r>a$

EE 333 Assignment 3

1-38



$v_0 = 10^7\ \text{m/sec}$ $a = 1\times10^{-3}$

$I_0 = 10^{-2}\ A$

$$I_0 = J\pi a^2 = \rho_V 10^7 \pi a^2 \quad \text{or} \quad \boxed{\rho_V = \frac{10^{-2}}{10^7\pi \times 10^{-6}} = \frac{10^{-3}}{\pi}}$$

using a circular cylindrical Gaussian surface of length L we obtain

$$\epsilon_0 E_\rho \pi 2\rho \ell = \rho_V \pi \rho^2 \ell \quad \text{for } r<a$$

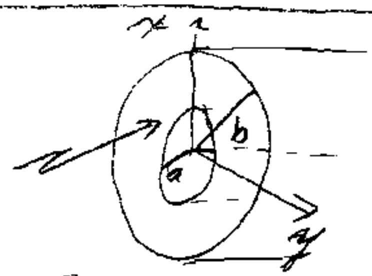
$$\boxed{E_\rho = \frac{\rho_V \rho}{\epsilon_0 2} = \frac{10^{-3}\rho}{2\pi\epsilon_0} \quad \text{for } r<a}$$

$$E_\rho = \frac{10^{-3}\rho\, 36\pi}{2\pi\, 10^{-9}} = 18\times10^6 \rho$$

$$\epsilon_0 E_\rho \pi 2\rho \ell = \rho_V \pi a^2 \ell \quad \text{for } r>a$$

$$\boxed{E_\rho = \frac{\rho_V a^2}{\epsilon_0 2\rho} = \frac{10^{-3}\times10^{-6}\, 36\pi}{\pi\, 2\times10^{-9}\rho} = 18/\rho \ \text{V/m}}$$

1-44



$\bar{J} = 2\rho \bar{a}_z$

$$\oint \frac{\bar{B}}{\mu_0}\cdot d\bar{\ell} = \int \bar{J}\cdot d\bar{s}$$

a) for $\rho < a$ $\int \bar{J}\cdot d\bar{s} = 0 \therefore B_\phi = 0$

b) $$\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_0}\rho\, d\phi = \int_{\rho=a}^{\rho}\int_{\phi=0}^{2\pi} 2\rho\bar{a}_z \cdot \rho\, d\phi\, d\rho\, \bar{a}_z = 4\pi \frac{\rho^3}{3}\Big|_a^\rho = \frac{4\pi}{3}\left[\rho^3 - a^3\right]$$

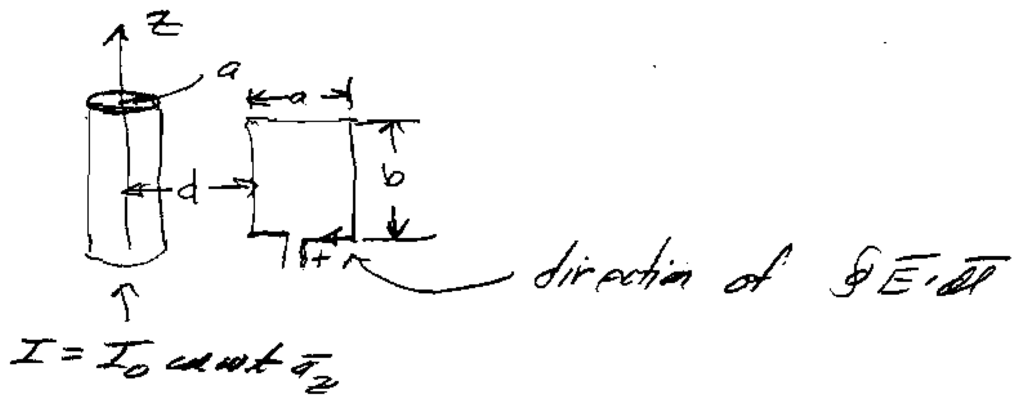
so $$B_\phi = \frac{2\mu_0}{3}\left[\rho^2 - \frac{a^3}{\rho}\right] \quad a<\rho<b$$

c) $$2\pi\rho B_\phi/\mu_0 = \frac{4\pi}{3}\left[b^3 - a^3\right]$$

$$B_\phi = \frac{2\mu_0}{3\rho}\left[b^3 - a^3\right] \quad \rho > b$$

EE 333 Assignment 3 (page 2)

1-45



a) for $\rho > a$ $\boxed{B_\phi = \frac{\mu_0 I_0 \cos\omega t}{2\pi\rho}}$

b) [i] ψ = magnetic flux through loop $= \int_{\rho=d}^{d+a}\int_{z=0}^{b} \frac{\mu_0 I_0 \cos\omega t}{2\pi\rho}\,dz\,d\rho$

$$\boxed{\psi = \frac{b\,\mu_0 I_0 \cos\omega t}{2\pi}\ln\left(\frac{d+a}{d}\right)}$$

$$\boxed{emf = \oint \bar{E}\cdot d\bar{l} = -\frac{d\psi}{dt} = \frac{\omega b \mu_0 I_0 \sin\omega t}{2\pi}\ln\left(\frac{d+a}{d}\right)}$$

EE 333 Homework 4 (solution)

2.17 $\bar{B} = \frac{1}{r^2}\sin\phi\cos^2\theta\,\bar{a}_r$ $\nabla\times\bar{B} = \mu_0\bar{J}$

$\nabla\cdot\bar{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(\sin\phi\cos^2\theta) = 0$ ∴ could be $\bar{B}$ field

$$\bar{J} = \frac{1}{\mu_0}\left\{\frac{\cos\phi\cos^2\theta}{r^3\sin\theta}\bar{a}_\theta + \frac{2\sin\phi\cos\theta\sin\theta}{r^3}\bar{a}_\phi\right\}$$

2.28 a) λ = length of one cycle in space $\lambda = \frac{2\pi}{\beta_0}$

phase factor $e^{\pm j\beta_0 z}$ ... phase as a function of z

phase velocity = velocity of a constant phase point $= \frac{\omega}{\beta_0}$

intrinsic impedance $\frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\epsilon_0}} = \eta_0$

b) $\bar{E}(z,t) = 37.7\cos(6\pi\times10^8 t + 2\pi z)\,\bar{a}_x$

i) $\omega = 6\pi\times10^8 = 2\pi f$ ∴ $\boxed{f = 3\times10^8 \text{ Hertz}}$

ii) $\lambda = \frac{2\pi}{\beta} = \boxed{1 \text{ meter}}$

iii) $v_{phase} = \frac{\omega}{\beta} = \frac{6\pi\times10^8}{2\pi} = \boxed{3\times10^8 \text{ m/sec}}$

iv) negative z direction

v) $\bar{H}(z,t) = 0.1\cos(6\pi\times10^8 t + 2\pi z)(-a_y)$

EE 333 Homework 5

2.29 $\overline{H} = \frac{1}{3\pi}\cos(\omega t - 30z)\,\overline{a_y}$

free space so $\eta = \eta_0 = 120\pi$ and $\boxed{v_p = \frac{\omega}{\beta_0} = 3\times10^8}$

$\therefore\ \overline{E} = 40\cos(\omega t - 30z)\,\overline{a_x}$

$$\omega = 3\times10^8\beta = 9\times10^9 = 2\pi f$$

$$\boxed{f = \frac{9\times10^9}{2\pi}}$$

$$\boxed{\lambda = \frac{2\pi}{30} = \frac{\pi}{15}\,m}$$

$$\boxed{\begin{aligned}\overline{H} &= \frac{1}{3\pi}\cos(9\times10^9 t - 30z)\,\overline{a_y}\\ \overline{E} &= 40\cos(9\times10^9 t - 30z)\,\overline{a_x}\end{aligned}}$$

EE 333 Homework 5 (page 2)

2.32 200MHz, positive z direction, E_x only
maximum when $z=1, t=0$ of 150 V/m, free space

$$\bar{E} = 150 \cos\left(2\pi \times 2\times10^8 t - \frac{4\pi\times10^8}{3\times10^8} z + \frac{4\pi}{3}\right)\bar{a}_x$$

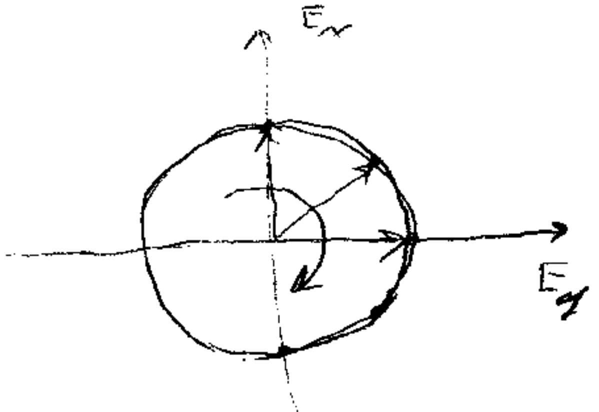
$$\bar{E} = 150 \cos\left(4\pi\times10^8 t - \frac{4\pi}{3} z + \frac{4\pi}{3}\right)\bar{a}_x$$

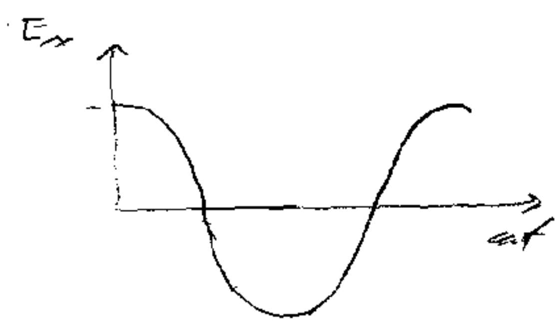
Special: $\hat{\bar{E}} = 500\, e^{-j\beta_0 z}(\bar{a}_x - j\bar{a}_y)$

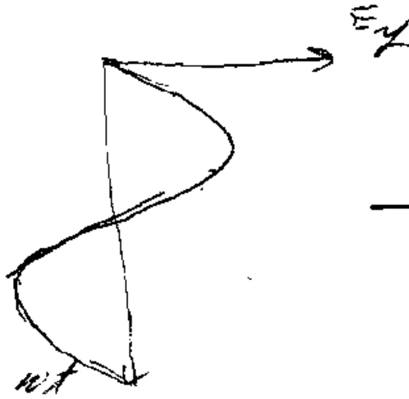
a) $\bar{E} = 500\cos(\omega t - \beta_0 z)\bar{a}_x + 500\sin(\omega t - \beta_0 z)\bar{a}_y$

b) $\hat{H} = \frac{500}{\eta} e^{-j\beta_0 z}(\bar{a}_y + j\bar{a}_x)$

c)







Right hand circular polarization

RHCP

EE 333 Assignment 6

3.1 $\bar{E} = 3z^2 y \cos(10^8 t)\bar{a}_x$ Lucite $\epsilon_r = 2.56$

a) $\bar{P} = ?$ $\epsilon_0 \bar{E} + \bar{P} = \bar{D} = \epsilon_0 \epsilon_r \bar{E}$

$\therefore \bar{P} = \epsilon_0 \epsilon_r \bar{E} - \epsilon_0 \bar{E} = \epsilon_0 \bar{E}(\epsilon_r - 1)$

$$\bar{P} = \frac{1.56 \times 10^{-9} \times 3}{36\pi} z^2 y \cos(10^8 t)\bar{a}_x \leftarrow$$

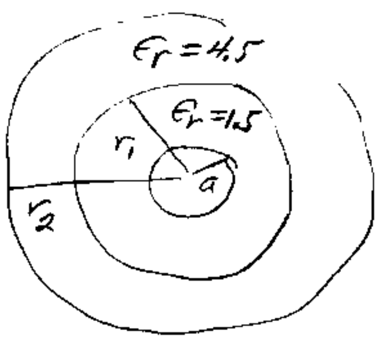
$$\bar{P} = 4.14 \times 10^{-11} z^2 y \cos(10^8 t)\bar{a}_x \leftarrow$$

b) $\rho_p = -\nabla \cdot \bar{P} = 0$ $\leftarrow$

$\bar{P}$ not a function of x and there is only a P_x (uniform polarization leads to no ρ_p)

c) $\bar{J}_p = \frac{\partial \bar{P}}{\partial t} = -4.14 \times 10^{-3} z^2 y \sin 10^8 t \, \bar{a}_x \leftarrow$

3.2 a)



"power coax"

ρ_ℓ C/m ; for circular cylindrical surface.

a) $\oint \bar{D} \cdot d\bar{s} = \rho_\ell L$

or $D_\rho 2\pi\rho \cancel{L} = \rho_\ell \cancel{L}$

$$\boxed{D_\rho = \frac{\rho_\ell}{2\pi\rho} \text{ everywhere}}$$

$$E_\rho = \frac{\rho_\ell}{1.5\epsilon_0 2\pi\rho} \quad a \le \rho \le r_1$$

$$E_\rho = \frac{\rho_\ell}{4.5\epsilon_0 2\pi\rho} \quad r_1 \le \rho \le r_2$$

$$P_\rho = \epsilon_0(\epsilon_r - 1)\bar{E} \quad \begin{cases} P_\rho = \dfrac{\rho_\ell}{6\pi\rho} & a \le \rho \le r_1 \\ P_\rho = \dfrac{7\rho_\ell}{18\pi\rho} & r_1 \le \rho \le r_2 \end{cases}$$

3.2 a) (continued) Outside of cable $\bar{E} = \bar{D} = \bar{P} = 0$

c) ρ_p for $r_1 < \rho \leq r_2$

$$\rho_p = -\nabla \cdot \bar{P} = -\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{7\rho}{18\pi}\right) = 0 \; !$$

Special Cu 10^{29} electrons/m^3, $\sigma = 5.8 \times 10^7$ mho/m

a) $\mu_e = -\frac{e\tau_c}{m}$; $\sigma = \frac{ne^2\tau_c}{m} = -ne\mu_e$

$$\therefore \mu_e = \frac{\sigma}{ne} = \frac{+5.8\times10^7}{10^{29}\times1.6\times10^{-19}} = +3.625\times10^{-3}$$

b) $\rho_V = 10^{29}(-1.6\times10^{-19}) = -1.6\times10^{10}\ C/m^3 = -16\ C/mm^3$

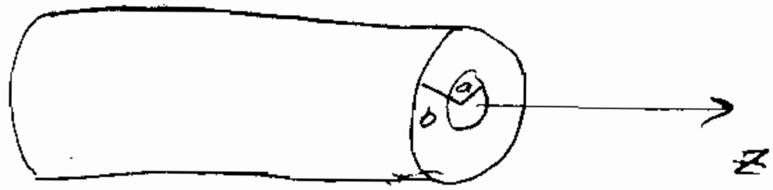
c) $\bar{v}_d = -\mu_e \bar{E} = -3.625\times10^{-3}\bar{a}_x$

d) $\bar{J} = \rho_V \bar{v}_d = 1.6\times10^{10}\times3.625\times10^{-3}\bar{a}_x = 5.8\times10^7\ A/m^2\ \bar{a}_x$

$$\bar{J} = 58\ A/mm^2\ \bar{a}_x$$

EE 333 Assignment 7

3.5



for $\rho \le a$ $\bar{J} = \frac{1}{2}\bar{a}_z$

for $a < \rho \le b$ $\bar{J} = -\frac{\rho}{2a}\bar{a}_z$

for $\rho \le a$ $\mu = \mu_0 \mu_{r_1}$; $a < \rho \le b$ $\mu = \mu_0 \mu_{r_2}$

a) Find $\bar{H}$ $\oint \bar{H} \cdot d\bar{l} = \int \bar{J} \cdot d\bar{s}$

$\boxed{\text{for } \rho \le a}$ $2\pi\rho H_{\phi_1} = \int_{\rho=0}^{\rho}\int_{\phi=0}^{2\pi} \frac{1}{2} \rho \, d\phi \, d\rho = \frac{\cancel{2\pi}\rho^{\cancel{2}}}{4}$

so $\boxed{H_{\phi_1} = \rho/4}$

$\boxed{\text{for } a < \rho \le b}$ $2\pi\rho H_{\phi_2} = \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} \frac{1}{2} \rho \, d\phi \, d\rho - \int_{\phi=0}^{2\pi}\int_{\rho=a}^{\rho} \frac{\rho}{2a} \rho \, d\phi \, d\rho$

$= 2\pi \frac{a^2}{4} - 2\pi \frac{\rho^3}{6a}\Big|_a^{\rho} = 2\pi \frac{a^2}{4} - 2\pi \frac{\rho^3}{6a} + 2\pi \frac{a^2}{6}$

$\frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12}$

$\therefore H_{\phi_2} = \frac{a^2}{4\rho} + \frac{a^2}{6\rho} - \frac{\rho^2}{6a} = \boxed{\frac{a^2}{\rho}\left(\frac{5}{12}\right) - \frac{\rho^2}{6a}}$

b) $\left.\begin{aligned} &\rho \le a \quad B_{\phi_1} = \mu_0 \mu_{r_1} H_{\phi_1} \\ &a < \rho \le b \quad B_{\phi_2} = \mu_0 \mu_{r_2} H_{\phi_2} \end{aligned}\right\}$

c) $\bar{M} = \chi_m \bar{H}$ $1 + \chi_m = \mu_r$ or $\chi_m = \mu_r - 1$

$\left.\begin{aligned} &\therefore 0 < \rho \le a \quad \bar{M} = (\mu_{r_1} - 1)\rho/4 \\ &\bar{J}_m = \nabla \times \bar{M} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\frac{\rho^2}{4}\right]\bar{a}_z = \frac{1}{2}\bar{a}_z \end{aligned}\right\}$ not required

c) (continued)

$$\overline{M_2} = (\mu_{r_2} - 1) H_{\phi_2} \overline{a_\phi} \qquad a < \rho \le b$$

$$\frac{\overline{J_{m_2}}}{(\mu_{r_2}-1)} = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left[-\frac{\rho^3}{6a}\right]\overline{a_z} = -\frac{3\rho^{\cancel{2}}}{6a\cancel{\rho}}\overline{a_z} = -\frac{3\rho}{6a}\overline{a_z}$$

$$\overline{J_{m_2}} = -(\mu_{r_2}-1)\frac{\rho}{2a}\overline{a_z}$$

d) $H_{\phi_1}(\rho = a) \overset{?}{=} H_{\phi_2}(\rho = a)$

$$\frac{a}{4} \overset{?}{=} \frac{5}{12}a - \frac{1}{6}a$$

$$\frac{a}{4} = \frac{a}{4} \quad ! \quad Q.E.D.$$

3.8 a) 1) conductors – free charge $\overline{J} = \sigma \overline{E}$

2) electric dipoles – bound charges separated when $\bar{E}$ applied

$$\overline{J_p} = \frac{\partial \overline{P}}{\partial t}, \quad \rho_p = -\nabla \cdot \overline{P}$$

3) magnetic dipoles – magnetic dipole alligned by applied $\bar{B}$ field

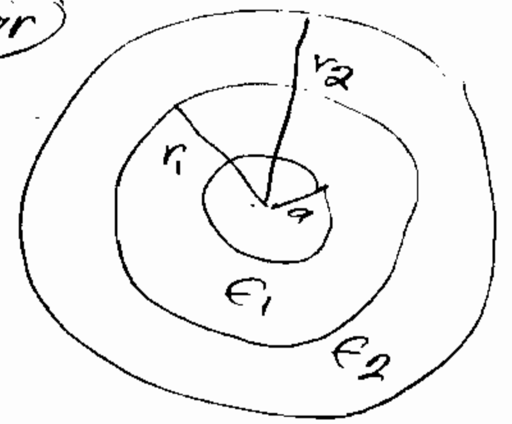
$$\overline{J_m} = \nabla \times \overline{M}$$

Must include the new ρ & $\overline{J}$ terms in Maxwell's Eq's

(881-3082 Mark)

EE 333 Assignment 7 (page 3)

3.8 b) charge Q on center sphere of radius "a"



$\oint \bar{D}\cdot d\bar{S} = Q$ for $r>a$

i.e. $4\pi r^2 D_r = Q$ or $\boxed{D_r = \frac{Q}{4\pi r^2}}$

$\boxed{a < r \le r_1}$ $E_r = \frac{Q}{4\pi \epsilon_1 r^2}$

$\bar{P} = \bar{D} - \epsilon_0 \bar{E} = \left(\frac{Q}{4\pi r^2} - \frac{Q}{4\pi \epsilon_{r_1} r^2}\right)\bar{a}_r = \frac{Q}{4\pi r^2}\left(1 - \frac{1}{\epsilon_{r_1}}\right)\bar{a}_r$

$\rho_p = -\nabla\cdot\bar{P} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{Q}{4\pi}\left(1-\frac{1}{\epsilon_{r_1}}\right)\right] = 0$

$r_1 < r \le r_2$ $E_r = \frac{Q}{4\pi \epsilon_2 r^2}$

$\bar{P} = \frac{Q}{4\pi r^2}\left(1 - \frac{1}{\epsilon_{r_2}}\right)\bar{a}_r$

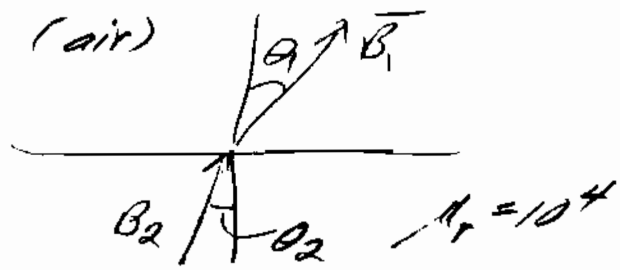
$\rho_p = 0$

$r_2 < r$ $E_r = \frac{Q}{4\pi \epsilon_0 r}$

$\bar{P} = \rho_p = 0$

ρ_{ps} @ $r = r_1$ $\rho_{ps} = \underbrace{-\bar{n}}_{\bar{a}_r}\cdot(\bar{P}_1 - \bar{P}_2)\Big|_{r=r_1} = \frac{Q}{4\pi r_1^2}\left[\frac{\epsilon_{r_1}-1}{\epsilon_{r_1}} - \frac{\epsilon_{r_2}-1}{\epsilon_{r_2}}\right]$

Special Problem



$$\tan\theta_1 = \frac{B_{t1}}{B_{n1}} \; ; \; \tan\theta_2 = \frac{B_{t2}}{B_{n2}}$$

but $B_{n1} = B_{n2}$ $\therefore$ $\frac{B_{t1}}{\tan\theta_1} = \frac{B_{t2}}{\tan\theta_2}$ or $\frac{\mu_1 H_{t1}}{\tan\theta_1} = \frac{\mu_2 H_{t2}}{\tan\theta_2}$

but $H_{t1} = H_{t2}$ so $\boxed{\tan\theta_2 = \frac{\mu_2}{\mu_1}\tan\theta_1}$

$$\theta_1 = \tan^{-1}\left[\frac{\mu_1}{\mu_2}\tan\theta_2\right] = \tan^{-1}\left[10^{-4}\tan\theta_2\right]$$

θ_2	θ_1
0°	0°
45°	$5.7\times10^{-3}\,^\circ$
89°	0.328°
89.9°	3.27°

EE 333 Assignment 8

3.15 a) for conductor with E_x only

$$\hat{E}_x = \hat{E}_m^+ e^{-\alpha z} e^{-j\beta z} + \hat{E}_m^- e^{+\alpha z} e^{+j\beta z}$$

$$\hat{H}_y = \frac{\hat{E}_m^+}{\hat{\eta}} e^{-\alpha z} e^{-j\beta z} - \frac{E_m^-}{\hat{\eta}} e^{+\alpha z} e^{+j\beta z}$$

1. Waves attenuate as they propagate.
2. E and H not in time phase.
3. $\bar{E} \times \bar{H}$ is in direction of propagation
4. $v_{ph} = \frac{\omega}{\beta}$ + $\lambda = \frac{2\pi}{\beta}$ for all media

$$P_{ave_{\text{free space}}} = \frac{1}{2} \frac{|\hat{E}_m|^2}{\eta_0} \quad W/m^2$$

$$P_{ave_{\text{conductor}}} = \frac{1}{2} \frac{|\hat{E}_m|^2}{|\hat{\eta}|} e^{-2\alpha z} \cos(\theta)$$

b) Sea water $\mu_r = 1$, $\epsilon_r = 79$, $\sigma = 3$ S/m

E_{tang} only ; $|E|_{at\ surface} = 10$ V/m

i) need 10µV/m @ receiver ; 20kHz

$$\frac{\sigma}{\omega\epsilon} = \frac{3 \times 36\pi}{2\pi \times 2\times 10^4 \times 79 \times 10^{-9}} = 0.342 \times 10^5 >> 1$$

$$\therefore \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi \times 10^4 \times 4\pi \times 10^{-7} \times 3 \times 2}{2}} = \sqrt{0.236} = \underline{0.486}$$

$$10^{-6} = e^{-0.486 z} \; ; \; \boxed{z = 28.4\ m}$$

3.15 b) ii) $f = 20\ GHz$

$$\frac{\sigma}{\omega\epsilon} = \frac{3 \times 36\pi \times 10^{9}}{4\pi \times 2\times10^{10} \times 10^{-9} \times 79} = 0.0342 \quad << 1$$

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so $$\alpha = \frac{\omega\sqrt{\mu\epsilon'}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2} = \frac{2\pi\times2\times10^{10}\sqrt{79}}{\sqrt{2}\ 3\times10^{8}}\left[0.02416\right]$$

or $\alpha = 26.326\times10^{2} \times 0.02416 = 63.6$ ←

$10^{-6} = e^{-\alpha z}$ or $\boxed{z = 21.7\ cm}$ ←

c) i) $$\hat{\eta}(20KHz) = \sqrt{\frac{\mu}{\epsilon(1 - j\frac{\sigma}{\omega\epsilon})}} \cong \sqrt{\frac{4\pi\times10^{-7}\times36\pi}{10^{-9}(-j\,2.34\times10^{5})79}} = \sqrt{\frac{52.9\times10^{-3}}{-j}}$$

$$\hat{\eta} = e^{j\frac{\pi}{4}} \times 0.23$$

$\therefore$ $$\boxed{P_{ave} = \frac{1}{2}\times10^{-10}\times\frac{1}{0.23}\cos\frac{\pi}{4} = 1.537\times10^{-10}\ W/m^2}$$ ←

ii) $$\hat{\eta} \cong \sqrt{\frac{\mu}{\epsilon}} = 120\pi\sqrt{\frac{1}{79}} = 42.41$$

$$\boxed{P_{ave} = \frac{1}{2}\times\frac{10^{-10}}{42.41} = 1.18\times10^{-12}\ W/m^2}$$ ←

3.18 $$P_{ave} = \frac{1}{2}Re\left[\hat{E}\times\hat{H}^*\right] = \frac{+E_0^2\sin^2\left(\frac{\pi x}{a}\right)}{2Z}\,a_z$$ ←

$$P_{total_z} = \int_{y=0}^{b}\int_{x=0}^{a} P_{ave}\,dx\,dy = \int_{y=0}^{b}\int_{x=0}^{a}\frac{E_0^2\sin^2\left(\frac{\pi x}{a}\right)}{2Z}\,dx\,dy = \frac{E_0^2 b}{2Z}\int_0^a \sin^2\left(\frac{\pi x}{a}\right)dx$$

$\rho = \frac{\pi x}{a}$: $d\rho = \frac{\pi}{a}dx$ then $$P_{total} = \frac{E_0^2 ba}{2Z\pi}\int_0^{\pi}\sin^2\rho\,d\rho$$

$$P_{total_z} = \frac{E_0^2 ab}{2\pi Z}\left[\frac{\rho}{2} - \frac{\sin 2\rho}{4}\right]_0^{\pi} = \boxed{\frac{E_0^2 ab}{4Z}}$$ ←

EE 333 Assignment 9

3.19 $\hat{E}_z = 50 e^{-j\beta x}$

$\lambda = 0.25 m$, $v_{ph} = 2\times10^8$ m/sec

a) $\lambda = v_{ph} T = \frac{v_{ph}}{f}$; $f = \frac{v_{ph}}{\lambda} = \frac{2\times10^8}{1/4} = \boxed{8\times10^8 Hz}$

$v_{ph} = \frac{1}{\sqrt{\mu\epsilon}} = \frac{3\times10^8}{\sqrt{\epsilon_r}}$; $\epsilon_r = \left(\frac{3\times10^8}{2\times10^8}\right)^2 = \boxed{2.25}$

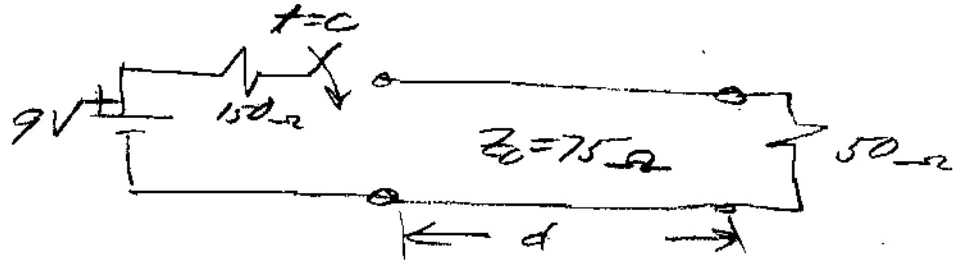
b) $\boxed{\hat{\bar{E}} = 50 e^{-j\beta x} \bar{a}_z}$; $\boxed{\bar{E} = 50\cos(\omega t - \beta x)\bar{a}_z}$

$\hat{H} = \frac{50}{\sqrt{\frac{\mu}{\epsilon}}} e^{-jx}(-\bar{a}_y) = \frac{-50\sqrt{2.25}}{120\pi} e^{-jx}\bar{a}_y$

or $\boxed{\bar{H} = -0.2\cos(\omega t - \beta x)\bar{a}_y}$

EE 333 Assignmat 10

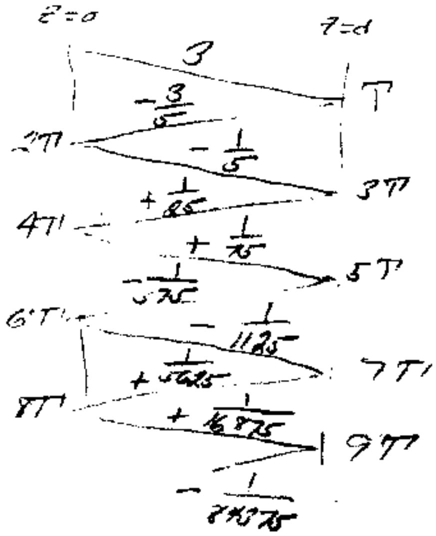
7.4

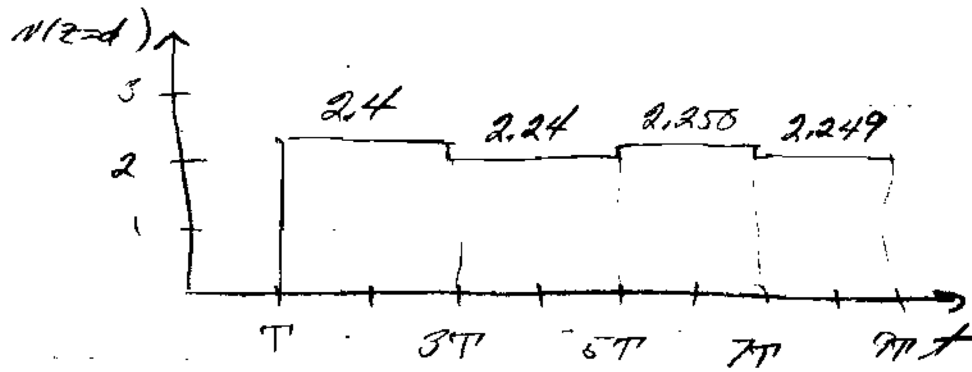


$\boxed{T = \frac{d}{u}}$

a) $\Gamma_g = \frac{150-75}{150+75} = \frac{75}{225} = \frac{1}{3}$; $\Gamma_T = \frac{50-75}{50+75} = \frac{-25}{125} = -\frac{1}{5}$

$v^+(z=0) = 9\,\frac{75}{225} = 3\,V$; $i^+(z=0) = \frac{3}{75} = \frac{1}{25}$

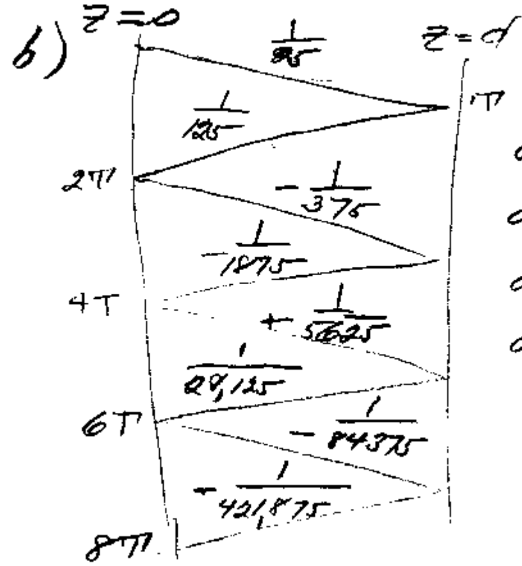


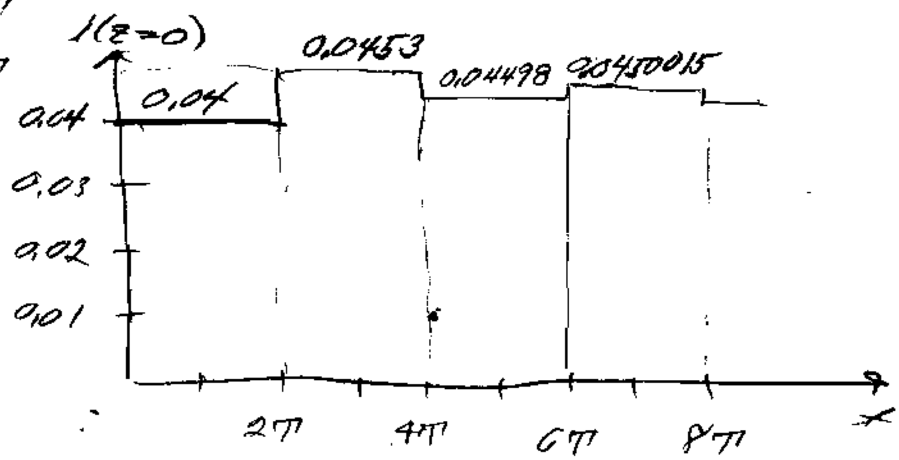


c) final value $= 9\,\frac{50}{200} = \frac{9}{4} = 2.25\,V$

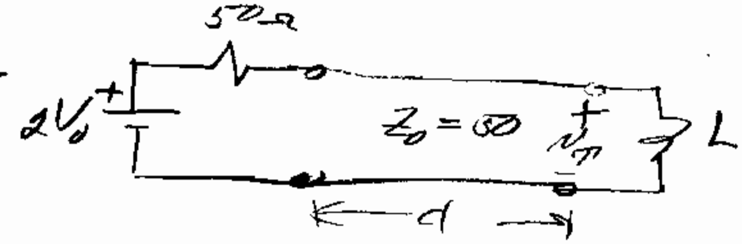
$i_{final} = \frac{9}{4\times 50} = \frac{9}{200} = 45\,mA$

b)





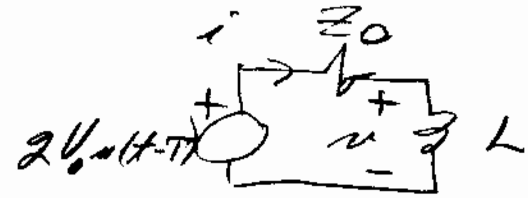
7.5



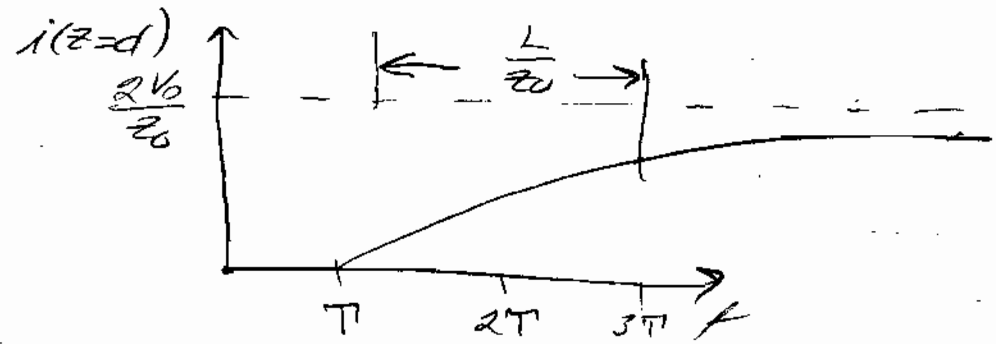
find $v_T(t)$!

$v^+ = V_0$

@ $z=d$;



$$i(t) = \frac{2V_0}{Z_0}\left(1-e^{-\frac{t-T}{L/Z_0}}\right)u(t-T)$$

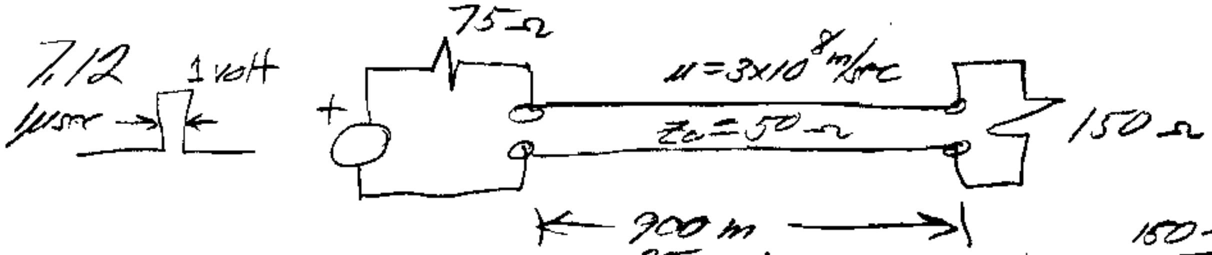


initial $i=0$ and it can't change instantaneously

after a long time inductor looks like a short circuit and $i = \frac{2V_0}{Z_0}$.

EE 333 Assignment 11 7.17

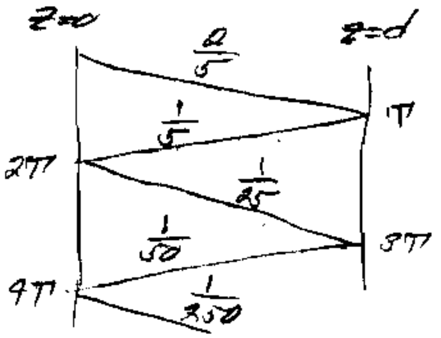
7.12

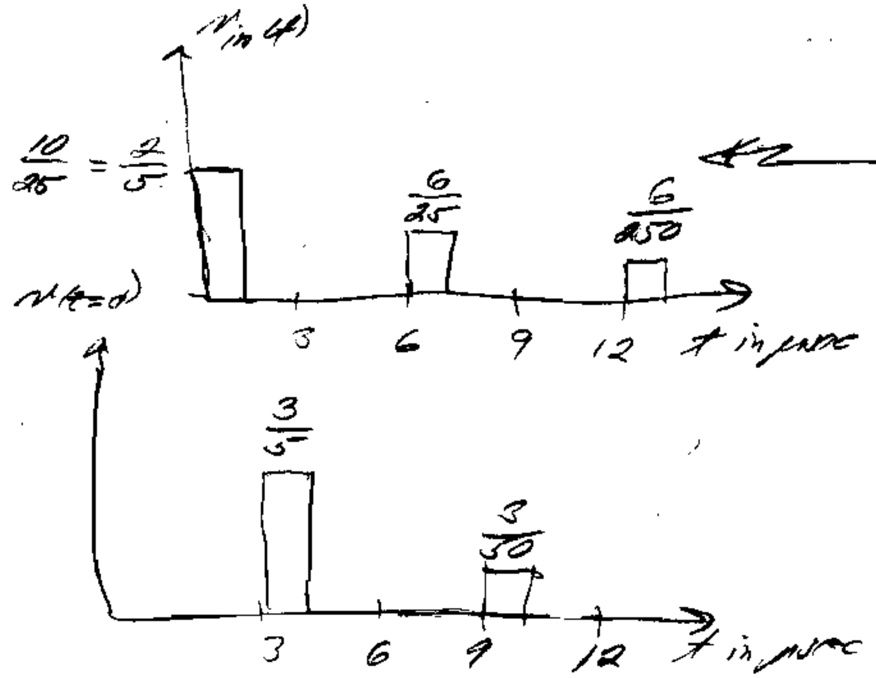


$\Gamma(0) = \frac{25}{125} = \frac{1}{5}$ $\Gamma(d) = \frac{150-50}{200} = \frac{1}{2}$

a) $\left[T = \frac{900}{3\times10^8} = 300\times10^{-8} = 3\,\mu\text{sec}\right] \leftarrow$

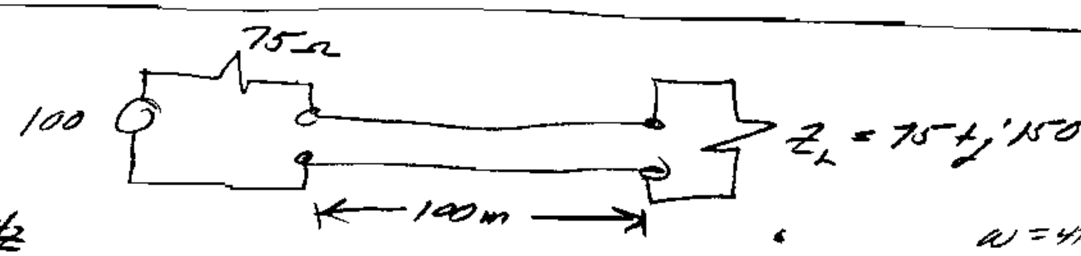
$v^+ = 1\,\frac{50}{125} = \frac{2}{5}$





b)

7.17



$f = 2$ MHz $\omega = 4\pi\times10^6$

$R = 150\ \Omega/\text{km},\ L = 1.4\ \text{mH/km},\ C = 88\ \text{nF/km},\ G = 0.8\ \mu\text{S/km}$

a) $Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{150+j4\pi\times10^3\times1.4}{0.8\times10^{-6}+j4\pi\times88\times10^{-3}}} = \sqrt{\frac{17{,}593\,e^{j89.51^\circ}}{1.10580\,e^{j90^\circ}}} = 126\,e^{-j0.245}\ \Omega \left[126-j0.54\right] \leftarrow$

$\gamma = \sqrt{17593\times1.1058\,e^{j179.51^\circ}} = 139.48\,e^{j89.75} = \boxed{0.6+j\,139.5}\ /\text{km} \leftarrow$

$\lambda = \frac{2\pi}{0.1395} = 45\,m$ so $\ell = 2.22\ldots\lambda$

$\therefore v_{ph} = \frac{\omega}{\beta} = \frac{4\pi\times10^6\times10^3}{1395} = 0.09\times10^9 = \underline{0.9\times10^8\ \text{m/sec}}$

7.17 b) $\Gamma_{Load} \cong \dfrac{75+j150-126}{75+j150+126} = \dfrac{-51+j150}{201+j150} = \dfrac{158.4\,e^{j108.8^\circ}}{250.8\,e^{j36.73^\circ}}$

or $\boxed{\Gamma_L = 0.63\,e^{j72^\circ} = 0.195+j0.6}$

$\therefore \Gamma(0) = \Gamma_L\, e^{-j2\beta l}\, e^{-2\alpha l} = 0.63\,e^{j72^\circ}\, e^{-j2\times 0.1395\times 100}\, e^{-2\times 0.6\times 10^{-3}\times 100}$

or $\Gamma(0) = 0.63\, e^{j(72^\circ - 1598.6^\circ)}\, \underbrace{e^{-0.12}}_{0.8869}$

$\Gamma(0) = 0.5588\,e^{-j86.55^\circ}$ $\boxed{= 0.0336 - j0.5578}$

$Z(0) \cong 126\,\dfrac{1+\Gamma(0)}{1-\Gamma(0)} = \dfrac{1.0336 - j0.5578}{0.9664 + j0.5578}\,126 = 126\,\dfrac{1.175\,e^{-j28.35^\circ}}{1.116\,e^{+j29.99^\circ}} = 132.7\,e^{-j58.3}$

or $\boxed{Z(0) = 69.65 - j112.95}$

c) $P_{ave} = \frac{1}{2}\mathrm{Re}\{|\hat{I}_L|^2 Z_L\} = \frac{1}{2}|\hat{I}_L|^2\, 75$

$\hat{I}(0) = \dfrac{100}{75+69.65-j112.95} = \dfrac{100}{144.65-j112.95} = \dfrac{100}{183.52\,e^{-j37.98^\circ}}$

$\hat{I}(0) = 0.5449\,e^{j37.98^\circ}$

$\dfrac{\hat{I}_L}{\hat{I}(0)} = \dfrac{V^+ e^{-j\beta l} e^{-\alpha l}}{V^+ e^{0}} \cdot \dfrac{1-\Gamma_L}{1-\Gamma(0)} = e^{-j0.1395\times 100}\, e^{-0.06}\, \dfrac{0.805 - j0.6}{0.9664 + j0.5578}$

$-799.27^\circ = -79.28^\circ$

or $\hat{I}_L = 0.5449\,e^{j37.98^\circ} \times e^{-j79.28^\circ} \times 0.942 \times \dfrac{1.004\,e^{-j36.7^\circ}}{1.116\,e^{j29.99^\circ}}$

$\hat{I}_L = 0.461\,e^{-j107.99^\circ}$

$\therefore P_{ave,\,load} = \frac{1}{2}|0.461|^2 \times 75 = \boxed{7.97\ \text{Watts}}$

Iskander Problem 7-17

$R := 150 \quad L := 1.4\cdot 10^{-3} \quad C := 88\cdot 10^{-9} \quad G := 0.8\cdot 10^{-6} \quad \omega := 4\cdot\pi\cdot 10^{6} \quad j := \sqrt{-1}$

$\gamma := \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)}$ $\quad \gamma = 0.595 + 139.482i$

$Zo := \sqrt{\dfrac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}}$ $\quad Zo = 126.132 - 0.538i$

$\Gamma L := \dfrac{75 + j\cdot 150 - Zo}{75 + j\cdot 150 + Zo}$ $\quad \Gamma L = 0.195 + 0.604i$

$\Gamma o := \Gamma L\cdot e^{-2\cdot\gamma\cdot 0.1}$ $\quad \Gamma o = 0.037 - 0.562i$

$Zin := Zo\cdot\dfrac{1 + \Gamma o}{1 - \Gamma o}$ $\quad Zin = 68.802 - 114.39i$

$Io := \dfrac{100}{75 + Zin}$ $\quad Io = 0.426 + 0.339i$

$IL := e^{-\gamma\cdot 0.1}\cdot Io\cdot\left(\dfrac{1 - \Gamma L}{1 - \Gamma o}\right)$ $\quad IL = -0.142 - 0.441i$

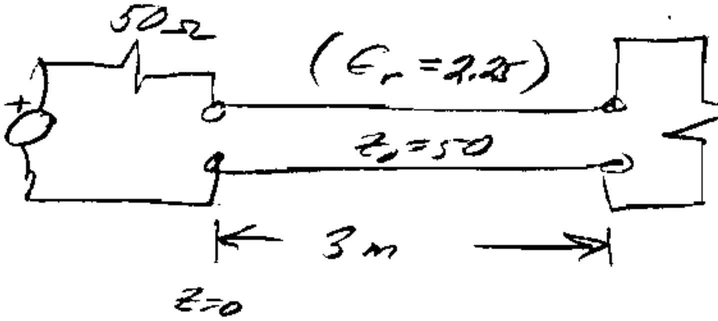
$Pave := \dfrac{1}{2}\cdot(|IL|)^2\cdot 75$ $\quad Pave = 8.042$

EE 333 Assignment 11 10.3, 10.15

10.1

$\hat{V}_g = 200$

$f = 50\,\text{MHz}$



a) $j\beta = j\frac{2\pi}{\lambda} = j\frac{2\pi\sqrt{\epsilon_r}\,f}{3\times10^8} = j\frac{2\pi\times1.5\times0.5\times10^8}{3\times10^8} = j\frac{\pi}{2}$

$v_{phase} = \frac{3\times10^8}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{1.5} = 2\times10^8\ \text{m/sec} = \frac{2}{3}c$

$\therefore$ line length in wavelengths $= \frac{3}{4}$

b) $\Gamma_{load} = \frac{100+j50-50}{100+j50+50} = \frac{50+j50}{150+j50} = \frac{1+j}{3+j} = \frac{\sqrt{2}\,e^{j45^\circ}}{\sqrt{10}\,e^{j18.43^\circ}}$

$= 0.447\,e^{j26.57^\circ}$

$\Gamma(0) = 0.447\,e^{j26.57^\circ}\underbrace{e^{-j\pi 3}}_{-1} = -0.447\,e^{j26.57^\circ}$

(44.7% voltage reflection)

c) $Z_{in} = Z(0) = 50\,\frac{1+\Gamma(0)}{1-\Gamma(0)} = 50\,\frac{1-\frac{1+j}{3+j}}{1+\frac{1+j}{3+j}} = 50\,\frac{3+j-1-j}{3+j+1+j} = 50\,\frac{2}{4+2j}$

$$\boxed{Z_{in} = \frac{\cancel{100}^{\,5}(4-2j)}{\cancel{20}} = 20-10j}$$

d) $\hat{I}_{in} = \frac{200}{50+Z_{in}} = \frac{200}{70-10j} = \frac{2\cancel{00}(7\cancel{0}+j1\cancel{0})}{5\cancel{000}} = \boxed{\frac{14}{5}+j\frac{2}{5}}$ $2.8+j0.4$

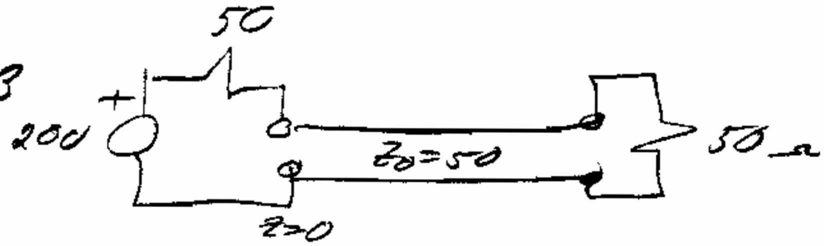
so $P_{ave_{in}} = \frac{1}{2}|I_{in}|^2 20 = \boxed{80\ \text{Watts}}$

e) $\hat{I}_{load} = e^{-j\frac{\pi}{2}\times3}\,\hat{I}_{in}\,\frac{1-\Gamma_{load}}{1-\Gamma(0)} = \frac{2}{5}+j\frac{6}{5}$ $0.4+j1.2$ $1.265\,e^{j71.56^\circ}$

$P_{ave_{load}} = \frac{1}{2}|I_L|^2\,100 = 80\ \text{Watts}$

$\boxed{\text{no line loss so } P_{in} = P_{load}}$

10.3



a) $\Gamma_{load} = 0 \quad \therefore \Gamma(z) = 0$ ←

$$Z(z) = \frac{V(z)}{I(z)} = Z_0$$ ←

b) $\hat{V}(z) = \hat{V}_m^+ e^{-j\beta z}$; $\hat{I}(z) = \frac{\hat{V}_m^+}{Z_0} e^{-j\beta z}$ ← $\boxed{\beta = \frac{\pi}{2}}$

$$\left.\begin{aligned}\hat{V}(z) &= \hat{V}_m^+ \\ \hat{I}(z) &= \hat{I}_m^+\end{aligned}\right\} @ \ \beta z = n2\pi \quad n = 0,1,2,3\ldots$$ ←

c) $\hat{I}_m = \frac{200}{50+50} = 2 = \hat{I}_m^+$ ←

$\therefore \hat{V}_m^+ = 100$ ←

$$\left.\begin{aligned}\hat{V}(z) &= 100 e^{-j\frac{\pi}{2}z} \\ \hat{I}(z) &= 2 e^{-j\frac{\pi}{2}z}\end{aligned}\right\}$$ ←

d) $P_{ave_{in}} = \frac{1}{2} |I(0)|^2 50 = 100$ Watts

$P_{ave_{load}} = \frac{1}{2} |I(z=3)|^2 50 = 100$ Watts

$|\hat{I}| \neq f(z)$!

10.15 $z_{L_{norm}} = 2 + j1 \quad (0.75 - 0.2865)\lambda = 0.4635\lambda$

from chart $z_{in_{norm}} = 0.4 - j0.2$! ←

$\Gamma_{load} = 0.45 e^{j26^\circ}$

$\Gamma_{in} = 0.45 e^{-j153.5^\circ}$

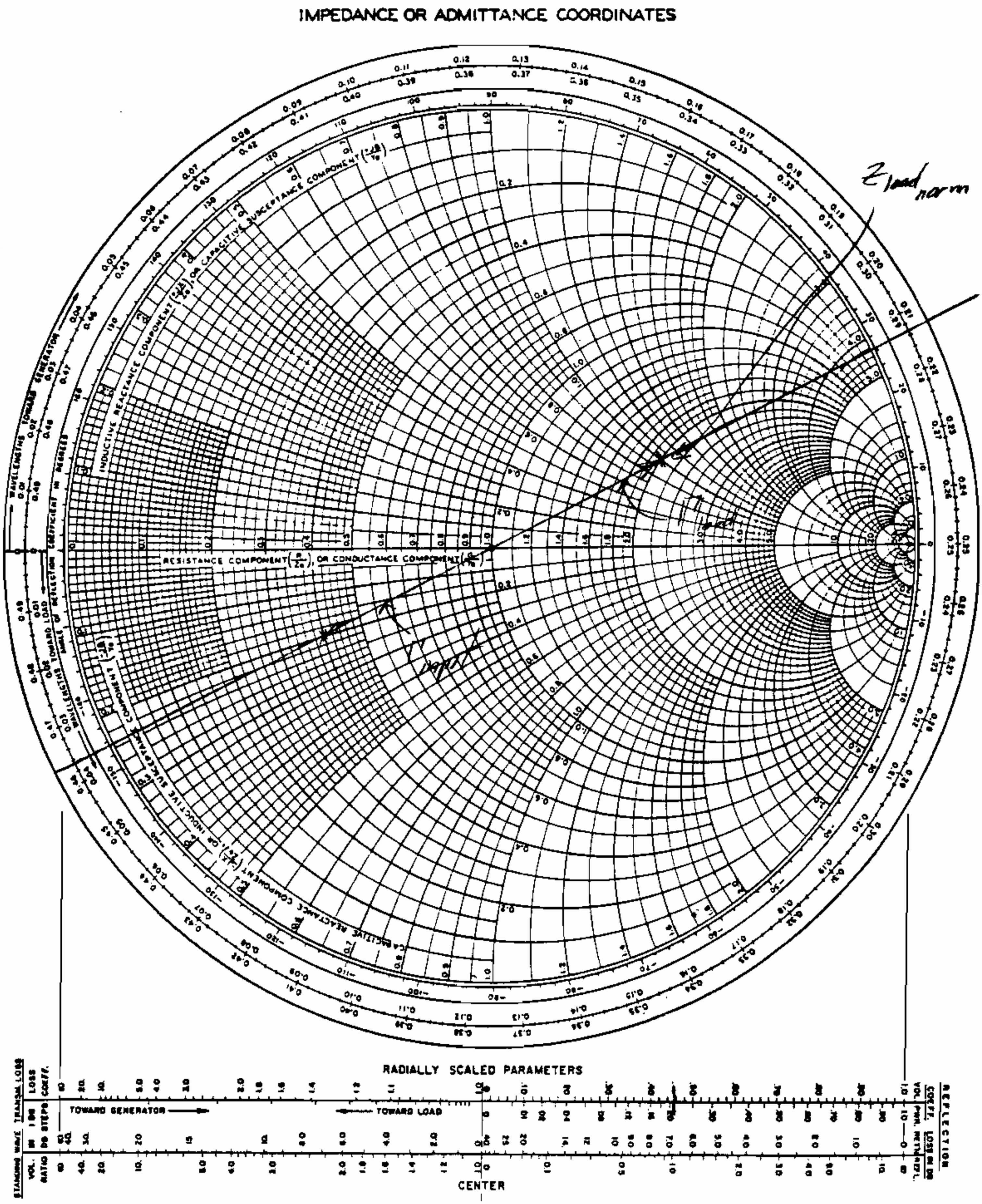


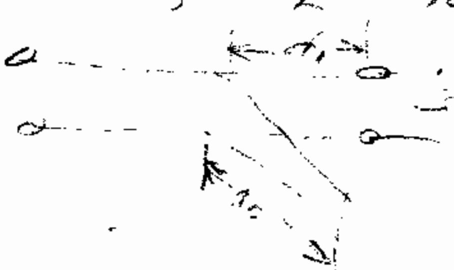
Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 333 Assignment 13

7.25 $f = 1\,GHz$, $Z_L = \frac{Z_0}{10}$

for air line

$\lambda = \frac{3\times10^8}{10^9} = 0.3\,m$



from Smith Chart $Y_{@stub_{norm}} = 1 - j2.$ $\therefore$ $\boxed{d_1 = 0.05\lambda \text{ or } d_1 = 1.5\,cm}$

$Y_{stub_N} = +j2.8$ $\boxed{\therefore d_2 = 0.25 + 0.196 = 0.446\lambda = 0.134\,m}$

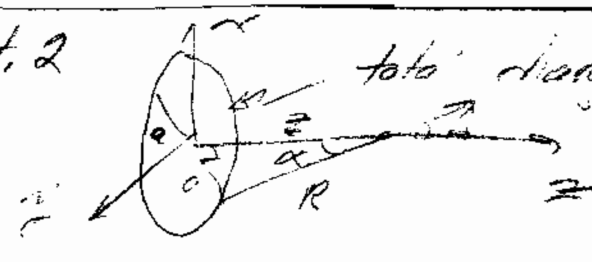
4.1

a) $\Phi_{\text{due to } Q_a} = \frac{Q_a}{4\pi\epsilon r}$

$\boxed{\therefore \Phi(r=a) = \frac{Q_a}{4\pi\epsilon a}}$

b) $\Phi = \Phi_{Q_a} + \Phi_{Q_b} = \frac{Q_a}{4\pi\epsilon b} + \frac{Q_b}{4\pi\epsilon b}$

4.2



a) $\boxed{\Phi = \frac{Q}{4\pi\epsilon R} = \frac{Q}{4\pi\epsilon\sqrt{a^2+z^2}}}$

b) $\bar{E} = -\nabla\Phi = -\frac{\partial\Phi}{\partial\rho}\bar{a}_\rho - \frac{1}{\rho}\frac{\partial\Phi}{\partial\phi}\bar{a}_\phi - \frac{\partial\Phi}{\partial z}\bar{a}_z = +\frac{Q\,2z}{8\pi\epsilon(a^2+z^2)^{3/2}}\bar{a}_z$ 4

c) $dE_z = \frac{Q\,a\,d\phi\cos\alpha}{2\pi a\;4\pi\epsilon R^2} = \frac{Q\,z\,d\phi}{8\pi^2\epsilon R^3} = \frac{Q\,z\,d\phi}{8\pi^2\epsilon(a^2+z^2)^{3/2}}$

$E_z = \int_{\phi=0}^{2\pi} dE_z = \frac{Q\,z}{4\pi\epsilon(a^2+z^2)^{3/2}}$, check.

Problem 7.25

IMPEDANCE OR ADMITTANCE COORDINATES

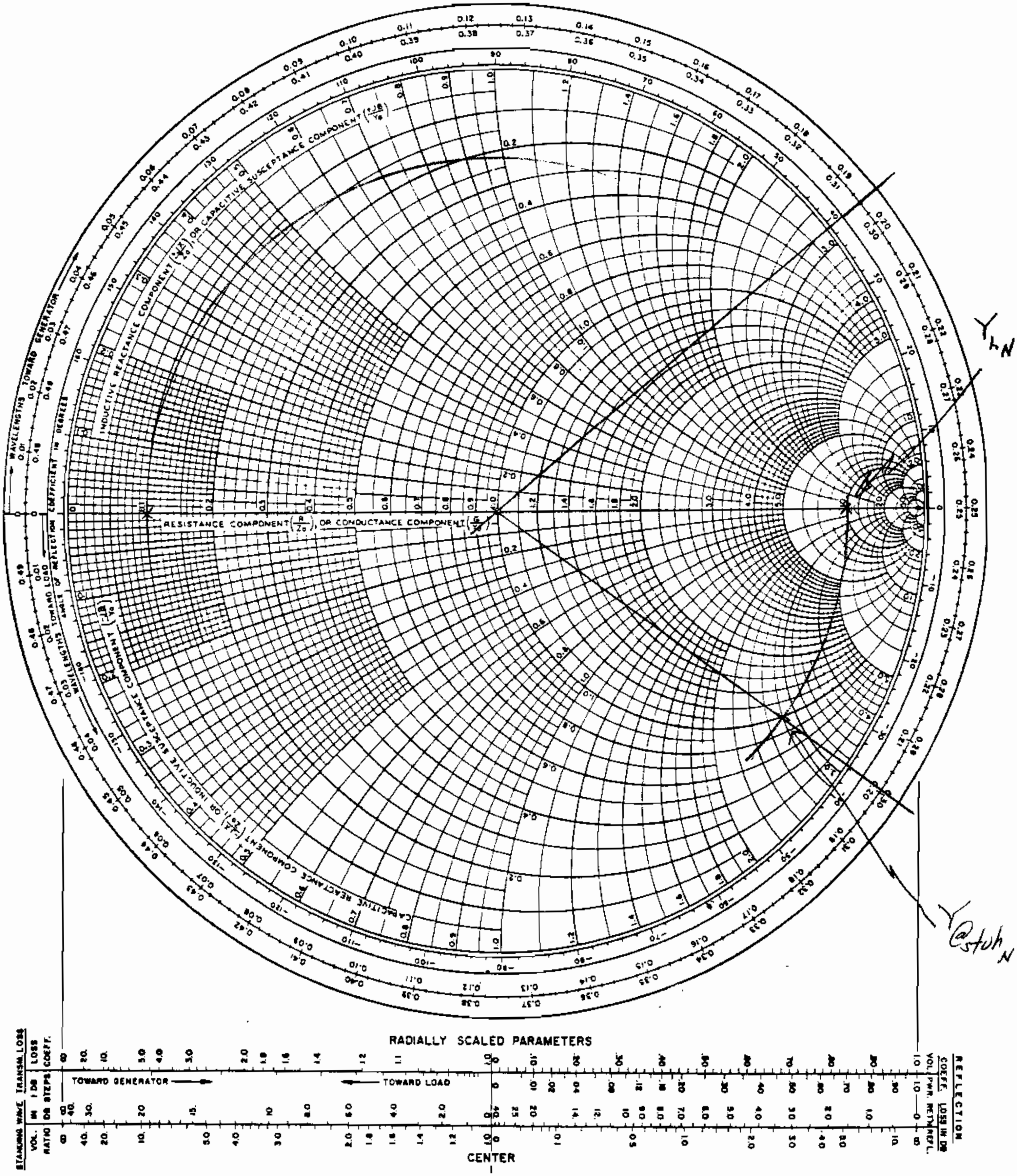
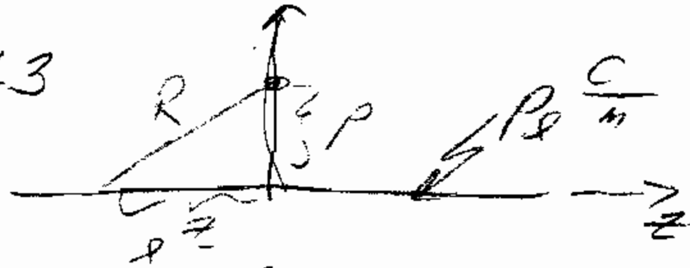


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 333 Homework 14

4.3



a) $\Phi = \int_{-\infty}^{\infty} \frac{\rho_\ell \, dz}{4\pi\epsilon R}$

$$\Phi = \lim_{\ell \to \infty} \int_{-\ell}^{\ell} \frac{\rho_\ell \, dz}{4\pi\epsilon\sqrt{\rho^2+z^2}} = \frac{\rho_\ell}{4\pi\epsilon} \ln\left(z+\sqrt{\rho^2+z^2}\right)\Big|_{-\ell}^{\ell}$$

lim as $\ell \to \infty$

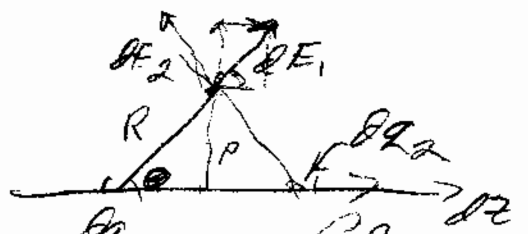
$$\Phi = \lim_{\ell \to \infty} \frac{\rho_\ell}{4\pi\epsilon} \ln\left[\frac{(\ell+\sqrt{\rho^2+\ell^2})}{(-\ell+\sqrt{\rho^2+\ell^2})}\right] \Rightarrow \infty$$

or $\Phi = \lim_{\ell \to \infty} \frac{\rho_\ell}{2\pi\epsilon} \ln \frac{\sqrt{\rho^2+\ell^2}+\ell}{\rho}$

$E = -\nabla\Phi$ so $E_\rho = -\frac{d\Phi}{d\rho} = \lim_{\ell \to \infty} \frac{\rho_\ell}{2\pi\epsilon} \frac{\ell}{\rho\sqrt{\rho^2+\ell^2}}$

or $\boxed{E_\rho = \frac{\rho_\ell}{2\pi\epsilon\rho}}$

c)



$$dE_\rho = \frac{\rho_\ell \, dz \sin\theta}{4\pi\epsilon R^2}$$

$$dE_\rho = \frac{\rho_\ell \, dz \, \rho}{4\pi\epsilon R^3} = \frac{\rho_\ell \, dz \, \rho}{4\pi\epsilon(z^2+\rho^2)^{3/2}}$$

so $E_\rho = \int_{-\infty}^{\infty} \frac{\rho_\ell \rho \, dz}{4\pi\epsilon(z^2+\rho^2)^{3/2}} = \frac{\rho_\ell \rho}{4\pi\epsilon}\left\{\frac{1}{\rho^2} \cdot \frac{z}{\sqrt{z^2+\rho^2}}\right\}_{-\infty}^{\infty}$

$$\boxed{E_\rho = \frac{\rho_\ell}{2\pi\epsilon\rho}}$$

total charge Q   Homework 14   (page 2)

4.7

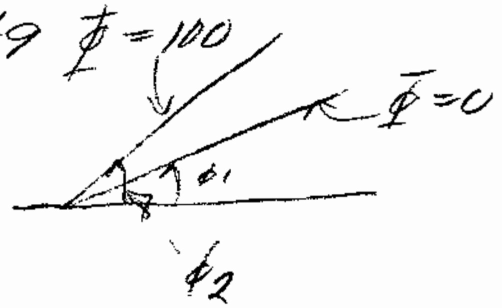
for $r>R$ $E = \frac{Q}{4\pi\epsilon r^2}$

energy density $= \frac{1}{2}\epsilon E^2 = \frac{1}{2}\epsilon \frac{Q^2}{16\pi^2\epsilon^2 r^4}$

$$\text{total energy} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=R}^{\infty} \frac{Q^2}{32\pi^2\epsilon r^4}\, r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$= -\frac{2\pi\times 2Q^2}{32\pi^2\epsilon}\cdot \frac{1}{r}\Big|_R^{\infty} = \boxed{\frac{Q^2}{8\pi\epsilon R}}$$

4.9



a) $\nabla^2 \Phi = 0 \overset{?}{=} \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \phi^2}$

so $\Phi = A\phi + B$

$0 = A\phi_1 + B \Rightarrow B = -A\phi_1$

applying boundary conditions: $100 = A\phi_2 + B$; $100 = A\phi_2 - A\phi_1$

or $A = \frac{100}{\phi_2 - \phi_1}$; $B = -\frac{100}{\phi_2-\phi_1}\phi_1$

$$\boxed{\Phi = \frac{100}{\phi_2-\phi_1}\cdot\phi - \frac{100}{\phi_2-\phi_1}\phi_1 = \frac{100}{\phi_2-\phi_1}\left[\phi - \phi_1\right]}$$

b) $\bar{E} = -\nabla\Phi = -\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\bar{a}_\phi = \boxed{-\frac{1}{\rho}\cdot\frac{100}{\phi_2-\phi_1}\bar{a}_\phi}$

c) for plate @ $\phi=\phi_1$ $\boxed{\rho_s = \bar{n}\cdot\bar{D} = \left(-\frac{100\epsilon}{\phi_2-\phi_1}\right)\cdot\frac{1}{\rho}}$

for plate @ $\phi=\phi_2$ $\boxed{\rho_s = +\frac{100\epsilon}{\phi_2-\phi_1}\cdot\frac{1}{\rho}}$