EE 333 Homework 1

1-2 $\bar{A}\cdot\bar{B} = -3 - 12 - 2 = -17 = |\bar{A}||\bar{B}|\cos\theta$

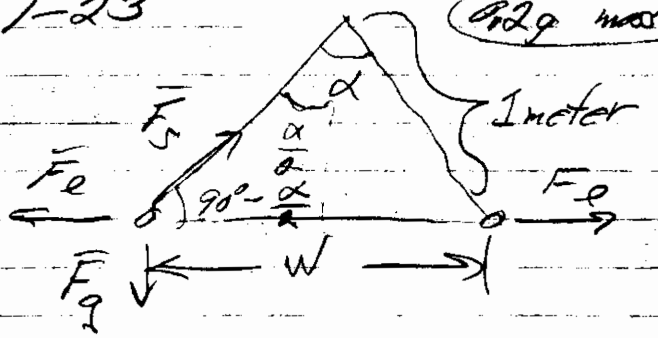
$|\bar{A}| = \sqrt{1+9+4} = \sqrt{14} = 3.7417$

$|\bar{B}| = \sqrt{9+16+1} = \sqrt{26} = 5.099$

$\therefore \cos\theta = \frac{17}{3.7414 \times 5.099} = 0.891$ or $\boxed{\theta \cong 27^\circ}$

parts b), c), and d) are on page 3

1-23



0.2g mass $\quad \bar{F}_s + \bar{F}_e + \bar{F}_g = 0$

$$\left\{ \begin{array}{l} F_s \cos\frac{\alpha}{2} = 0.2 \times 980 \times 10^{-5} \\ F_s \sin\frac{\alpha}{2} = Q\frac{Q}{4\pi\epsilon_0 W^2} \end{array} \right\}$$

(1) or $\cot\frac{\alpha}{2} = \left[\frac{0.2 \times 980 (4\pi\epsilon_0 W^2) \times 10^{-5}}{Q^2}\right]$ $\quad \alpha = 45^\circ$

$$Q^2 = \frac{0.2 \times 980 (\cancel{4\pi} \times \frac{1}{\cancel{36\pi}_9} \times 10^{-9} \times 10^{-5}) W^2}{2.415}$$

but $w = 2 \times 1 \times \sin\left(\frac{45}{2}\right) = 0.7667$

so $Q^2 = \frac{0.2 \times 980 \times 10^{-14} \times 0.5867}{9 \times 2.415} = 5.28 \times 10^{-14}$

or $\boxed{Q = 2.30 \times 10^{-7}}$

b) $\cot\frac{\alpha}{2} = \frac{0.2 \times 980 \times \cancel{4\pi} \times \frac{1}{\cancel{36\pi}} \times 10^{-9} \times 10^{-5} \times w^2}{9 \times 0.5^2 \times 10^{-12}}$ $\quad - 4\sin^2\frac{\alpha}{2}$

but $w = 2\sin\frac{\alpha}{2}$

or $\frac{\cos\frac{\alpha}{2}}{\sin^3\frac{\alpha}{2}} = 348.444 \times \frac{10^{-14}}{10^{-12}} = 3.4844$

$\boxed{\alpha = 76.71}$

76.71116

1-25 $\bar{F} = q(\bar{u}\times\bar{B} + \bar{E}) = q[u_0(3\bar{a}_x - \bar{a}_y + 2\bar{a}_z)\times B_0(\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z)$
$+\bar{E}] = 0$

or $u_0 B_0[6\bar{a}_z + 12\bar{a}_y + \bar{a}_z + \cancel{4\bar{a}_x} + 2\bar{a}_y - \cancel{4\bar{a}_x}] + \bar{E} = 0$

$$\boxed{u_0 B_0[14\bar{a}_y + 7\bar{a}_z] = -\bar{E}}$$

1-26 (z direction) magnetic force in $-x$ direction

1-30 $F = qE = m\frac{du}{dt} \Rightarrow du = \frac{q}{m}E\,dt$

@ $t=0$ $u=0$ $\therefore$ $u(t) = \frac{q}{m}Et \Rightarrow dx = \frac{q}{m}E\,t\,dt$

integrating: $x = \frac{q}{m}E\frac{t^2}{2}$ for $x=0$ @ $t=0$

$\therefore$ t for transit through accelerating region is:

$$t = \sqrt{\frac{2mx}{qE}} \quad \text{but } E = \frac{V}{x}$$

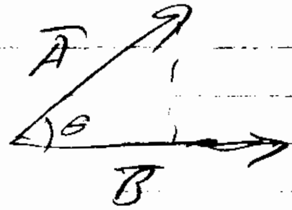
so $t = \sqrt{\frac{2mx^2}{qV}}$ and $u_{@\,exit} = \frac{qV\cancel{x}}{m\cancel{x}}\sqrt{\frac{2m}{qV}}$

a) $$\boxed{\text{or } u = \sqrt{\frac{qV2}{m}}}$$

b) circle (constant force normal to velocity)

c) ? $\bar{B}$ field does not change magnitude of $\bar{u}$!

1-2 b)



projection of $\bar{A}$ onto $\bar{B} = \bar{A}\cdot\bar{B}/|\bar{B}|$

$$\boxed{= \frac{17}{\sqrt{26}} = 3.334}$$

c) $\perp$ vector $= \bar{A}\times\bar{B} = 4\bar{a}_z + \bar{a}_y - 9\bar{a}_z + 3\bar{a}_x - \bar{a}_y - 8\bar{a}_x$

or $\bar{A}\times\bar{B} = -5\bar{a}_x - 5\bar{a}_y - 5\bar{a}_z$

unit $\perp$ vector $= -\dfrac{5\bar{a}_x + 5\bar{a}_y + 5\bar{a}_z}{\sqrt{75}} = -\dfrac{1}{\sqrt{3}}\left(\bar{a}_x + \bar{a}_y + \bar{a}_z\right)$

d) $\bar{A}\cdot\bar{a}_r = A_r$ component of $\bar{A}$ in r direction

$A_r = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$

$$\boxed{\text{so } \bar{A}\cdot\bar{a}_r = \sin\theta\cos\phi - 3\sin\theta\sin\phi + 2\cos\theta}$$

EE 333 Homework 2

1-48 $\rho_v = \rho_0 \left(1 - \frac{r^2}{a^2}\right) \quad r < a$

$\rho_v = 0 \quad r > a$

for $r < a$ $$\int_{\phi=0}^{2\pi}\int_{\theta=0}^{2\pi} \epsilon_0 E_r r^2 \sin\theta \, d\theta \, d\phi = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{r} \rho_0 \left(1 - \frac{r^2}{a^2}\right) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\cancel{4\pi} r^2 \epsilon_0 E_r = \cancel{4\pi} \rho_0 \left[\frac{r^3}{3} - \frac{r^5}{5a^2}\right]$$

$$\therefore \quad E_r = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{r^3}{5a^2}\right] \longleftarrow$$

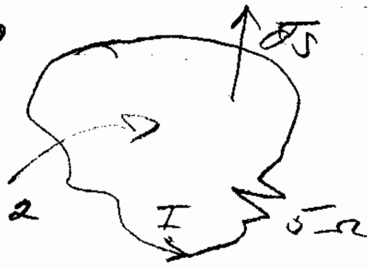
for $r > 0$

$$4\pi r^2 \epsilon_0 E_r = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{a} \rho_0 \left(1 - \frac{r^2}{a^2}\right) r^2 \sin\theta \, dr \, d\theta \, d\phi$$

$$\cancel{4\pi} r^2 \epsilon_0 E_r = \cancel{4\pi} \rho_0 \left[\frac{a^3}{3} - \frac{a^3}{5}\right]$$

$$\text{or} \quad E_r = \frac{\rho_0}{\epsilon_0 r^2}\left[\frac{a^3}{3} - \frac{a^3}{5}\right] = \frac{2\rho_0 a^3}{15 \epsilon_0 r^2} \longleftarrow$$

1-50



$A = 0.1 \, m^2$

$\bar{B} = 0.2 \sin 10^3 t \, \bar{a}_z$

Use Faraday's Law

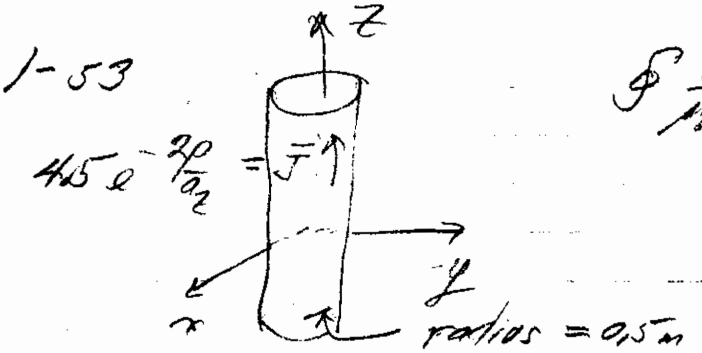
$$\oint \bar{E} \cdot d\bar{l} = -\frac{d}{dt}\int \bar{B} \cdot d\bar{s} \quad = \text{emf around the loop!}$$

$$\text{emf} = -\frac{d}{dt}\left(0.2 \sin 10^3 t \, \bar{a}_z \cdot 0.1 \, \bar{a}_z\right)$$

$$\text{emf in volts} = -0.02 \times 10^3 \cos 10^3 t$$

$$\therefore \quad I = \frac{\text{emf}}{5} = -4 \cos 10^3 t \longleftarrow$$

1-53



$$\oint \frac{\bar B}{\mu_0}\cdot d\bar l = \int \bar J \cdot d\bar s$$ for no time variation

for $\rho < 0.5\,m$
$$\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_0}\bar a_\phi \cdot \rho\, d\phi\, \bar a_\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} 4.5\, e^{-2\rho}\rho\, d\rho\, d\phi$$

$$\frac{1}{\mu_0}\cdot 2\pi\rho B_\phi = 2\pi\times 4.5\int_0^\rho \rho e^{-2\rho}\,d\rho = 9\pi\left\{e^{-2\rho}\left[\frac{\rho}{-2}-\frac{1}{4}\right]\right\}_0^\rho$$

$$B_\phi = \frac{9\mu_0}{2\rho}\left\{e^{-2\rho}\left[-\frac{\rho}{2}-\frac{1}{4}\right]+\frac{1}{4}\right\}$$

$$B_\phi = \frac{9\mu_0}{4\cdot 2\rho}\left\{-e^{-2\rho}\rho\left[\frac{1}{2}+\frac{1}{4\rho}\right]4+1\right\}$$

$$B_\phi = 1.125\times\frac{\mu_0}{\rho}\left\{1-2\rho e^{-2\rho}-e^{-2\rho}\right\}$$

for $\rho > 0.5\,m$

$$\frac{1}{\mu_0}2\pi\rho B_\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\frac{1}{2}} 4.5\, e^{-2\rho}\rho\, d\rho\, d\phi$$

$$B_\phi = \frac{\mu_0 9\pi}{2\pi\rho}\int_0^{\frac{1}{2}}\rho e^{-2\rho}\,d\rho = \frac{9\mu_0}{2\rho}\left\{e^{-1}\left[-\frac{1}{2}\right]+\frac{1}{4}\right\}$$

$$B_\phi = \frac{9\mu_0}{8\rho}\left\{1-2e^{-1}\right\} = 1.125\mu_0(0.2642)\frac{1}{\rho}$$

$$\boxed{B_\phi = 0.297\mu_0\times\frac{1}{\rho}}$$

EE 333 (Homework 3)

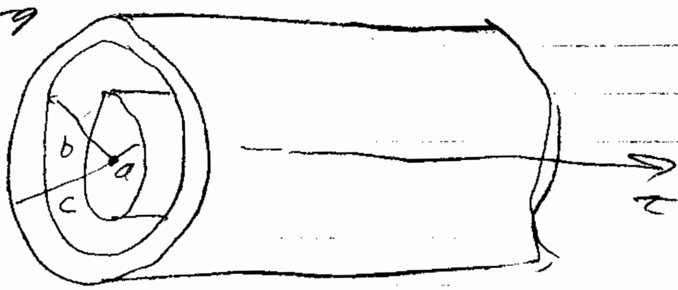
1-55 $B_\phi = \frac{\mu_0 I \cos\omega t}{2\pi\rho}$ for $\rho > a$ (from class notes)

(i) total flux through loop $= \int_{\rho=d}^{d+a}\int_{z=0}^{b} \frac{\mu_0 I}{2\pi\rho}\, dz\, d\rho \cos\omega t$

total flux $= \frac{\mu_0 I b}{2\pi}\ln\frac{d+a}{a}\cos\omega t$

(ii) emf $= -\frac{\partial}{\partial t}(\text{total flux}) = \frac{\omega\mu_0 I b}{2\pi}\ln\frac{d+a}{a}\sin\omega t$

1-59



$J_z = \frac{I}{\pi a^2}$ for $\rho < a$

$J_z = -\frac{I}{\pi(c^2-b^2)}$ for $b < \rho < c$

zero everywhere else

for $\rho < a$ $\quad \int_0^{2\pi}\frac{B_\phi}{\mu_0}\rho\, d\phi = I\frac{\pi\rho^2}{\pi a^2} = \frac{2\pi B_\phi \rho}{\mu_0}$ so $\boxed{B_\phi = \frac{\mu_0 I \rho}{2\pi a^2}}$

for $a < \rho < b$ $\quad \frac{2\pi\rho B_\phi}{\mu_0} = I$ giving $\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}}$

for $b < \rho < c$ $\quad \frac{2\pi\rho B_\phi}{\mu_0} = I\left\{1 - \frac{\pi(\rho^2-b^2)}{\pi(c^2-b^2)}\right\} = I\left\{\frac{\pi(c^2-b^2-\rho^2+b^2)}{\pi(c^2-b^2)}\right\}$

$$\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}\left\{\frac{c^2-\rho^2}{c^2-b^2}\right\}}$$

for $\rho > c$ $\quad \boxed{B_\phi = 0}$

EE 333 Homework 3 (page 2)

2-1 $\psi = E_0 [1-(\frac{a}{\rho})^3] z \cos\phi$

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\bar{a}_\phi + \frac{\partial\psi}{\partial z}\bar{a}_z$$

$$\therefore \nabla\psi = +E_0 a^3 z\cos\phi\left(\frac{1}{\rho^4}\right)\bar{a}_\rho - E_0\left[1-\left(\frac{a}{\rho}\right)^3\right] z\sin\phi\,\bar{a}_\phi + E_0\left[1-\left(\frac{a}{\rho}\right)^3\right]\cos\phi\,\bar{a}_z$$

2-2 a) $\nabla\cdot\bar{A} = 0$

b) $\nabla\cdot\bar{B} = 0$

c) $\nabla\cdot\bar{C} = \frac{1}{r^2}\frac{\partial(r^2\cdot r)}{\partial r} = 3$

d) $\nabla\cdot\bar{D}\big|_{r=3} = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\cdot 2r^2)\Big|_{r=3} = \frac{8r^3}{r^2}\Big|_{r=3} = 24$

e) $\nabla\cdot\bar{E} = 3+1-1 = 3$

2-12 $\bar{E} = \frac{\rho_V r}{3\epsilon_0}\bar{a}_r$

for static field $\nabla\times\bar{E} = 0$; the curl of the above is zero because it has only E_r which is not a function of θ or ϕ

$$\nabla\cdot\epsilon_0\bar{E} = \rho_V = \epsilon_0\left\{\frac{1}{r^2}\frac{\partial\left(\frac{\rho_V r^3}{3\epsilon_0}\right)}{\partial r}\right\} = \epsilon_0\frac{3r^2\rho_V}{3\epsilon_0 r^2} = \rho_V$$

2-14 $\bar{B} = \frac{1}{r^2}\sin\phi\cos^2\theta\,\bar{a}_r$ static field??

$\nabla\cdot\bar{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(\sin\phi\cos^2\theta) = 0$! $\nabla\times\frac{\bar{B}}{\mu_0} = \bar{J}$

$$\text{so } \bar{J} = \frac{1}{\mu_0 r\sin\theta}\cdot\frac{1}{r^2}\cos\phi\cos^2\theta\,\bar{a}_\theta + \frac{1}{\mu_0 r}\cdot\frac{2}{r^2}\sin\phi\cos\theta\sin\theta\,\bar{a}_\phi$$

$$\bar{J} = \frac{1}{\mu_0 r^3}\left\{\cos\phi\frac{\cos^2\theta}{\sin\theta}\bar{a}_\theta + 2\sin\phi\cos\theta\sin\theta\,\bar{a}_\phi\right\}$$

EE 333 Homework 4

2-26 $\hat{\bar{E}} = 50 e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{3}z} \bar{a}_x$

a) $\boxed{\bar{a}_x}$

b) $\boxed{+\bar{a}_z}$

c) $\beta = \frac{\pi}{3}$ $\therefore$ $\frac{\pi}{3}\lambda = 2\pi$ or $\boxed{\lambda = 6}$

d) $\beta = \omega\sqrt{\mu_0\epsilon_0} = \frac{2\pi f}{c} = \frac{\pi}{3}$ $\therefore$ $\boxed{f = \frac{c}{6} = \frac{1}{2}\times 10^8}$

e) $\boxed{\hat{\bar{H}} = \frac{50}{120\pi} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{3}z} \bar{a}_y}$

f) $\bar{E} = Re\{50 e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{3}z} e^{j\omega t}\} \bar{a}_x$

$$\boxed{\bar{E} = 50\cos\left(\omega t - \frac{\pi}{3}z + \frac{\pi}{4}\right)\bar{a}_x}$$

EE 333 Homework 5

2-33 $\overline{H} = 0.15 \cos(\omega t + \frac{2\pi}{7} z + \frac{\pi}{13})\, \overline{a}_y$

a) $\beta_0 = \frac{2\pi}{7}$; $\therefore \beta_0 \lambda = 2\pi$ or $\boxed{\lambda = 7\,m}$
also $\beta_0 = 2\pi f / c$ so $\frac{2\pi}{7} = 2\pi f / c$ or $\boxed{f = \frac{c}{7} = 0.428 \times 10^8}$

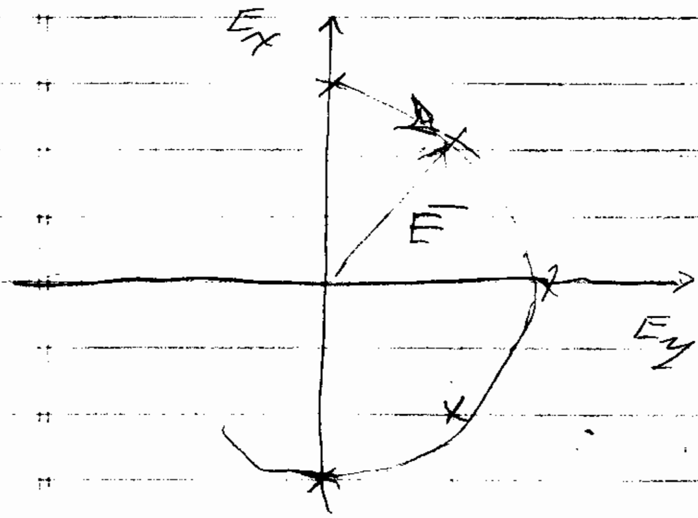
b) $\boxed{|\overline{H}| = 0.15}$ $|\overline{E}| = 120\pi |\overline{H}| = 56.55$

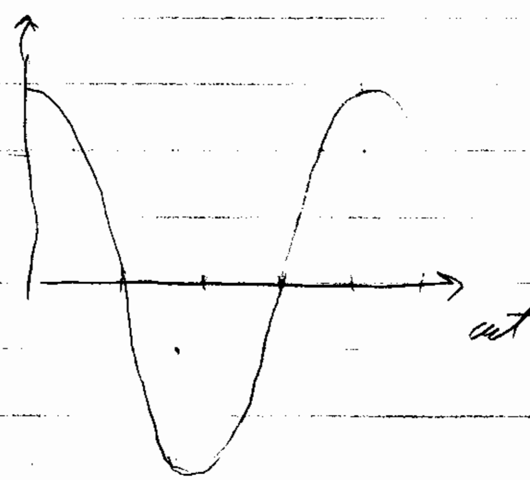
c) $\boxed{\hat{E} = 56.55\, e^{j(\frac{2\pi}{7} z + \frac{\pi}{13})} (-\overline{a}_x)}$

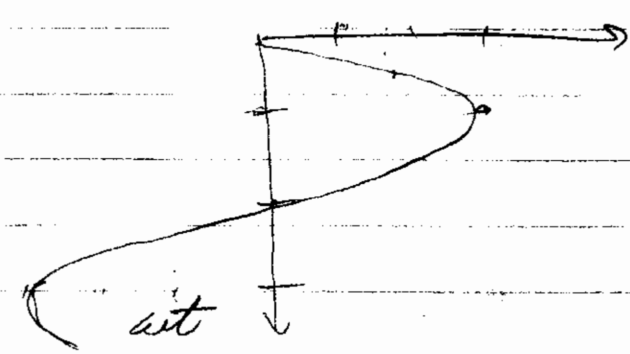
Special $\hat{E} = 500\, e^{-j\beta_0 z} (\overline{a}_x - j\overline{a}_y)$

a) $\overline{E} = 500 \cos(\omega t - \beta_0 z)\, \overline{a}_x + 500 \sin(\omega t - \beta_0 z)\, \overline{a}_y$

b) $\hat{H} = \frac{500}{120\pi} e^{-j\beta_0 z} (\overline{a}_y + j\overline{a}_x)$







CW or right hand circular polarization

EE 333 Homework 6

3-13 $\bar{E} = 10\cos\omega t\, \bar{a}_z$, $A = 10^{-2}$

a) $\bar{J}_c = \sigma \bar{E} = 0$

$\bar{J}_{displacement} = \dfrac{\partial(\epsilon_0 \bar{E})}{\partial t} = \boxed{-10\omega\epsilon_0 \sin\omega t\, \bar{a}_z}$

$\boxed{\text{so } I_{total} = -0.1\,\omega\epsilon_0 \sin\omega t}$

b) $\epsilon = 80\epsilon_0$, $\sigma = 4\,S/m$, $f = 10^8$

$\dfrac{|\bar{J}_c|}{|\bar{J}_d|} = \dfrac{\overset{\text{"}\sigma\text{"}}{\cancel{4}} \times 36\cancel{\pi} \times \cancel{10}}{\cancel{10} \times 2\cancel{\pi} \times 10^8 \times 10^{-9} \times \underset{20}{\cancel{80}}} = \dfrac{36 \times 10}{40} \doteq 9$

$\bar{I}_{total} = 10^{-2}(\bar{J}_c + \bar{J}_d) = 10^{-2}(40\cos\omega t - 10\omega\epsilon_0 \times 80 \sin\omega t)\,\bar{a}_z$

$\bar{I}_{total} = \left[0.4\cos\omega t - 10^{-1}\dfrac{2\cancel{\pi} \times 10^8 \times 80 \times 10^{-9}}{\underset{18}{\cancel{36\pi}}} \sin\omega t\right]\bar{a}_z$

$\bar{I}_{total} = [0.4\cos\omega t - 4.44 \times 10^{-2} \sin\omega t]\,\bar{a}_z$

3-21 $\epsilon_r = 6.3$, $\mu_r = 1.98$, $f = 10^9$, assume $\sigma = 0$

$\bar{E}(z,t) = 100\cos(\omega t - \beta z)\,\bar{a}_x$

a) $\beta = \omega\sqrt{\mu\epsilon} = \dfrac{2\pi \times 10^9 \times \sqrt{6.3 \times 1.98}}{3 \times 10^8} = \boxed{7.397 \times 10 \text{ rad/sec}}$

$\boxed{\lambda = \dfrac{2\pi}{\beta} = 8.49\,cm}$

$v_{phase} = \dfrac{\omega}{\beta} = \dfrac{2\pi \times 10^9}{73.97} = \boxed{0.0849 \times 10^9 \text{ m/sec}}$

b) $\eta = \sqrt{\dfrac{\mu}{\epsilon}} = 120\pi\sqrt{\dfrac{1.98}{6.3}} = \boxed{211\,\Omega}$

c) $\bar{H}(z,t) = \underset{\frac{\text{"}|E_x|\text{"}}{\eta}}{0.473} \cos(2\pi \times 10^9 t - 73.97z)\,\bar{a}_y$

3-22 sea water $\sigma = 4\ S/m$, $\mu_r = 1$, $\epsilon_r = 81$

$$100 < \left|\frac{\bar{J}_c}{\bar{J}_d}\right| = \left|\frac{\sigma \bar{E}}{j\omega\epsilon_0\epsilon_r\bar{E}}\right| = \frac{\sigma}{2\pi f\epsilon_0\epsilon_r}$$

$$\therefore f \le \frac{\sigma}{2\pi\epsilon_0\epsilon_r \times 100} = \frac{\cancel{4}^{2} \times 36\cancel{\pi}}{\cancel{2\pi} \times 10^{-9} \times 81 \times 10^2} = 0.888 \times 10^7$$

$$\boxed{\text{or } f \le 8.88\ MHz}$$

3-26 a) $f = 24\ MHz$, $\phi = 1\ rad/m$
decrease by 1/2 for each meter travelled

$\therefore e^{-\alpha \times 1} = 0.5$ $\boxed{\alpha = -\ln 0.5 = 0.693}$

skin depth $\boxed{\delta = \frac{1}{\alpha} = 1.44m}$

$\boxed{\beta = \text{phase shift per meter} = 1\ rad/m}$

$\boxed{\lambda = \frac{2\pi}{\beta} = 2\pi\ m}$

$$u_{phase} = \frac{\omega}{\beta} = \frac{2\pi \times 24 \times 10^6}{1} = \boxed{150.8 \times 10^6\ m/sec}$$

EE 333 Homework 7

3-1 $\bar{E} = 3z^2 y \cos 10^8 t\, \bar{a}_x$, $\epsilon_r = 2.56$

a) $\bar{D} = \epsilon_0 \bar{E} + \bar{P} = \epsilon_r \epsilon_0 \bar{E}$

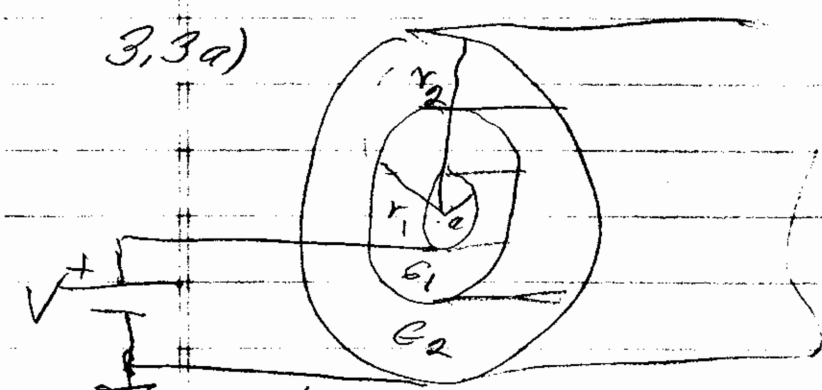
$\therefore \bar{P} = \epsilon_0 \bar{E}(\epsilon_r - 1) = \dfrac{3 \times 1.56 \times 10^{-9}}{36\pi} z^2 y \cos 10^8 t\, \bar{a}_x$

$$\boxed{\bar{P} = 4.138 \times 10^{-11} z^2 y \cos 10^8 t\, \bar{a}_x} \longleftarrow$$

b) $\rho_p = -\nabla \cdot \bar{P} = 0$ Field has only an x component that does not vary with x!

c) $\bar{J}_p = \dfrac{\partial \bar{P}}{\partial t} = -4.138 \times 10^{-3} z^2 y \sin 10^8 t\, \bar{a}_x$

3.3a)



$\epsilon_1 = 1.5\epsilon_0$, $\epsilon_2 = 4.5\epsilon_0$

ρ_l = charge/unit length

$$\boxed{\oint \bar{D} \cdot d\bar{s} = \int \rho_v\, dv}$$

Use circular cylindrical surface

a) $\displaystyle\int_{z=0}^{L} \int_{\phi=0}^{2\pi} D_\rho\, \rho\, d\phi\, dz = \rho_l L = 2\pi L D_\rho \rho$

$\therefore D_\rho = \dfrac{\rho_l}{2\pi\rho}$ for $a < \rho < r_2$ $\longleftarrow$

$E_\rho = \dfrac{\rho_l}{3\epsilon_0 \pi \rho}$ for $a < \rho < r_1$ $\longleftarrow$

$E_\rho = \dfrac{\rho_l}{9\epsilon_0 \pi \rho}$ for $r_1 < \rho < r_2$ $\longleftarrow$

$$P_\rho = \epsilon_0 E(\epsilon_r - 1) \begin{cases} \dfrac{\rho_l}{6\pi\rho} & \text{for } a < \rho < r_1 \\ \dfrac{\rho_l\, 3.5}{9\pi\rho} & r_1 < \rho < r_2 \end{cases} \longleftarrow$$

3.3 continued: Outside of cable charge enclosed is zero $\therefore \bar{D} = \bar{E} = \bar{P} = 0$

c) $\rho_p = -\nabla \cdot \bar{P} = -\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{7\rho\ell}{\pi m}\right) = 0$

Special Problem: Cu, $\sigma = 5.8 \times 10^7$ mho/m

$n = 10^{29}$ electrons/m^3

a) $\mu_e = -\frac{e\tau_c}{m}$; $\sigma = \frac{ne^2\tau_c}{m} = -ne\mu_e$

$\therefore \mu_e = \frac{\sigma}{ne} = \frac{5.8 \times 10^7}{10^{29} \times 1.6 \times 10^{-19}} = 3.625 \times 10^{-3}$

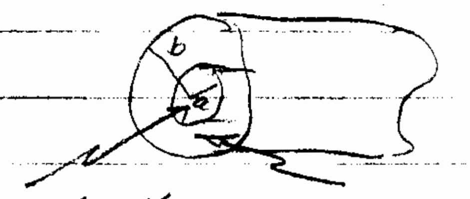
b) $\rho_v = 10^{29}(-1.6 \times 10^{-19}) = -1.6 \times 10^{10}$ C/m^3 $= -16$ C/mm^3

c) $\bar{u}_d = -\mu_e \bar{E} = -3.625 \times 10^{-3}\, \bar{a}_x$ m/sec

d) $\bar{J} = \rho_v \bar{u}_d = +1.6 \times 10^{10} \times 3.625 \times 10^{-3} = 5.8 \times 10^7$ A/m^2

or $\bar{J} = 58$ A/mm^2

EE 333 Homework 8



3-11 $\bar{J} = \frac{1}{2}\bar{a}_z$ for $\rho<a$

$\bar{J} = \frac{1}{2}\rho/a\,(-\bar{a}_z)$ for $a<\rho<b$

$\mu_1=\mu_{r_1}\mu_0 \quad \mu_2=\mu_{r_2}\mu_0$

a) $2\pi\rho H_\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \frac{1}{2}\bar{a}_z\cdot \rho\, d\phi\, d\rho\, \bar{a}_z = \frac{\pi\rho^2}{2}$ for $\rho<a$

$\boxed{\text{so } H_{\phi_1} = \rho/4 \text{ for } \rho<a}$

for $a<\rho<b$ $\quad 2\pi\rho H_\phi = \frac{1}{2}\pi a^2 - \int_{\phi=0}^{2\pi}\int_{\rho=a}^{\rho} \frac{\rho}{2a}\rho\, d\phi\, d\rho = \frac{\pi a^2}{2} - \frac{\pi}{a}\left[\frac{\rho^3}{3}-\frac{a^3}{3}\right]$

or $H_{\phi_2} = \frac{a^2}{4\rho} - \frac{\rho^2}{6a} + \frac{a^2}{6\rho} = \boxed{\frac{a^2}{\rho}\times\frac{5}{12} - \frac{\rho^2}{6a}}$

b) for $\rho<a$ $\quad B_{\phi_1} = \mu_{r_1}\mu_0 \times \rho/4$

for $a<\rho<b$ $\quad B_{\phi_2} = \mu_{r_2}\mu_0\left[\frac{5}{12}\cdot\frac{a^2}{\rho} - \frac{\rho^2}{6a}\right]$

c) $\bar{M} = \chi_m \bar{H}$ and $\chi_m = \mu_r - 1$

$\boxed{\text{so } \bar{M}_2 = (\mu_{r_2}-1)H_{\phi_2}\bar{a}_\phi} \quad a<\rho<b$

$\frac{\bar{J}_{m_2}}{\mu_{r_2}-1} = \nabla\times \bar{M}_2 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[-\frac{\rho^3}{6a}\right]\bar{a}_z = \boxed{-\frac{3\rho}{6a}\bar{a}_z}$

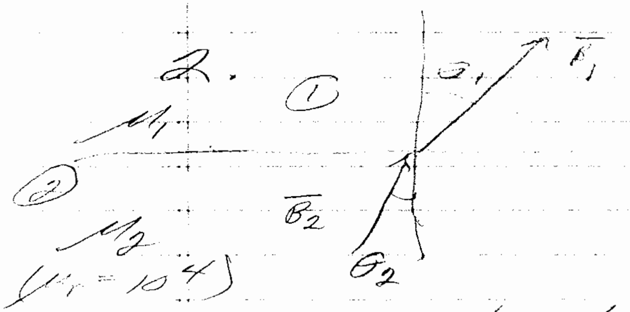
so $\bar{J}_{m_2} = \boxed{(\mu_{r_2}-1)\left[-\frac{\rho}{2a}\right]\bar{a}_z}$

d) $H_{\phi_1}(\rho=a) \stackrel{?}{=} H_{\phi_2}(\rho=a)$

$\frac{a}{4} = a\left[\frac{5}{12}-\frac{1}{6}\right] = a\left[\frac{3}{12}\right] = \frac{a}{4}$!

EE 333 Homework 8 (page 2)

2.



$\tan\theta_1 = \frac{B_{tan_1}}{B_{n_1}}$; $\tan\theta_2 = \frac{B_{tan_2}}{B_{n_1}}$

However $B_{n_1} = B_{n_2}$

$\therefore \frac{B_{tan_1}}{\tan\theta_1} = \frac{B_{tan_2}}{\tan\theta_2} \Rightarrow \frac{\mu_1 H_{tan_1}}{\tan\theta_1} = \frac{\mu_2 H_{tan_2}}{\tan\theta_2}$

but $H_{tan_1} = H_{tan_2}$ so $\boxed{\tan\theta_2 = \frac{\mu_2}{\mu_1}\tan\theta_1}$

$\theta_1 = \tan^{-1}\left[\frac{\mu_1}{\mu_2}\tan\theta_2\right]$

θ_2	θ_1
$0°$	$0°$
$45°$	5.7×10^{-3} °
$89°$	$0.328°$
$89.9°$	$3.27°$

EE 333 Homework 9

3-24 a)

free space	Conductive ~~Media~~
no attenuation	attenuation
$\bar{E}$ + $\bar{H}$ in phase (η real)	$\hat{\eta}$ $\bar{E}+\bar{H}$ out of phase
$v_{ph} = 3\times10^8$ m/sec	v_{ph} reduced
$P_{ave} = \frac{E_m^{+2}}{2\eta_0}$	$P_{ave} = \frac{E_m^{+2} e^{-2\alpha z}}{2\lvert\hat{\eta}\rvert}\cos\theta$
	where $\hat{\eta} = \lvert\hat{\eta}\rvert e^{j\theta}$

b) $\mu_r = 1$, $\epsilon_r = 79$, $\sigma = 3$ S/m, $E(z=0) = 10$ V/m

i) $f = 2\times10^4$; $\frac{\sigma}{\omega\epsilon} = \frac{3\times36\pi}{2\pi\times2\times10^4\times79\times10^{-9}} = 0.34177\times10^5$

conduction current dominate (good conductor) &

so $\alpha = \beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \frac{4\pi\times10^4\sqrt{79}}{\sqrt{2}\times3\times10^8}\sqrt{0.34177\times10^5}$

$\alpha = 4867.7\times10^{-4} = \boxed{0.48677}$

$10e^{-\alpha z} = 10^{-5}$; $e^{-\alpha z} = 10^{-6}$

$-\alpha z = -13.8155$ $\therefore z = \frac{13.8155}{0.48677} = \boxed{28.38 \text{ m}}$

ii) $\frac{\sigma}{\omega\epsilon} = \frac{3\times36\pi}{2\pi\times2\times10^{10}\times10^{-9}\times79} = 0.34177\times10^{-1}$

$\therefore \alpha = \frac{2\pi\times2\times10^{10}\sqrt{79}}{\sqrt{2}\,3\times10^8}\left[0.024\right] = 0.632\times10^2 = 63.2$

$z = \frac{13.8155}{63.2} = \boxed{0.218 \text{ m}}$

EE 333 Homework 9 (page 2)

3-24 c) @ $f = 2\times10^4$; $\hat{\eta} = \sqrt{\dfrac{\mu_0}{\epsilon(1-j\frac{\sigma}{\omega\epsilon})}}$

or $\eta = 120\pi \sqrt{\dfrac{1}{79(-j0.34177\times10^5)}} = 0.229\sqrt{j} = 0.229\, e^{j\frac{\pi}{4}}$

$P_{ave} = \dfrac{E_m^2}{2|\hat{\eta}|}\cos\theta = \dfrac{10^{-10}}{2\times0.229}\cos\dfrac{\pi}{4} = \boxed{1.544\times10^{-10}\ \text{watts}/m^2}$ ←

@ $f = 2\times10^{10}$ $\hat{\eta} = 120\pi\sqrt{\dfrac{1}{79(1-j0.034177)}}$

$\hat{\eta} = 120\pi\sqrt{\dfrac{1}{79\times1.000\, e^{-j1.957^\circ}}} = 42.4\, e^{j0.978^\circ}$

so $P_{ave} = \dfrac{10^{-10}}{2\times42.4}\cos(0.978^\circ) = \boxed{1.179\times10^{-12}\ W/m^2}$ ←

3-34 $P_{ave_{max}} = 10\times10^{-3}\ W/cm^2 \times \dfrac{10^4\ cm^2}{m^2} = 10^2\ W/m^2$

$\therefore \frac{1}{2}E_m^2\left(\frac{1}{\eta_0}\right)\cos0^\circ = 10^2$

$E_m^2 = 2\times120\pi\times10^2 = 7.53\times10^4$

or $\boxed{E_m \cong 275\ V/m}$ ←

EE 333 Homework 10

5-1 $\lambda/2 = 2m$ but $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon}} = \frac{c}{f}$

$\therefore f = \frac{c}{\lambda} = \frac{3\times10^8}{2} = \boxed{1.5\times10^8\ Hz}$ ←

5-4 $f = 8\times10^8$; $\lambda/2 = 6.25\times10^{-2}m$

a) $c = \frac{3\times10^8}{\sqrt{\epsilon_r}} = \lambda f$; $\epsilon_r = \left(\frac{3\times10^8}{12.5\times10^{-2}\times8\times10^8}\right)^2$

$\boxed{\epsilon_r = 0.9\times10^{-3}\times10^4 = 9}$ ←

b) $H=0$ @ $\boxed{z = -\lambda/4 = -3.125\times10^{-2}m}$ ←

c) $H = \frac{2E_m}{\eta}\cos\beta z\cos\omega t$

@ $z=-0.4m$; $H = \frac{2\times220\sqrt{9}}{120\pi}\cos\left(\frac{8\pi\times0.4}{12.5\times10^{-2}}\right)\cos\omega t$

$H = 3.5\times0.309\cos\omega t = \boxed{1.08\cos\omega t\ A/m}$ ←

$\bar{J} = \bar{n}\times\bar{H} = -\bar{a}_z\times3.5\,\bar{a}_y\cos\omega t = \boxed{\bar{a}_x\ 3.5\cos\omega t}$ ←

5-6

①	②
$\epsilon_{r_1}=8$	$\epsilon_{r_2}=2$
$\mu_{r_1}=2$	$\mu_{r_2}=2$
$\sigma_1=0$	$\sigma_2=0$

$\hat{E}_1 = 250\,e^{-j\beta_1 z}\,\bar{a}_x$

$f = 2.5\times10^9\ Hz$

a) $\Gamma(0^-) = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{120\pi\sqrt{\frac{2}{2}} - 120\pi\sqrt{\frac{1}{4}}}{120\pi\sqrt{1}+120\pi\sqrt{\frac{1}{4}}} = \frac{1-0.5}{1+0.5} = \boxed{\frac{1}{3}}$ ←

EE 333 Homework 10 (page 2)

5-6 b) $\hat{E}_{m_1}^{-}(0) = \frac{250}{3} = \boxed{83.33}$

c) $\hat{E}_{m_2}(0^+) = \hat{T}(0) = E_{m_1}^{+} = \frac{2 \times 1 \times 250}{1 + \frac{1}{2}} = \frac{4}{3} \times 250 = \boxed{333.3}$

d) $\hat{H}_{m_2}^{+} = \frac{333.3}{377} = \boxed{0.884}$

5-10 a) $\epsilon_r = 4.9$, $\mu = \mu_0$, $\sigma = 0$ radome

thickness $= \lambda/2$ for no reflection

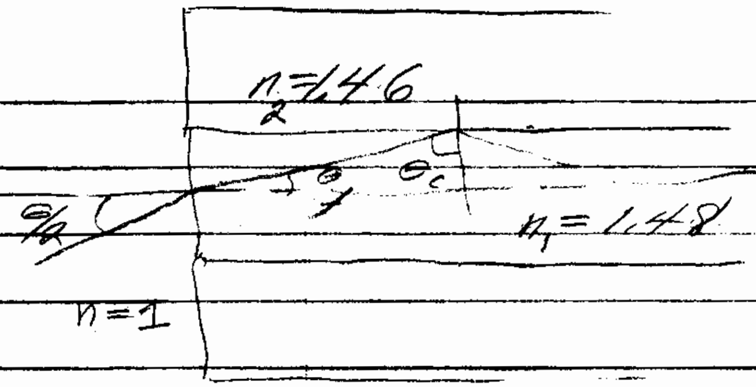
$f = 10^{10}$ Hz

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{2\pi f \sqrt{\mu\epsilon}} = \frac{3 \times 10^{8}}{10^{10}\sqrt{4.9}} = 1.355 \times 10^{-2} \text{ m}$$

$\lambda/2$ = thickness $= 0.678 \times 10^{-2}$ m $= \boxed{0.678 \text{ cm}}$

EE 333 Homework 11

1. $\dfrac{\sin\theta_t}{\sin\left(\frac{\theta}{2}\right)} = \dfrac{1}{n_1}$



$\sin\theta_c = \dfrac{n_2}{n_1}$

$\sin\left(\frac{\theta}{2}\right) = n_1 \sin\theta_t \quad ; \quad \theta_t = 90^\circ - \theta_c$

1, θ_t, θ_c, $\sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$, n_2/n_1 $\quad$ so $\sin\left(\frac{\theta}{2}\right) = n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$

or $\boxed{\sin\left(\frac{\theta}{2}\right) = \sqrt{n_1^2 - n_2^2}}$

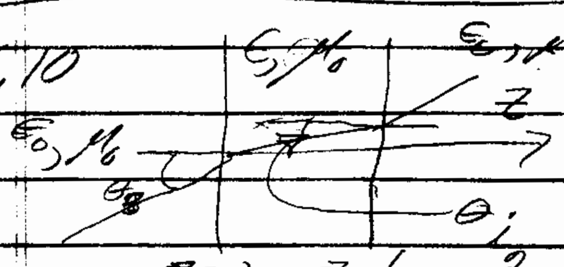
$\therefore \sin\left(\frac{\theta}{2}\right) = 0.242 \quad ; \quad \boxed{\theta = 28^\circ}$

Review Question 6-8. $\bar{E} = \hat{E}_m e^{-j\beta \bar{n}_\beta \cdot \bar{r}}\, \bar{a}_0$ $\qquad \bar{E}$, $\bar{n}_\beta$

if we write $\bar{H} = \dfrac{\bar{n}_\beta \times \bar{E}}{\eta}$ we have the proper magnitude and direction

i.e. $\bar{n}_\beta \times \bar{E}$ is $\perp$ $\bar{E}$ and directed so that $\bar{E} \times \bar{H}$ is in the direction of $\bar{n}_\beta$ (the direction of propagation

6.10



$\boxed{\theta_{t_1} = \theta_{i_2}}$

$\sin\theta_{t_1} = \sin\theta_B \sqrt{\dfrac{\epsilon_0}{\epsilon}} = \dfrac{1}{\sqrt{1+\epsilon/\epsilon_0}}$

$\sqrt{\frac{\epsilon}{\epsilon_0}+1}$, $\sqrt{\frac{\epsilon}{\epsilon_0}}$, θ_B, 1 $\qquad$ $\sqrt{1+\frac{\epsilon}{\epsilon_0}}$, 1, θ_{i_2}, $\sqrt{\frac{\epsilon}{\epsilon_0}}$

$\boxed{\text{or } \tan\theta_{i_2} = \sqrt{\dfrac{\epsilon_0}{\epsilon}}}$

θ_{i_2} is the Brewster $\angle$ at $z=d$. !

EE 333 Homework 11 (page 2)

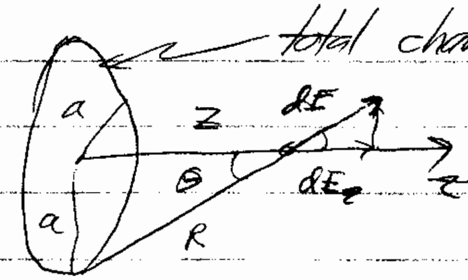
6-19 $\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}$; $n_1 = 1$, $n_2 = 1.5 + \frac{0.5\times10^{-14}}{\lambda^2}$

$\sin\theta_t = \frac{1}{n_2}\sin\theta_i$ but $\theta_i = 30^\circ$ so $\theta_t = \sin^{-1}\left(\frac{1}{2n_2}\right)$

$400\,nm = 400\times10^{-9} = 4\times10^{-7}$

λ	θ_t	
400 nm	19.058°	violet
450	19.143°	blue
550	19.25°	green
600	19.28°	yellow
650	19.31°	orange
700	19.33°	red

4-2 total charge Q ($\rho_\ell = \frac{Q}{2\pi a}$)



$R = \sqrt{a^2+z^2}$

$d\Phi = \frac{dQ}{4\pi\epsilon R}$

a) $\Phi = \int_{\phi=0}^{2\pi}\frac{dQ}{4\pi\epsilon R} = \int_{\phi=0}^{2\pi}\frac{Q\,a\,d\phi}{2\pi a\,4\pi\epsilon\sqrt{a^2+z^2}} = \boxed{\frac{Q}{4\pi\epsilon\sqrt{a^2+z^2}}}$

b) $\bar{E} = -\nabla\Phi = -\frac{\partial\Phi}{\partial z}\bar{a}_z = \boxed{\frac{2Qz}{8\pi\epsilon(a^2+z^2)^{3/2}}\bar{a}_z}$

c) $dE_z = \frac{dQ\cos\theta}{4\pi\epsilon R^2} = \frac{Q\,a\,d\phi\; z}{2\pi a\,(a^2+z^2)^{3/2}\,4\pi\epsilon}$

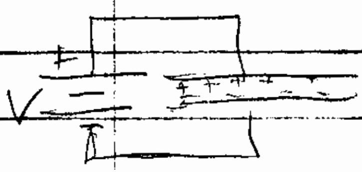
$E_z = \int_{\phi=0}^{2\pi} dE_z = 2\pi\, dE_z = \frac{Qz}{4\pi\epsilon(a^2+z^2)^{3/2}}$!

EE 333 Homework 12

4-7 $W=\frac{1}{2}\int_{vol}\Phi\rho_v\,dv=\frac{1}{2}\oint\Phi\rho_s\,ds$

$\Phi(R)=\frac{Q}{4\pi\epsilon R}$ ∴ $\boxed{W=\frac{1}{2}\frac{Q}{4\pi\epsilon R}\cdot Q=\frac{Q^2}{8\pi\epsilon R}}$

4-8 a) $W=\frac{1}{2}\int\rho_v\Phi\,dv=\frac{V}{2}\int\rho_s\,ds=\frac{QV}{2}$

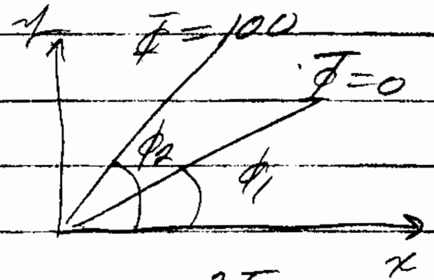


b) $V=\frac{Q}{C}$ ∴ from a) $\boxed{W=\frac{Q^2}{2C}}$

c) $C=\frac{\epsilon A}{d}$ d is increased by a factor of 3
so C decreases by a factor of 3

Q is constant so from b) $\boxed{\text{W increased by a factor of 3}}$

4-9



$\nabla^2\Phi=0=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial\Phi}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}+\cdots$

or for this problem $\Phi=f(\phi)$

a) $\nabla^2\Phi=\frac{1}{\rho^2}\frac{\partial^2\Phi}{\partial\phi^2}=0$ ∴ $\Phi=C_1\phi+C_2$

$0=C_1\phi_1+C_2$; $100=C_1\phi_2+C_2=C_1\phi_2-C_1\phi_1$

so $C_1=\frac{100}{\phi_2-\phi_1}$ and $C_2=-C_1\phi_1=\frac{-100\phi_1}{\phi_2-\phi_1}$

and $\Phi=\frac{100}{\phi_2-\phi_1}\phi-\frac{100}{\phi_2-\phi_1}\phi_1=\boxed{\frac{100}{\phi_2-\phi_1}(\phi-\phi_1)}$

4-9 b) $\bar{E} = -\nabla \Phi = -\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\bar{a}_\phi$

$$\boxed{\text{or } \bar{E} = -\frac{100}{\phi_2-\phi_1}\cdot\frac{1}{\rho}\bar{a}_\phi}$$

c) $\rho_s = \bar{n}\cdot\bar{D}$; $\bar{D} = \epsilon\bar{E}$

for plate @ $\phi = \phi_1$ this gives:

$$\rho_s = \bar{a}_\phi \cdot \frac{\epsilon 100}{\phi_1-\phi_2}\cdot\frac{1}{\rho}\bar{a}_\phi = \frac{\epsilon 100}{\rho(\phi_1-\phi_2)}$$

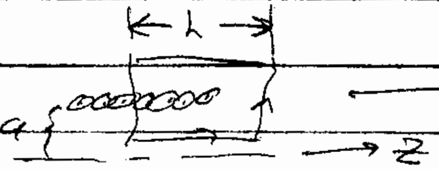
for plate @ $\phi = \phi_2$

$$\rho_s = -\bar{a}_\phi \cdot \frac{-\epsilon 100}{\phi_2-\phi_1}\cdot\frac{1}{\rho}\bar{a}_\phi = \frac{\epsilon 100}{\rho(\phi_2-\phi_1)}$$

EE 333 Homework 13

4-27 a) $\int \bar{B}\cdot d\bar{s} = \int \nabla\times\bar{A}\cdot d\bar{s} = \oint \bar{A}\cdot d\bar{l}$

Stokes Theorem

b) 

$\oint \bar{H}\cdot d\bar{l} = NLI = H_z L$ for $\rho \le a$

∴ B_z for $\rho < a = \mu N I$

only B_z ∴ $\bar{B} = \nabla\times\bar{A} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \frac{\partial A_\rho}{\partial\phi}\right]\bar{a}_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\phi)\,\bar{a}_z$

∴ only $\bar{A}$ only has a ϕ component.

from part a) using a circular contour we have:

$\rho < a$ $\quad \int_{\phi=0}^{2\pi} A_\phi \rho\, d\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \mu N I \rho\, d\phi\, d\rho = 2\pi\mu N I \frac{\rho^2}{2}$

or $\cancel{2\pi}\rho A_\phi = \cancel{2\pi}\mu N I \frac{\rho^{2}}{2}$ or $\boxed{A_\phi = \frac{\mu N I \rho}{2}}$

$B_z = 0$ for $\rho > a$ giving $\cancel{2\pi}A_\phi \rho = \cancel{2\pi}\mu N I \frac{a^2}{2}$

or $\boxed{A_\phi = \frac{\mu N I a^2}{2\rho}}$

4-29 $B = 1\ \text{W/m}^2$ ∴ $H \cong 0.065$ in iron

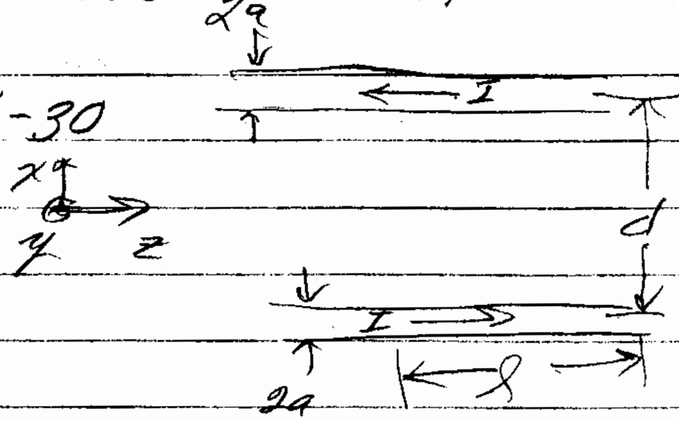
B_n is the same in the gap and in the magnetic material

∴ $H_{gap} = \frac{B}{\mu_0}$

$NI = \oint \bar{H}\cdot d\bar{l} = 0.065\times 0.4 + \frac{5\times10^{-4}}{4\pi\times10^{-7}} = 0.026 + 0.398\times10^{3}$

or $\boxed{NI \cong 398 \text{ Ampere Turns}}$

4-30



$\bar{H}_{\text{outside lower conductor}} = \frac{I}{2\pi x}\bar{a}_y$

$\therefore \psi_m \cong \frac{\mu I}{2\pi}\int_{x=a}^{d}\int_{z=0}^{l}\frac{1}{x}\,dz\,dx$ for length l

for $d \gg a$

$$\boxed{\psi_m = \frac{\mu I l}{2\pi}\ln\left(\frac{d}{a}\right) \quad \text{for } d \gg a}$$

b) $\bar{B}_{\text{between conductors}} = \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}$

$\therefore$ flux linkage for length l $= \int_{z=0}^{l}\int_{x=a}^{d-a}\left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}\right]dx\,dz$

or $\lambda = \frac{l\mu_0 I}{2\pi}\left[\ln\left(\frac{d-a}{a}\right)\right] + \frac{\mu_0 I l}{2\pi}\underbrace{\int_a^{d-a}\frac{dx}{d-x}}$

let $u = d-x$

$du = -dx$

$\lambda = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{d-a}{a}\right) + \frac{\mu_0 I l}{2\pi}\int_{d-a}^{a}\frac{-du}{u}$

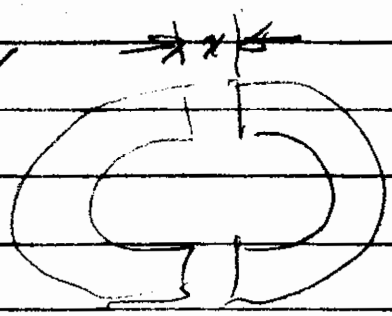
$\lambda = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{d-a}{a}\right) - \frac{\mu_0 I l}{2\pi}\ln\left(\frac{a}{d-a}\right) = \frac{\mu_0 I l}{\pi}\ln\left(\frac{d-a}{a}\right)$

$$\boxed{L = \frac{\lambda}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right) \approx \frac{\mu_0 l}{\pi}\ln\left(\frac{d}{a}\right)}$$

for length l !

for $d \gg a$

4-34



Energy Density $= \frac{1}{2}\mu H^2$

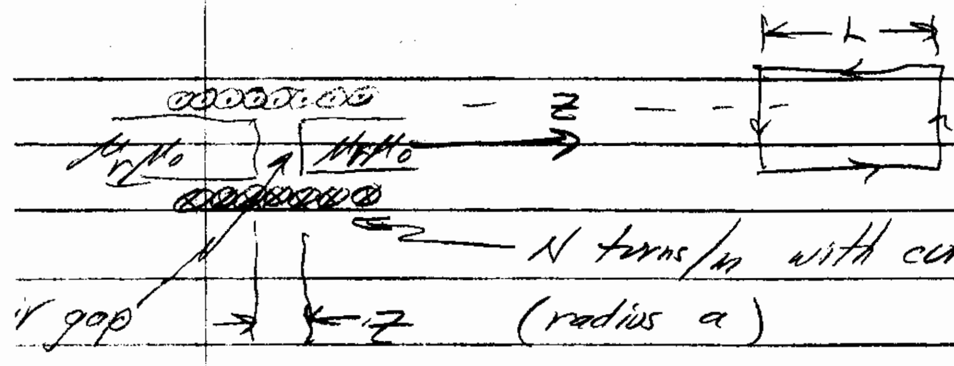
∴ energy stored in gaps is:

$$W_m = \frac{1}{2}\mu_0 H^2 (2Sx)$$

two gaps

$$\boxed{F_x = \frac{\partial W_m}{\partial x} = \mu_0 S H^2 \text{ (Newtons)}}$$

EE 333 Homework 14



$\oint \bar{H} \cdot d\bar{l} = NIL$

$\therefore H_z L = NIL$

$\boxed{H_z = NI}$

constant current $\therefore F_z = \nabla W_m$

→ energy change is in air gap and magnetic material

$B_{z\,\text{everywhere}} = \mu_0 \mu_r NI$ (Normal B is continuous)

so $H_{z_{gap}} = \mu_r NI$

If we pick our length L to include the gap and calculate the magnetic energy density over this length we obtain:

$$W_m = \pi a^2 z\left(\frac{1}{2}\mu_0 \mu_r^2 N^2 I^2\right) + \pi a^2 (L-z)\left(\frac{1}{2}\mu_0 \mu_r N^2 I^2\right)$$

$$W_m = \frac{\pi a^2 N^2 I^2}{2}\left[z \mu_0 \mu_r^2 + (L-z)\mu_0 \mu_r\right]$$

$$W_m = \frac{\pi a^2 N^2 I^2 \mu_r \mu_0}{2}\left[z \mu_r + (L-z)\right]$$

$$\frac{\partial W_m}{\partial z} = \frac{\pi a^2 N^2 I^2 \mu_r \mu_0}{2}(\mu_r - 1) = F_z$$