EE 333 Homework 1

1.2 a) $\bar{A} = \bar{a}_x - 3\bar{a}_y + 2\bar{a}_z$

$\bar{B} = -3\bar{a}_x + 4\bar{a}_y - \bar{a}_z$

$\bar{A}\cdot\bar{B} = AB\cos\theta = \sqrt{14}\sqrt{26}\cos\theta = -3-12-2 = -17$

$\therefore \theta = \cos^{-1}\left(-\frac{17}{\sqrt{14}\sqrt{26}}\right) = \cos^{-1}\left(-\frac{17}{3.7416\cdot 5.099}\right) = \cos^{-1}(-0.89$

$\boxed{\theta \simeq 153^\circ}$

b) want $A\cos\theta = \left|\frac{\bar{A}\cdot\bar{B}}{B}\right| = \left|\frac{-17}{5.099}\right| = \boxed{3.334}$

c) $\bar{A}\times\bar{B}$ is $\perp$ to the plane of $\bar{A}$ + $\bar{B}$

$\bar{A}\times\bar{B} = 4\bar{a}_z + 2\bar{a}_y - \bar{a}_z 9 + 3\bar{a}_x - 6\bar{a}_y - 8\bar{a}_x$

$= -5\bar{a}_x - 5\bar{a}_y - 5\bar{a}_z$

unit vector in this plane $= \frac{-5(\bar{a}_x + \bar{a}_y + \bar{a}_z)}{\sqrt{3\times 25}} = \boxed{-\frac{\bar{a}_x + \bar{a}_y + \bar{a}_z}{\sqrt{3}}}$

d) $\bar{A}\cdot\bar{a}_r = ?$

from page 28 in text

$A_r = A_x\cos\phi\sin\theta + A_y\sin\phi\sin\theta - A_z\cos\theta = \bar{A}\cdot\bar{a}_r$

$\boxed{\therefore \bar{A}\cdot\bar{a}_r = \cos\phi\sin\theta - 3\sin\phi\sin\theta + 2\cos\theta}$

This is projection of $\bar{A}$ onto $\bar{a}_r$

not a constant because $\bar{a}_r$ is not constant!

EE 333 Homework 2

1.25 $\bar{F} = q(\bar{E} + \bar{u} \times \bar{B})$

$q[u_0(3\bar{a}_x - \bar{a}_y + 2\bar{a}_z) \times B_0(\bar{a}_x + 2\bar{a}_y - 4\bar{a}_z) + \bar{E}] = 0$

or $u_0 B_0[6\bar{a}_z + 12\bar{a}_y + \bar{a}_z + 4\bar{a}_x + 2\bar{a}_y - 4\bar{a}_x] + \bar{E} = 0$

$\therefore\ -\bar{E} = u_0 B_0 [7\bar{a}_z + 14\bar{a}_y]$

1.26 (z direction) magnetic force in $-x$ direction

1.30 $F = q\bar{E} = m\frac{dv}{dt} \Rightarrow dv = \frac{q}{m} E\,dt$

@ $t=0$ $v=0$ $\therefore$ $v(t) = \frac{q}{m} E t \Rightarrow dx = \frac{q}{m} E t\,dt$

integrating $\left(x = \frac{qE}{m}\frac{t^2}{2}\right)$ for $x=0$ @ $t=0$

$\therefore$ t for transit through accelerating region is:

$$t = \sqrt{\frac{2mx}{qE}}$$

However $E = \frac{V}{x}$ $\therefore$ $t = \sqrt{\frac{2mx^2}{qV}}$

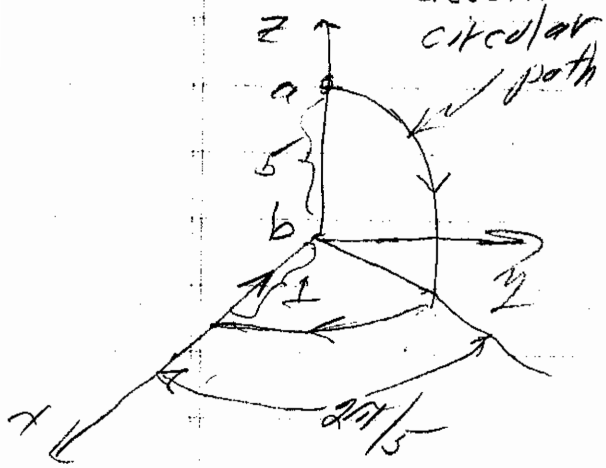
a) $\therefore$ from above v @ exit $= \frac{qVx}{mx}\sqrt{\frac{2m}{qV}} = \sqrt{\frac{2qV}{m}}$

b) circle (constant force normal to velocity)

c) $\bar{B}$ field does not change magnitude of $\bar{v}$

EE 333 Homework 2 (page 2)

1.37 $\bar{E} = K \frac{\cos\theta}{r^2}\bar{a}_r + \frac{K}{2}\frac{\cos\theta}{r^3}\bar{a}_\theta$



work $= q\int \bar{E}\cdot \bar{dl}$

$\therefore$ work $= \int_{\theta=0}^{\frac{\pi}{2}} \frac{K}{2}\frac{\cos\theta}{5^2}\bar{a}_\theta \cdot d\theta\, 5\,\bar{a}_\theta$

+ zero for second arc because E_ϕ and $E_r = 0$

$+ \int_{r=1}^{0} K \frac{\cos(\frac{\pi}{2})}{4}\bar{a}_r \cdot dr\,\bar{a}_r \quad = 0$

$\therefore$ work $= \int_{\theta=0}^{\frac{\pi}{2}} \frac{K\cdot 5}{5^3\times 2}\cos\theta\, d\theta = \frac{K5}{2\times 5^3}\sin\theta \Big|_0^{\frac{\pi}{2}} = \boxed{\frac{K}{2\times 5^2}}$

EE 333 Homework 3

1.48 $\rho_V = \rho_0\left(1-\frac{r^2}{a^2}\right)$ $r<a$

$\rho_V = 0$ $r>a$

for $r<a$ $\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\epsilon_0 E_r r^2 \sin\theta\, d\theta\, d\phi = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{r}\rho_0\left(1-\frac{r^2}{a^2}\right) r^2 \sin\theta\, dr\, d\theta\, d\phi$

$\therefore$ $4\pi r^2 \epsilon_0 E_r = 4\pi\rho_0\left[\frac{r^3}{3}-\frac{r^5}{5a^2}\right]$

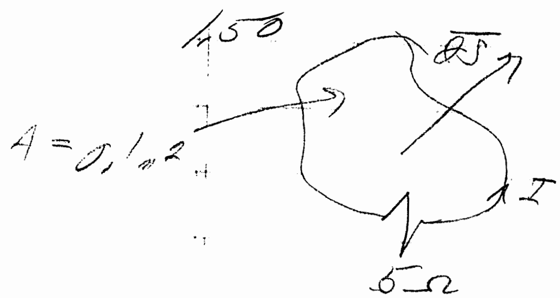
or $E_r = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3}-\frac{r^3}{5a^2}\right]$ ←

for $r>a$ $4\pi r^2\epsilon_0 E_r = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{a}\rho_0\left[1-\frac{r^2}{a^2}\right] r^2\sin\theta\, dr\, d\theta\, d\phi$

$4\pi r^2\epsilon_0 E_r = 4\pi\rho_0\left[\frac{a^3}{3}-\frac{a^3}{5}\right]$

or $E_r = \frac{a^3\rho_0}{\epsilon_0 r^2}\overbrace{\left[\frac{1}{3}-\frac{1}{5}\right]}^{2/15} = \frac{2a^3\rho_0}{15\epsilon_0 r^2}$ ←

1.50



Use Faraday's Law

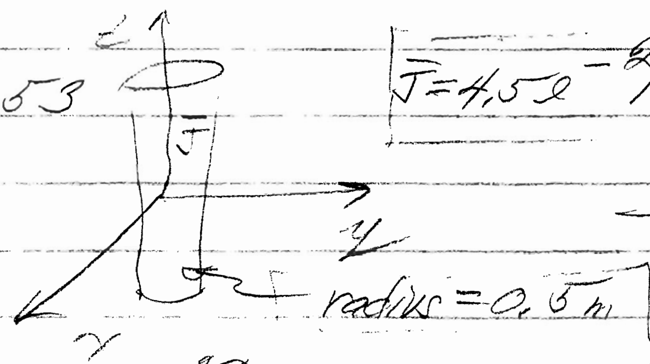
$\oint \bar{E}\cdot d\bar{l} = -\frac{d}{dt}\int \bar{B}\cdot d\bar{S}$ = emf around loop

emf $= -\frac{d}{dt}\left(0.2\sin 10^3 t\,\bar{a}_z \cdot 0.1\,\bar{a}_z\right)$

or emf in volts $= -0.02\times 10^3 \cos 10^3 t$ ←

$\therefore$ $I = \frac{emf}{R} = -4\cos 10^3 t$ ←

1.53



$$\boxed{\bar{J} = 4.5\, e^{-2\rho}\, \bar{a}_z}$$

$$\rightarrow \oint \frac{\bar{B}}{\mu_0}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{s}$$

for $\rho < 0.5\,m$

$$\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_0}\bar{a}_\phi \cdot \rho\, d\phi\, \bar{a}_\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} 4.5\, e^{-2\rho}\rho\, d\rho\, d\phi$$

$$\frac{B_\phi}{\mu_0}\cdot \cancel{2\pi}\rho = \cancel{2\pi}\times 4.5\left\{e^{-2\rho}\left[-\frac{\rho}{2}-\frac{1}{4}\right]\right\}_0^{\rho}$$

$$B_\phi = \mu_0 \times 4.5\, \frac{1}{\rho}\left\{e^{-2\rho}\left[-\frac{\rho}{2}-\frac{1}{4}\right]+\frac{1}{4}\right\}$$

$$\text{or } \underline{B_\phi = \frac{9}{8}\cdot\frac{\mu_0}{\rho}\left\{1-2\rho\, e^{-2\rho}-e^{-2\rho}\right\}}$$

for $\rho > 0.5\,m$

$$\frac{2\pi\rho\, B_\phi}{\mu_0} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\frac{1}{2}} 4.5\, e^{-2\rho}\rho\, d\rho\, d\phi$$

$$B_\phi = \frac{\mu_0\times 4.5\times \cancel{2\pi}}{\cancel{2\pi}\rho}\int_{\rho=0}^{\frac{1}{2}}\rho\, e^{-2\rho}\, d\rho = \frac{9\mu_0}{2\rho}\left\{e^{-1}\left[-\frac{1}{2}\right]+\frac{1}{4}\right\}$$

$$B_\phi = \frac{9\mu_0}{8\rho}\left\{1-2e^{-1}\right\} \simeq \underline{0.297\,\frac{\mu_0}{\rho}}$$

EE 333 Homework 4 (page 2)

2.1 $\psi = E_0\left[1-\left(\frac{a}{\rho}\right)^3\right] z\cos\phi$

$$\nabla\psi = \frac{\partial\psi}{\partial\rho}\bar{a}_\rho + \frac{1}{\rho}\frac{\partial\psi}{\partial\phi}\bar{a}_\phi + \frac{\partial\psi}{\partial z}\bar{a}_z$$

$$\therefore \nabla\psi = 3E_0 a^3 z\cos\phi\left(\frac{1}{\rho^4}\right)\bar{a}_\rho - \frac{E_0}{\rho}\left[1-\left(\frac{a}{\rho}\right)^3\right] z\sin\phi\ \bar{a}_\phi$$
$$+ E_0\left[1-\left(\frac{a}{\rho}\right)^3\right]\cos\phi\ \bar{a}_z$$

2.2 a) $\nabla\cdot\bar{A} = 0$

b) $\nabla\cdot\bar{B} = 0$

c) $\nabla\cdot\bar{C} = \frac{1}{r^2}\cdot\frac{\partial(r^2\cdot r)}{\partial r} = 3$

d) $\nabla\cdot\bar{D}\Big|_{r=3} = \frac{1}{r^2}\frac{\partial}{\partial r}(2r^4) = \frac{2\cdot 4r^3}{r^2}\Big|_{r=3} = 24$

e) $\nabla\cdot\bar{E} = 3+1-1 = 3$

2.12 $\bar{E} = \frac{\rho_v r}{3\epsilon_0}\bar{a}_r$

for static E field $\nabla\times\bar{E} = 0$ $\left\{\begin{array}{l}\text{The curl of the above function} \\ \text{is } \phi \text{ because it has only } E_r \\ \text{that is not a function of } \theta \text{ or } \phi\text{!}\end{array}\right.$

$$\nabla\cdot(\epsilon_0\bar{E}) = \rho_v = \epsilon_0\left\{\frac{1}{r^2}\frac{\partial\left(\frac{\rho_v r^3}{3\epsilon_0}\right)}{\partial r}\right\} = \epsilon_0\frac{3r^2\rho_v}{3\epsilon_0 r^2} = \rho_v$$

$\therefore$ charge density for this $\bar{E}$ is ρ_v!

2.14 $\bar{B} = \frac{1}{r^2}\sin\phi\cos^2\theta\,\bar{a}_r$

$\nabla\cdot\bar{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(\sin\phi\cos^2\theta) = 0$ $\therefore$ could be a static B field!

$$\bar{J} = \nabla\times\left(\frac{\bar{B}}{\mu_0}\right) = \frac{1}{r\mu_0\sin\theta}\cdot\frac{1}{r^2}\cos\phi\cos^2\theta\,\bar{a}_\theta + \frac{1}{\mu_0 r}\frac{2}{r^2}\sin\phi\cos\theta\sin\theta\,\bar{a}_\phi$$

$$\bar{J} = \frac{1}{\mu_0 r^3}\left\{\frac{\cos\phi\cos^2\theta}{\sin\theta}\bar{a}_\theta + 2\sin\phi\cos\theta\sin\theta\,\bar{a}_\phi\right\}$$

EE 333 Homework 4

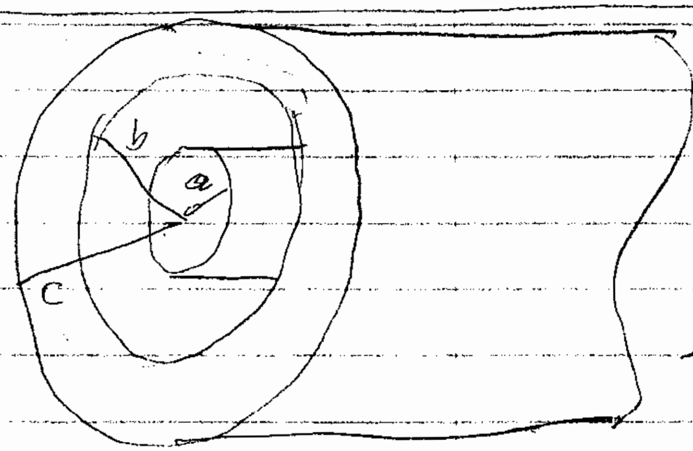
1.56 from class notes $B_\phi = \frac{\mu_0 I \cos\omega t}{2\pi\rho}$ for $\rho > a$

(i.) Total flux through loop $= \int_{\rho=d}^{d+a}\int_{z=0}^{b} \frac{\mu_0 I \cos\omega t}{2\pi\rho}\, dz\, d\rho$

Total flux $= \frac{\mu_0 I b}{2\pi} \ln\left(\frac{d+a}{a}\right) \cos\omega t$

(ii) $emf = -\frac{d}{dt}(\text{total flux}) = \boxed{\frac{\omega \mu_0 I b}{2\pi} \ln\left(\frac{d+a}{a}\right) \sin\omega t}$

1.59



$J_z = \frac{I}{\pi a^2}$ for $\rho < a$

$J_z = -\frac{I}{\pi(c^2-b^2)}$ $b < \rho < c$

zero everywhere else

$\rho < a$ $\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_0} \rho\, d\phi = I \frac{\pi\rho^2}{\pi a^2} = \frac{2\pi B_\phi \rho}{\mu_0}$ or $\boxed{B_\phi = \frac{\mu_0 I \rho}{2\pi a^2}}$

$a < \rho < b$ $\frac{2\pi\rho B_\phi}{\mu_0} = I$ $\therefore$ $\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}}$

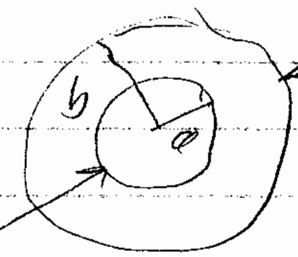
$b < \rho < c$ $\frac{2\pi\rho B_\phi}{\mu_0} = I\left\{1 - \frac{\pi(\rho^2-b^2)}{\pi(c^2-b^2)}\right\} = I\left\{\frac{\cancel{\pi}(c^2 \cancel{-b^2} - \rho^2 \cancel{+b^2})}{\cancel{\pi}(c^2-b^2)}\right\}$

so $\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}\left\{\frac{c^2-\rho^2}{c^2-b^2}\right\}}$

for $\rho > c$ $\boxed{B_\phi = 0}$

EE 333 Homework 5

1.46



ρ_s ρ_s

use Gauss's law

a) $r < a$

charge enclosed $= 0$

$\therefore$ $\boxed{E_r = 0}$

$a < r < b$ $\oint \epsilon_0 \bar{E} \cdot d\bar{s} = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} \epsilon_0 E_r r^2 \sin\theta \, d\phi \, d\theta = \epsilon_0 E_r r^2 4\pi$

$\int \rho_v dv \Rightarrow \oint \rho_s ds = 4\pi a^2 \rho_s \quad \therefore \rho_s a^2 = \epsilon_0 E_r r^2$

or $\boxed{E_r = \frac{\rho_s a^2}{\epsilon_0 r^2}}$

for $r > b$ $4\pi \epsilon_0 r^2 E_r = \rho_s (4\pi a^2 - 4\pi b^2)$

or $\boxed{E_r = \frac{\rho_s (a^2 - b^2)}{\epsilon_0 r^2}}$

8.26 $\hat{E} = 50 e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{3}z} a_x$; $\beta = \frac{\pi}{3} = \frac{\omega}{v_{phase}}$

a) polarized in x direction

b) positive z

c) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi 3}{\pi} = 6\,m$

d) $f = \frac{\omega}{2\pi}$; $v = \frac{\omega}{\beta} = \frac{2\pi f 3}{\pi} = 6f$

given that the wave is propagating in free space $v = c$ $\therefore$ $\boxed{f = \frac{v}{6} = 50\,MHz}$

e) $\frac{\hat{E}}{\hat{H}} = \eta_0 \quad \therefore \quad \hat{H} = \frac{50}{120\pi} e^{j\frac{\pi}{4}} e^{-j\frac{\pi}{3}z} a_y$ $\quad \hat{E} \times \hat{H} =$ direction of propagation

f) $\bar{E} = 50 \cos(\omega t - \frac{\pi}{3} z + \frac{\pi}{4}) \bar{a}_x$

EE 333 Homework 6

2.33 $\bar{H} = 0.15\cos\left(\omega t + \frac{2\pi z}{7} + \frac{\pi}{13}\right)\bar{a}_y$

a) $\beta_0 = \frac{2\pi}{7}$; $\lambda\beta_0 = 2\pi$ so $\boxed{\lambda = \frac{2\pi 7}{2\pi} = 7m}$

b) $\boxed{|\bar{H}| = 0.15}$ $|\bar{E}| = 120\pi|\bar{H}| = 56.55$

c) $\boxed{\hat{\bar{E}} = 56.55\, e^{j\left(\frac{2\pi z}{7} + \frac{\pi}{13}\right)}(-\bar{a}_x)}$

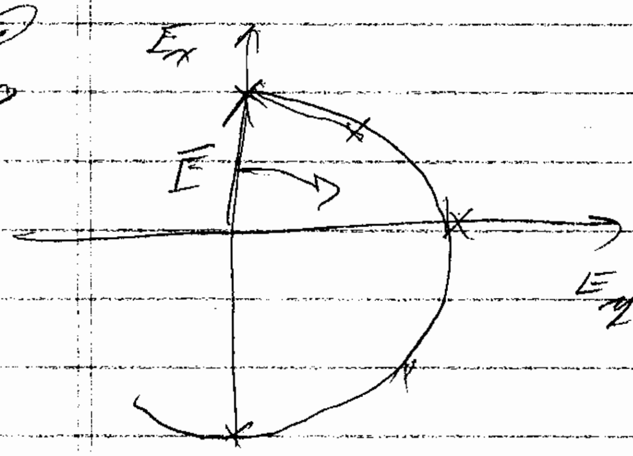
Special $\hat{\bar{E}} = 500\, e^{-j\beta_0 z}(\bar{a}_x - j\bar{a}_y)$

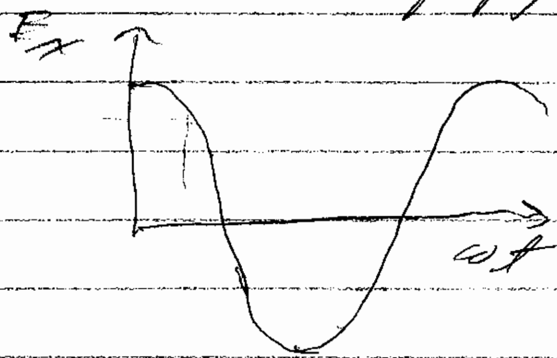
a) $\bar{E} = 500\cos(\omega t - \beta_0 z)\bar{a}_x + 500\sin(\omega t - \beta_0 z)\bar{a}_y$

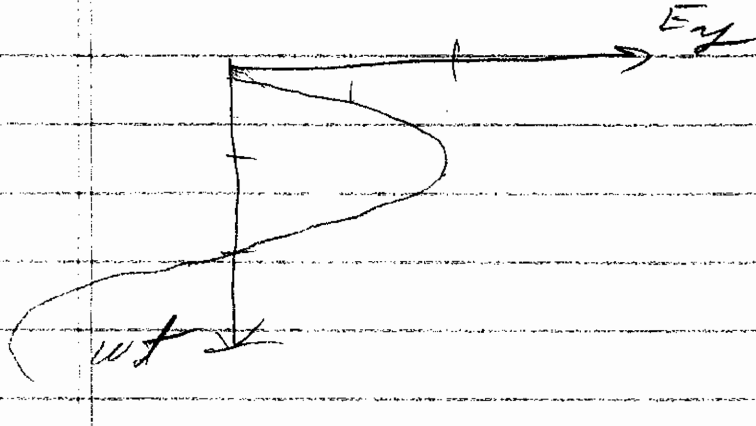
b) $\hat{\bar{H}} = \frac{500}{120\pi}\, e^{-j\beta_0 z}\underbrace{(\bar{a}_y + j\bar{a}_x)}$

obtained by remembering {$E \times H$ gives direction of propagation}

look @ z=0







{CW or right hand circular polarization.}

3.13 $\bar{E} = 10\cos\omega t\,\bar{a}_z$, $A = 10^{-2}$

$\boxed{\bar{J}_c = \sigma\bar{E} = 0}$

$\bar{J}_{displacement} = \dfrac{\partial(\epsilon_0 \bar{E})}{\partial t} = -10\omega\epsilon_0 \sin\omega t\,\bar{a}_z$

$\therefore \boxed{I_{total} = |\bar{J}| \times A = -0.1\omega\epsilon_0 \sin\omega t}$

b) sea water $\epsilon = 80\epsilon_0$, $\sigma = 4\,S/m$, $f = 10^8$

$\dfrac{|\bar{J}_c|}{|\bar{J}_d|} = \dfrac{\overset{\sigma}{\overset{2}{\cancel{4}}} \times \cancel{10} \times 36\cancel{\pi}}{\cancel{10} \times \cancel{2\pi} \times 10^8 \times 10^{-9} \times 80} = \dfrac{72}{80} \times 10 = 9$

$\bar{I}_{total} = 10^{-2}(\bar{J}_c + \bar{J}_d) = 10^{-2}(40\cos\omega t - 10\omega\epsilon_0 \times 80\sin\omega t)\bar{a}_z$

$\bar{I}_{total} = [0.4\cos\omega t - 4.44 \times 10^{-2}\sin\omega t]\,\bar{a}_z$

EE 333 Homework 7

3.21 $\epsilon_r = 6.3$, $\mu_r = 1.98$, $f = 10^9$, assume $\sigma = 0$

$E(z,t) = \cos(\omega t - \beta z)\bar{a}_x$

a) $\beta = \omega\sqrt{\mu\epsilon} = \frac{2\pi \times 10^9 \times \sqrt{6.3 \times 1.98}}{3 \times 10^8} = \boxed{7.397 \times 10 \ \text{rad}/m}$

$\boxed{\lambda = \frac{2\pi}{\beta} = 8.49\,cm}$

$u_{phase} = \frac{\omega}{\beta} = \frac{2\pi \times 10^9}{73.96} = \boxed{0.849 \times 10^8 \ m/sec}$

b) $\eta = \sqrt{\frac{\mu}{\epsilon}} = 120\pi\sqrt{\frac{1.98}{6.3}} = \boxed{211\ \Omega}$

c) $\bar{H}(z,t) = \underbrace{0.473}_{\frac{|E_x|}{\eta}} \cos(2\pi \times 10^9 t - 73.97z)\,\bar{a}_y$

3.22 sea water $\sigma = 4$ S/m, $\mu_r = 1$, $\epsilon_r = 81$

$100 < \left|\frac{\bar{J}_c}{\bar{J}_d}\right| = \left|\frac{\sigma|E|}{j\omega\epsilon_0\epsilon_r|E|}\right| = \frac{\sigma}{2\pi f\epsilon_0\epsilon_r}$

$\therefore f \le \frac{\sigma}{2\pi\epsilon_0\epsilon_r \times 100} = \frac{4 \times 36\pi}{2\pi \times 10^{-9} \times 81 \times 10^2} = 0.888 \times 10^7$

$\boxed{\text{or } f \le 8.88\ MHz}$

EE 333 Homework 7 (page 2)

3.26 a) $f = 24\,MHz$, $\phi = 1$ rad/m

amplitude decreases by ½ for each meter travelled

$\therefore\ e^{-\alpha \times 1} = 0.5$ or $\boxed{\alpha = -\ln 0.5 = 0.693}$

skin depth $\boxed{\delta = \frac{1}{\alpha} = 1.44\,m}$

$\boxed{\beta = \text{phase shift per meter} = 1\ \text{rad/m}}$

$\boxed{\lambda = \frac{2\pi}{\beta} = 2\pi\,m}$

$v_{phase} = \frac{\omega}{\beta} = \frac{2\pi \times 24 \times 10^6}{1} = 150.8 \times 10^6\ m/sec$

or $\boxed{v_{phase} = 1.508 \times 10^8\ m/sec}$

EE 333 Homework 8

3.1 $\bar{E} = 3z^2 y \cos 10^8 t\, \bar{a}_x$, $\epsilon_r = 2.56$

a) $\bar{D} = \epsilon_0 \bar{E} + \bar{P} = \epsilon_r \epsilon_0 \bar{E}$

$$\therefore \bar{P} = \epsilon_0 \bar{E}(\epsilon_r - 1) = \frac{3 \times 1.56 \times 10^{-9}}{36\pi} z^2 y \cos 10^8 t\, \bar{a}_x$$

$$\boxed{\bar{P} = 4.138 \times 10^{-11} z^2 y \cos 10^8 t\, \bar{a}_x}$$

b) $\rho_p = -\nabla \cdot \bar{P} = 0$ $\left\{ \begin{array}{l} \text{Field has only an } x \text{ component} \\ \text{that does not vary with } x! \end{array} \right\}$

c) $$\boxed{\bar{J}_p = \frac{\partial \bar{P}}{\partial t} = -4.138 \times 10^{-3} z^2 y \cos 10^8 t\, \bar{a}_x}$$

3.2 a) $\bar{E} = \left(3\rho^2 \cot\phi\, \bar{a}_\rho + \frac{\cos\phi}{\rho} \bar{a}_\phi\right) \sin 3 \times 10^8 t$

from above $\bar{P} = \epsilon_0 \bar{E}(\epsilon_r - 1)$; $\bar{J}_p = \frac{\partial \bar{P}}{\partial t}$; $\rho_p = -\nabla \cdot \bar{P}$

i) Teflon $\epsilon_r = 2.1$ $\left\{ \begin{array}{l} \bar{P} = 1.1 \epsilon_0 \bar{E} \\ \bar{J}_p = 3.3 \times 10^8 \epsilon_0 \left[3\rho^2 \cot\phi\, \bar{a}_\rho + \frac{\cos\phi}{\rho} \bar{a}_\phi\right] \cos 3 \times 10^8 t \end{array} \right\}$

$$\rho_p = -\nabla \cdot \bar{P} = -\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho P_\rho) - \frac{1}{\rho}\frac{\partial}{\partial \phi} P_\phi$$

$$= 1.1\epsilon_0 \left[-9\rho \cot\phi + \frac{\sin\phi}{\rho^2}\right] \sin 3 \times 10^8 t$$

ii) Glass $\epsilon_r = 6.3$ multiply above by $\frac{5.3}{1.1}$!

iii) Sea water $\epsilon_r = 81$ multiply result in part i) by 72.72

$\rightarrow \frac{80}{1.1}$

Special Problem Cu ; $\sigma = 5.8 \times 10^{7}$; $n = 10^{29}$

mobility $= -\frac{e\tau_c}{m}$; $\sigma = \frac{ne^2\tau_c}{m} = -ne(\text{mobility})$

a) $\therefore$ mobility $= \frac{-\sigma}{ne} = + \frac{5.8 \times 10^{7}}{10^{29} \times 1.6 \times 10^{-19}} = \underline{+3.625 \times 10^{-3}}$

b) $\rho_v = 10^{29} \times (-1.6 \times 10^{-19}) = \boxed{-1.6 \times 10^{10}\ C/m^3 = -16\ C/mm^3}$

c) $\bar{u}_d = -(\text{mobility}) \times \bar{E}$

for $\bar{E} = 1\bar{a}_x$ $\boxed{\bar{u}_d = -3.625 \times 10^{-3}\ m/sec\ \bar{a}_x}$

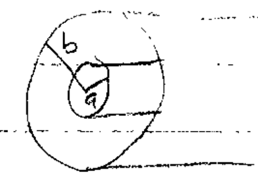
d) $\bar{J} = \rho_v \bar{u}_d = -1.6 \times 10^{10} \times 3.625 \times 10^{3}$

or $\boxed{\bar{J} = 5.8 \times 10^{7}\ A/m^2 = 58\ A/mm^2}$

EE 333 Homework 9

$\bar{J} = \frac{1}{2}\bar{a}_z$ for $\rho < a$

3.11 $\bar{J} = \frac{1}{2}\,\rho/a\,(-\bar{a}_z)$ for $a<\rho<b$



a) $2\pi\rho H_\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \frac{1}{2}\bar{a}_z \cdot \rho\, d\phi\, d\rho\, \bar{a}_z = \frac{\pi\rho^2}{2}$ for $\rho<a$

$\boxed{\text{so } H_{\phi_1} = \frac{\rho}{4} \text{ for } \rho<a}$

for $a<\rho<b$ $2\pi\rho H_\phi = \frac{1}{2}\pi a^2 - \int_{\phi=0}^{2\pi}\int_{\rho=a}^{\rho}\frac{\rho}{2a}\rho\,d\phi\,d\rho = \frac{\pi a^2}{2} - \frac{\pi}{a}\left[\frac{\rho^3}{3} - \frac{a^3}{3}\right]$

or $H_\phi = \frac{a^2}{4\rho} - \frac{\rho^2}{6a} + \frac{a^2}{6\rho} = \boxed{\frac{5}{12}\cdot\frac{a^2}{\rho} - \frac{1}{6}\frac{\rho^2}{a}}$

b) for $\rho<a$ $B_{\phi_1} = \mu_{r_1}\mu_0 \cdot \frac{\rho}{4}$

for $a<\rho<b$ $B_{\phi_2} = \mu_{r_2}\mu_0\left[\frac{5}{12}\cdot\frac{a^2}{\rho} - \frac{1}{6}\cdot\frac{\rho^2}{a}\right]$

c) $\bar{M} = \chi_m \bar{H}$ and $\chi_m = \mu_r - 1$

$\therefore \boxed{\bar{M}_2 = (\mu_{r_2}-1)H_{\phi_2}\bar{a}_\phi}$ $a<\rho<b$

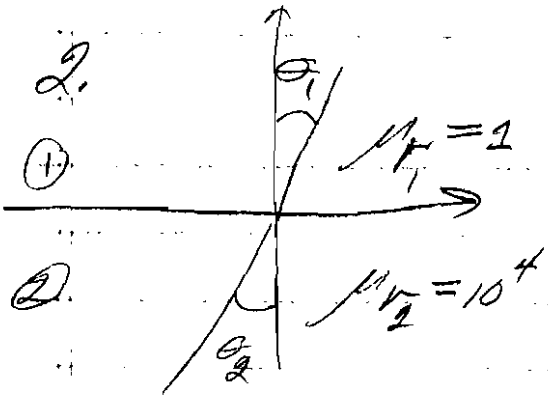
$\boxed{\bar{J}_{m_2} = \nabla\times\bar{M}_2 = (\mu_{r_2}-1)\left[-\frac{\rho}{2a}\right]\bar{a}_z}$

d) $H_{\phi_1}(\rho=a) \stackrel{?}{=} H_{\phi_2}(\rho=a)$

$\frac{a}{4} = a\left[\frac{5}{12}-\frac{1}{6}\right] = a\frac{3}{12} = \frac{a}{4}$

EE 333 Homework 9 (page 2)

2.



$$\tan\theta_1 = \frac{B_{tang_1}}{B_{norm_1}} \; ; \; \tan\theta_2 = \frac{B_{tang_2}}{B_{norm_2}}$$

but $B_{n_1} = B_{n_2}$

$$\therefore \frac{B_{tang_1}}{\tan\theta_1} = \frac{B_{tang_2}}{\tan\theta_2}$$

or $\dfrac{\mu_1 H_{tang_1}}{\tan\theta_1} = \dfrac{\mu_2 H_{tang_2}}{\tan\theta_2}$ However $H_{tang_1} = H_{tang_2}$

so $\boxed{\tan\theta_2 = \frac{\mu_2}{\mu_1}\tan\theta_1}$

$$\theta_1 = \tan^{-1}\left[\frac{\mu_1}{\mu_2}\tan\theta_2\right]$$

θ_2	θ_1
0°	0°
45°	$5.7 \times 10^{-3\,\circ}$
89°	0.328°
89.9°	3.27°

EE 333 Homework 10

3-24 a)

free space	Conductive Media
no attenuation	attenuation
E + H in phase (η real)	$\hat{\eta}$ $\bar{E}$ + $\bar{H}$ out of phase
$v_{ph} = 3\times10^8$ m/sec	v_{ph} reduced
$P_{ave} = \frac{E_m^{+2}}{2\eta_0}$	$P_{ave} = \frac{E_m^{+2} e^{-2\alpha z}}{2\lvert\hat{\eta}\rvert}\cos\theta$
	where $\hat{\eta} = \lvert\hat{\eta}\rvert e^{j\theta}$

b) $\mu_r = 1$, $\epsilon_r = 79$, $\sigma = 3$ S/m, $E(z=0) = 10$ V/m

i) $f = 2\times10^4$; $\frac{\sigma}{\omega\epsilon} = \frac{3\times36\pi}{2\pi\times2\times10^4\times79\times10^{-9}} = 0.34177\times10^5$

conduction current dominates (good conductor)

so $\alpha = \beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \frac{4\pi\times10^4\sqrt{79}}{\sqrt{2}\times3\times10^8}\sqrt{0.34177\times10^5}$

$\alpha = 4867.7\times10^{-4} = \boxed{0.48677}$

$10e^{-\alpha z} = 10^{-5}$; $e^{-\alpha z} = 10^{-6}$

$-\alpha z = -13.8155$ $\therefore z = \frac{13.8155}{0.48677} = \boxed{28.38\text{ m}}$

ii) $\frac{\sigma}{\omega\epsilon} = \frac{3\times36\pi}{2\pi\times2\times10^{10}\times10^{-9}\times79} = 0.34177\times10^{-1}$

$\therefore \alpha = \frac{2\pi\times2\times10^{10}\sqrt{79}}{\sqrt{2}\,3\times10^8}[0.024] = 0.632\times10^2 = 63.2$

$z = \frac{13.8155}{63.2} = \boxed{0.218\text{ m}}$

EE 333 Homework 10 (page 2)

3-24 c) @ $f = 2\times10^4$; $\hat{\eta} = \sqrt{\dfrac{\mu_0}{\epsilon(1 - j\frac{\sigma}{\omega\epsilon})}}$

or $\hat{\eta} = 120\pi \sqrt{\dfrac{1}{79(-j\,0.34177\times10^5)}} = 0.229\sqrt{j} = 0.229\, e^{j\frac{\pi}{4}}$

$P_{ave} = \dfrac{E_m^2}{2|\hat{\eta}|}\cos\theta = \dfrac{10^{-10}}{2\times0.229}\cos\dfrac{\pi}{4} = \boxed{1.544\times10^{-10}\ \text{watts}/m^2}$ ←

@ $f = 2\times10^{6}$ $\hat{\eta} = 120\pi\sqrt{\dfrac{1}{79(1 - j\,0.034177)}}$

$\hat{\eta} = 120\pi \sqrt{\dfrac{1}{79\times1.000\, e^{-j1.957^\circ}}} = 42.4\, e^{j0.978^\circ}$

so $P_{ave} = \dfrac{10^{-10}}{2\times42.4}\cos(0.978^\circ) = \boxed{1.179\times10^{-12}\ W/m^2}$ ←

3-34 $P_{ave_{max}} = 10\times10^{-3}\ W/cm^2 \times \dfrac{10^4\ cm^2}{m^2} = 10^2\ W/m^2$

$\therefore \frac{1}{2}E_m^2\left(\frac{1}{\eta_0}\right)\cos0^\circ = 10^2$

$E_m^2 = 2\times120\pi\times10^2 = 7.53\times10^4$

or $\boxed{E_m \cong 275\ V/m}$ ←

EE 333 Homework 11

5.1 $\lambda/2 = 1\ m$ but $\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{\omega\sqrt{\mu\epsilon_0}} = \frac{c}{f}$

$\therefore f = \frac{c}{\lambda} = \frac{3\times10^8}{2} = \boxed{1.5\times10^8\ Hz}$

5.4 $f = 8\times10^8$; $\lambda/2 = 6.25\times10^{-2}\ m$

a) $c = \frac{3\times10^8}{\sqrt{\epsilon_r}} = \lambda f$; $\therefore \epsilon_r = \left(\frac{3\times10^8}{12.5\times10^{-2}\times8\times10^8}\right)^2$

$\boxed{\epsilon_r = 0.9\times10^{-3}\times10^4 = 9}$

b) $H = 0$ @ $\boxed{z = -\lambda/4 = -3.125\times10^{-2}\ m}$

c) $H = \frac{2E_m}{\eta}\cos\beta z\cos\omega t$

@ $z = -0.4m$; $H = \frac{2\times200\sqrt{9}}{120\pi}\cos\left(\frac{2\pi\times0.4}{12.5\times10^{-2}}\right)\cos\omega t$

$H = 3.5\times0.309\cos\omega t = \boxed{1.08\cos\omega t}$

$\bar{J} = \bar{n}\times\bar{H} = -\bar{a}_z\times3.5\bar{a}_y\cos\omega t = \boxed{\bar{a}_x 3.5\cos\omega t}$

5.6

①	②
$\epsilon_{r_1} = 8$	$\epsilon_{r_2} = 2$
$\mu_{r_1} = 2$	$\mu_{r_2} = 2$

$\sigma_1 = \sigma_2 = 0$; $\hat{E}_1 = 250\, e^{-j\beta_1 z}\bar{a}_x$

$f = 2.5\times10^9$

a) $\Gamma(0) = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{120\pi\sqrt{\frac{2}{2}} - 120\pi\sqrt{\frac{1}{4}}}{120\pi\sqrt{1} + 120\pi\sqrt{\frac{1}{4}}} = \frac{1-0.5}{1+0.5} = \boxed{\frac{1}{3}}$

b) $\hat{E}_{m_1}(0^-) = \frac{250}{3} = \boxed{83.33}$

c) $\hat{E}_{m_2}(0^+) = \hat{T}(0)E^+_{m_1} = \frac{2\times1\times250}{1+\frac{1}{2}} = \frac{4}{3}\times250 = \boxed{333.3}$

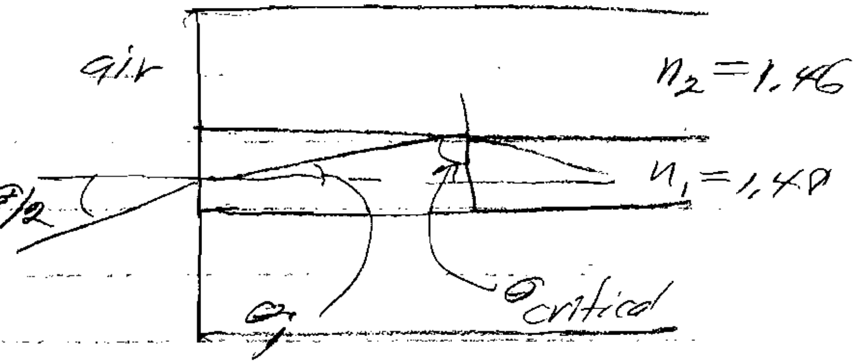
d) $\hat{H}^+_{m_2} = \frac{333.3}{377} = \boxed{0.884}$

? e) $Z = \eta_1\frac{1+\Gamma}{1-\Gamma} = \frac{120\pi}{2}\left[\frac{1+\Gamma(0^-)e^{+j2\beta(8.25\times10^{-2})}}{1-\Gamma(0^-)e^{-j2\beta(8.25\times10^{-2})}}\right] = 60\pi\left[\frac{3-1}{3+1}\right] = 30\pi$

or $\boxed{Z \cong 94.25\ \Omega}$

EE 333 Homework 12

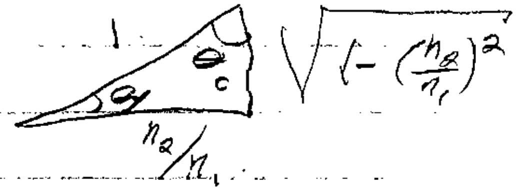
1. $\dfrac{\sin\theta_t}{\sin\left(\frac{\theta}{2}\right)} = \dfrac{1}{n_1}$



$\sin\theta_c = \dfrac{n_2}{n_1}$

$\sin\left(\frac{\theta}{2}\right) = n_1 \sin\theta_t$

but $\theta_t = 90^\circ - \theta_c$

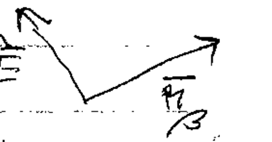


from figure $\sin\left(\frac{\theta}{2}\right) = n_1 \sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$

$$\boxed{\text{or } \sin\left(\frac{\theta}{2}\right) = \sqrt{n_1^2 - n_2^2}}$$

$\therefore \sin\left(\frac{\theta}{2}\right) = 0.242 \Rightarrow \boxed{\theta = 28^\circ}$

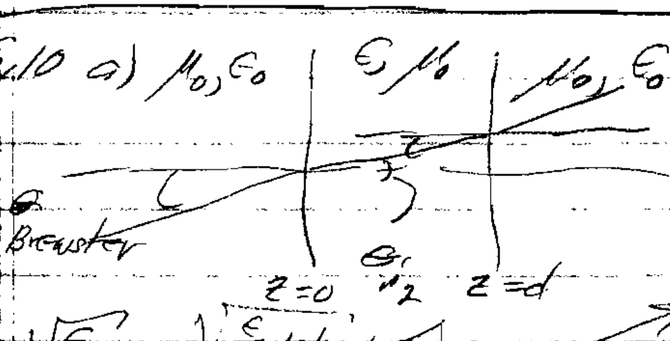
Revised question 6-2 $\quad \hat{\bar{E}} = \hat{E}_m e^{-j\beta \bar{n}_\beta \cdot \bar{r}}$ $\quad \bar{a}_\theta$



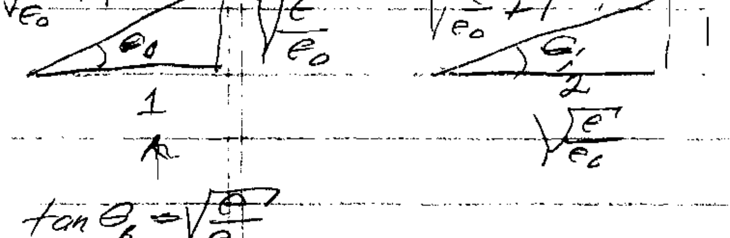
if we write $\hat{H} = \dfrac{\bar{n}_\beta \times \hat{\bar{E}}}{\eta}$ we have the correct magnitude and direction

i.e. $\bar{n}_\beta \times \hat{\bar{E}}$ is $\perp$ $\hat{\bar{E}}$ and directed such that $\hat{\bar{E}} \times \hat{H}$ is in the $\bar{n}_\beta$ direction (direction of propagation)

Problem 6.10 a) $\mu_0, \epsilon_0 \quad | \quad \epsilon, \mu_0 \quad | \quad \mu_0, \epsilon_0$ $\qquad \boxed{\theta_{t_1} = \theta_{i_2}}$



$\sin\theta_{t_1} = \sin\theta_B \sqrt{\dfrac{\epsilon_0}{\epsilon}} = \dfrac{1}{\sqrt{1+\epsilon/\epsilon_0}}$



$\tan\theta_b = \sqrt{\dfrac{\epsilon}{\epsilon_0}}$

$\Rightarrow \tan\theta_{i_2} = \tan\theta_{t_1} = \sqrt{\dfrac{\epsilon_0}{\epsilon}}$

$\left\{ \text{i.e. } \theta_{i_2} \text{ is the Brewster angle at } z=d\ ?\ \right\}$

EE 333 Homework 12 (page 2)

6.19 $\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}$ $\left\{ \begin{array}{l} n_1 = 1 \\ n_2 = 1.5 + \frac{0.5 \times 10^{-4}}{\lambda^2} \end{array} \right\}$

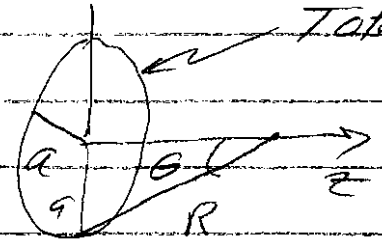
or $\sin\theta_t = \frac{1}{n_2}\sin\theta_i$ but $\theta_i = 30^\circ$

$\therefore \theta_t = \sin^{-1}\left(\frac{1}{2n_2}\right)$

λ	θ_t	
400 nm	19.058°	violet
450 nm	19.143°	blue
550 nm	19.25°	green
600 nm	19.28°	yellow
650 nm	19.31°	orange
700 nm	19.33°	red

EE 333 Homework 13

4.2 Total charge = Q $\left(\rho_\ell = \frac{Q}{2\pi a}\right)$



$R = \sqrt{a^2+z^2}$

$d\Phi = \frac{dQ}{4\pi\epsilon R}$

a) $\Phi = \int_{\phi=0}^{2\pi} \frac{dQ}{4\pi\epsilon R} = \int_{\phi=0}^{2\pi} \frac{Q a\, d\phi}{2\pi a\; 4\pi\epsilon \sqrt{a^2+z^2}} = \frac{Q}{4\pi\epsilon\sqrt{a^2+z^2}}$

b) $E = -\nabla\Phi = -\frac{\partial\Phi}{\partial z}\bar{a}_z = \boxed{\frac{2Qz\;\bar{a}_z}{8\pi\epsilon(a^2+z^2)^{3/2}}}$

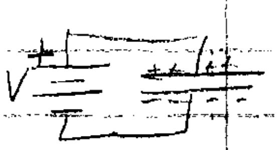
c) $dE_z = \frac{dQ\cos\theta}{4\pi\epsilon R^2} = \frac{Q a\, d\phi}{2\pi a} \cdot \frac{z}{4\pi\epsilon(a^2+z^2)^{3/2}}$

$E_z = \int_{\phi=0}^{2\pi} dE_z = 2\pi\, dE_z = \frac{Qz}{4\pi\epsilon(a^2+z^2)^{3/2}}$

4.7 $W = \frac{1}{2}\int_{vol} \Phi\rho_v\, dv = \frac{1}{2}\int \Phi\rho_s\, ds$

$\Phi(R) = \frac{Q}{4\pi\epsilon R}$ ∴ $W = \frac{1}{2}\frac{Q}{4\pi\epsilon R}\cdot Q = \boxed{\frac{Q^2}{8\pi\epsilon R}}$

4.8 a) $W = \frac{1}{2}\int \rho_v \Phi\, dv = \frac{V}{2}\int \rho_s\, ds = \boxed{\frac{QV}{2}}$



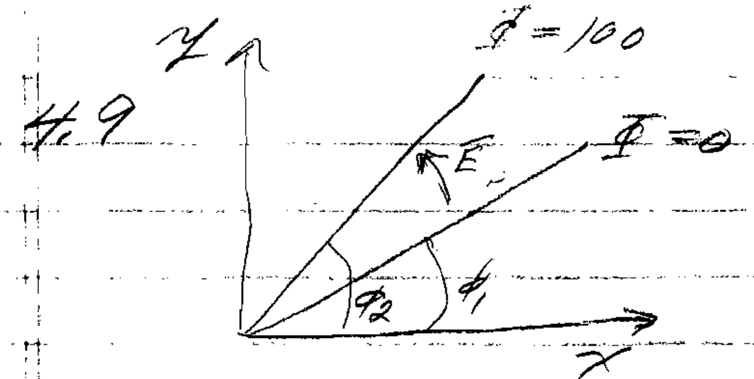
b) $V = \frac{Q}{C}$ ∴ from a) $\boxed{W = \frac{Q^2}{2C}}$

c) $C = \frac{\epsilon A}{d}$ $\left\{ \begin{array}{l} \text{if } d \text{ is increased by factor of 3} \\ C \text{ decreases by a factor 3} \end{array} \right\}$

Q is constant so from b) W increases by a factor of 3.

Homework 13 (page 2)

4.9



a) $\nabla^2 \Phi = 0 = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho \frac{\partial \Phi}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \phi^2}$

for this problem $\Phi = f(\phi)$

$\therefore \nabla^2 \Phi = \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \phi^2} = 0$

so $\Phi = C_1 \phi + C_2$

applying boundary conditions we have:

$0 = C_1 \phi_1 + C_2$; $100 = C_1 \phi_2 + C_2$

$C_2 = -C_1 \phi_1$; $100 = C_1 \phi_2 - C_1 \phi_1$ or $\boxed{C_1 = \frac{100}{\phi_2 - \phi_1}}$

$\boxed{C_2 = -\frac{100\phi_1}{\phi_2 - \phi_1}}$

$\therefore \boxed{\Phi = \frac{100}{\phi_2 - \phi_1}\left[\phi - \phi_1\right]}$

b) $\bar{E} = -\nabla \Phi = -\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\bar{a}_\phi = -\frac{100}{\phi_2 - \phi_1}\cdot\frac{1}{\rho}\bar{a}_\phi$

c) for plate @ $\phi = \phi_2$ $\rho_s = \epsilon|E| = \frac{100\,\epsilon}{\rho(\phi_2 - \phi_1)}$

$[\rho_s = \hat{n}\cdot\bar{D}$; $\bar{D} = \epsilon\bar{E}]$ @ $\phi = \phi_1$ $\rho_s = \frac{100\epsilon}{\rho(\phi_1 - \phi_2)}$

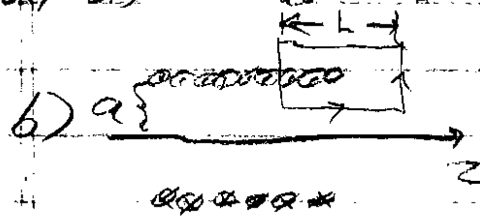
EE 333 Homework 14

$$C_1 = \frac{\epsilon A}{d} = \frac{\epsilon x w}{d}$$

Special $\quad W_e = \cancel{2}\times \frac{1}{\cancel{2}} C_1 V^2 = \frac{\epsilon x w}{d} V^2$

constant voltage so $\bar{F} = +\nabla W_e = +\frac{\partial W_e}{\partial x} \boxed{= \frac{\epsilon w V^2}{d}}$

4.27 a) $\int \bar{B}\cdot \overline{ds} = \int \nabla\times\bar{A}\cdot\overline{ds} = \oint \bar{A}\cdot\overline{dl}$ → Stokes Theorem

b) 

$\oint \bar{H}\cdot\overline{dl} = NLI = H_z L$ for $\rho < a$

so $\boxed{B_z = \mu N I}$ for $\rho < a$ and zero for $\rho > a$

we demonstrated early in the course that only B_z exists

$$\bar{B} = \nabla\times\bar{A} = \frac{1}{\rho}\left[\frac{\partial}{\partial\rho}(\rho A_\phi) - \underbrace{\frac{\partial A_\rho}{\partial\phi}}_{\text{zero}}\right]\bar{a}_z = \frac{1}{\rho}\frac{\partial}{\partial\rho}(\rho A_\phi)\bar{a}_z$$

we see that B_z is a function of A_ϕ only (i.e. only A_ϕ exists,

from part a) using a circular contour centred on z axis we have:

$$\text{for } \rho < a \quad \int_{\phi=0}^{2\pi} A_\phi \rho\, d\phi = \int_{\phi=0}^{2\pi}\int_{\rho=0} \overbrace{\mu N I}^{B_z} \rho\, d\phi\, d\rho = \frac{2\pi \mu N I \rho^2}{2}$$

$$\text{or } 2\pi\rho A_\phi = 2\pi\frac{\mu N I \rho^2}{2} \Rightarrow \boxed{A_\phi = \frac{\mu N I \rho}{2}}$$

$B_z = 0$ for $\rho > a$ so if we use a circular contour with $\rho > a$ we obtain:

$$2\pi A_\phi \rho = 2\pi\frac{\mu N I a^2}{2} \text{ or } \boxed{A_\phi = \frac{\mu N I a^2}{2\rho}}$$

EE 333 (Last Homework)

Special (force per unit area on solenoid)

From last weeks problem or early work in semester

$$H_{inside\ solenoid} = NI\bar{a}_z \quad \text{or} \quad B_z = \mu NI$$

$$\bar{F} = \nabla W_m \quad ; \quad W_m/_{m^3} = \frac{1}{2}\mu H^2$$

in a length L of solenoid of radius a $\quad W_m = \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi}\int_{z=0}^{L} \frac{1}{2}\mu N^2 I^2 \, dz\, \rho d\phi\, d\rho$

$$W_m = \frac{\mu I^2 N^2}{2} \cdot \frac{\cancel{2}\pi L a^2}{\cancel{2}} = \frac{\mu I^2 N^2 \pi L a^2}{2}$$

$\therefore$ Force on coils over a length $L = \frac{\partial W}{\partial a} = \mu I^2 N^2 \pi L a$

area of length $L = 2\pi a L$

$$\text{so } \bar{F}/_{\text{~~unit area~~}} = \frac{\mu I^2 N^2 \pi L a}{2\pi L a} = \boxed{\frac{1}{2}\mu I^2 N^2 \bar{a}_\rho}$$

4.29 $B = 1 \ \text{w}/\text{m}^2$ from curve $\boxed{H \cong 0.065 \text{ in iron}}$

ψ_m is the same in the gap as in the magnetic material

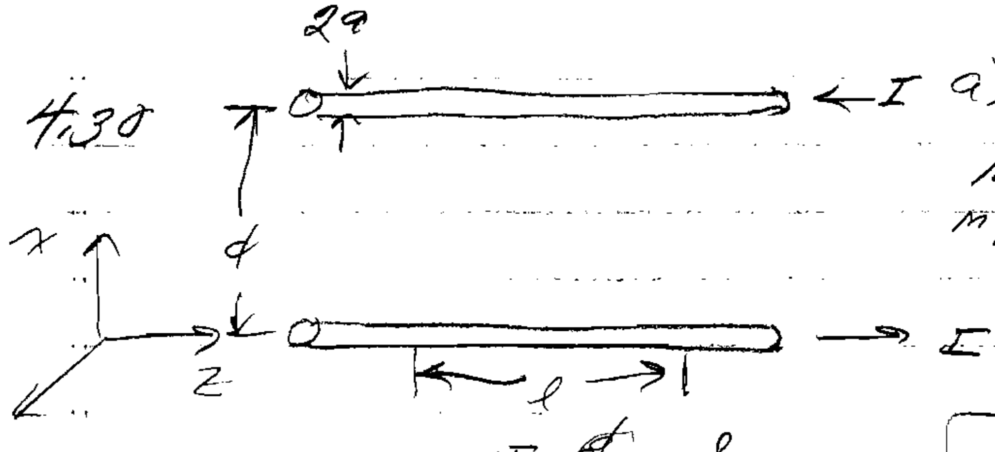
$$\therefore H_{gap} = \frac{B}{\mu_0}$$

$$NI = \oint \bar{H}\cdot d\bar{l} = 0.065 \times 0.4 + \frac{5\times10^{-4}}{4\pi\times10^{-7}} \cdot 1 = 0.026 + 0.398\times10^{3}$$

$$\boxed{\text{or } NI \cong 398 \text{ ampere-turns}}$$

EE 333 Last Homework (page 2)

4.30



a) $\overline{H} = \frac{I}{2\pi x}\,\overline{a_y}$ from lower current in plane of paper

from above ψ_m due to lower current over length l $= \frac{\mu I}{2\pi}\int_{x=a}^{d}\int_{z=0}^{l}\frac{1}{x}\,dz\,dx = \boxed{\frac{\mu I l}{2\pi}\ln\left(\frac{d}{a}\right)}$

valid for $d \gg a$

b) $\overline{B}$ between conductors $= \frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}$

$\therefore$ flux linkage for length l $= \int_{z=0}^{l}\int_{x=a}^{d-a}\left[\frac{\mu_0 I}{2\pi x} + \frac{\mu_0 I}{2\pi(d-x)}\right]dx\,dz$

or flux linkage $\lambda = \frac{\mu_0 I l}{2\pi}\ln\left(\frac{d-a}{a}\right) - \frac{\mu_0 I l}{2\pi}\ln\left(\frac{a}{d-a}\right)$

or $\lambda = \frac{\mu_0 I l}{\pi}\ln\left(\frac{d-a}{a}\right)$

L for length l $= \frac{\lambda}{I} = \frac{\mu_0 l}{\pi}\ln\left(\frac{d-a}{a}\right) \approx \boxed{\frac{\mu_0 l}{\pi}\ln\left(\frac{d}{a}\right)}$

for $d \gg a$

b) or $L/m = \frac{\mu_0}{\pi}\ln\left(\frac{d}{a}\right)$