EE 333 — Test 1 — Solution

1. $\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \epsilon_0 E_r\, r^2 \sin\theta\, d\theta\, d\phi = \underline{4\pi r^2 \epsilon_0 E_r} = \int_{r=0}^{r}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0 \sqrt{\frac{a}{r}}\, r^2 \sin\theta\, d\theta\, d\phi\, dr$

above correct for $r<a$ ∴ $4\pi r^2 \epsilon_0 E_r = 4\pi\rho_0\sqrt{a}\,\frac{r^{5/2}}{5/2}$

or $\boxed{E_r = \frac{2\rho_0\sqrt{a}}{5\epsilon_0}\, r^{1/2} \quad \text{for } r<a}$ ←

for $r>a$ $\boxed{E_r = \frac{2\rho_0 a^3}{5\epsilon_0 r^2}}$ ←

2. $\oint \frac{\bar B}{\mu_0}\cdot d\bar l = \int \bar J\cdot d\bar s$

for $\rho \le a$ $2\pi\rho\,\frac{B_\phi}{\mu_0} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} J_0\rho\,\bar a_z\cdot \rho\, d\phi\, d\rho\, \bar a_z = 2\pi J_0 \frac{\rho^3}{3}$

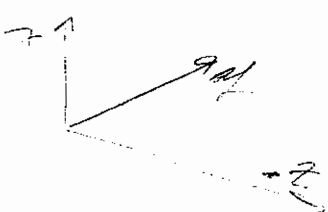
∴ $\boxed{B_\phi = \frac{\mu_0 J_0 \rho^2}{3}}$ ←

for $\rho > a$ $2\pi\rho\,\frac{B_\phi}{\mu_0} = 2\pi J_0 \frac{a^3}{3}$ ∴ $\boxed{B_\phi = \frac{\mu_0 J_0 a^3}{3\rho}}$ ←

3. @ surface $E_r = \frac{Q}{4\pi\epsilon_0 r^2}$

so $Q_{max} = 4\pi\epsilon_0 r^2 \times 3\times 10^6 = 4\pi\,\frac{10^{-9}}{36\pi}\times 25\times10^{-2}\times 3\times10^{6} = \boxed{8.333\times10^{-5}\ C}$ ←

4.



$\bar B = B_0\,\bar a_y$ $\bar J = \frac{I}{A}(-\bar a_z)$ ∴ $\bar v = K(\bar a_z)$

∴ $\bar F_{electrons} = +e\,K'\,\bar a_x$

so have net negative charge on top surface and it would take work to move a positive charge from top to bottom so bottom is at a higher potential and voltmeter would read a <u>negative</u> value.

5. $v_{phase} = \frac{\omega}{\omega k} = \frac{1}{k} = 340$ m/sec

∴ $\boxed{k = \frac{1}{340} = 2.94\times10^{-3}\ \text{sec/m}}$ ←