EE 333 Test 1 (Solution) (17)

1. a) $\nabla \cdot \overline{B} = \nabla \cdot (x\overline{a}_x) = 1$ ∴ not a possible $\overline{B}$ field

b) $\overline{F} = q(\overline{E} + \overline{v} \times \overline{B}) = q[10\overline{a}_x + 2\overline{a}_y + (v_x\overline{a}_x + v_y\overline{a}_y) \times 2\overline{a}_z]$

$\overline{F} = q[10\overline{a}_x + 2\overline{a}_y - 2v_x\overline{a}_y + 2v_y\overline{a}_x] = 0$ (possible)

$\boxed{v_x = 1,\ v_y = -5}$

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2. a) Can make up this spherically symmetric charge distribution from rings of charge with $\rho = K$ ∴ $\overline{E} = E_r\overline{a}_r$

b) $\iint \epsilon_0 \overline{E} \cdot \overline{dS} = \iiint \rho_v\, dv$ (use spherical surface)

for $r \le a$ $\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \epsilon_0 E_r r^2 \sin\theta\, d\theta\, d\phi = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi}\int_{r=0}^{r} \frac{r}{4\pi} r^2 \sin\theta\, dr\, d\theta\, d\phi$

or $4\pi r^2 \epsilon_0 E_r = \frac{r^4 \cancel{4\pi}}{4 \times 4\pi}$ or $\boxed{E_r = \frac{r^2}{16\pi\epsilon_0} \text{ for } r \le a}$

for $r > a$ must integrate ρ_v from $r=0$ to a (total charge is enclosed)

so $4\pi r^2 \epsilon_0 E_r = \frac{\cancel{4\pi} a^4}{4 \times \cancel{4\pi}}$; $\boxed{E_r = \frac{a^4}{16\pi\epsilon_0 r^2} \text{ for } r > a}$

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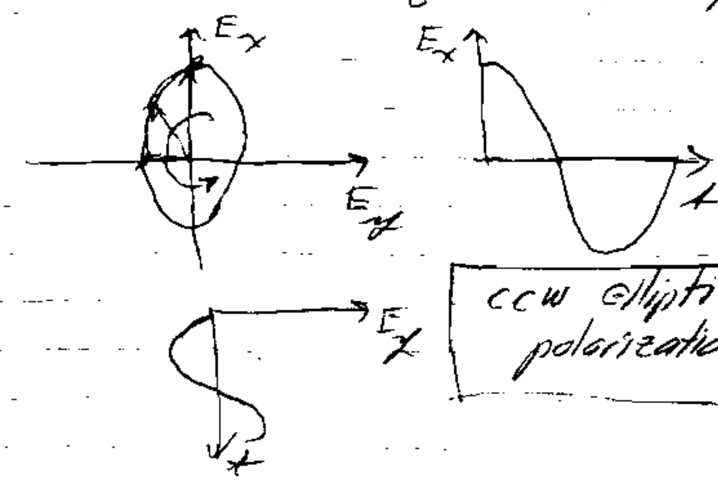
3. a) $\beta = \pi \times 10^6$ so $\overline{v}_{phase} = \frac{2\pi \times 10^6}{\pi \times 10^6}\overline{a}_z = \boxed{2\overline{a}_z \text{ m/sec}}$

b) $\lambda = \frac{2\pi}{\beta} = \boxed{2 \times 10^{-6} \text{ m}}$

c) $\overline{E} = Re\{\hat{\overline{E}} e^{j\omega t}\} = 2\cos(\omega t - \beta z)\overline{a}_x - \sin(\omega t - \beta z)\overline{a}_y$



ccw elliptic polarization

d) For each wave $\hat{\overline{E}} \times \hat{\overline{H}} = ?\,\overline{a}_z$

and $\frac{\hat{\overline{E}}}{\hat{\overline{H}}} = \eta_0$

$\boxed{∴\ \hat{\overline{H}} = \frac{1}{\eta_0}(2\overline{a}_y - j\overline{a}_x)e^{-j\beta z}}$