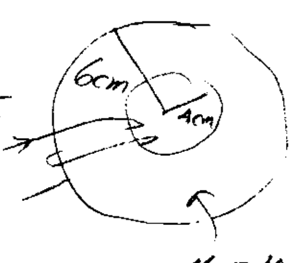
EE 434 Homework 1 5-16

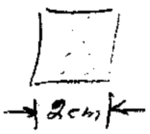
5-4

$\underline{0.1 = I}$

$\underline{n = 100}$



cross section



a) $\mathcal{R} = \dfrac{2\pi 5\times10^{-2}}{10^3 4\pi\times10^{-7}\times4\times10^{-4}}$

$\mathcal{R} = \dfrac{10^{-1}}{16\times10^{-8}} = 0.0625\times10^{7}$

$\therefore\ \psi_m = \dfrac{nI}{\mathcal{R}} = B_{ave}A = \mu H_{ave}A$

or $H_{ave} = \dfrac{nI}{\mathcal{R}\mu A} = \dfrac{nI\times A}{\ell\ \mu A} = \dfrac{nI}{\ell} = \dfrac{10}{2\pi5\times10^{-2}}$ $\boxed{= 0.318\times10^{2}}$

b) $\oint \overline{H}\cdot d\overline{\ell} = nI$ or $H_\phi = \dfrac{nI}{\ell}$ (the same as above)

with air gap don't have a path with constant $\overline{H}$ so cannot take $\overline{H}$ outside of the integral to find its value

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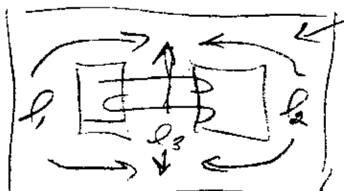
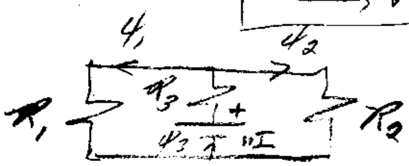
5-5



$\mu_r = 10^4,\ I = 0.1,\ n = 80$

$\ell_3 = 4cm,\ \ell_1 = \ell_2 = 12cm$

$A = 2cm^2 = 2\times10^{-4}m^2$



$\boxed{\psi_1 = \psi_2 = \dfrac{\psi_3}{2}}$

$\psi_3 = \dfrac{nI}{\mathcal{R}_3 + \dfrac{\mathcal{R}_1}{2}}$

$\psi_3 = \dfrac{8}{\dfrac{4\times10^{-2}}{4\pi\times10^{-7}\times10^{4}\times2\times10^{-4}} + \dfrac{6\times10^{-2}}{4\pi\times10^{-7}\times10^{4}\times2\times10^{-4}}} = \dfrac{8\times4\pi\times10^{-7}\times10^{4}\times2\times10^{-4}}{10^{-1}}$

$\boxed{\psi_3 = 201.06\times10^{-6} = 0.201\,mW = 2\psi_{1,2}}$

$B_{ave_3} = \dfrac{\psi_3}{A} = \dfrac{0.201\times10^{-3}}{2\times10^{-4}} = 0.1\times10$ $\boxed{\cong 1\ Weber/m^2}$

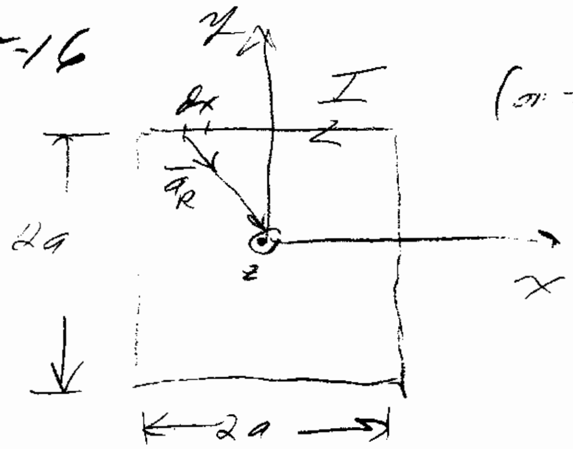
$B_{ave_1} = B_{ave_2} = \dfrac{\psi_1}{A} \cong$ $\boxed{0.5\ Weber/m^2}$

$H_{ave_3} = 2H_{ave_1} = 2H_{ave_2} = \dfrac{B_{ave_3}}{\mu_0\mu_r} = \dfrac{1}{4\pi\times10^{-7}\times10^{4}}$ $\boxed{\approx 80}$

$[H_{ave_1}\ell_1 + H_{ave_3}\ell_3 = nI]$? $\Rightarrow 40\times0.12 + 80\times0.04 = 8$
$4.8 + 3.2 = 8$ } QED

EE 434 Homework 1 (Solutions page 2)

5-16



(on top)

$$\overline{dB} = \frac{\mu I\, dx\, \bar a_x \times \bar a_R}{4\pi (x^2+a^2)}$$

$$\bar a_R = \frac{-x\bar a_x - a\bar a_y}{\sqrt{x^2+a^2}}$$

$$\therefore \quad \overline{dB} = \frac{\mu I\, dx\,(-a\bar a_z)}{4\pi (x^2+a^2)^{3/2}}$$

$$B_{z_{top}} = \int_a^{-a} \frac{-\mu I a\, dx}{4\pi (x^2+a^2)^{3/2}} = \frac{-\mu I a}{4\pi}\left\{\frac{x}{a^2(x^2+a^2)^{1/2}}\right\}_a^{-a}$$

$$B_{z_{top}} = \frac{-\mu I}{4\pi}\left\{\frac{-a}{a\sqrt{2}\,a} - \frac{a}{a\sqrt{2}\,a}\right\} = \frac{2\mu I}{4\pi\sqrt{2}\,a}$$

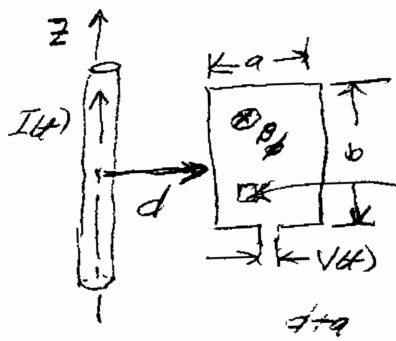
$$B_{z_{top}} = \frac{\mu I}{2\pi a\sqrt{2}} = \frac{\mu I\sqrt{2}}{4\pi a}$$

by symmetry other 3 sides contribute the same

$$\therefore \quad B_{z\,@\,center} = 4B_{z_{top}} = \frac{\mu I\sqrt{2}}{\pi a}$$

EE 434 Homework 2

5.19



$I(t) = I_m \sin \omega t$

a) $H_\phi = \dfrac{I_m \sin \omega t}{2\pi\rho}$ from $\oint \bar{H}\cdot d\bar{l} = I$

$d\bar{S} = dz\, d\rho\, \bar{a}_\phi$

b) $\oint \bar{E}\cdot d\bar{l} = V(t) = -\dfrac{d}{dt}\displaystyle\int_{\rho=d}^{d+a}\int_{z=0}^{b} \dfrac{\mu_0 I_m \sin \omega t}{2\pi\rho}\, dz\, d\rho$

$V(t) = -\dfrac{d}{dt}\left\{\dfrac{\mu_0 I_m \sin \omega t\; b}{2\pi} \ln\left(\dfrac{d+a}{d}\right)\right\} = -\dfrac{\mu_0 I_m \omega b \cos \omega t}{2\pi} \ln\left(\dfrac{d+a}{d}\right)$

(right hand terminal positive) $\boxed{\text{see A page 282}}$

or ... because $\bar{F}$ on positive charges would be in the z direction on both long sides of the loop but of greater magnitude on the left hand side driving a net positive charge to the right hand terminal.

for $I_m = 10A$, $f = 20kHz$, $d = 4\times10^{-3}\,m$, $a = b = 0.1\,m$

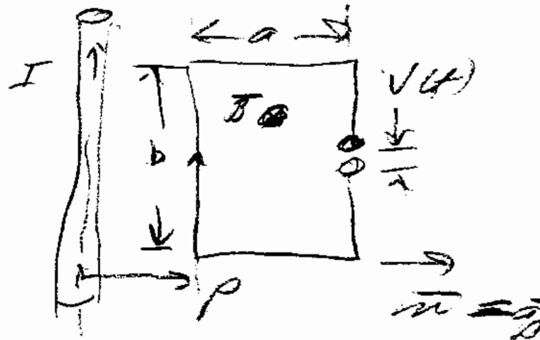
$V(t) = -\dfrac{4\pi\times10^{-7}\times10\times\cancel{2\pi}\times2\times10^{4}\times10^{-1}\ln\left(1+\dfrac{0.1}{4\times10^{-3}}\right)\cos(2\pi\times2\times10^{4}t)}{\cancel{2\pi}}$

$V(t) = -81.87\times10^{-3}\cos(4\pi\times10^{4}t) = \boxed{-81.9\cos(4\pi\times10^{4}t)\ mV}$

c) 2 times above with opposite polarity

EE 434 Homework 2 (solutions - page 2)

5.21



$V(t) = \oint \bar E\cdot d\bar l = \int \bar u \times \bar B \cdot d\bar l$

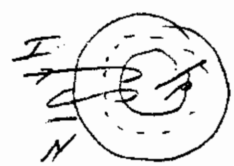
$\bar B = \frac{\mu_0 I}{2\pi\rho}\bar a_\phi$

$\bar u = \bar a_\rho u_\rho$

so $V(t) = \int_0^b \frac{\mu_0 I u}{2\pi\rho}\bar a_z \cdot dz\,\bar a_z + \int_b^0 \frac{\mu_0 I u}{2\pi(\rho+a)}\,dz = \frac{\mu_0 I u}{2\pi}\left\{\frac{b}{\rho} - \frac{b}{\rho+a}\right\}$

top terminal positive

5.23 $W_m = \frac{1}{2}\int_V \bar B\cdot\bar H\,dv$



$\oint \bar H\cdot d\bar l = NI$

$H_\phi = \frac{NI}{2\pi\rho}$

a) so $W_m = \frac{1}{2}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{d} \frac{\mu N^2 I^2}{4\pi^2\rho^2}\,dz\,\rho\,d\phi\,d\rho = \frac{\mu N^2 I^2\, 2\pi d}{2\times 4\pi^2}\ln\frac{b}{a}$

or $W_m = \frac{\mu d N^2 I^2}{4\pi}\ln\frac{b}{a} = \frac{1}{2}L I^2$

$\therefore \boxed{L = \frac{2W_m}{I^2} = \frac{\mu d N^2}{2\pi}\ln\frac{b}{a}}$ same as Example 5-17

b) $I = 2$, $\mu_r = 4\times10^3$, $a = 10^{-2}$, $b = 3\times10^{-2}$, $d = 2\times10^{-2}$, $N = 150$

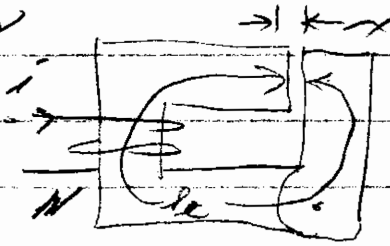
$W_m = \frac{4\times10^3\times4\pi\times10^{-7}\times2\times10^{-2}\times4\times1.5^2\times10^4}{4\pi} = 72\times10^{-2}\ln 3 = \boxed{79.1\times10^{-2}\ J}$

so $L = \frac{2W_m}{I^2} = \frac{2\times79.1\times10^{-2}}{4} = \boxed{0.395\ H}$

c) if $\mu_r = f(I)$ or $f(H)$ which would be true for ferromagnetic material?

EE 434 Homework 3

5.48



a) $\phi_m = \frac{NI}{R_{iron}+R_{gap}} = \frac{NI}{R_{total}}$

b) $L = \frac{\lambda}{I} = \frac{N\phi_m}{I} = \frac{N^2}{R_{total}}$

$\therefore U_m = \frac{1}{2} L I^2 = \frac{1}{2} \frac{N^2}{R} I^2$

found that $F_x = \frac{\partial W_m}{\partial x} = \frac{I^2}{2}\frac{\partial L}{\partial x} = \frac{I^2}{2}\frac{d}{dx}\left(\frac{N^2}{R}\right) = \frac{-N^2 I^2}{2R^2}\frac{\partial R}{\partial x}$

or $F_x = -\frac{1}{2}\phi_m^2 \frac{\partial R}{\partial x}$

so for $l_c = 0.12$, $A_c = 4\times10^{-4}$, $x = 1.5\times10^{-3}$, $I = 1.25A$, $N = 200$
and $\mu = 10^5\mu_0$

$R_{total} = \frac{0.12}{10^5\times4\pi\times10^{-7}\times4\times10^{-4}} + \frac{\overset{10^{-3}}{x}}{4\pi\times10^{-7}\times4\times10^{-4}} = \boxed{2.387\times10^3 + \underset{\uparrow}{0.0298\times10^8}} \quad 1.99\times10^9 x$

$\phi_m = \frac{200\times1.25}{R_{total}} = \frac{2.5\times10^2}{2.98\times10^6} = \boxed{0.839\times10^{-4} \text{ Webers}}$

$\therefore F_x = -\frac{1}{2}(0.839\times10^{-4})^2\times1.99\times10^9 = \boxed{-7 \text{ Newtons}}$

$B_{ave} = \frac{\phi_m}{A} = \frac{0.839\times10^{-4}}{4\times10^{-4}} = \boxed{0.21\ W/m^2}$

$H_{ave_{gap}} = \frac{B_{ave}}{\mu} = \frac{0.21}{4\pi\times10^{-7}} = \boxed{1.67\times10^5}$

$H_{ave_{iron}} = \frac{0.21}{4\pi\times10^{-7}\times10^5} = \boxed{1.67}$

$L = \frac{4\times10^4}{2.98\times10^6} = 1.31\times10^{-2} \boxed{\approx 13\ mH}$

$U_m = \frac{1}{2}LI^2 = \frac{1}{2}\,1.31\times10^{-2}\times1.25^2 = \boxed{1.02\times10^{-2}\ J}$

If $x = 0.75$ mm $R_{new} \approx \frac{R_{old}}{2}$ so $\phi_{m_{new}} = 2\phi_{m_{old}}$ and $\boxed{F_{x_{new}} = 4F_{x_{old}}}$

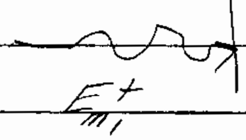
EE 434 Homework 3 (solutions - page 2)

5.48 $x=0$ $\frac{d\mathcal{R}}{dx}$ is same as before but $\mathcal{R}=\mathcal{R}_{1mm}=2.387\times10^{3}$

so $\phi_m = \frac{2.5\times10^{2}}{2.387\times10^{3}} = 1.047\times10^{-1}$

and $F_x = -\frac{1}{2}(1.047\times10^{-1})^2\times1.99\times10^{9} = \boxed{-1.09\times10^{7}\ N}$

6-5 air | $\epsilon=\epsilon_r\epsilon_0$



$$\frac{\hat{E}_{m_2}^{+}}{\hat{E}_{m_1}^{+}} = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2\eta_0\frac{1}{\sqrt{\epsilon_r}}}{\eta_0+\eta_0\frac{1}{\sqrt{\epsilon_r}}}$$

a) so $\boxed{\frac{\hat{E}_{m_2}^{+}}{\hat{E}_{m_1}^{+}} = \frac{2}{1+\sqrt{\epsilon_r}}}$

$$\boxed{\frac{\hat{E}_{m_1}^{-}}{\hat{E}_{m_1}^{+}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{1}{\sqrt{\epsilon_r}}-1}{\frac{1}{\sqrt{\epsilon_r}}+1} = \frac{1-\sqrt{\epsilon_r}}{1+\sqrt{\epsilon_r}}}$$

b) $\hat{E}_{m_1}^{+}=100$, $\epsilon_r=2.25$ $\sqrt{\epsilon_r}=1.5$

$$\hat{E}_{m_2}^{+} = \frac{100\times2}{2.5} = \boxed{80}$$

$$\hat{E}_{m_1}^{-} = 100\,\frac{-0.5}{2.5} = \boxed{-20}$$

c) $\epsilon_r=81$; $\sqrt{81}=9$

so $\hat{E}_{m_2}^{+} = \frac{100\times2}{10} = \boxed{20\ \frac{V}{m}}$

$E_{m_1}^{-} = 100\,\frac{-8}{10} = \boxed{-80\ \frac{V}{m}}$

6-6 $v_{phase} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$ ←

index of refraction $n = \sqrt{\epsilon_r}$ so $\boxed{n = \frac{c}{v_{ph}}}$ ←

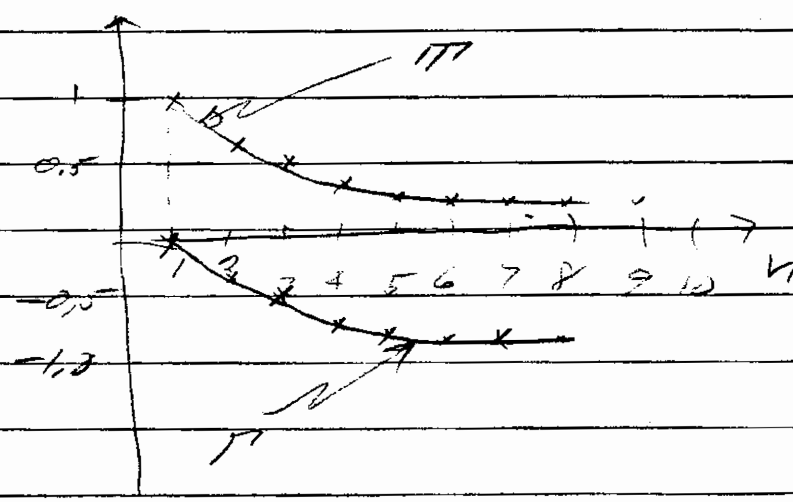
$n_{polyethylene} = \sqrt{2.25} = 1.5$ ←

$n_{water} = \sqrt{81} = 9$ ←

$\Gamma = \frac{1-n}{1+n}$; $T = \frac{2}{1+n}$

$\boxed{T - \Gamma = \frac{2-1+n}{1+n} = 1}$ ← {E field is continuous at boundary}

n	T	Γ
1	1	0
2	0.67	-0.33
3	0.5	-0.5
4	0.4	-0.6
5	0.33	-0.67
6	0.29	-0.71
7	0.25	-0.75
8	0.22	-0.77
9	0.2	-0.8
10	0.18	-0.82



EE 434 Homework 4

6-17 $\hat{\eta}_1$ | $\hat{\eta}_2$ | $\hat{\eta}_3$ $\quad \gamma_2 \to j\beta_2$

$\leftarrow \lambda/4 \rightarrow$ $\quad e^{\gamma_2 d} \to e^{j\frac{2\pi}{\lambda_2}\cdot\frac{\lambda_2}{4}} = j$

then $\hat{Z}_1(0) = \eta_2 \dfrac{j(\hat{\eta}_3+\eta_2) - j(\hat{\eta}_3-\eta_2)}{j(\hat{\eta}_3+\eta_2) + j(\hat{\eta}_3-\eta_2)} = \dfrac{2j\eta_2\cdot\eta_2}{2j\cdot\hat{\eta}_3} = \dfrac{\eta_2^2}{\hat{\eta}_3}$ ←

6-18 $\epsilon_{r_3} = 2.56$ in above problem we need $\hat{Z}_1(0) = \eta_1$

$\therefore \eta_2 = \sqrt{\eta_1\eta_3}$ or for all $\mu = \mu_0$

we want $\dfrac{\mu_0}{\epsilon_2} = \sqrt{\dfrac{\mu_0^2}{\epsilon_1\epsilon_3}}$; $\dfrac{1}{\epsilon_{r_2}} = \dfrac{1}{\sqrt{\epsilon_{r_1}\epsilon_{r_3}}}$

$\boxed{\epsilon_{r_2} = \sqrt{1\times 2.56} = 1.6}$ ←

reciprocal because get same value for η_2 if η_1 + η_3 are interchanged ?

6-20 $\hat{\eta}_1$ | η_2 | $\hat{\eta}_3$ $\quad e^{\gamma_2 d} \to e^{j\pi} = -1$

$\leftarrow \lambda/2 \rightarrow$

so $\hat{Z}_1(0) = \eta_2 \dfrac{-(\hat{\eta}_3+\eta_2) - (\hat{\eta}_3-\eta_2)}{-(\hat{\eta}_3+\eta_2) + (\hat{\eta}_3-\eta_2)} = \dfrac{\eta_2(-2\hat{\eta}_3)}{-2\eta_2} = \hat{\eta}_3$

same result whenever $e^{j\frac{2\pi}{\lambda}\cdot d} = -1$

or $d = n\dfrac{\lambda}{2}$; $n = 1, 2, 3, 4 \cdots$

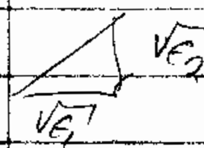
EE 43A   Homework 4   (Solutions - page 2)

6-35 ① μ_0, ϵ_0 | ② $\mu_0, 3\epsilon_0$ $\theta_{i_{\text{Brewster}}} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$\therefore$ from ① → ② $\theta_{i_B} = 60°$

② → ① $\theta_{i_B} = 30°$

must be // polarized

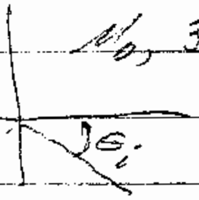


$\Longleftrightarrow \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 90° - \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

so $\theta_{i_{\text{Brewster}}}$ angles from opposite directions are always compliments!

EE 434 Homework 5

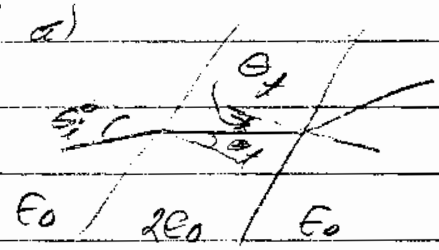
6.36 μ_0, ϵ_0 | $\mu_0, 3\epsilon_0$



$\theta_{critical} = \sin^{-1}\sqrt{\frac{1}{3}} = \underline{\underline{35.26^\circ}}$

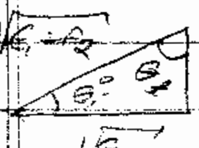
polarization not important

6.37 a)



$\theta_i^\circ = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$



$\sin\theta_t = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1+\epsilon_2}} \cdot \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1+\epsilon_2}}$

thus we see that $\boxed{\theta_t = 90^\circ - \theta_i^\circ}$

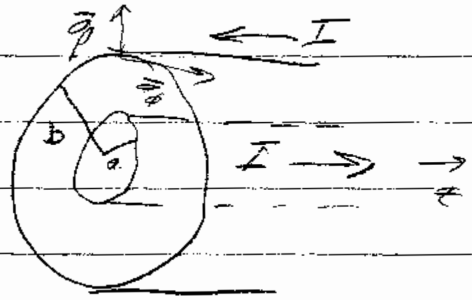
We showed in problem 6.35 that the Brewster angle for waves incident on the interface from either side are complementary angles. The angle of incidence at the second surface is θ_t so their will be no reflection at the surface because $\theta_t = 90^\circ - \theta_i^\circ$ as shown above

b) $\Gamma_{\parallel} = 0$; $\theta_i^\circ = \tan^{-1}\sqrt{2} = 54.73^\circ$

$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_t}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_t}$; $\theta_t = \sin^{-1}\left(\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right) = 35.26^\circ$

$\Gamma_\perp = \frac{0.5774 - 1.155}{0.5774 + 1.155} = \frac{-0.577}{1.732} = \boxed{-0.333}$

7.2



$$E_z = \frac{-J_z}{\sigma} \; ; \; J_z = \frac{I}{\pi c^2 - \pi b^2}$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\underset{@\rho = b}{\bar{E}\times\bar{H}} = \frac{J_z I}{\sigma 2\pi b}\,\bar{a}_\rho = \frac{I^2}{2\pi^2 b\sigma(c^2-b^2)}\,\bar{a}_\rho$$

$$\underset{\text{outer conductor}}{P_{\text{into}}} = \int_{z=0}^{L}\int_{\phi=0}^{2\pi} \frac{I^2\,\bar{a}_\rho}{2\pi^2 b\sigma(c^2-b^2)}\cdot b\,d\phi\,dz$$

$$P = \frac{2\pi L I^2}{2\pi^2\sigma(c^2-b^2)} = I^2\,\frac{L}{\sigma\pi(c^2-b^2)} = I^2 R \;\checkmark$$

EE 434 Homework 6

12.21

7-13 $\hat{E}_x = \hat{E}_m^+ e^{-j\beta z}[1+\Gamma]$ $\hat{H}_y = \frac{\hat{E}_m^+}{\eta} e^{-j\beta z}[1-\Gamma]$

$$\bar{P}_{ave} = \frac{1}{2}Re\{\hat{\bar{E}} \times \hat{\bar{H}}^*\} = \frac{1}{2}Re\left\{\frac{|E_m^+|^2}{\eta}(1+\Gamma)(1-\Gamma^*)\right\}\bar{a}_z$$

$$\bar{P}_{ave} = \frac{|E_m^+|^2}{2\eta} Re\{1+\cancel{\Gamma_r}+j\Gamma_i-\cancel{\Gamma_r}+j\Gamma_i-|\Gamma|^2\}\bar{a}_z$$

$$\bar{P}_{ave} = \frac{|E_m^+|^2}{2\eta}[1-|\Gamma|^2]\bar{a}_z = \bar{P}_{ave}^+ - \bar{P}_{ave}^-$$

$$\therefore \frac{|\bar{P}_{ave}^-|}{|\bar{P}_{ave}^+|} = |\Gamma|^2$$

7-17 $P_{ave} = \frac{1}{2}Re\left\{\frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{\eta} e^{j\theta}[1+\Gamma][1-\Gamma^*]\right\}$

$$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} e^{-2\alpha z} Re\{[\cos\theta + j\sin\theta][1+\cancel{\Gamma_r}+j\Gamma_i-\cancel{\Gamma_r}+j\Gamma_i-|\Gamma|^2]\}$$

$$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} e^{-2\alpha z}\{\cos\theta[1-|\Gamma|^2] - \sin\theta\, 2\Gamma_i\} \quad (1)$$

for $\Gamma = 0$ this becomes $\boxed{P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} e^{-2\alpha z}\cos\theta}$

b) lossless case $\hat{\eta} \to \eta$ i.e. $\theta \to 0$ and $\alpha \to 0$

giving: $\boxed{P_{ave} = \frac{|E_m^+|^2}{2\eta}[1-|\Gamma|^2] \text{ as in 7-13}}$

$$P_{ave\ \text{positive wave}} = \frac{1}{2}Re\left\{\frac{\hat{E}_m^+ e^{-\alpha z} \cancel{e^{-j\beta z}} e^{+j\theta} \hat{E}_m^{+*} e^{-\alpha z} \cancel{e^{+j\beta z}}}{\eta}\right\} = \frac{|\hat{E}_m^+| e^{-2\alpha z}\cos\theta}{\eta}$$

$$P_{ave\ \text{negative wave}} = \frac{1}{2}Re\left\{\frac{\hat{E}_m^+ e^{-\alpha z} \cancel{e^{-j\beta z}}\, \Gamma\, \hat{E}_m^{+*} e^{-\alpha z} \cancel{e^{+j\beta z}} e^{j\theta}(-\Gamma^*)}{\eta}\right\} = \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}|\Gamma|^2}{2\eta}\cos\theta$$

summing these two terms would miss the $-\sin\theta[2\Gamma_i]$ term in the above expression ?

EE 434 Homework 6 (solution - page 2)

7-21 P_{av} of sun @ earth $= 1340\ W/m^2$

assuming single frequency wave we have:

$$P_{av} = \frac{|\hat{E}_m^+|^2}{2\eta} \; ; \text{ or } |\hat{E}_m| = \sqrt{2 \times 377 \times 1340} = 1.005 \times 10^3\ V/m$$

$$|\hat{H}_m| = \frac{|\hat{E}_m|}{\eta} = 2.66\ A/m$$

P_{total} from sun $= 4\pi (1.48 \times 10^{11})^2\, 1.34 \times 10^3 = \boxed{36.88 \times 10^{25} \text{ Watts}}$

EE 434 Homework 7

A. 1" radius ⊙ waveguide

$$f_{c_{lowest}} = \frac{c}{2\pi}\frac{P_{11}'}{a} = \frac{3\times10^8 \times 1.841\times10^2}{2\pi\times2.54} = \underline{3.46\times10^9\,Hz}$$

B. P_{max} for $0.4\times0.9"$ waveguide @ 10 GHz

$$\hat{E}_y = -j K \frac{\omega\mu a}{\pi}\sin\frac{\pi}{a}x = E_{y\,max}\sin\frac{\pi}{a}x$$

$$\hat{H}_x = K_1'\frac{\beta_{10}a}{\pi}\sin\frac{\pi}{a}x = -E_{y\,max}\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\frac{\pi}{a}x$$

$$P_{avg/m^2} = \frac{1}{2}\operatorname{Re}\{\hat{E}\times\hat{H}^*\} = \frac{1}{2}\operatorname{Re}\{\hat{a}_z E_{y\,max}^2\sin^2\frac{\pi}{a}x\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\}$$

$$P_{avg} = \frac{1}{2}E_{y\,max}^2\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\int_{y=0}^{b}\underbrace{\int_{x=0}^{a}\sin^2\frac{\pi}{a}x\,dx}_{\frac{1}{2}\left[1-\cos\frac{2\pi x}{a}\right]}\,dy$$

$$\text{so } P_{avg} = \frac{1}{2}E_{y\,max}^2\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\,\frac{ba}{2} \qquad \text{(Equation 8-90)}$$

$$P_{avg} = 1.16\times10^{-7}E_{y\,max}^2$$

but $E_{y\,max} \cong 3\times10^6$ V/m so $P_{avg_{max}} = 1.16\times10^{-7}\times9\times10^{12} \cong \underline{1.04\times10^6\,W}$

8.23 $f_c = \frac{1}{2\sqrt{\mu\epsilon}\,a}$ TE_{10} mode

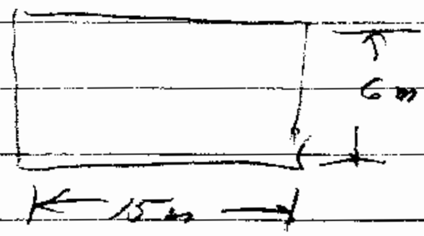
a) 5 GHz; $a_{min} = \frac{1}{2\sqrt{\mu\epsilon}\,f} = \frac{3\times10^8}{10\times10^9} = \underline{3\times10^{-2}\,m}$ ←

b) 5 MHz; $a_{min} = \frac{3\times10^8}{10\times10^6} = \underline{30\,m}$ ←

c) 5 KHz; $a_{min} = \frac{3\times10^8}{10\times10^3} = \underline{3\times10^4\,m}$ ←

EE 434 Homework 7 (solutions - page 2)

8.27



TE_{10} mode

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}\,15} = \frac{3\times10^8}{30} = \underline{10\,MHz}$$

<u>Vertical polarization</u>

AM will not propagate } 535 → 1605 KHz
FM will propagate } 88 → 108 MHz

EE 434 Homework 8

11-11 $\Delta z = 0.01\lambda$; $\hat{I} = 5A$; $f = 300MHz$

a) $\lambda = \frac{3\times10^8}{3\times10^8} = 1m$ $\therefore$ $\boxed{\Delta z = 0.01m}$

(far zone if $\beta_0 r >> 1$ from (11-30)
so $\beta_0 r = 20$ is reasonably in the far zone)
this is $\left[r = \frac{20}{\beta_0} = \frac{20}{2\pi}\lambda = 3.18m \right]$

b) $d\hat{E}_\theta = \eta\, d\hat{H}_\phi = \frac{j\omega\mu \hat{I}\Delta z e^{-j\beta r}}{4\pi r}\sin\theta$

$\sin\theta = 1$ @ $\theta = 90°$

c) $\bar{P} = \frac{1}{2}Re\{d\hat{E}_\theta \bar{a}_\theta \times d\hat{H}_\phi^* \bar{a}_\phi\} = \frac{1}{2}\cdot\frac{\omega^2\mu^2 \hat{I}^2 \Delta z^2}{\eta\, 16\pi^2 r^2}\bar{a}_r$

$\bar{P} = \frac{\cancel{4\pi^2} f^2 \mu^2 \hat{I}^2 \Delta z^2 \sqrt{\epsilon}\sqrt{\epsilon}}{\sqrt{\mu}\, \cancel{32\pi^2}_{8}\, r^2 \sqrt{\epsilon}}\bar{a}_r = \frac{\mu \hat{I}^2 \Delta z^2}{8\sqrt{\mu\epsilon}\,\lambda^2 r^2}\bar{a}_r$

$\boxed{\bar{P} = \frac{\eta \hat{I}^2}{8 r^2}\left(\frac{\Delta z}{\lambda}\right)^2} = \frac{377\times25\times10^{-4}}{8\times3.18^2} = \boxed{0.0117\ Watts/m^2}$

11-14 $P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin^2\theta \frac{\eta \hat{I}^2}{8r^2}\left(\frac{\Delta z}{\lambda}\right)^2 r^2 \sin\theta\, d\theta\, d\phi$

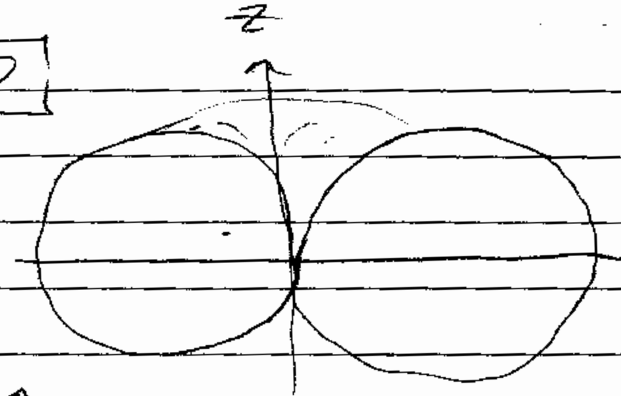
$\left[P_{rad} = \frac{2\pi\eta\hat{I}^2}{8_4}\left(\frac{\Delta z}{\lambda}\right)^2 \underbrace{\int_0^{\pi}\sin^3\theta\, d\theta}_{4/3} = \frac{\pi\eta\hat{I}^2}{3}\left(\frac{\Delta z}{\lambda}\right)^2 \right]$

a) so $P_{rad} = \frac{\pi\times377\times25\times10^{-4}}{3} = \boxed{0.99\ Watts}$

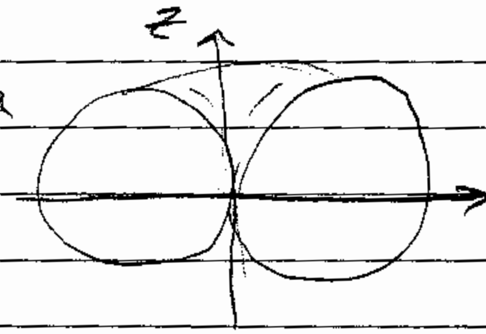
b) if $\Delta z \to 0.02m$ $\quad P_{rad} = 4\times0.99 = \underline{3.95\ Watts}$

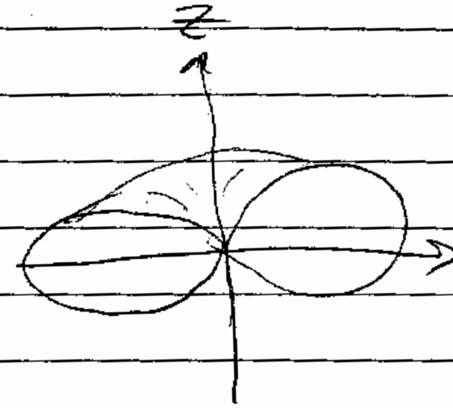
if $\Delta z = 0.005m$ $\quad P_{rad} = \frac{0.99}{4} = \underline{0.25\ Watts}$

11-19 a) $\boxed{L = \frac{\lambda}{2} = 2\ell}$



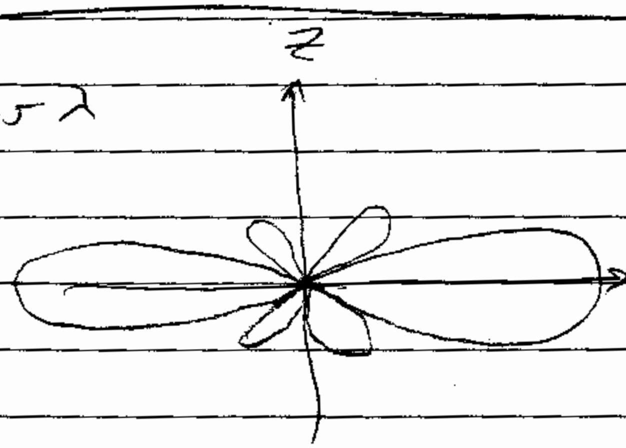
b) $L = \frac{5}{8}\lambda$





c) $L = \lambda$

11-20 a) $L = 1.5\lambda$



b) $L = 2\lambda$