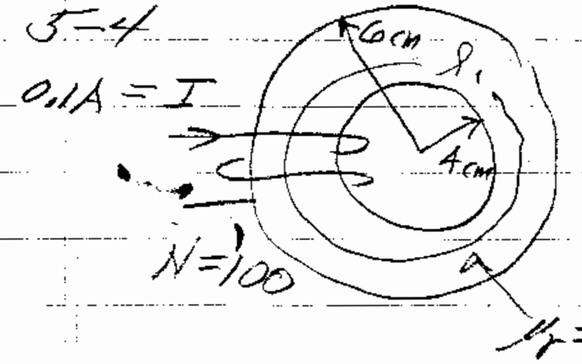
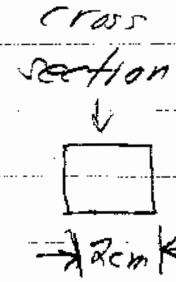
5-5
5-16

EE 434 Homework 1

5-4





a) $R = \dfrac{2\pi 5\times10^{-2}}{10^3\times4\pi\times10^{-7}\times4\times10^{-4}}$

$R = 0.0625\times10^7$

$\therefore \psi = \frac{nI}{A} = B_{ave}A = \mu H_{ave} A$

$\psi = \frac{NI}{R}$

or $H_{ave} = \frac{\psi}{\mu A} = \frac{NI}{\mu A R} = \frac{NI\cancel{\mu A}}{\cancel{\mu A}l} = \frac{NI}{l} = \frac{10^{-1}\times10^2}{2\pi\times5\times10^{-2}} = \boxed{31.8}$

b) $\oint \bar{H}\cdot d\bar{l} = NI = Hl$ or $\boxed{H = \frac{NI}{l} \text{ as above}}$

With a gap we don't have a path with constant $\bar{H}$ so we can not take $\bar{H}$ outside the line integral and calculate its value

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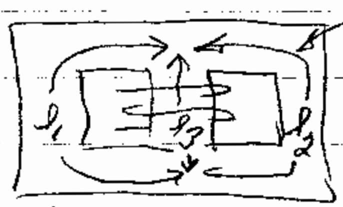
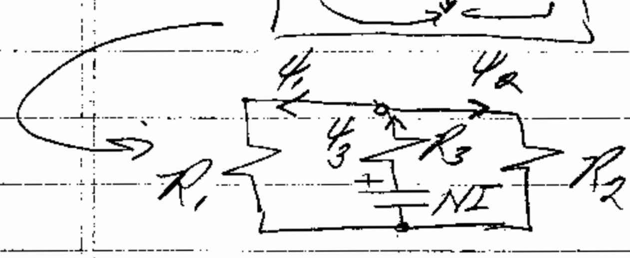
5-5



$\mu_r = 10^4$, $I = 0.1A$, $N = 80$

$l_3 = 4cm$, $l_1 = l_2 = 12cm$

$A = 2cm^2$



$\boxed{\psi_1 = \psi_2 = \frac{\psi_3}{2}}$

$\psi_3 = \dfrac{NI}{R_3 + \frac{R_1}{2}} = \dfrac{8}{\dfrac{4\times10^{-2}}{4\pi\times10^{-7}\times10^4\times2\times10^{-4}} + \dfrac{6\times10^{-2}}{4\pi\times10^{-7}\times10^4\times2\times10^{-4}}}$

$\boxed{\psi_3 = 201.6\times10^{-6} = 0.202 m \text{ Webers} = 2\psi_{1,2}}$

$B_{AVE_3} = \frac{\psi_3}{A} = \frac{0.202\times10^{-3}}{4\times10^{-4}} \cong 1 \text{ Weber/m}^2$

$H_{ave_3} = 2H_{ave_1} = 2H_{ave_2} = \frac{B_{ave_3}}{\mu_0\mu_r} \boxed{\simeq 80}$

EE 434 Homework 1 (page 2)

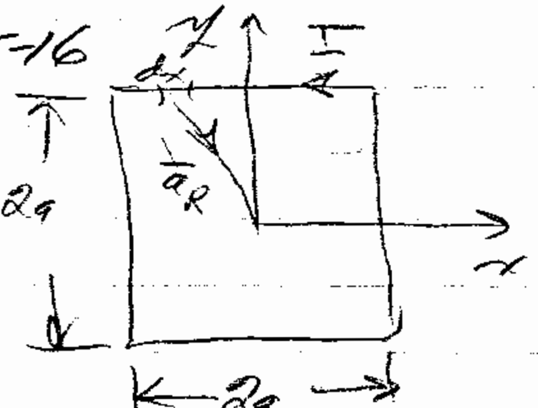
5-5 (continued) $H_{ave_1} l_1 + H_{ave_3} l_3 = nI$?

$$40\times 0.12 + 80\times 0.04 \stackrel{?}{=} 8$$

$$4.8+3.2 \stackrel{?}{=} 8$$

$$\boxed{8=8} \; !$$

5-16



For top of square we have:

$$\overline{dB} = \frac{\mu I\,dx\,\bar a_x \times \bar a_R}{4\pi(x^2+a^2)}$$

$$\bar a_R = \frac{-x\bar a_x - a\bar a_y}{\sqrt{x^2+a^2}}$$

$$\therefore \overline{dB} = \frac{\mu I\,dx\,(-a\bar a_z)}{4\pi(x^2+a^2)^{3/2}}$$

$$B_{z_{top}} = \int_a^{-a} \frac{-\mu I\,dx\,a}{4\pi(x^2+a^2)^{3/2}} = -\frac{\mu I a}{4\pi}\left\{\frac{x}{a^2(x^2+a^2)^{1/2}}\right\}_a^{-a}$$

$$B_{z_{top}} = -\frac{\mu I}{4\pi}\left\{\frac{-a}{a\sqrt{2}\,a} - \frac{a}{a\sqrt{2}\,a}\right\}$$

$$B_{z_{top}} = \frac{\mu I}{2\pi a\sqrt{2}} = \frac{\mu I\sqrt{2}}{4\pi a}$$

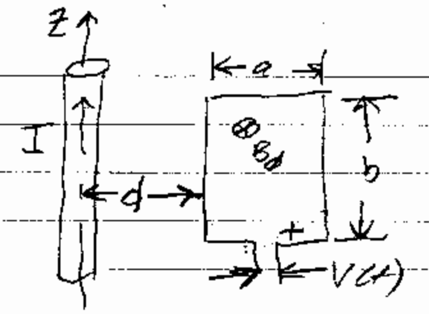
From symmetry all sides contribute the same

$$\boxed{\therefore B_{z_{total}} = \frac{\sqrt{2}\,\mu I}{\pi a}}$$

EE 434 Homework 2

quasi-static

5-19



$I(t) = I_m \sin\omega t$; $\oint \bar H \cdot d\bar l = I$

a) $H_\phi = \frac{I}{2\pi\rho} = \frac{I_m \sin\omega t}{2\pi\rho}$

clockwise integration

b) $\oint \bar E\cdot d\bar l = -\frac{d}{dt}\left\{\int_{\rho=d}^{d+a}\int_{z=0}^{b} \frac{\mu_0 I_m \sin\omega t}{2\pi\rho}\, dz\, d\rho\right\}$

$\oint \bar E\cdot d\bar l = -\frac{d}{dt}\left\{\frac{\mu_0 I_m \sin\omega t}{2\pi}\, b \ln\frac{d+a}{d}\right\} = -\frac{\omega\mu_0 I_m b \ln\left[\frac{d+a}{d}\right]}{2\pi}\cos\omega t$

(right hand terminal positive)

(case A page 282 of text)

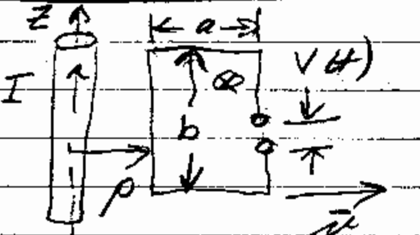
For $I_m = 10A$, $f = 20kHz$, $d = 4\times10^{-3}m$, $a=b=0.1m$

$V(t) = -\frac{2\pi\times2\times10^4\times4\pi\times10^{-7}\times10\times10^{-1}}{2\pi}\ln\left[1+\frac{0.1}{4\times10^{-3}}\right]\cos(2\pi\times2\times10^4 t)$

or $V(t) = -81.9\cos(4\pi\times10^4 t)$ mV

c) 2 times above result with opposite polarity

5-21

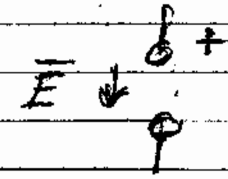


$\oint \bar E\cdot d\bar l = \oint \bar v\times\bar B\cdot d\bar l$

$\bar B = \frac{\mu_0 I}{2\pi\rho}\bar a_\phi$ (integrate clockwise)

$\oint \bar E\cdot d\bar l = \int \bar v\times\bar B\cdot d\bar l = \int_0^b v\,\bar a_\rho\times\frac{\mu_0 I}{2\pi\rho}\bar a_\phi\cdot dz\,\bar a_z + \int_b^0 v\,\bar a_\rho\times\frac{\mu_0 I}{2\pi(\rho+a)}\bar a_\phi\cdot dz\,\bar a_z$

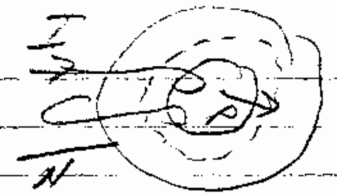
$\oint \bar E\cdot d\bar l = \frac{v\mu_0 I}{2\pi}\left\{\frac{b}{\rho}-\frac{b}{\rho+a}\right\}$



(top terminal positive)

EE 43A Homework 2 (page 2)

5-23 $U_m = \frac{1}{2}\int_{vol} \bar{B}\cdot\bar{H}\,dv$



a) $\oint \overline{H\cdot dl} = NI$ $\therefore$ $H_\phi = \frac{NI}{2\pi\rho}$

so $U_m = \frac{1}{2}\int_{z=0}^{d}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi} \frac{\mu N^2 I^2}{4\pi^2\rho^2}\,\rho\,d\rho\,d\phi\,dz$

$$U_m = \frac{\mu N^2 I^2\, 2\pi\, d}{2\times 4\pi^2}\ln\left(\frac{b}{a}\right) = \frac{1}{2}LI^2$$

$$\therefore L = \frac{2U_m}{I^2} = \frac{2\mu N^2 I^2 d \ln\left(\frac{b}{a}\right)}{4\pi I^2}$$

$$\text{or } L = \frac{\mu d N^2}{2\pi}\ln\left(\frac{b}{a}\right)$$

b) $I = 2$, $\mu_r = 4\times10^3$, $a = 10^{-2}$, $b = 3\times10^{-2}$, $d = 2\times10^{-2}$

$N = 150$

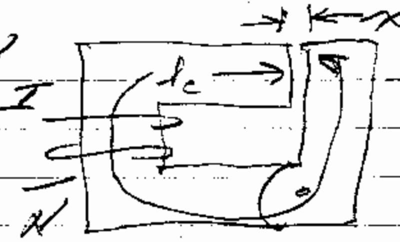
$$\boxed{U_m = 79.1\times10^{-2}\ J}$$

$$\boxed{L = \frac{2U_m}{I^2} = 0.395\ H}$$

c) if $\mu_r = f(I)$ or $f(H)$ which would be true for ferromagnetic material?

EE 434 Homework 3

5-48



a) $\phi_m = \dfrac{NI}{\mathcal{R}_{iron} + \mathcal{R}_{gap}}$

b) $L = \dfrac{\lambda}{I} = \dfrac{N\phi_m}{I} = \dfrac{N^2}{\mathcal{R}_{total}}$ ←

$\therefore U_m = \frac{1}{2} L I^2 = \frac{1}{2} \dfrac{N^2 I^2}{\mathcal{R}_{total}}$ ←

in general $F_x = \dfrac{\partial U_m}{\partial x} = \dfrac{I^2}{2}\dfrac{\partial L}{\partial x} = -\dfrac{\phi_m^2}{2}\dfrac{\partial \mathcal{R}}{\partial x}$ ←

$\left\{ \begin{array}{l} \text{for } l_c = 0.12,\ A_c = 4\times10^{-4},\ x = 1.5\times10^{-3},\ I = 1.25A,\ N = 200; \\ \mu = 10^5 \mu_0 \end{array} \right.$

$$\mathcal{R}_{total} = \frac{0.12}{10^5 \times 4\pi\times10^{-7}\times4\times10^{-4}} + \frac{x}{4\pi\times10^{-7}\times4\times10^{-4}} = 2.387\times10^{3} + 19.89\times10^{8} x$$

for the above x $\mathcal{R}_{total} = 2.387\times10^3 + 1.989\times10^9 \times 1.5\times10^{-3}$

$$\therefore \phi_m = \frac{200\times1.25}{29.84\times10^5} = 8.37\times10^{-5} \text{ Webers} \leftarrow$$

$$\text{so } F_x = -\tfrac{1}{2}(8.37\times10^{-5})^2 \times 19.89\times10^8 = \boxed{-6.97 \text{ Newtons}} \leftarrow$$

$$B_{ave} = \frac{\phi_m}{A} = \boxed{0.21 \ W/m^2} \leftarrow$$

$$H_{ave_{gap}} = \frac{B_{ave}}{\mu_0} = \boxed{1.67\times10^5}$$

$$H_{ave_{iron}} = \frac{0.21}{4\pi\times10^{-7}\times10^5} = \boxed{1.67}$$

$$L = \frac{N^2}{\mathcal{R}_{total}} = \frac{4\times10^4}{2.98\times10^6} \cong \boxed{13 mH}$$

$U_m = \frac{1}{2} L I^2 =$ [illegible] J

if $x = 0.75$ mm $\mathcal{R}_{new} = \dfrac{\mathcal{R}_{total}}{2}$; $\phi_{m_{new}} = 2\phi_{m_{old}}$ so $\boxed{F_{x_{new}} = 4F_x}$ ←

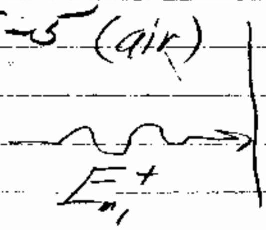
5.48 (continued) if $x \to 0$ $\frac{\partial R}{\partial x}$ is the same

but $R = R_{iron} = 2.387 \times 10^{8}$

so $\psi_{m} = \frac{2.5 \times 10^{8}}{2.387 \times 10^{8}} = 1.047 \times 10^{-1}$

$F_x = -\frac{1}{2}(1.047 \times 10^{-1})^2 \times 1.99 \times 10^{9} = \boxed{-1.09 \times 10^{7}\ N}$

6-5 (air)



$\epsilon = \epsilon_r \epsilon_0$ $\quad \frac{\hat{E}_{m_2}^+}{\hat{E}_{m_1}^+} = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\eta_0 \frac{1}{\sqrt{\epsilon_r}}}{\eta_0 + \eta_0 \frac{1}{\sqrt{\epsilon_r}}}$

so $\boxed{\frac{\hat{E}_{m_2}^+}{\hat{E}_{m_1}^+} = \frac{2}{1+\sqrt{\epsilon_r}}}$

$\frac{\hat{E}_{m_1}^-}{\hat{E}_{m_1}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \boxed{\frac{1-\sqrt{\epsilon_r}}{1+\sqrt{\epsilon_r}}}$

b) $\hat{E}_{m_1}^+ = 100$, $\epsilon_r = 2.25$, $\sqrt{\epsilon_r} = 1.5$

so $\hat{E}_{m_2}^+ = 100 \frac{2}{2.5} = \boxed{80}$

$\hat{E}_{m_1}^- = 100 \frac{-0.5}{2.5} = \boxed{-20}$

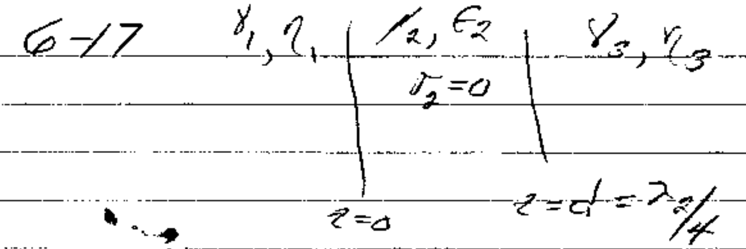
c) $\epsilon_r = 81$; $\sqrt{81} = 9$

so $\hat{E}_{m_2}^+ = \frac{100 \times 2}{10} = \boxed{20 \frac{V}{m}}$

$\hat{E}_{m_1}^- = (100 \times -8)/10 = \boxed{-80\ V/m}$

EE 434 Homework 4

6-17



$\gamma_2 = j\beta_2$

$e^{\gamma_2 d} = e^{j\beta_2 d} = e^{j\frac{2\pi}{\lambda_2}\cdot\frac{\lambda_2}{4}} = j$

$\therefore$ from 6-99a $\quad Z(0) = \eta_2 \dfrac{j(\hat\eta_3+\hat\eta_2)+j(\hat\eta_2-\hat\eta_3)}{j(\hat\eta_3+\hat\eta_2)+j(\hat\eta_3-\hat\eta_2)} = \dfrac{\cancel{2}\hat\eta_2^2}{\cancel{2}\hat\eta_3} = \dfrac{\eta_2^2}{\eta_3}$

6-18 $\epsilon_{r_3} = 2.56$ in the above problem we need $Z(0) = \eta_1$

$\therefore \eta_2 = \sqrt{\eta_1 \eta_3}$ assuming $\mu_1 = \mu_2 = \mu_3 = \mu_0$

this becomes $\dfrac{1}{\epsilon_2} = \dfrac{1}{\sqrt{\epsilon_1\epsilon_3}}$; $\epsilon_{r_2} = \sqrt{\epsilon_{r_1}\epsilon_{r_2}}$

$$\boxed{\epsilon_{r_2} = \sqrt{2.56} = 1.6}$$

reciprocal because we obtain the same value of η_2 if η_1 and η_3 are interchanged!

6-20 problem 6-17 with $d = \frac{\lambda}{2}$; $e^{j\beta_2 d} = e^{j\frac{2\pi}{\lambda_2}\frac{\lambda_2}{2}} = -1$

$\therefore Z_0 = \eta_2 \dfrac{-\hat\eta_3 - \cancel{\eta_2} - \hat\eta_3 + \cancel{\eta_2}}{-\cancel{\hat\eta_3} - \eta_2 + \cancel{\hat\eta_3} - \eta_2} = \dfrac{\cancel{\eta_2}(-2\hat\eta_3)}{-2\cancel{\eta_2}} = \boxed{\hat\eta_3}$

same result whenever $e^{j\frac{2\pi}{\lambda_2}d} = -1$

or $d = n\dfrac{\lambda_2}{2} \Rightarrow e^{j\frac{2\pi}{\lambda_2}\frac{n\lambda_2}{2}} = e^{jn\pi}$

$\rightarrow$ n odd

6-35

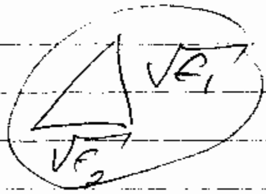
① μ_0, ϵ_0 | ② $\mu_0, 3\epsilon_0$

$\theta_i^\circ = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$\therefore$ from ① → ② $\theta_i^\circ = \tan^{-1}\sqrt{3} = 60^\circ$

from ② → ① $\theta_i^\circ = \tan^{-1}\sqrt{\frac{1}{3}} = 30^\circ$

must be ∥ polarized $\left\{ \text{there is no Brewster angle for } \perp \text{ polarization} \right\}$

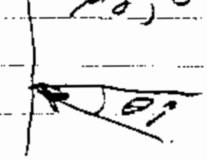


$$\boxed{\Rightarrow \tan^{-1}\sqrt{\frac{\epsilon_1}{\epsilon_2}} = 90^\circ - \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}}$$

so we see that θ_i°'s from opposite direction are always compliments

EE 434 Homework 5

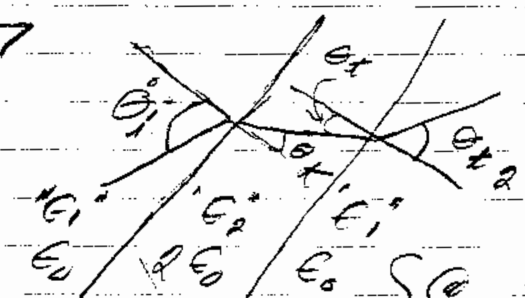
6-36 μ_0, ϵ_0 | $\mu_0, 3\epsilon_0$



$$\theta_{i\,critical} = \sin^{-1}\sqrt{\tfrac{1}{3}} = 35.26°$$

polarization not a factor

6-37

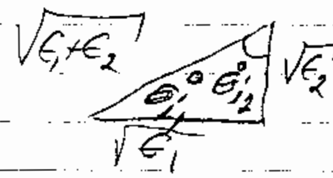


$$\theta_i^{\circ} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

{ @ first interface

$$\sin\theta_t = \sin\theta_{i_1}\sqrt{\frac{\epsilon_1}{\epsilon_2}} \quad (1)$$



{ this θ_t is the angle of incidence at the second interface }

$$\therefore \frac{\sin\theta_{t2}}{\sin\theta_i\sqrt{\frac{\epsilon_1}{\epsilon_2}}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

or $\boxed{\sin\theta_{t_2} = \sin\theta_i}$ $\boxed{\theta_{t_2} = \theta_1}$

Must show that $\theta_t = \theta_{i_2}^{\circ} = \tan^{-1}\sqrt{\frac{\epsilon_1}{\epsilon_2}}$

from (1) and the triangle shown above:

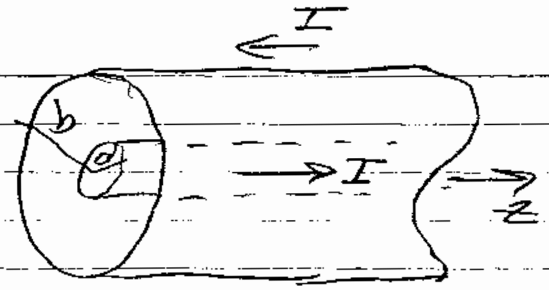
$$\sin\theta_t = \sin\theta_i\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1+\epsilon_2}}\sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sqrt{\frac{\epsilon_1}{\epsilon_1+\epsilon_2}}$$

or $\sin\theta_t = \sin\theta_{i_2} \Rightarrow \theta_t = \theta_{i_2}$

So there is no reflection from the 2nd surface

EE 434 Homework 6

7-2



in outer conductor

$$E_z = -\frac{J_z}{\sigma}$$

$$J_z = \frac{I}{\pi c^2 - \pi b^2}$$

$$H_\phi = \frac{I}{2\pi\rho} \longleftarrow \text{for } a < \rho < b \text{ (found many times)}$$

$$\overline{E} \times \overline{H} \Big|_{@\rho = b} = \frac{J_z}{\sigma} \cdot \frac{I}{2\pi b}\,\overline{a}_\rho = \frac{I^2}{2\pi^2 b\sigma (c^2 - b^2)}\,\overline{a}_\rho$$

in general $R = \frac{L}{\sigma A}$

$$P_{\text{into outer conductor / length } L} = \int_{z=0}^{L}\int_{\phi=0}^{2\pi} \frac{I^2\,\overline{a}_\rho \cdot b\,d\phi\,dz}{2\pi^2 b\sigma (c^2 - b^2)} = \frac{I^2 L}{\pi\sigma(c^2 - b^2)} = I^2 R \leftarrow$$

7-13 $\hat{E}_x = \hat{E}_m^+ e^{-j\beta z}[1 + \Gamma]$; $\hat{H}_y = \frac{\hat{E}_m^+}{\eta} e^{-j\beta z}[1 - \Gamma]$

$$\overline{P}_{ave} = \frac{1}{2}\mathrm{Re}\{\hat{E} \times \hat{H}^*\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|\hat{E}_m^+|^2}{\eta}(1+\Gamma)(1-\Gamma^*)\right\}\overline{a}_z$$

$$\overline{P}_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta}\mathrm{Re}\{1 + \underbrace{\Gamma - \Gamma^*}_{2j\Gamma_{imaginary}} - |\Gamma|^2\}\overline{a}_z$$

$$\therefore \overline{P}_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta}\{1 - |\Gamma|^2\}\overline{a}_z = \overline{P}_{ave}^+ + \overline{P}_{ave}^- \longleftarrow$$

P_{ave} through area A $= \overline{P}_{ave} A = P_{ave}^+ + P_{ave}^-$

$$\therefore \frac{|P_{ave}^-|}{|P_{ave}^+|} = |\Gamma|^2 \longleftarrow$$

$$\boxed{\text{return loss} = 10\log_{10}|\Gamma|^2}$$

7-17

a) $\mathcal{P}_{ave} = \frac{1}{2}\mathrm{Re}\left\{ \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{\eta} e^{+j\theta} [1+\Gamma][1-\Gamma^*] \right\}$

$\mathcal{P}_{ave} = \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{2\eta} \mathrm{Re}\left\{ [\cos\theta + j\sin\theta][1 + \cancel{\Gamma_r} + j\Gamma_i - \cancel{\Gamma_r} + j\Gamma_i - |\Gamma|^2] \right\}$

$\Rightarrow \mathcal{P}_{ave} = \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{2\eta} \left\{ \cos\theta [1-|\Gamma|^2] - \sin\theta [2\Gamma_i] \right\} \leftarrow (1)$

b) for $\Gamma = 0$ this becomes $\mathcal{P}_{ave} = \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{2\eta} \cos\theta$

and for a lossless region $\alpha = 0, \theta = 0$

so $\boxed{\mathcal{P}_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta}}$ (Eq. 7-59 in text)

c) $\mathcal{P}_{ave}$ positive wave only $= \frac{1}{2}\mathrm{Re}\left\{ \frac{\hat{E}_m^+ e^{-\alpha z} \cancel{e^{-j\beta z}} e^{+j\theta} \hat{E}_m^{+*} e^{-\alpha z} \cancel{e^{+j\beta z}}}{\eta} \right\} = \boxed{\frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{2\eta} \cos\theta}$

$\mathcal{P}_{ave}$ negative wave only $= \frac{1}{2}\mathrm{Re}\left\{ \frac{\hat{E}_m^+ e^{-\alpha z} \cancel{e^{-j\beta z}} \Gamma \hat{E}_m^{+*} e^{-\alpha z} \cancel{e^{+j\beta z}} (-\Gamma^*)}{\eta e^{-j\theta}} \right\} = \boxed{\frac{-|\hat{E}_m^+|^2 e^{-2\alpha z}}{2\eta} |\Gamma|^2}$

{ Summing the above two results would miss the $-\sin\theta\,[2\Gamma_i]$ term in equation (1) above ? }

EE 434 Homework 7

7-21 $P_{ave_{sun\ @\ earth}} = 1340\ W/m^2$

assuming a single frequency wave we have:

$$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} \quad \text{or} \quad |\hat{E}_m^+| = \sqrt{2\times377\times1340} = \boxed{1\times10^3\ V/m}$$

$$\text{and } |\hat{H}_m^+| = \frac{|\hat{E}_m^+|}{\eta} = \boxed{2.65\ A/m}$$

$$P_{ave_{total}} = 4\pi(1.48\times10^{11})^2\times1.34\times10^3 = \boxed{36.88\times10^{25}\text{ Watts}}$$
from sun

P_{max} for X-band waveguide @ 10 GHz

$$\hat{E}_y = E_{y_{max}} \sin\frac{\pi}{a}x \quad ; \quad \hat{H}_x = -E_{y_{max}}\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\frac{\pi}{a}x$$

$$\bar{\mathcal{P}}_{ave} = \frac{1}{2}\mathcal{R}e\{\hat{\bar{E}}\times\hat{\bar{H}}^*\} = \frac{1}{2}\mathcal{R}e\left\{\bar{a}_z E_{y_{max}}^2 \sin^2\left(\frac{\pi}{a}x\right)\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right\}$$

$$P_{ave} = \frac{1}{2}E_{y_{max}}^2\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\int_{y=0}^{b}\int_{x=0}^{a}\underbrace{\sin^2\left(\frac{\pi}{a}x\right)}_{\frac{1}{2}\left[1-\cos\left(\frac{2\pi x}{a}\right)\right]}dx\,dy$$

$$P_{ave} = \frac{1}{2}E_{y_{max}}^2\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\,\frac{ba}{2} \quad \text{. (Equation 8-90)}$$

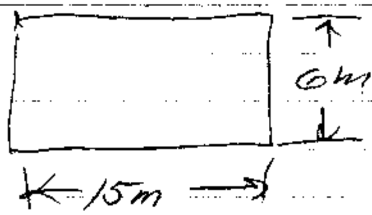
$$\text{or } P_{ave} = 1.16\times10^{-7}E_{y_{max}}^2$$

For air filled waveguide $E_{max} \cong 3\times10^6\ V/m$

$$\therefore P_{ave_{max}} = 1.16\times10^{-7}\times9\times10^{12} = \boxed{1.04\times10^6\text{ Watts}}$$

EE 434 Homework 7 (page 2)

8-27



TE_{10} mode

$$\boxed{f_c = \frac{1}{2\sqrt{\mu\epsilon}\,15} = \frac{3\times10^8}{30} = 10\,MHz}$$

$\boxed{\text{Vertical polarization}}$

AM will not propagate 535 → 1605 KHz

FM will propagate 88 → 108 MHz

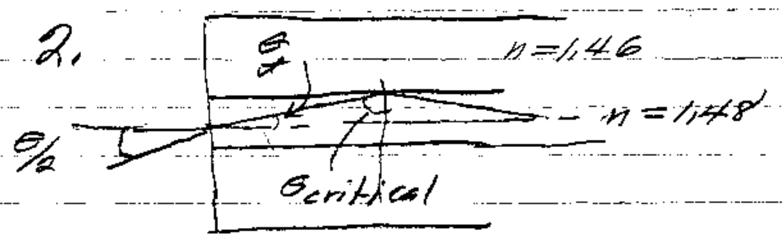
EE 434 Homework 8

1. 1" radius circular waveguide

TE_{11} cutoff frequency $f_c = \frac{3\times10^8\times10^2}{2\pi\times2.54} p'_{1,1}$

$f_c = \frac{3\times10^{10}}{2\pi\times2.54}\times1.841 = \boxed{3.46\times10^9\ Hz}$

2.



$\theta_c = \sin^{-1}\left(\frac{1.46}{1.48}\right) = 80.57^\circ$

$\therefore \theta_t = 90 - \theta_c = 9.43^\circ$

$\sin\theta_t = \sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\left(\frac{\theta}{2}\right)$; $\sin\frac{\theta}{2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}\sin\theta_t = 1.48\sin 9.43^\circ = 0.2425$

so $\theta = 2\sin^{-1}(0.2425) = 28^\circ$

EE 434 Homework 9

11.11 $\Delta z = 0.01\lambda$; $I = 5A$; $f = 300MHz$

a) $\lambda = \frac{3\times10^8}{3\times10^8} = 1m \quad \therefore \boxed{\Delta z = 1cm}$

$\beta_o r = 20 = \frac{2\pi r}{\lambda_o}$; $\frac{r}{\lambda_o} = \frac{20}{2\pi} = 3.18$ or $\boxed{r = 3.18m}$

far zone for $r >> \frac{\lambda}{2\pi} = \boxed{0.16m}$

b) $|\hat{dE_\theta}|_{@\,\theta=90^\circ} = \frac{\omega\mu I dz}{4\pi r} = \frac{2\pi\times3\times10^8\times5\times10^{-2}\times4\pi\times10^{-7}}{2\times4\pi\times3.18} = \boxed{29.638\times10^{-1}}$

$|\hat{dH_\phi}|_{@\,\theta=90^\circ} = \frac{|d\hat{E}_\theta|}{\eta_o} = \boxed{7.86\times10^{-3}}$

c) $P_{ave} = \frac{1}{2}Re\{\hat{E}\times\hat{H}^*\} = \frac{2.9638\times7.86\times10^{-3}}{2} = \boxed{11.66\ mW/m^2}$

11.14 $P_{rad} = \frac{\pi\eta I^2}{3}\left(\frac{\Delta z}{\lambda}\right)^2$ from notes

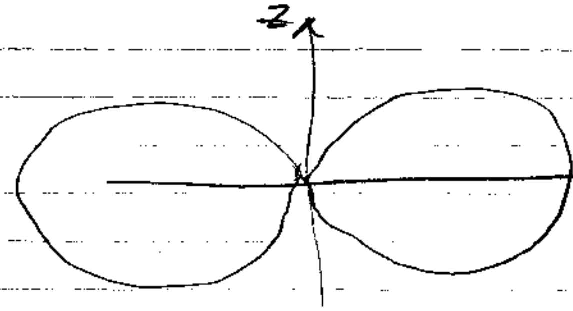
a) $\therefore P_{rad} = \frac{\pi^2\times120\times25\times10^{-4}}{3} = \boxed{0.987\ Watts}$

b) if $\Delta z = 0.02m$ $\quad P_{rad} = 4\times0.987 = \boxed{3.95\ Watts}$

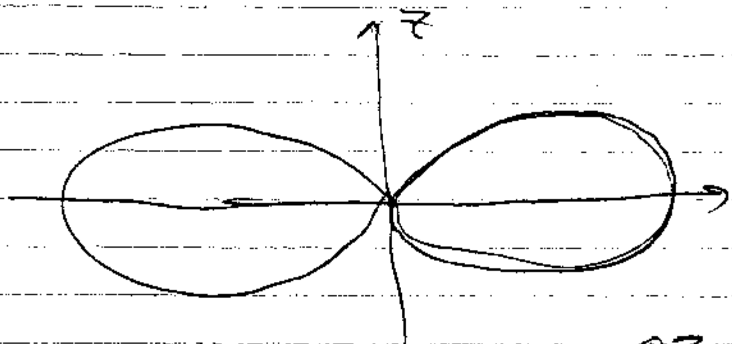
if $\Delta z = 0.005m$ $\quad P_{rad} = \frac{0.987}{4} = \boxed{0.25\ Watts}$

EE 43A Homework 9 (solution - page 2)

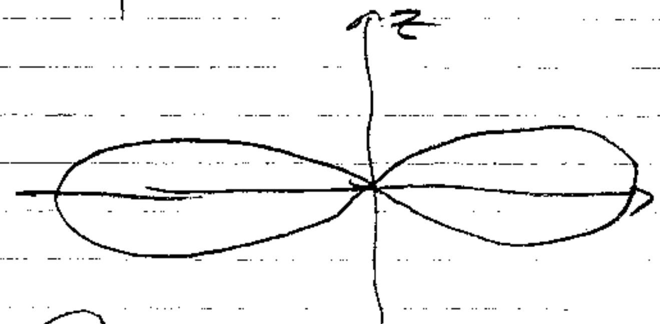
11-19 a) $L = \frac{\lambda}{2}$



b) $L = \frac{5}{8}\lambda$

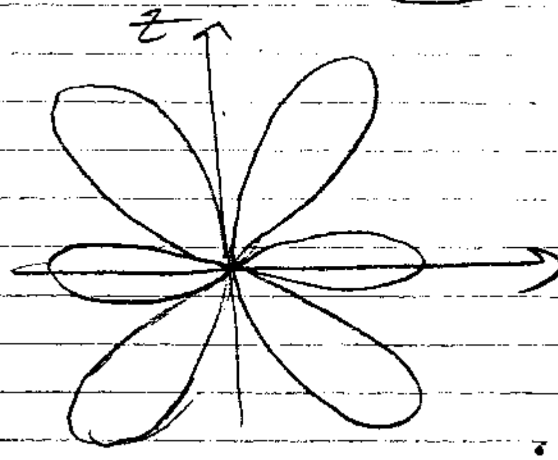


c) $L = \lambda$

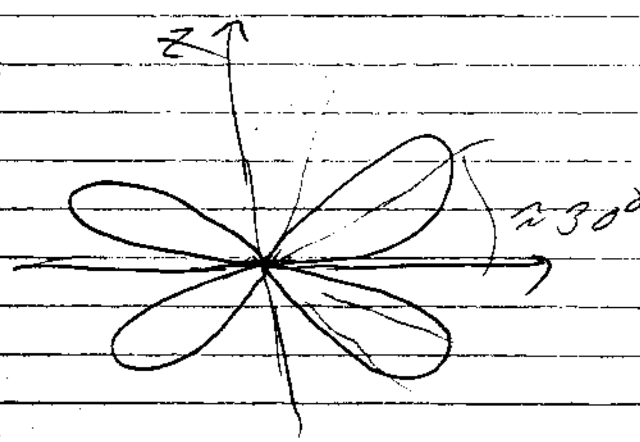


11-20 a)

$L = 1.5\lambda$



b) $L = 2\lambda$



EE 434 Homework 10

9-1 "dz"; $I = 10A$; $\left(\frac{dz}{\lambda}\right) = \frac{1}{50}$; $r = 10^3 m$

$$P_{max} = \frac{1}{2} Re\left\{ \frac{\beta I^2 dz^2 \omega\mu}{16\pi^2 r^2} \right\} = \frac{\cancel{2\pi f}\sqrt{\mu\epsilon}\, I^2 dz^2 \cancel{2\pi f}\mu\epsilon}{\cancel{32\pi^2} r^2\, 2\epsilon}$$

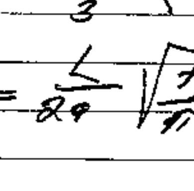
$$P_{max} = \frac{I^2 dz^2 \sqrt{\mu\epsilon}}{\lambda^2 8 r^2 \epsilon} = \frac{I^2 dz^2 \eta}{8 \lambda^2 r^2} = \frac{10^2 \times 120\pi}{50^2 \times 8 \times 10^6} = \boxed{1.88 \times 10^{-6}\ W/m^2}$$

9-2 $L = 1m$; $f = 10^6$ ∴ $\lambda = \frac{3\times10^8}{10^6} = 300m$ (short dipole)

$r = 5\times10^3 m$; $I = 12A$

$$P(\theta = 30^\circ) = \frac{I^2 \eta}{8 r^2}\left(\frac{dz}{\lambda}\right)^2 \sin^2 30^\circ = \frac{12^2 \times 120\pi}{8\times 25\times10^6}\left(\frac{1}{300}\right)^2 \sin^2 30^\circ$$

$$P(\theta = 30^\circ) = 7.54\times10^{-10}\ W/m^2$$

9-5 

$f = 1 MHz$; C_u wire radius $= 10^{-3} m$ ($\sigma = 5.8\times10^7$)

a) $\lambda = \frac{3\times10^8}{10^6} = 300m$ (short dipole)

$$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{dz}{\lambda}\right)^2 = \frac{2\pi\times377}{3}\left(\frac{2}{300}\right)^2 = \boxed{0.0351\ \Omega}$$

$$R_{loss} = \frac{L}{2a}\sqrt{\frac{f\mu}{\pi\sigma}} = \frac{2}{2\times10^{-3}}\sqrt{\frac{10^6\times4\pi\times10^{-7}}{\pi\times5.8\times10^7}} = \boxed{0.083\ \Omega}$$

$$\therefore\ \xi = \frac{0.0351}{0.0351 + 0.083} = 0.297 \quad \boxed{29.7\%}$$

b) $G = \xi D = 0.297\times1.5 = 0.446$ or $\boxed{-3.51 dB}$

c) $20 = \frac{I^2 \pi\eta}{3}\left(\frac{dz}{\lambda}\right)^2$ so $I = \sqrt{\frac{60}{\pi\eta}}\left(\frac{\lambda}{dz}\right)$

or $I = \sqrt{\frac{60}{120\pi^2}}\left(\frac{\lambda}{dz}\right) = \frac{1}{\sqrt{2}\pi}\cdot150 = \boxed{33.76 A}$

$$P_{trans} = \frac{P_{rad}}{\xi} = \frac{20}{0.297} = \boxed{67.34\ Watts}$$

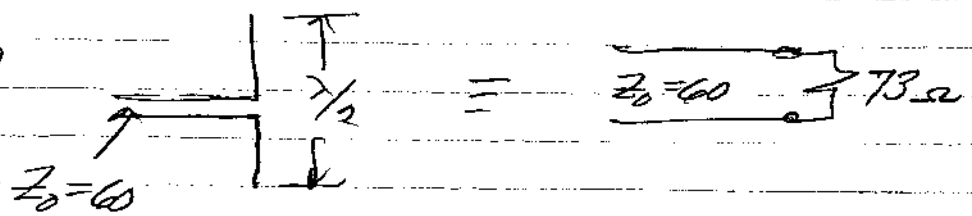
EE 434 Homework 10 (solution page 2)

9.8 $\xi = 0.9$; $D_{max} = 6.7\,dB$; $G = ?$

$$D_{dB} = 10\log_{10} D \quad \therefore \quad D = 10^{\frac{D_{dB}}{10}} = 4.677$$

$$G = \xi D = 4.2 \quad \text{or} \quad \boxed{G_{in\,dB} = 6.24}$$

9.10



$$\Gamma_{load} = \frac{73-60}{73+60} = \frac{13}{133} = 0.0977$$

$$SWR = \frac{|\hat{V}_{max}|}{|\hat{V}_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.0977}{1-0.0977} = \boxed{1.217}$$

EE 434 Homework 11 16, 21, 23

9.15 3 GHz, 1 m diameter dishes, (require 10^{-9} Watts @ receiver)

separation = 40 km ; $P_t = ?$; $\lambda = \frac{3\times10^8}{3\times10^9} = 0.1\,m$

$$10^{-9} = \left(\frac{1}{4\times10^4\times10^{-1}}\right)^2 \left(\frac{\pi}{4}\right)^2 P_t$$

$$\text{so } P_t = \frac{16\times10^{-9}\times16\times10^6}{\pi^2} = \boxed{25.94\times10^{-3}}$$

9.16 1 kW transmitter @ 50 MHz ; $\lambda = \frac{3\times10^8}{5\times10^7} = 0.6\times10$

$\lambda/2$ dipole $\boxed{D = 1.64}$; $G_{receiver} = 13\,dB = 10\log_{10} G$

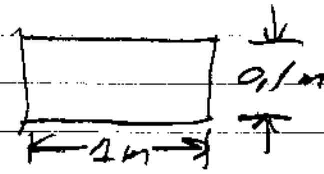
separation = 36 km

$$P_r = \left(\frac{\lambda}{4\pi r}\right)^2 D_r D_t P_t = \left(\frac{60}{4\pi\times3\times10^4}\right)^2 1.64\times19.95$$

$$\boxed{P_r = 8.29\times10^{-6}\text{ Watts}}$$

9.21 94 GHz,

$\lambda = \frac{3\times10^8}{9.4\times10^{10}} = 0.319\times10^{-2}$

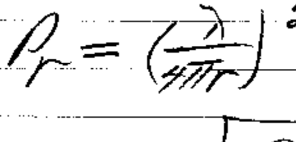


a) vertical $BW = \frac{0.319\times10^{-2}}{0.1}$

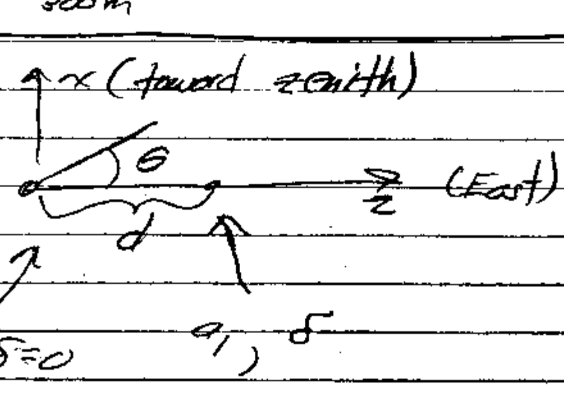
$\boxed{BW = 0.0319 = 1.83^\circ}$

$\boxed{\text{horizontal } BW = 0.183^\circ}$

b) @ 300 m beam is $\boxed{300\times0.183\frac{\pi}{180} = 0.958\,m}$



9-23



$F_a = a_0 e^{j\psi_0} + a_1 e^{j\psi_1}$

$\psi = n\beta d\cos\theta - \delta_n$

EE 434 Homework 11 (solutions page 2)

a) $a_0 = a_1 = 1$, $\delta = \frac{\pi}{4}$, $d = \frac{\lambda}{2}$

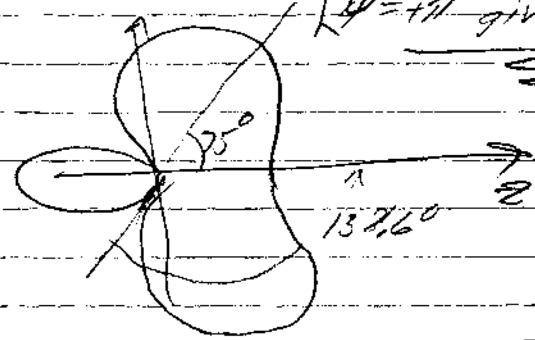
$\therefore \psi_0 = 0$, $\psi_1 = 2\pi\left(\frac{d}{\lambda}\right)\cos\theta - \frac{\pi}{4} = \pi\cos\theta - \frac{\pi}{4}$

$|F_a| = a\dfrac{\sin\psi}{\sin\frac{\psi}{2}}$ (uniform array)

$\boxed{\text{maximum @ } \psi = 0}$; $\cos\theta = \frac{1}{4} \Rightarrow \boxed{\theta = \pm 75.52^\circ}$

1^{st} zero @ $\psi = -\pi = \pi\cos\theta - \frac{\pi}{4}$ or $\cos\theta = -\frac{3}{4}$

$[\psi = +\pi$ gives $\cos\theta = \frac{5}{4}]$ $\theta_{zero} = \pm 138.6^\circ$



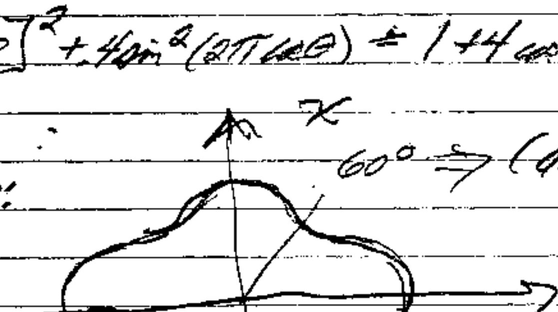
b) $a_0 = 1$, $a_1 = 2$, $\delta = 0$, $d = \lambda$

$F_a = 1 + 2e^{j\left(2\pi\frac{\lambda}{\lambda}\cos\theta\right)} = 1 + 2e^{j2\pi\cos\theta}$

$F_a = 1 + 2\left[\cos(2\pi\cos\theta) + j\sin(2\pi\cos\theta)\right]$

$|F_a| = \left[1 + 2\cos(2\pi\cos\theta)\right]^2 + 4\sin^2(2\pi\cos\theta) = 1 + 4\cos(2\pi\cos\theta) + 4$

computer plot gives:



EE 434 Homework 12 18, 20, 24

9.18 $P_t = 10W$ $f = 6GHz$ $G_t = 20dB$

$R = 70km$ $\lambda = \frac{3\times10^8}{6\times10^9} = 0.05$ $G_r = 23dB$

a) $G(\theta\phi) = \frac{P\ w/m^2}{P_{rad}/4\pi r^2}$ so $P_r = \frac{10^2\times10}{4\pi\times4.9\times10^9} = \boxed{1.99\times10^{-7} W/m^2}$

b) $P_r = P_r A_{eff_r} = P_r \frac{\lambda^2}{4\pi} D_r = \frac{1.99\times10^{-7}\times25\times10^{-4}\times199.5}{4\pi} = 7.9\times10^{-9} W$ (Or)

c) $T_{sys} = 1,000K$ bandwidth = 10MHz

$S/N = \frac{P_r}{kTB} = \frac{7.9\times10^{-9}}{1.38\times10^{-23}\times10^3\times10^7} = \boxed{5.72\times10^4 = 47.6dB}$

9-20 $BW = 1.5°$ @ 20GHz (circular aperature)

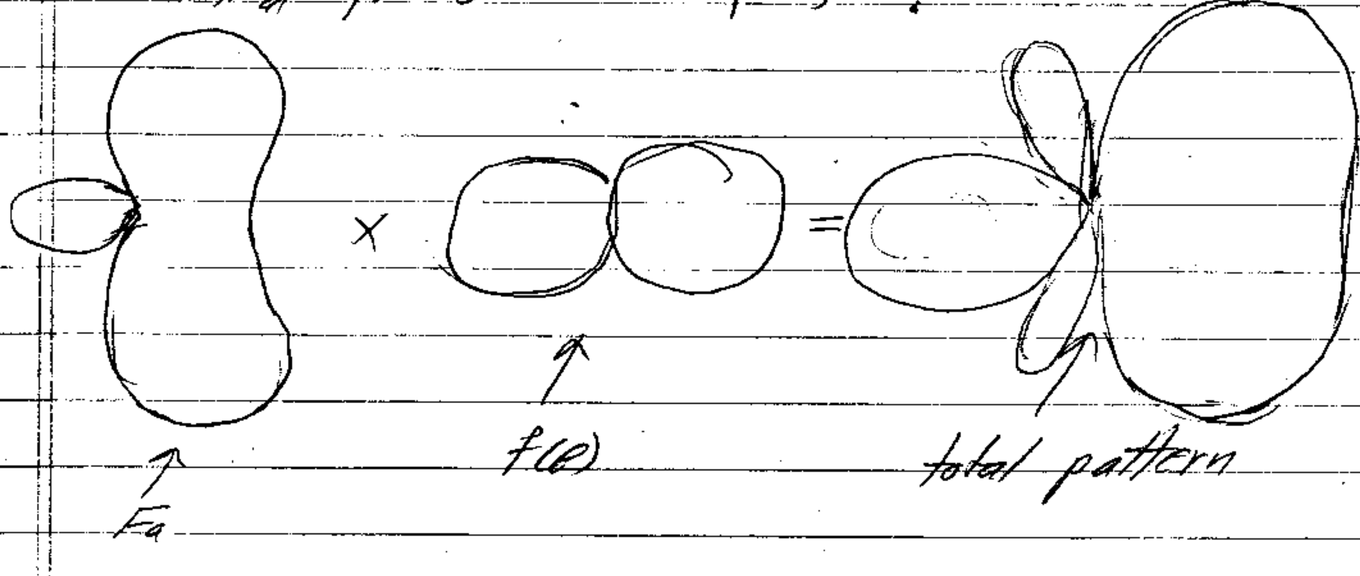
a) $BW \cong \frac{\lambda}{dia}$; $D = \frac{4\pi}{\lambda^2}A_{eff} = \frac{4\pi}{\lambda^2}\cdot\frac{\pi dia^2}{4} = \frac{\pi^2}{\lambda^2}\left(\frac{\lambda}{BW}\right)^2$

so $D = \left(\frac{\pi}{BW}\right)^2 = \left(\frac{\pi\times180}{1.5\times\pi}\right)^2 = \boxed{1.44\times10^4 = 41.58dB}$

b) Area doubled $\boxed{D \text{ is doubled, } BW \text{ divided by } \sqrt{2}}$

c) frequency doubled $\boxed{BW \text{ divided by } 2,\ D \text{ multiplied by } 4}$

9.24 $|F_a| = |1 + e^{j(\pi\cos\theta - \frac{\pi}{4})}|$; $f(\theta) = K\cos\theta$



[Ulaby 9.24]

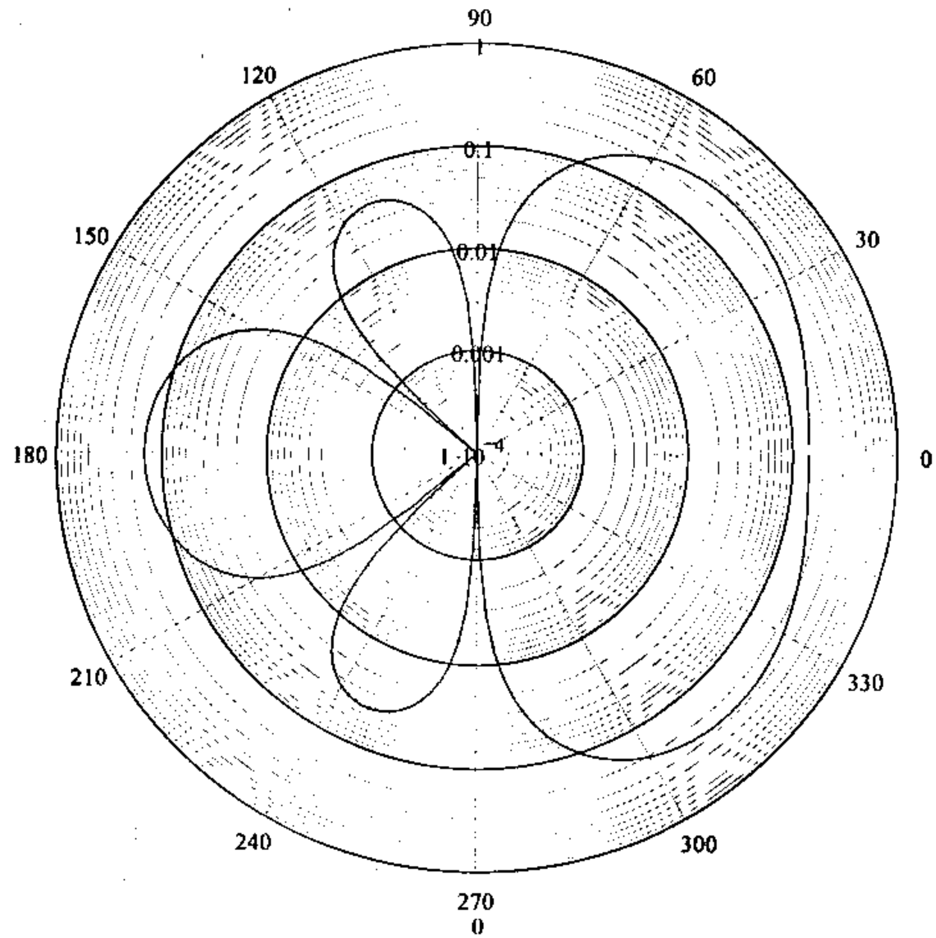
ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2 \quad d := \frac{1}{2}; \quad \phi := \frac{\pi}{4}; \qquad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \qquad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^1$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.

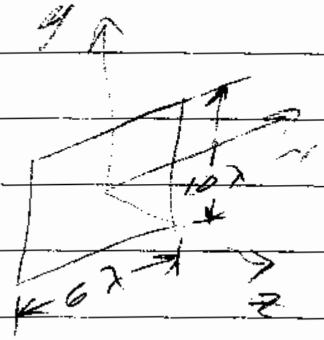
$(|F(\theta)\cdot\cos(\theta)|)^2$



Problems

Homework 12

11-26



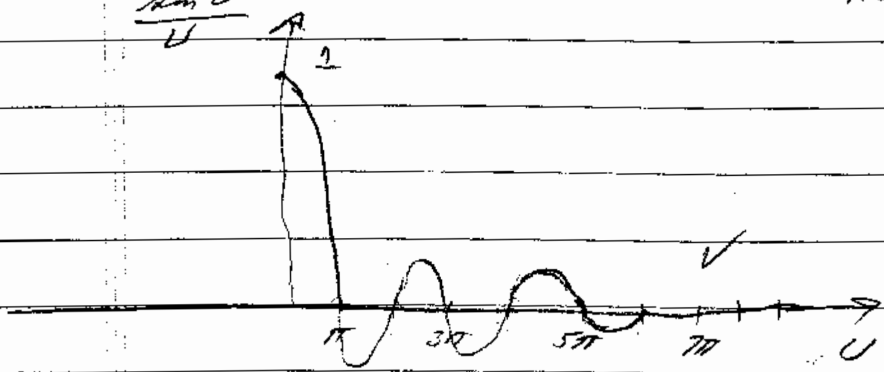
$\phi = 0$ pattern (xz plane)

$$\hat{E} = \frac{j\beta E_m^+}{4\pi r} ab(1+\cos\theta)e^{-j\beta r}\frac{\sin(\frac{\beta a}{2}\sin\theta)}{\frac{\beta a}{2}\sin\theta}\hat{a}_\theta$$

$\frac{\sin U}{U} \rightarrow \frac{\sin(\frac{\beta a}{2}\sin\theta)}{\frac{\beta a}{2}\sin\theta}$

(aperture is 6λ wide in this plane)

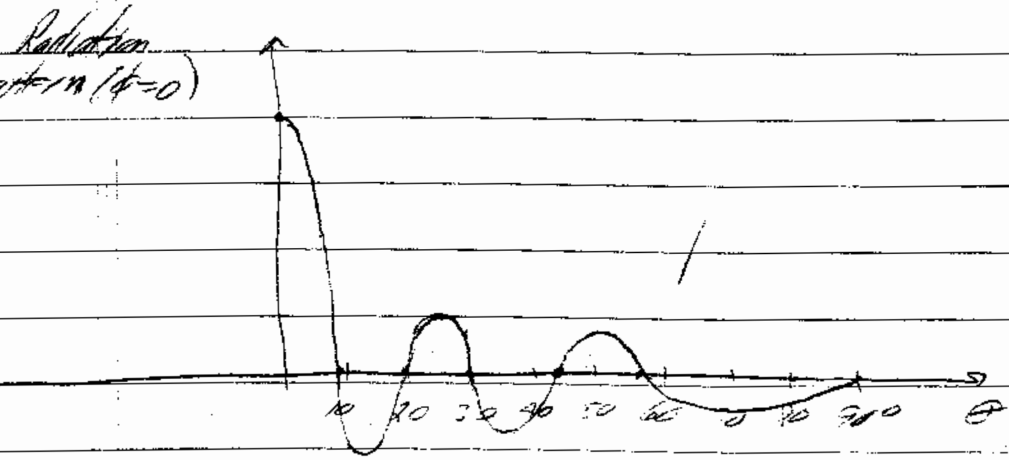
$\therefore \frac{\beta a}{2} = \frac{2\pi}{\lambda}\cdot\frac{6\lambda}{2} = 6\pi$ which is the visible range of U (i.e. $0 < U < 6\pi$ for $0 < \theta < \frac{\pi}{2}$)

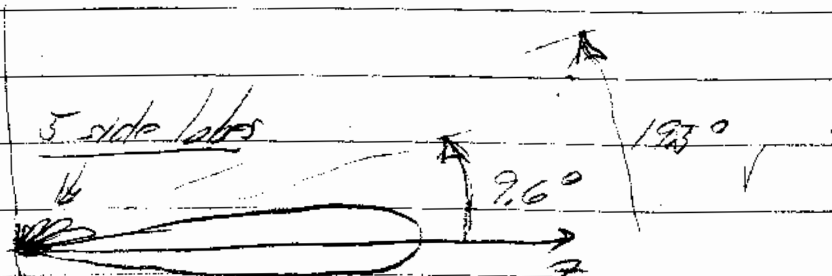


Zeros [$6\pi\sin\theta = n\pi$]

U	θ
π	9.6°
2π	19.5°
3π	30.°
4π	42°
5π	56°
6π	90°

Radiation Pattern ($\phi=0$)





From table 11 $\theta_{half\,power} \cong \frac{50°}{6} = 8.33°$ ←

First sidelobe down 13.2 dB ←