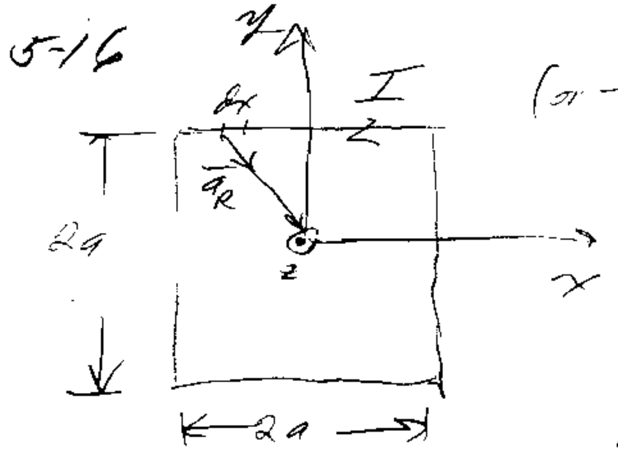
EE 434 Homework 1

5-16



(x-top)

$$\overline{dB} = \frac{\mu I\, dx\, \bar{a}_x \times \bar{a}_R}{4\pi (x^2+a^2)}$$

$$\bar{a}_R = \frac{-x\bar{a}_x - a\bar{a}_y}{\sqrt{x^2+a^2}}$$

$$\therefore\ \overline{dB} = \frac{\mu I\, dx\,(-a\bar{a}_z)}{4\pi (x^2+a^2)^{3/2}}$$

$$B_{z_{top}} = \int_a^{-a} \frac{-\mu I a\, dx}{4\pi (x^2+a^2)^{3/2}} = \frac{-\mu I a}{4\pi}\left\{\frac{x}{a^2(x^2+a^2)^{1/2}}\right\}_a^{-a}$$

$$B_{z_{top}} = \frac{-\mu I}{4\pi}\left\{\frac{-a}{a\sqrt{2}\,a} - \frac{a}{a\sqrt{2}\,a}\right\} = \frac{2\mu I}{4\pi\sqrt{2}\,a}$$

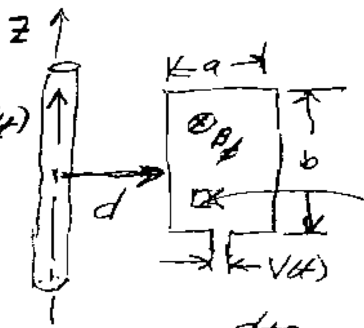
$$B_{z_{top}} = \frac{\mu I}{2\pi a\sqrt{2}} = \frac{\mu I\sqrt{2}}{4\pi a}$$

by symmetry other 3 sides contribute the same

$$\therefore\ B_{z\ @\ center} = 4B_{z_{top}} = \frac{\mu I\sqrt{2}}{\pi a}$$

EE 434 Homework 1 (solutions page 2)

5.19



$I(t) = I_m \sin \omega t$

a) $H_\phi = \dfrac{I_m \sin \omega t}{2\pi\rho}$ from $\oint \bar{H} \cdot d\bar{l} = I$

$d\bar{S} = dz\, d\rho\, \bar{a}_\phi$

b) $$\oint \bar{E} \cdot d\bar{l} = V(t) = -\frac{d}{dt} \int_{\rho=d}^{d+a} \int_{z=0}^{b} \frac{\mu_0 I_m \sin \omega t}{2\pi\rho}\, dz\, d\rho$$

$$V(t) = -\frac{d}{dt}\left\{\frac{\mu_0 I_m \sin \omega t\, b}{2\pi} \ln\left(\frac{d+a}{d}\right)\right\} = -\frac{\mu_0 I_m \omega b \cos \omega t}{2\pi} \ln\left(\frac{d+a}{d}\right)$$

(right hand terminal positive) $\boxed{\text{see A page 282}}$

or ... because $\bar{F}$ on positive charges would be in the z direction on both long sides of the loop but of greater magnitude on the left hand side driving a net positive charge to the right hand terminal.

for $I_m = 10A$, $f = 20kHz$, $d = 4 \times 10^{-3} m$, $a = b = 0.1 m$

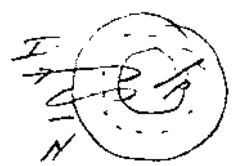
$$V(t) = -\frac{4\pi \times 10^{-7} \times 10 \times \cancel{2\pi} \times 2 \times 10^{4} \times 10^{-1} \ln\left(1 + \frac{0.1}{4 \times 10^{-3}}\right)}{\cancel{2\pi}} \cos(2\pi \times 2 \times 10^{4} t)$$

$$V(t) = -81.88 \times 10^{-3} \cos(4\pi \times 10^{4} t) = \boxed{-81.9 \cos(4\pi \times 10^{4} t)\ mV}$$

c) 2 times above with opposite polarity

EE 434 Homework 2

5.23 $U_m = \frac{1}{2}\int_V \vec{B}\cdot\vec{H}\,dv$



$\oint \vec{H}\cdot d\vec{l} = NI$

$H_\phi = \frac{NI}{2\pi\rho}$

a) so $U_m = \frac{1}{2}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{d} \frac{\mu N^2 I^2}{4\pi^2\rho^2}\,dz\,\rho\,d\phi\,d\rho = \frac{\mu N^2 I^2\, 2\pi d}{2\times 4\pi^2}\ln\frac{b}{a}$

or $U_m = \frac{\mu d N^2 I^2}{4\pi}\ln\frac{b}{a} = \frac{1}{2}LI^2$

$\therefore \boxed{L = \frac{2U_m}{I^2} = \frac{\mu d N^2}{2\pi}\ln\frac{b}{a}}$ same as Example 5-17

b) $I = 2$, $\mu_r = 4\times10^3$, $a = 10^{-2}$, $b = 3\times10^{-2}$, $d = 2\times10^{-2}$, $N = 150$

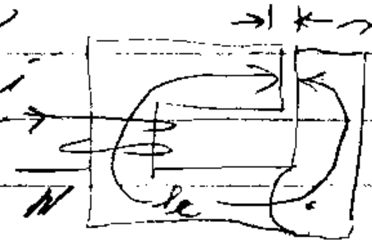
$U_m = \frac{4\times10^3\times 4\pi\times10^{-7}\times 2\times10^{-2}\times 4\times 1.5^2\times10^4}{4\pi} = 72\times10^{-2}\ln 3 = \boxed{79.1\times10^{-2}\,J}$

so $L = \frac{2U_m}{I^2} = \frac{2\times 79.1\times10^{-2}}{4} = \boxed{0.395\ H}$

c) if $\mu_r = f(I)$ or $f(H)$ which would be true for ferromagnetic material?

EE 434 Homework 2 (Solutions - page 2)

5.48



a) $\phi_m = \frac{NI}{R_{iron}+R_{gap}} = \frac{NI}{R_{total}}$

b) $L = \frac{\lambda}{I} = \frac{N\phi_m}{I} = \frac{N^2}{R_{total}}$

$\therefore W_m = \frac{1}{2} L I^2 = \frac{1}{2}\frac{N^2}{R} I^2$

found that $F_x = \frac{\partial W_m}{\partial x} = \frac{I^2}{2}\frac{\partial L}{\partial x} = \frac{I^2}{2}\frac{d}{dx}\left(\frac{N^2}{R}\right) = \frac{-N^2 I^2}{2R^2}\frac{\partial R}{\partial x}$

or $F_x = -\frac{1}{2}\phi_m^2 \frac{\partial R}{\partial x}$

so for $l_c = 0.12$, $A_c = 4\times10^{-4}$, $x = 1.5\times10^{-3}$, $I = 1.25A$, $N = 200$ and $\mu = 10^5 \mu_0$

$$R_{total} = \frac{0.12}{10^5\times4\pi\times10^{-7}\times4\times10^{-4}} + \frac{x}{4\pi\times10^{-7}\times4\times10^{-4}} = 2.387\times10^3 + 0.0298\times10^8$$

($x = 10^{-3}$; $1.99\times10^9 x$)

$$\phi_m = \frac{200\times1.25}{R_{total}} = \frac{2.5\times10^2}{2.98\times10^6} = \boxed{0.839\times10^{-4}\text{ Webers}}$$

$$\therefore F_x = -\frac{1}{2}(0.839\times10^{-4})^2\times1.99\times10^9 = \boxed{-7\text{ Newtons}}$$

$$B_{ave} = \frac{\phi_m}{A} = \frac{0.839\times10^{-4}}{4\times10^{-4}} = \boxed{0.21\ W/m^2}$$

$$H_{ave_{gap}} = \frac{B_{ave}}{\mu} = \frac{0.21}{4\pi\times10^{-7}} = \boxed{1.67\times10^5}$$

$$H_{ave_{iron}} = \frac{0.21}{4\pi\times10^{-7}\times10^5} = \boxed{1.67}$$

$$L = \frac{4\times10^4}{2.98\times10^6} = 1.31\times10^{-2} \boxed{\approx 13\text{ mH}}$$

$$W_m = \frac{1}{2}LI^2 = \frac{1}{2}\times1.31\times10^{-2}\times1.25^2 = \boxed{1.02\times10^{-2}\text{ J}}$$

If $x = 0.75$ mm $R_{new} = \frac{R_{total}}{2}$ so $\phi_{m_{new}} = 2\phi_{m_{old}}$ and $\boxed{F_{x_{new}} = 4F_{x_{old}}}$

5.48 $x=0$ $\frac{d\mathcal{R}}{dx}$ is same as before but $\mathcal{R}=\mathcal{R}_{1mm}=2.387\times10$

so $\frac{\mathcal{F}}{\mathcal{R}} = \frac{2.5\times10^{2}}{2.387\times10^{3}} = 1.047\times10^{-1}$

and $F_x = -\frac{1}{2}(1.047\times10^{-1})^2\times1.99\times10^{9} = \boxed{-1.09\times10^{7}\,N}$ ←

6-5 air | $\epsilon=\epsilon_r\epsilon_0$ $\quad \frac{\hat{E}^+_{m_2}}{\hat{E}^+_{m_1}} = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2\eta_0\frac{1}{\sqrt{\epsilon_r}}}{\eta_0+\eta_0\frac{1}{\sqrt{\epsilon_r}}}$

$E^+_{m_1}$

a) so $\boxed{\frac{\hat{E}^+_{m_2}}{\hat{E}^+_{m_1}} = \frac{2}{1+\sqrt{\epsilon_r}}}$ ←

$\boxed{\frac{\hat{E}^-_{m_1}}{\hat{E}^+_{m_1}} = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\frac{1}{\sqrt{\epsilon_r}}-1}{\frac{1}{\sqrt{\epsilon_r}}+1} = \frac{1-\sqrt{\epsilon_r}}{1+\sqrt{\epsilon_r}}}$ ←

b) $\hat{E}^+_{m_1} = 100$, $\epsilon_r = 2.25$ $\quad \sqrt{\epsilon_r} = 1.5$

$\hat{E}^+_{m_2} = \frac{100\times2}{2.5} = \boxed{80}$ ←

$\hat{E}^-_{m_1} = 100\,\frac{-0.5}{2.5} = \boxed{-20}$ ←

c) $\epsilon_r = 81$; $\sqrt{81} = 9$

so $\hat{E}^+_{m_2} = \frac{100\times2}{10} = \boxed{20\,\frac{V}{m}}$ ←

$E^-_{m_1} = 100\,\frac{-8}{10} = \boxed{-80\,\frac{V}{m}}$ ←

EE 434 Homework 3

6-6 $v_{phase} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$ ←

index of refraction $n = \sqrt{\epsilon_r}$ so $\boxed{n = \frac{c}{v_{ph}}}$ ←

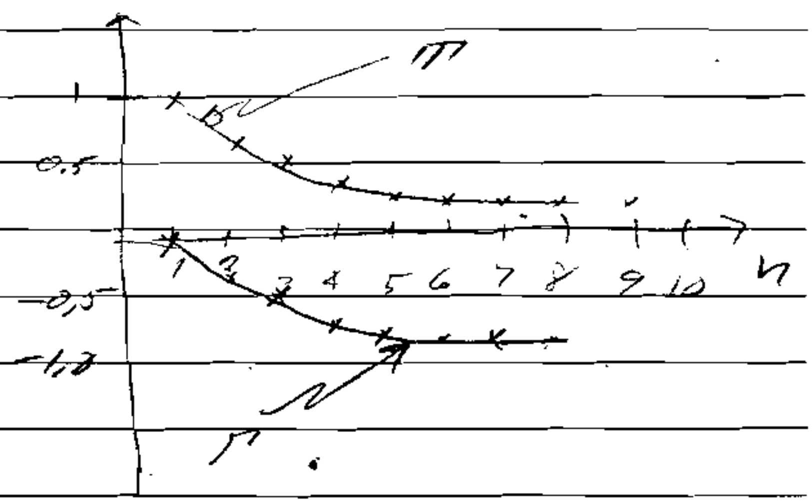
$n_{polyethylene} = \sqrt{2.25} = 1.5$ ←

$n_{water} = \sqrt{81} = 9$ ←

$\Gamma = \frac{1-n}{1+n}$; $T = \frac{2}{1+n}$

$\boxed{T - \Gamma = \frac{2-1+n}{1+n} = 1}$ ← {E field is continuous at boundary}

n	T	Γ
1	1	0
2	0.67	-0.33
3	0.5	-0.5
4	0.4	-0.6
5	0.33	-0.67
6	0.29	-0.71
7	0.25	-0.75
8	0.22	-0.77
9	0.2	-0.8
10	0.18	-0.82



6-17 $\hat{\eta}_1$ | $\hat{\eta}_2$ | $\hat{\eta}_3$ $\gamma_2 \to j\beta_2$

$\leftarrow \lambda/4 \rightarrow$ $\quad e^{\gamma_2 d} \to e^{j\frac{2\pi}{\lambda_2}\cdot\frac{\lambda_2}{4}} = j$

then $$\hat{Z}_1(0) = \eta_2 \frac{j(\hat{\eta}_3+\eta_2) - j(\hat{\eta}_3-\eta_2)}{j(\hat{\eta}_3+\eta_2) + j(\hat{\eta}_3-\eta_2)} = \frac{2j\eta_2\cdot\eta_2}{2j\hat{\eta}_3} = \frac{\eta_2^2}{\hat{\eta}_3}$$

6-18 $\epsilon_{r_3} = 2.56$ in above problem we need $\hat{Z}_1(0) = \eta_1$

$\therefore$ $\eta_2 = \sqrt{\eta_1\eta_3}$ or for all $\mu = \mu_0$

we want $\frac{\mu_0}{\epsilon_2} = \sqrt{\frac{\mu_0^2}{\epsilon_1\epsilon_3}}$; $\frac{1}{\epsilon_{r_2}} = \frac{1}{\sqrt{\epsilon_{r_1}\epsilon_{r_3}}}$

$\therefore$ $\boxed{\epsilon_{r_2} = \sqrt{1\times 2.56} = 1.6}$

reciprocal because got same value for η_2 if η_1 + η_3 are interchanged!

6-20 $\hat{\eta}_1$ | η_2 | $\hat{\eta}_3$ $\quad e^{\gamma_2 d} \to e^{j\pi} = -1$

$\leftarrow \lambda/2 \rightarrow$

so $$\hat{Z}_1(0) = \eta_2 \frac{-(\hat{\eta}_3+\eta_2) - (\hat{\eta}_3-\eta_2)}{-(\hat{\eta}_3+\eta_2) + (\hat{\eta}_3-\eta_2)} = \frac{\eta_2(-2\hat{\eta}_3)}{-2\eta_2} = \hat{\eta}_3$$

same result whenever $e^{j\frac{2\pi}{\lambda}\cdot d} = -1$

or $d = n\frac{\lambda}{2}$; $\cancel{n = 1, 2, 3, 4 \cdots}$

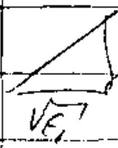
n odd

6-35 ① μ_0, ϵ_0 | ② $\mu_0, 3\epsilon_0$ $\quad \theta_{i_{Brewster}} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

∴ from ① → ② $\theta_{i_B} = 60^\circ$

② → ① $\theta_{i_B} = 30^\circ$

must be // polarized



$\Longleftrightarrow \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 90^\circ - \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

so $\theta_{i_{Brewster}}$ angles from opposite directions are always complements.

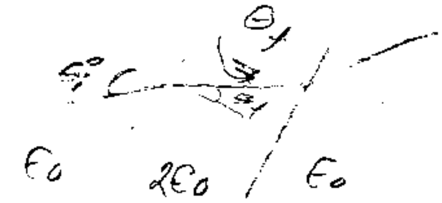
EE 434 Homework 5

6.36 μ_0, ϵ_0 | $\mu_0, 3\epsilon_0$

$\theta_{critical} = \sin^{-1}\sqrt{\frac{1}{3}} = \underline{\underline{35.26^\circ}}$

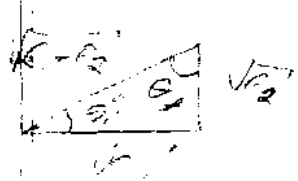
polarization not important

6.37 a)



$\theta_i^\circ = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$

$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$



$\sin\theta_t = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1+\epsilon_2}} \cdot \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1+\epsilon_2}}$

thus we see that $\boxed{\theta_t = 90^\circ - \theta_i^\circ}$

We showed in problem 6.35 that the Brewster angle for waves incident on the interface from either side are complementary angles. The angle of incidence at the second surface is θ_t so there will be no reflection at this surface because $\theta_t = 90^\circ - \theta_i^\circ$ as shown above

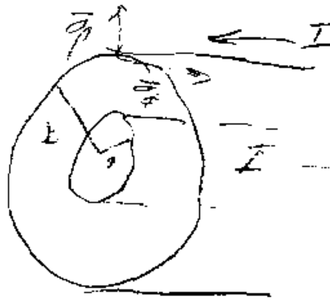
b) $\Gamma_\parallel = 0$; $\theta_i^\circ = \tan^{-1}\sqrt{2} = 54.73^\circ$

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_t}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_t} \; ; \; \theta_t = \sin^{-1}\left(\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right) = 35.26^\circ$$

$$\Gamma_\perp = \frac{0.5774 - 1.155}{0.5774 + 1.155} = \frac{-0.577}{1.732} = \boxed{-0.333}$$

EE 434 Homework 5 (Solutions - page 6)

7.2



$$E_z = \frac{J_z}{\sigma} \; ; \; J_z = \frac{I}{\pi c^2 - \pi b^2}$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\bar{E} \times \bar{H} \,@\, \rho = b = \frac{J_z I}{\sigma 2\pi b}\,\bar{a}_\rho = \frac{I^2}{2\pi^2 b \sigma (c^2 - b^2)}\,\bar{a}_\rho$$

$$P_{\text{into outer conductor}} = \int_{z=0}^{L} \int_{\phi=0}^{2\pi} \frac{I^2\,\bar{a}_\rho}{2\pi^2 b \sigma (c^2 - b^2)} \cdot b\,d\phi\,dz$$

$$P = \frac{2\pi L I^2}{2\pi^2 \sigma (c^2 - b^2)} = I^2 \frac{L}{\sigma \pi (c^2 - b^2)} = I^2 R \;!$$

EE 434 Homework 6

7-13 $\hat{E}_x = \hat{E}_m^+ e^{-j\beta z}[1+\Gamma]$ $\hat{H}_y = \frac{\hat{E}_m^+}{\eta} e^{-j\beta z}[1-\Gamma]$

$$P_{ave} = \frac{1}{2}\mathrm{Re}\{\hat{\bar{E}} \times \hat{\bar{H}}^*\} = \frac{1}{2}\mathrm{Re}\left\{\frac{|E_m^+|^2}{\eta}(1+\Gamma)(1-\Gamma^*)\right\}\bar{a}_z$$

$$\bar{P}_{ave} = \frac{|E_m^+|^2}{2\eta}\mathrm{Re}\{1+\cancel{\Gamma_r}+j\Gamma_i-\cancel{\Gamma_r}+j\Gamma_i-|\Gamma|^2\}\bar{a}_z$$

$$\bar{P}_{ave} = \frac{|E_m^+|^2}{2\eta}[1-|\Gamma|^2]\bar{a}_z = \bar{P}_{ave}^+ - \bar{P}_{ave}^-$$

$$\therefore \frac{|P_{ave}^-|}{|P_{ave}^+|} = |\Gamma|^2$$

7-17 $P_{ave} = \frac{1}{2}\mathrm{Re}\left\{\frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{\eta} e^{j\theta}[1+\Gamma][1-\Gamma^*]\right\}$

$$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta}e^{-2\alpha z}\mathrm{Re}\{[\cos\theta + j\sin\theta][1+\cancel{\Gamma_r}+j\Gamma_i-\cancel{\Gamma_r}+j\Gamma_i-|\Gamma|^2]\}$$

$$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta}e^{-2\alpha z}\{\cos\theta[1-|\Gamma|^2] - \sin\theta\, 2\Gamma_i\} \quad (1)$$

for $\Gamma = 0$ this becomes $\boxed{P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta}e^{-2\alpha z}\cos\theta}$

b) lossless case $\hat{\eta} \to \eta$ i.e $\theta \to 0$ and $\alpha \to 0$

giving: $\boxed{P_{ave} = \frac{|E_m^+|^2}{2\eta}[1-|\Gamma|^2] \text{ as in 7-13}}$

$$\underset{\text{positive wave}}{P_{ave}} = \frac{1}{2}\mathrm{Re}\left\{\frac{\hat{E}_m^+ e^{-\alpha z}e^{-j\beta z}e^{+j\theta}\hat{E}_m^{+*}e^{-\alpha z}e^{+j\beta z}}{\eta}\right\} = \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}\cos\theta}{\eta}$$

$$\underset{\text{negative wave}}{P_{ave}} = \frac{1}{2}\mathrm{Re}\left\{\frac{\hat{E}_m^+ e^{\alpha z}e^{+j\beta z}\,\Gamma\,\hat{E}_m^{+*}e^{+\alpha z}e^{-j\beta z}e^{j\theta}(-\Gamma^*)}{\eta}\right\} = \frac{|\hat{E}_m^+|^2 e^{2\alpha z}|\Gamma|^2}{2\eta}\cos\theta$$

summing these two terms would miss the $-\sin\theta\,[2\Gamma_i]$ term in the above expression !?

7-21 P_{ave} of sun @ earth $= 1340 \ W/m^2$

assuming single frequency wave we have:

$$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} \;; \text{ or } |\hat{E}_m| = \sqrt{2 \times 377 \times 1340} = \underline{\underline{1.005 \times 10^3 \ V/m}}$$

$$|\hat{H}_m| = \frac{|\hat{E}_m|}{\eta} = \underline{\underline{2.66}} \ A/m$$

$$P_{total \text{ fm sun}} = 4\pi (1.48 \times 10^{11})^2 \, 1.34 \times 10^3 = \boxed{36.88 \times 10^{25} \ Watts}$$

EE 434 Homework 7

11-11 $\Delta z = 0.01\lambda$; $\hat{I} = 5A$; $f = 300 MHz$

a) $\lambda = \frac{3\times10^8}{3\times10^8} = 1m$ $\therefore$ $\boxed{\Delta z = 0.01 m}$

(far zone if $\beta_0 r >> 1$ from (11-30)
so $\beta_0 r = 20$ is reasonably in the far zone)

this is $\left[r = \frac{20}{\beta_0} = \frac{20}{2\pi}\lambda = 3.18m\right]$

b) $d\hat{E}_\theta = \eta\, d\hat{H}_\phi = j\frac{\omega\mu I \Delta z e^{-j\beta r}}{4\pi r}\sin\theta$

$\sin\theta = 1$ @ $\theta = 90°$

c) $\bar{P} = \frac{1}{2}Re\{d\hat{E}_\theta \bar{a}_\theta \times d\hat{H}_\phi^* \bar{a}_\phi\} = \frac{1}{2}\cdot\frac{\omega^2\mu^2 I^2 \Delta z^2}{\eta\, 16\pi^2 r^2}\bar{a}_r$

$\bar{P} = \frac{\cancel{4\pi^2 f^2}\mu^2 I^2 \Delta z^2 \sqrt{\epsilon}\sqrt{\epsilon}}{\sqrt{\mu}\; 32\pi^2 r^2 \sqrt{\epsilon}}\bar{a}_r = \frac{\mu I^2 \Delta z^2}{8\sqrt{\mu\epsilon}\,\lambda^2 r^2}\bar{a}_r$

$\boxed{P = \frac{\eta I^2}{8 r^2}\left(\frac{\Delta z}{\lambda}\right)^2} = \frac{377\times25\times10^{-4}}{8\times3.18^2} = \boxed{0.0117 \text{ Watts}/m^2}$

11-14 $P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin^2\theta \frac{\eta I^2}{8 r^2}\left(\frac{\Delta z}{\lambda}\right)^2 r^2 \sin\theta\, d\theta\, d\phi$

$\left[P_{rad} = \frac{2\pi\eta I^2}{8_4}\left(\frac{\Delta z}{\lambda}\right)^2 \underbrace{\int_0^\pi \sin^3\theta\, d\theta}_{4/3} = \frac{\pi\eta I^2}{3}\left(\frac{\Delta z}{\lambda}\right)^2\right]$

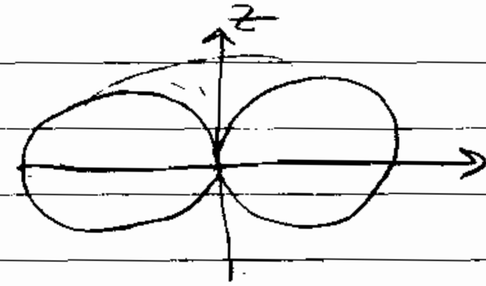
a) so $P_{rad} = \frac{\pi\times377\times25\times10^{-4}}{3} = \boxed{0.99 \text{ Watts}}$

b) if $\Delta z \to 0.02m$ $P_{rad} = 4\times0.99 = 3.95$ Watts

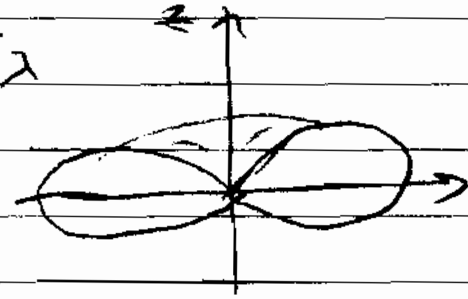
if $\Delta z = 0.005m$ $P_{rad} = \frac{0.99}{4} = 0.25$ Watts

EE 434 Homework 7 (solutions - page 2)

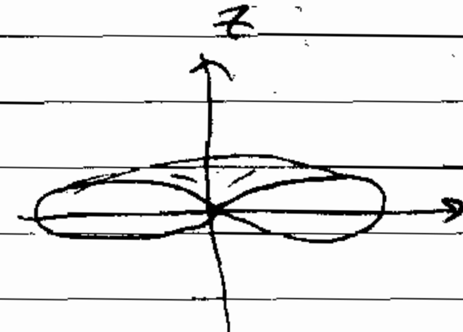
11-19 a) $L = \frac{\lambda}{2}$



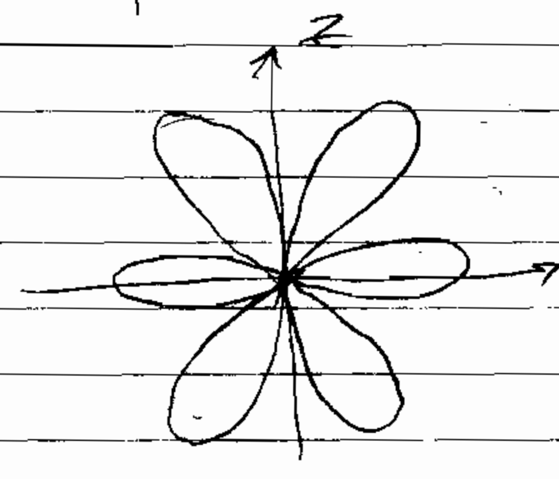
b) $L = \frac{5}{8}\lambda$



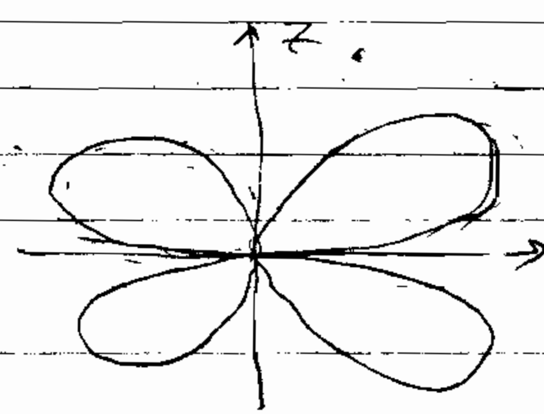
c) $L = \lambda$



11-20 a) $L = 1.5\lambda$



b) $L = 2\lambda$



EE 434 Homework 8

9.1, 9.2, 9.5
9.8, 9.10, 9.15

9.1 "dz" dipole ; $I = 10A$; $dz = \frac{\lambda}{50}$; $r = 10^3 m$

$$P_{max} = \frac{1}{2} R_e \left\{ \frac{\beta I^2 dz^2 \omega\mu}{16\pi^2 r^2} \right\} = \frac{2\pi I^2 dz^2 \omega\mu}{2\lambda\, 16\pi^2 r^2}$$

but $\left[\omega\mu = \frac{2\pi f \sqrt{\mu}\sqrt{\mu\epsilon}}{\sqrt{\epsilon}} = \frac{2\pi\eta}{\lambda} \right]$

$$\therefore P_{max} = \frac{I^2 dz^2 \eta}{\lambda^2 8 r^2} = \frac{10^2 \times 120\pi}{(50)^2 8 \times 10^6} = 1.89 \times 10^{-6}\ W/m^2$$

9.2 $L = 1m$; $f = 10^6$ $\therefore \lambda = \frac{3\times 10^8}{10^6} = 300m$ (short dipole)

$I = 12A$, $r = 5\times 10^3$

$$P(\theta = 30°) = \frac{I^2\eta}{8r^2}\left(\frac{dz}{\lambda}\right)^2 \sin^2 30° = \frac{12^2 \times 377}{8 \times 25\times 10^6}\left(\frac{1}{300}\right)^2 \sin^2 30° = 7.54 \times 10^{-10}\ W/m^2$$

9.5 $L = 2m$, $f = 10^6$ $\therefore \lambda = \frac{3\times 10^8}{10^6} = 300m$ (short dipole)

Cu antenna $\therefore \sigma = 5.8\times 10^7$ wire radius = 1mm

a) $$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{dz}{\lambda}\right)^2 = \frac{2\pi\times 377}{3}\left(\frac{2}{300}\right)^2 = \boxed{0.0351\ \Omega}$$

$$R_{loss} = \frac{L}{2a}\sqrt{\frac{f\mu}{\pi\sigma}} = \frac{2}{2\times 10^{-3}}\sqrt{\frac{10^6 \times 4\pi\times 10^{-7}}{\pi \times 5.8\times 10^7}} = \boxed{0.083\ \Omega}$$

so $$\xi = \frac{0.0351}{0.0351 + 0.083} = 0.297 \text{ or } \boxed{29.7\%}$$

b) $G = \xi D = 0.297 \times 1.5 = 0.446$ or $\boxed{-3.51 dB}$

c) $$20 = \frac{I^2 \pi\eta}{3}\left(\frac{dz}{\lambda}\right)^2 \text{ or } I = \sqrt{\frac{60}{\pi\eta}}\left(\frac{\lambda}{dz}\right)$$

or $$I = \sqrt{\frac{60}{120\pi^2}}\left(\frac{\lambda}{dz}\right) = \frac{1}{\sqrt{2}\pi}(150) = \boxed{33.76A}$$

$$P_{transmitter} = \frac{P_{rad}}{\xi} = \frac{20}{0.297} = \boxed{67.34\ Watts}$$

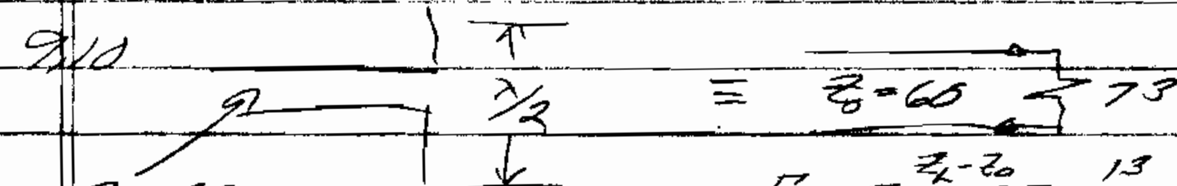
EE 434 Homework 8 (solution - page 2)

9.8 $\xi = 90\%$; $D_{dB} = 6.7\,dB$; $G = ?$

$$D_{dB} = 10\log_{10} D \quad \therefore D = 10^{\frac{D_{dB}}{10}} = 4.677; \; G = \xi D = 4.2$$

$$\boxed{\text{or } G_{dB} = 6.24\,dB}$$

9.10



$$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{13}{133} = 0.0977$$

$$SWR = \frac{|\hat{V}_{max}|}{|\hat{V}_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.0977}{1-0.0977} = \boxed{1.217}$$

9.15 3 GHz ; 1m dishes ; $P_r = 10^{-9}\,W$; $r = 4\times10^{4}$

$$\lambda = \frac{3\times10^{8}}{3\times10^{9}} = 0.1\,m$$

$$P_t = 10^{-9}(r\lambda)^2\left(\frac{1}{A_r A_t}\right) = \frac{10^{-9}\times16\times10^{8}\times10^{-2}\times16}{\pi^2} = \underline{25.94\times10^{-3}\,W}$$

EE 434 Homework 9

9.16 $\lambda/2$ dipole ; $P_t = 10^3$; $f = 50\,MHz$; $G_r = 13dB$

$R = 3\times10^4$; $D_r = 19.95$; $\lambda = \frac{3\times10^8}{5\times10^7} = 6m$

$\therefore P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi R}\right)^2 = 10^3 \times 19.95 \times 1.64 \left(\frac{6}{4\pi\times3\times10^4}\right)^2 = \underline{8.28\times10^{-6}\,W}$

9.21 94GHz ; antenna 1m × 0.1m ; $\lambda = \frac{3\times10^8}{94\times10^9} = 3.19\times10^{-3}\,m$

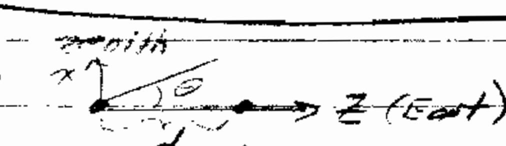
a) $BW_{elevation} \cong \frac{3.19\times10^{-3}}{0.1} = 3.19\times10^{-2}$ radians $= \boxed{1.8^\circ}$

$BW_{azimuth} \cong 3.19\times10^{-3}$ radians $= \boxed{0.18^\circ}$

b)

$d = 300\times3.19\times10^{-3} = \boxed{0.957\,m}$

9-23



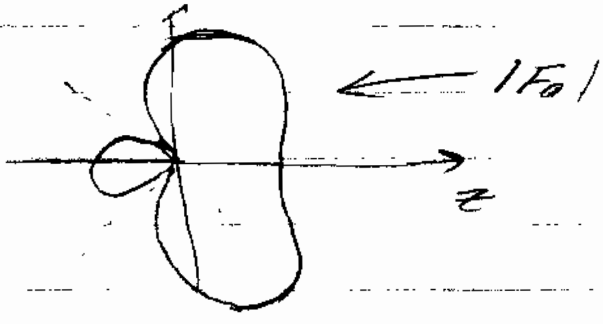
a) $a_0 = a_1 = 1$; $\delta = \frac{\pi}{4}$; $d = \frac{\lambda}{2}$

uniform array $\therefore$ $|F_a| = \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\frac{\psi}{2}}$ where $\psi = \frac{2\pi}{\lambda}d\cos\theta - \delta$

so $\boxed{\psi = \pi\cos\theta - \frac{\pi}{4}}$ and $|F_a| = \frac{\sin\psi)}{\sin\frac{\psi}{2}}$

maximum of array pattern @ $\psi = 0$ so $\left[\theta_{max} = \cos^{-1}\frac{1}{4} = 75.52^\circ\right]$

1^{st} zero when $\psi = -\pi = \pi\cos\theta - \frac{\pi}{4}$ or $\cos\theta = -\frac{3}{4}$; $\theta_{zero} = 138.6^\circ$



EE 434 Homework 9 (solutions - page 2)

9.23 b) $a_0=1$, $a_1=2$, $\delta=0$, $d=\lambda$

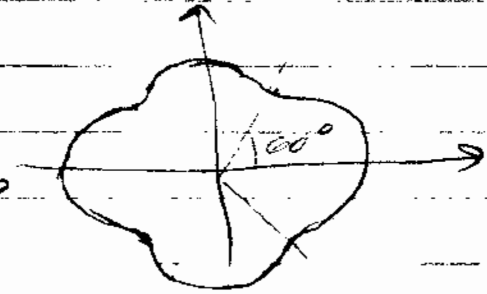
$F_a = 1+2e^{j\psi}$; $\psi = \frac{2\pi}{\lambda} d\cos\theta - \delta = 2\pi\cos\theta$

$\therefore F_a = 1+2e^{j2\pi\cos\theta} = 1+2\cos(2\pi\cos\theta)+j2\sin(2\pi\cos\theta)$

so $|F_a| = [1+2\cos(2\pi\cos\theta)]^2 + 4\sin^2(2\pi\cos\theta)$

$|F_a| = 1+4\cos(2\pi\cos\theta)+4 = 5+4\cos(2\pi\cos\theta)$

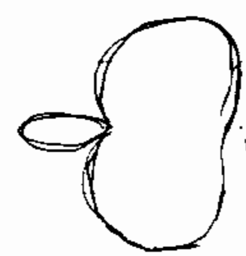
computer plot gives:



@ $\theta=0$ + 90° $|F_a|=9$

@ $\theta=60°$ $|F_a|=1$

9-24 from 9.23a)

$F_a =$ 

$f_{element} =$ 

total pattern $= f_{element} \times F_a$

$\therefore$ total pattern is 

EE 434 Homework 10

Broadcast Antenna Array $\left[F_a = (z-z_1)(z-z_2) \right]$
where $z = e^{j\psi} = e^{j\beta d \cos\theta}$

$\alpha = 0$ (fed in phase) will provide maximum @ $\theta = 90°$
want zeroes when $\theta = 26°$ and $154°$

$\therefore \psi_1 = \frac{2\pi d}{\lambda} \cos 26° = \frac{\pi}{2} \cos 26° = 0.449\pi$
$\psi_2 = \frac{\pi}{2} \cos 154° = -0.449\pi$

so $F_a = (z - e^{j0.449\pi})(z - e^{-j0.449\pi}) = z^2 - z(e^{j0.449\pi} + e^{-j0.449\pi}) + 1$

$\boxed{\text{or } F_a = z^2 - 0.319z + 1}$ $\boxed{z = e^{j\frac{\pi}{2}\cos\theta}}$

(see attached plot)

9.18 a) $P_t = 10W$, $f = 6GHz$, $G_t = 20dB$, $G_r = 23dB$, $R = 2\times10^4$

$D(\theta,\phi) = \frac{P\,W/m^2\; 4\pi R^2}{P_t}$; $\lambda = \frac{3\times10^8}{6\times10^9} = 0.05m$

a) $P_{@\,receiver} = \frac{G_t P_t}{4\pi R^2} = \frac{10^2 \times 10}{4\pi \times 4\times10^8} = 0.0199\times10^{-5} = \boxed{1.99\times10^{-7} W/m^2}$

b) $P_{receiver} = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2 = 1.99\times10^{-7} \times 25\times10^{-4} \times 199.53 \left(\frac{1}{4\pi}\right) = \boxed{7.899\times10^{-9} W}$

c) $N = kTB = 1.38\times10^{-23} \times 10^3 \times 10^7 = 1.38\times10^{-13} \therefore \boxed{S/N = 5.72\times10^4 \text{ or } 47.6 dB}$

9.20 Beamwidth $= 1.5°$ @ $20GHz$ $\quad 1.5° \times \frac{\pi}{180} = \frac{1.5\times10^{-2}}{dia}$; $dia = 0.573$

$\lambda = \frac{3\times10^8}{20\times10^9} = 1.5\times10^{-2} m$; $D = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi^2(0.573)^2}{2.25\times10^{-4}\times4} = 1.44\times10^4$

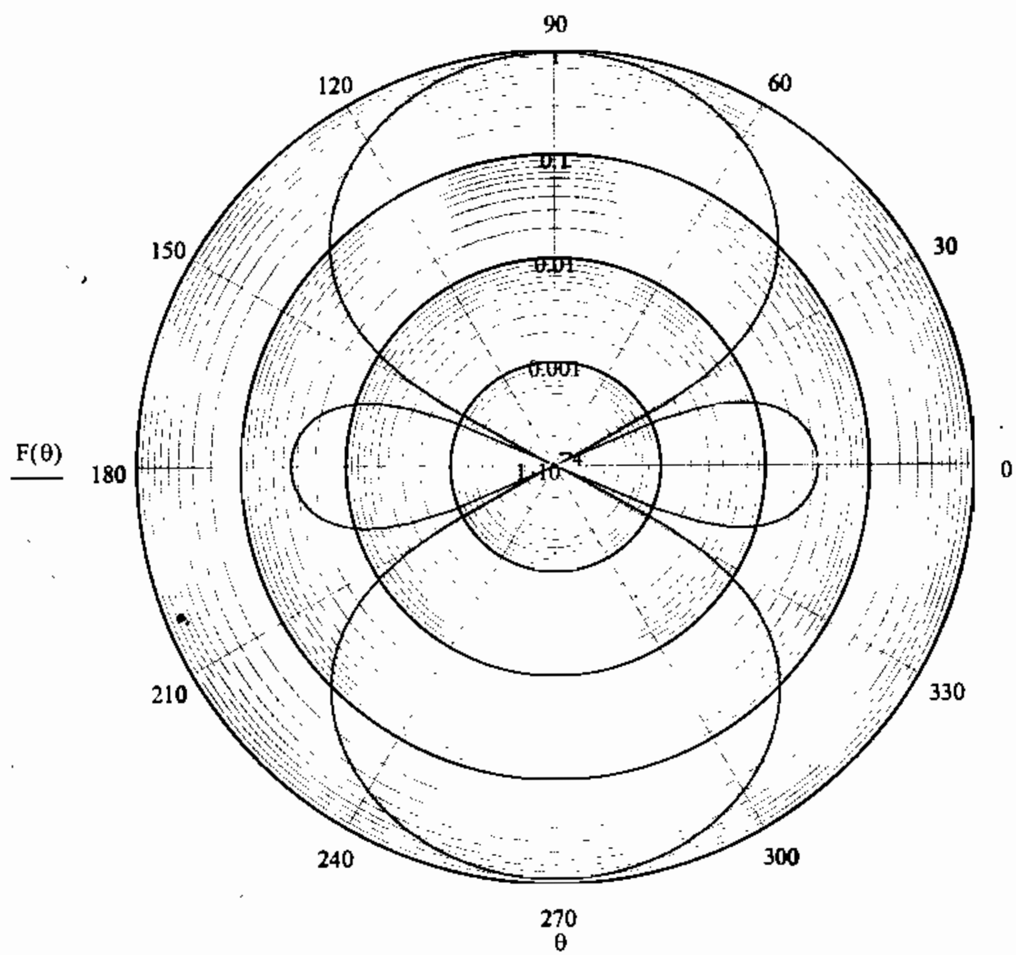
a) $\therefore \boxed{D = 1.44\times10^4 \text{ or } 41.6dB}$

b) for twice the area D will be multiplied by 2 and beamwidth $\div \sqrt{2}$

c) for twice the frequency $\boxed{\text{beamwidth} \div 2}$ and $\boxed{D \text{ multiplied by } 4}$

Broadcast Array Special Problem

$j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad z(\theta) := e^{j\cdot\frac{\pi}{2}\cdot\cos(\theta)} \quad z_1 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(26\cdot\frac{\pi}{180}\right)} \quad z_2 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(154\cdot\frac{\pi}{180}\right)} \quad F(\theta) := \left[\left|\frac{1}{\sqrt{3}}\cdot(z(\theta) - z_1)\cdot(z(\theta) - z_2)\right|\right]^2$



EE 434 Homework 11

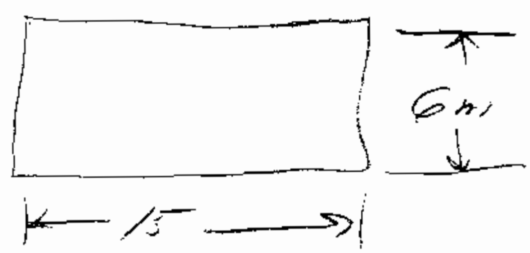
8-23 $f_c = \dfrac{1}{2\sqrt{\mu\epsilon}\,a}$ for TE_{10} mode

a) 5 GHz $\quad a_{min} = \dfrac{1}{2\sqrt{\mu\epsilon}\,f} = \dfrac{3\times10^8}{10\times10^9} = \boxed{3\times10^{-2}\,m}$

b) 5 MHz $\quad a_{min} = \dfrac{3\times10^8}{10\times10^6} = \boxed{30\,m}$

c) 5 kHz $\quad a_{min} = \dfrac{3\times10^8}{10^4} = \boxed{3\times10^4\,m}$

8-27



TE_{10} mode

$f_c = \dfrac{3\times10^8}{2\times15} = 10\,MHz$

→ Vertical polarization

AM will not propagate $\quad$ 535→1605 kHz

FM will propagate $\quad$ 88→108 MHz

EE 434 Homework 12

P_{max} for $0.4'' \times 0.9''$ waveguide @ 10 GHz

$$\hat{E}_y = E_{y_{max}} \sin \frac{\pi}{a} x$$
$$\hat{H}_x = -E_{y_{max}} \sqrt{\frac{\epsilon}{\mu}} \sqrt{1-\left(\frac{f_c}{f}\right)^2} \sin \frac{\pi}{a} x$$

$$\overline{P_{ave}}/_{m^2} = \frac{1}{2} Re\{\hat{E} \times \hat{H}^*\} = \frac{1}{2} Re\left\{\bar{a}_z E_{y_{max}}^2 \sin^2\left(\frac{\pi}{a}x\right) \sqrt{\frac{\epsilon}{\mu}} \sqrt{1-\left(\frac{f_c}{f}\right)^2}\right\}$$

$$\therefore P_{ave} = \frac{1}{2} E_{y_{max}}^2 \sqrt{\frac{\epsilon}{\mu}} \sqrt{1-\left(\frac{f_c}{f}\right)^2} \int_{y=0}^{b} \int_{x=0}^{a} \underbrace{\sin^2\left(\frac{\pi}{a}x\right)}_{\frac{1}{2}\left[1-\cos\left(\frac{2\pi x}{a}\right)\right]} dx\, dy$$

$$P_{ave} = \frac{1}{2} E_{y_{max}}^2 \sqrt{\frac{\epsilon}{\mu}} \sqrt{1-\left(\frac{f_c}{f}\right)^2} \frac{ba}{2}$$ (Equation 8-90)

or $\boxed{P_{ave} = 1.16 \times 10^{-7} E_{y_{max}}^2}$

but $E_{y_{max}} \cong 3 \times 10^6$ V/m in air

so $P_{ave_{max}} = 1.16 \times 10^{-7} \times 9 \times 10^{12} = \boxed{1.04 \times 10^6 \text{ W}}$ ←