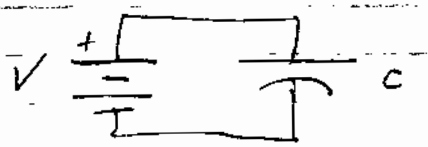
EE 434 Assignment 1

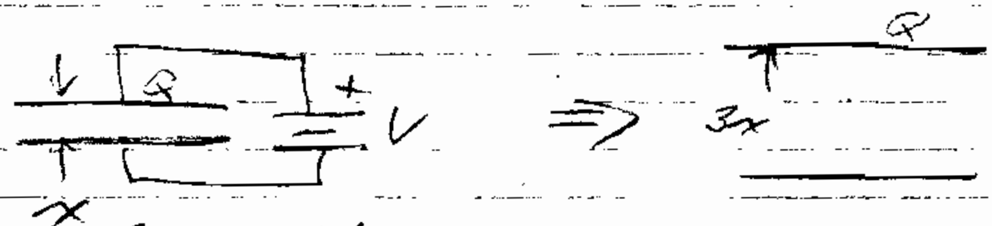
4.8 a) $W=\frac{1}{2}\int \rho_v \Phi \, dv = \frac{1}{2} V \int \rho_v \, dv = \boxed{\frac{1}{2}VQ}$



b) $W=\frac{1}{2}CV^2 = \frac{1}{2}C\frac{Q^2}{C^2} = \boxed{\frac{1}{2}\frac{Q^2}{C}}$

$\boxed{C=\frac{Q}{V}}$

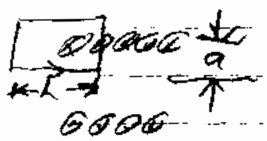
c)



$W=\frac{1}{2}\cdot\frac{Q^2}{C}$; $C=\frac{\epsilon A}{d}$ so C goes down by a factor of 3

$\boxed{\text{and } W \text{ is multiplied by } 3}$

4.25 Solenoid with N turns/m and current I



for $\rho > a$ $B_z = 0$

for $\rho < a$ $H_z L = NLI$ or $H_z = NI$

a) $\bar{B} = \nabla\times\bar{A}$; $\int \nabla\times\bar{A}\cdot d\bar{S} = \boxed{\oint \bar{A}\cdot d\bar{l} = \int \bar{B}\cdot d\bar{S}}$

b) use circular contours centered on the axis

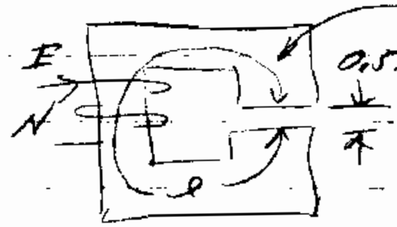
$\rho < a$ $A_\phi 2\pi\rho = NI\mu\pi\rho^2$ or $\boxed{A_\phi = \frac{NI\mu\rho}{2}}$

$\rho > a$ $A_\phi 2\pi\rho = \mu NI\pi a^2$ or $\boxed{A_\phi = \frac{\mu N I a^2}{2\rho}}$

EE 434 Homework 3

$B = \mu H$

4.27



Area $= 1\,cm^2$ $\quad l = 40\,cm$

[Want $B = 1\ W/m^2$ in air gap]

$\mu_{iron\,(not\,saturated)} = \frac{\Delta B}{\Delta H} = \frac{1.3}{0.025} = 52$

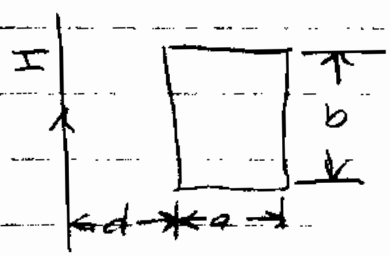
$$R_{gap} = \frac{5\times10^{-4}}{4\pi\times10^{-7}\times10^{-4}} = 0.398\times10^{7}$$

$$R_{iron} = \frac{0.4}{52\times10^{-4}} = 7.69\times10$$

$$\psi_m = B_{gap}\times10^{-4} = 10^{-4} = \frac{NI}{R_g + R_{iron}}$$

$$\therefore \left[NI = 10^{-4}(0.398\times10^{7} + 76.9) = 398 \text{ Ampere turns}\right]$$

4.30



$$L_{12} = \frac{\psi_{12}}{I}$$

$$\psi_{12} = \int_{\rho=d}^{d+a}\int_{z=0}^{b} \frac{\mu_0 I}{2\pi\rho}\,dz\,d\rho$$

$$\therefore L_{12} = \frac{\mu_0 b}{2\pi}\ln\left(\frac{d+a}{d}\right)$$

4.31 $\quad \bar{H} = NI\,\bar{a}_z$ for $\rho < a$; $\bar{H} = 0$ for $\rho > a$

N is turns/unit length

$$W_m/L = \frac{1}{2}\int \bar{B}\cdot\bar{H}\,dv = \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\int_{z=0}^{L} \mu N^2 I^2 \rho\,d\phi\,d\rho\,dz$$

$$\boxed{W_m/L = \frac{\mu N^2 I^2 L\, 2\pi a^2}{4\cdot 2}} = \frac{\mu (NI)^2 \pi a^2 L}{2}$$

Energy stored in length L with gap of length $l = \frac{(NI)^2\pi a^2}{2}\left[\mu_0\mu_r(L-l) + \mu_0 l\right]$

$$\boxed{F_l = \frac{\partial W_m}{\partial l} = \frac{(NI)^2\pi a^2}{2}(\mu_0 - \mu_0\mu_r) = \frac{\mu_0 (NI)^2 \pi a^2}{2}(1-\mu_r)}$$

EE 434 Homework 3

5.3 μ_0, $\epsilon_1 = 80$ | $\sigma = \infty$ $f = 150\,MHz$

a) $\bar{E} = 0$ @ $\lambda/2$ back from conducting plane

$\eta_1 = \sqrt{\frac{\mu_0}{\epsilon}}$ so $\epsilon = \frac{\mu_0}{\eta_1^2} = \frac{4\pi \times 10^{-7}}{64 \times 10^2} = 0.196 \times 10^{-10}$

$v_{phase} = \frac{1}{\sqrt{\mu_0 \epsilon}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1.96 \times 10^{-10}}} = 0.637 \times 10^8\ m/sec$

$\therefore \lambda/2 = \frac{0.637 \times 10^8}{2 \times 1.5 \times 10^8} = \boxed{0.212\ m}$

b) zero H @ $\lambda/4$ from conductor $\boxed{= 0.106\,m}$

c) $\hat{E}^i = 100 e^{j0°}$; $\hat{H}^i = 2 \frac{100}{\eta} = \frac{200}{80} = \boxed{2.5\ A/m}$

incident + reflected wave ?

$|\hat{H}|_{z=-2} = 2.5 \cos\beta_1(-2) = 2.5 \cos(-14.81 \times 2) = \boxed{0.558\ A/m}$

$1697°$

5.5 $\boxed{20\,KHz}$ a) $\lambda/2_{in\ air} = \frac{3 \times 10^8}{2 \times 2 \times 10^4} = \boxed{0.75 \times 10^4\ m}$

for sea water $\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{1/2}$

but $\frac{\sigma}{\omega\epsilon} = \frac{4 \times 36\pi}{2\pi \times 2 \times 10^4 \times 10^{-9} \times 81} = 0.444 \times 10^5$

so $\beta \simeq \frac{2\pi \times 2 \times 10^4 \times \sqrt{81}}{3 \times 10^8 \times \sqrt{2}} \sqrt{4.44 \times 10^4} = 56.198 \times 10^{-2}$

giving $\lambda/2_{sea\ water} = \frac{2\pi}{2\beta} = \boxed{5.59\,m}$

assume $\boxed{E_{incident} = E_0}$ then $E_t = E_0 \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2}$; $\boxed{\eta_1 \cong 377}$

$\hat{\eta}_2 = \sqrt{\frac{\mu}{\epsilon}} \sqrt{\frac{\omega\epsilon}{\sigma}}\, e^{j\frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)} \cong 0.198\, e^{j45°}$

5.5 continued

$$\text{so } \hat{T} = \frac{0.396\, e^{j45^\circ}}{377 + 0.198\, e^{j45^\circ}} \cong 1.05\times10^{-3} e^{j45^\circ}$$

$$\therefore E_{\substack{surface \\ transmitted}} = \boxed{1.05\times10^{-3} E_0}$$

We showed above that sea water is a very good conductor so $\alpha = \beta$. We need to find how far we penetrate before $|E| = 10^{-4} E_0$

$$\text{so } 1.05\times10^{-3} E_0\, e^{-0.562 z} = 10^{-4} E_0$$

$$e^{-0.562 z} = 0.0952$$

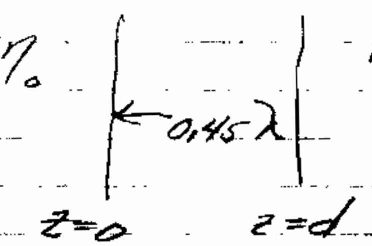
$$\boxed{z = \frac{\ln 0.0952}{-0.562} = 4.18\ m}$$

5.10 fiberglass radome: $\epsilon_r = 4.9$, $\mu = \mu_0$, $\sigma = 0$, $f = 10\,GHz$

a) for no reflection thickness $= \lambda/2 = \frac{3\times10^{8}}{\sqrt{4.9}\, 2\times10^{10}} = \boxed{0.678\times10^{-2} m}$

b) $\boxed{f = 9\,GHz}$ $\lambda/2_{new} = \frac{10}{9}\lambda_{original} = .753\times10^{-2} m$

$\therefore$ radome is now $0.678\times10^{-2} \times \frac{1}{2\times0.753\times10^{-2}} = \boxed{0.45\lambda}$



$\boxed{Z(d) = 377}$

$\eta_{radome} = \frac{377}{\sqrt{4.9}} = 170.31$

$Z(d^-)_{norm} = \sqrt{4.9} = 2.214$

$Z(0) = 274.2 + j143$

from Smith chart $Z(0)_{norm} = 1.61 + j0.83$; $Z(0)_{norm} = 0.727 + j0.379$

in general $\hat{E}_1 = \hat{E}^+_{m_1} e^{-j\beta_1 z}[1 + \Gamma_1(z)]$; $\hat{E}_2 = \hat{E}^+_{m_2} e^{-j\beta_2 z}[1 + \Gamma_2(z)]$

at $z = 0$ $\hat{E}_1 = \hat{E}_2$ $\therefore$ $\hat{E}^+_{m_1}[1 + \Gamma(0^-)] = \hat{E}^+_{m_2}[1 + \Gamma(0^+)]$

5.10 continued

$$\hat{E}^+_{m_2} = 100\,\frac{1+\Gamma(0^-)}{1+\Gamma(0^+)} = 100\,\underbrace{\frac{1+0.279e^{j113^\circ}}{1+0.380e^{j36^\circ}}}_{\text{from chart}}$$

$$\hat{E}^+_{m_2} = 100\,\frac{1+(-0.105+j0.249)}{1+(0.307+j0.223)} = 100\,\frac{0.895+j0.249}{1.307+j0.223} = 100\,\frac{0.929e^{j15.5}}{1.32e^{j9.68}}$$

$$\hat{E}^+_{m_2} = 75.21e^{j5.82^\circ}$$

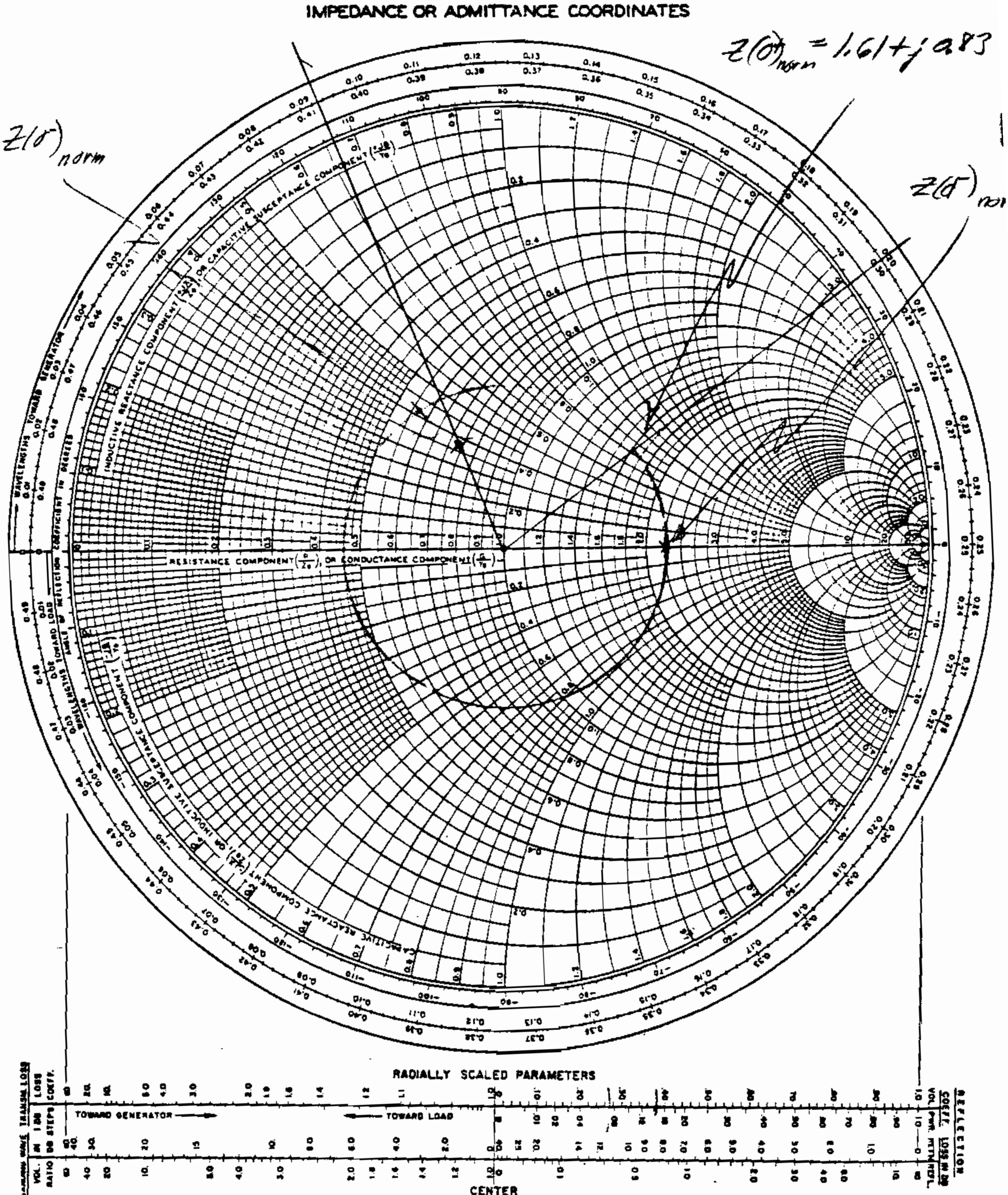
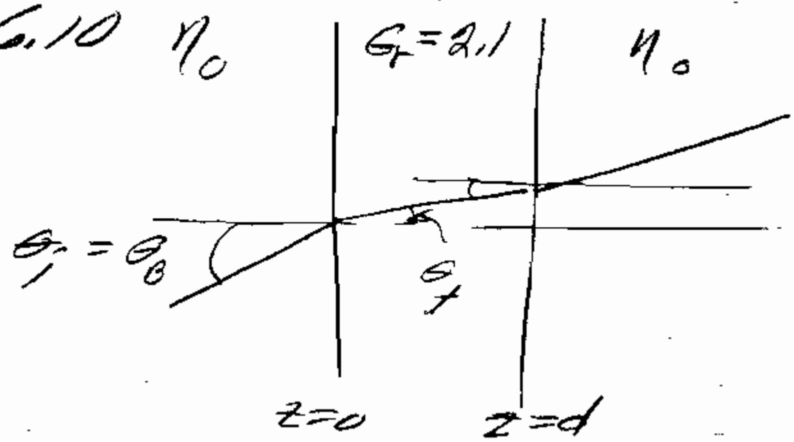


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 434 Homework 4

6.10



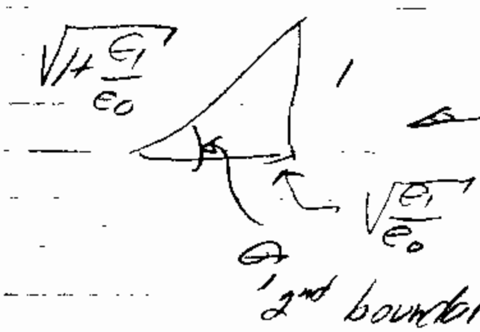
Snell's Law

$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

so $\sin\theta_t = \sqrt{\frac{\epsilon_0}{\epsilon_1}} \sin\theta_i = \sin\theta_i$ @ 2nd boundary

but $\theta_i = \theta_{Brewster}$ or $\tan\theta_B = \tan\theta_i = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$

$$\therefore \sin\theta_{i\,\text{2nd boundary}} = \sqrt{\frac{\epsilon_0}{\epsilon_1}} \sin\theta_B = \sqrt{\frac{\epsilon_0}{\epsilon_1}} \left(\frac{\sqrt{\frac{\epsilon_1}{\epsilon_0}}}{\sqrt{1+\frac{\epsilon_1}{\epsilon_0}}} \right) = \frac{1}{\sqrt{1+\frac{\epsilon_1}{\epsilon_0}}}$$



$$\tan\theta_{i\,\text{2nd boundary}} = \sqrt{\frac{\epsilon_0}{\epsilon_1}} = \tan\theta_B \text{ @ 2nd boundary}$$

$[\theta_i = \theta_B$ @ 2nd boundary$]$

$\therefore$ no reflection of $\parallel$ polarization at either boundary

EE 434 Homework 5 (solution)

6.19 $\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}$

for glass $n = A + \frac{B}{\lambda^2}$ with $A = 1.5$, $B = 5\times10^{-15}$

$\theta_i = 30^\circ$ so $\boxed{\sin\theta_t = \frac{1}{n}\sin\theta_i = \frac{1}{2n}}$

color	λ nm	n	θ_t
violet	400	1.5312	19.058°
blue	450	1.5247	19.14°
green	550	1.516	19.25°
yellow	600	1.5139	19.28°
orange	650	1.518	19.31°
red	700	1.5102	19.33°

8.6 $E_x = A\cos\left(\frac{\pi x}{a}\right)\sin\left(\frac{2\pi y}{b}\right)\sin(7\pi\times10^{10}t - \beta z)$

a) TE_{12} or TM_{12} mode

b) $f = \frac{7}{2}\times10^{10} = 35\,GHz$

c) $\beta_{1,2} = \sqrt{\frac{49\pi^2\times10^{20}}{9\times10^{16}} - \left(\frac{\pi}{2.3\times10^{-2}}\right)^2 - \left(\frac{2\pi}{1.2\times10^{-2}}\right)^2} = 4.95\times10^2/m$

d) $f_c = \frac{3\times10^8}{2\pi}\sqrt{29.26\times10^4} = 25.8\times10^9 = \boxed{25.8\,GHz}$

$\eta_{TE} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 559.9\,\Omega$

$\eta_{TM} = 120\pi\sqrt{1-\left(\frac{f_c}{f}\right)^2} = 254.7\,\Omega$

8.11 TE_{10} $a = 2b$ $1.5 f_{c_{10}} = 9\times10^9$

so $6\times10^9 = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{3\times10^8}{2a}$; $a = 2.5\,cm$

EE 434 Homework 6 (solutions)

1. a) rectangular tunnel 15 m by 6 m

lowest $f_{c_{TE_{10}}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{15}\right)^2} = \frac{3\times10^8}{30} = \boxed{10^7 \text{ Hertz}}$ ←

$\boxed{10 \text{ MHz}}$

b) Vertical polarization ←

c) AM 535–1605 KHz will not propagate ←

FM 88–108 MHz will propagate ←

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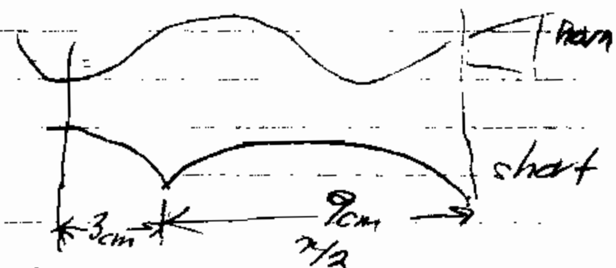
2. (8.15) $VSWR = 2.1$; $\frac{\lambda}{2} = 9\text{cm}$



$Z_{norm@V_{min}} = \frac{1}{VSWR} = 0.476$

on Smith chart move $\frac{3}{18}\lambda$ towards the load

this gives $\boxed{Z_{norm_{norm}} = 1.12 - j\,0.78}$ ←

$f_c = \frac{3\times10^8}{2\pi}\sqrt{\frac{\pi^2}{0.076^2}} = \boxed{1.974\,GH}$; $\eta_{TE_{10}} = \frac{377}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$

so we need f ; $\lambda = \frac{2\pi}{\beta} = \frac{\cancel{2\pi}\, c}{\cancel{2\pi} f\sqrt{1-\left(\frac{f_c}{f}\right)}} = \frac{c}{\sqrt{f^2 - f_c^2}}$

or $\sqrt{f^2 - f_c^2} = \frac{3\times10^8}{0.18} = 1.6\times10^9$

giving $f = \sqrt{(1.6\times10^9)^2 + (1.974\times10^9)^2} = \boxed{2.54\times10^9}$

$\eta_{TE_{10}} = \frac{377}{\sqrt{1 - 0.604}} = 599\,\Omega$

$Z_{norm} = 599\, Z_{norm_{norm}} = \boxed{671 - j\,467}$ ←

EE 434 Homework 6 (Solutions-page 2)

3. (8.8) $f = 1.3 f_c$, $E_{max} = 0.75 \times 2 \times 10^6$ V/m

rectangular waveguide $a = 2.3$ cm, $b = 1.2$ cm TE_{10} mode

from class notes

$\hookrightarrow \hat{P}_z = H_0^2 \frac{\omega \mu a^2 \beta_{10}}{\pi^2} \sin^2 \frac{\pi x}{a} \longrightarrow \frac{1}{2}\left[1 - \cos\left(\frac{2\pi x}{a}\right)\right]$; $H_0 = H_{z_{max}}$

$$P_{ave} = \frac{1}{2} \mathrm{Re} \left\{ \int_{y=0}^{b} \int_{x=0}^{a} \hat{P}_z \, dx \, dy \right\} = \frac{H_0^2 \omega \mu a^2 \beta_{10} a b}{2\pi^2 \, 2}$$

$$E_{max} = H_0 \frac{\omega \mu a}{\pi} \therefore P_{ave} = \frac{\cancel{\pi^2} E_{max}^2}{\omega^{\cancel{2}} \mu^{\cancel{2}} \cancel{a^2}} \cdot \frac{\cancel{\omega} \cancel{\mu} \cancel{a^2} \beta_{10} ab}{4 \cancel{\pi^2}} = \frac{E_{max}^2 \beta_{10} ab}{\omega \mu 4}$$

$$\text{or } P_{ave} = \frac{E_{max}^2 \, \omega \sqrt{\mu \epsilon} \sqrt{1 - \left(\frac{1}{1.3}\right)^2} \, ab}{4 \omega \mu}$$

$$P_{ave} = E_{max}^2 \frac{\sqrt{1 - \left(\frac{1}{1.3}\right)^2} \, ab}{4 \eta_0} = E_{max}^2 \, 1.16947 \times 10^{-7}$$

$$P_{ave_{max}} = 2.25 \times 10^{12} \times 1.169 \times 10^{-7} = \boxed{0.263 \times 10^6 \text{ Watts}}$$

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4. TE_{11} $f_c = \frac{3 \times 10^8}{2\pi} \left(\frac{p'_{11}}{2.54 \times 10^{-2}} \right) = \frac{3 \times 10^8}{2\pi} \left(\frac{1.841}{2.54 \times 10^{-2}} \right) = 3.46$ GHz

TM_{01} $f_c = \frac{3 \times 10^8}{2\pi} \left(\frac{\overset{= 2.405}{p_{01}}}{2.54 \times 10^{-2}} \right) = 4.5$ GHz

TE_{01} $f_c = \frac{3 \times 10^8}{2\pi} \left(\frac{p'_{01}}{2.54 \times 10^{-2}} \right) = 7.2$ GHz

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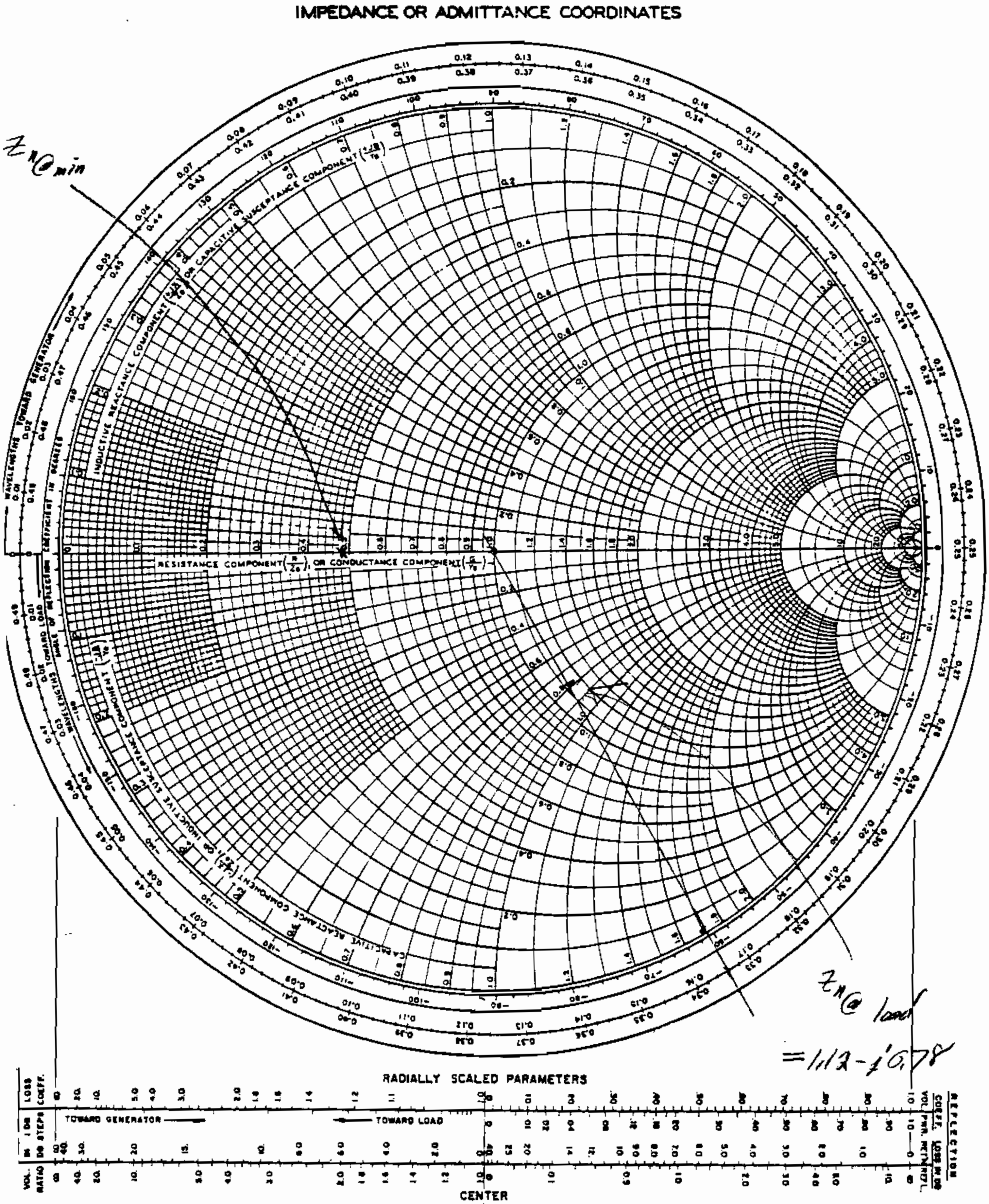


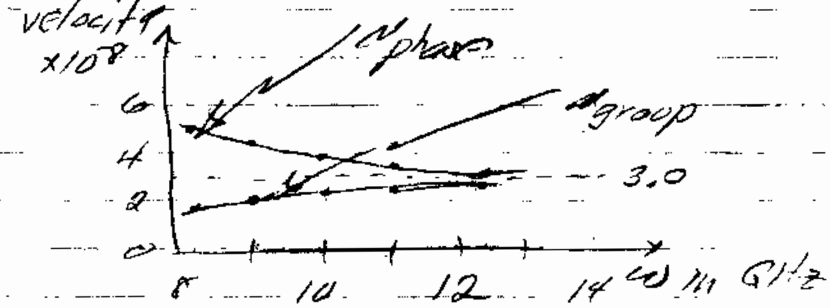
Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 434 Homework 7 solution

1. $\gamma = \sqrt{\omega_c^2\mu\epsilon - \omega^2\mu\epsilon} = \omega\sqrt{\mu\epsilon}\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$ ∴ $\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$v_{ph} = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$; $v_{group} = \frac{\partial\omega}{\partial\beta} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$ with $f_c = 6.557$ GHz

f	v_{ph}	v_g
8.2 GHz	5×10^8	1.8×10^8
9	4.38×10^8	2.05×10^8
10	3.97×10^8	2.26×10^8
11	3.73×10^8	2.41×10^8
12.4	3.53×10^8	2.54×10^8



2. a) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi\times3\times10^8}{2\pi\times10^{10}\underbrace{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}_{0.755}} = \boxed{3.97\times10^{-2}\text{ m}}$

$d = \lambda$

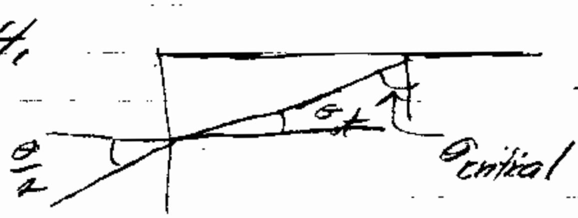
b) use $v_{group} = 3\times10^8\times0.755 = 2.26\times10^8$; transit time $= \frac{100}{2.26\times10^8} = \boxed{0.44\,\mu\text{sec}}$

c) change the dielectric so that $f_c < 6$ GHz

3. $f_c = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$; $f_{c_{TE_{10}}} = 6.557$ (E ~~field~~ electric vertical)

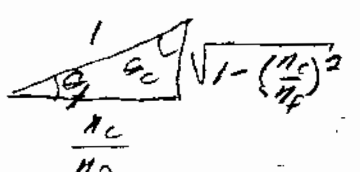
for the horizontal field the lowest cutoff frequency is for the TE_{01} mode which is more than twice the TE_{10} cutoff frequency (i.e. > 13 GHz) So we see the the horizontal E field part of our wave will not propagate.

4.



$\frac{\sin\theta_t}{\sin\left(\frac{\theta}{2}\right)} = \frac{1}{n_f}$; $\theta_t = 90° - \theta_c$

$\sin\theta_c = \frac{n_c}{n_f}$



$\sin\left(\frac{\theta}{2}\right) = n_f\sin\theta_t = n_f\sqrt{1-\left(\frac{n_c}{n_f}\right)^2} = \sqrt{n_f^2 - n_c^2}$

$\sin\left(\frac{\theta}{2}\right) = 0.242$ or $\boxed{\theta = 28°}$

Homework 8

EE434 (Solution)

9.2 $\bar{P}_{ave} = \frac{1}{2}\mathrm{Re}\{\hat{E}\times\hat{H}^*\} = \frac{1}{2}\mathrm{Re}\{\bar{a}_\theta\left[|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(-\frac{\beta_0}{r^2}+\frac{j}{r^3}\right)\cdot\left(\frac{j\beta_0}{r}+\frac{1}{r^2}\right)\right]$

$+\bar{a}_r\left[|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{j\beta_0^2}{r}+\frac{\beta_0}{r^2}-\frac{j}{r^3}\right)\cdot\left(\frac{-j\beta_0}{r}+\frac{1}{r^2}\right)\right]\}$

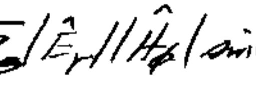
$=\frac{1}{2}\mathrm{Re}\{\bar{a}_\theta|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(\frac{j\beta_0^2}{r^3}-\cancel{\frac{\beta_0}{r^4}}+\cancel{\frac{\beta_0}{r^4}}+j\frac{1}{r^5}\right)$

$+\bar{a}_r|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta_0^3}{r^2}+\cancel{\frac{j\beta_0^2}{r^3}}-\cancel{\frac{j\beta_0^2}{r^3}}+\cancel{\frac{\beta_0}{r^4}}-\cancel{\frac{\beta_0}{r^4}}-\frac{j}{r^5}\right)\}$

only non-imaginary term

so $\bar{P}_{ave}=\frac{1}{2}|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta_0^3}{r^2}\right)\bar{a}_r$

9.3 $\bar{P}=K\frac{\sin\theta\cos\phi}{r^2}\bar{a}_r$ W/m² $\quad 0<\theta<\pi$, $-\frac{\pi}{2}<\phi<\frac{\pi}{2}$



a) elevation or θ BW $\frac{1}{2}\theta_{BW}=90°-\sin^{-1}(\frac{1}{2})$; $\theta_{BW}=120°$

azimuthal or φ BW $\cos(\frac{\phi_{BW}}{2})=\frac{1}{2}$; $\phi_{BW}=2\cos^{-1}(\frac{1}{2})=120°$

b) θ BW to 1st null $\sin\theta=0$ @ $\theta=0°$ (maximum @ $\theta=90°$) ∴ $BW_{null\ to\ null}=180°$

φ BW to 1st null $BW_{null\ to\ null}=180°$

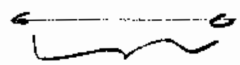
c) $P_{rad}=K\int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\theta=0}^{\pi}\frac{\sin\theta\cos\phi}{r^2}r^2\sin\theta\,d\theta\,d\phi=K\sin\phi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi}=K\,2\times\frac{\pi}{2}=K\pi$

$D(\theta,\phi)=\frac{K\sin\theta\cos\phi\,4\pi r^2}{r^2 K\pi}=\boxed{4\sin\theta\cos\phi}$

$\boxed{D=D(\theta,\phi)_{max}=4}$

Homework 9

9.7 a) 2 isotropic radiators



$\lambda/2$

$\psi = \pi$, $d = \lambda/2$

$X = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\phi - \pi = \pi(\cos\phi - 1)$; $|AF| = \frac{\sin X}{\sin \frac{X}{2}}$

ϕ	X	$\lvert AF \rvert$
0	0	2
$\pm 30°$	-0.42	2
$\pm 60°$	-1.57	1.99
$\pm 90°$	$-\pi$	0

[Plot on next page]

b) $d = \frac{\lambda}{4}$, $\psi = 90°$ so $X = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} \cos\phi - \frac{\pi}{2} = \frac{\pi}{2}(\cos\phi - 1)$

[Plot attached]

c) $\lambda = d$, $\psi = \pi$

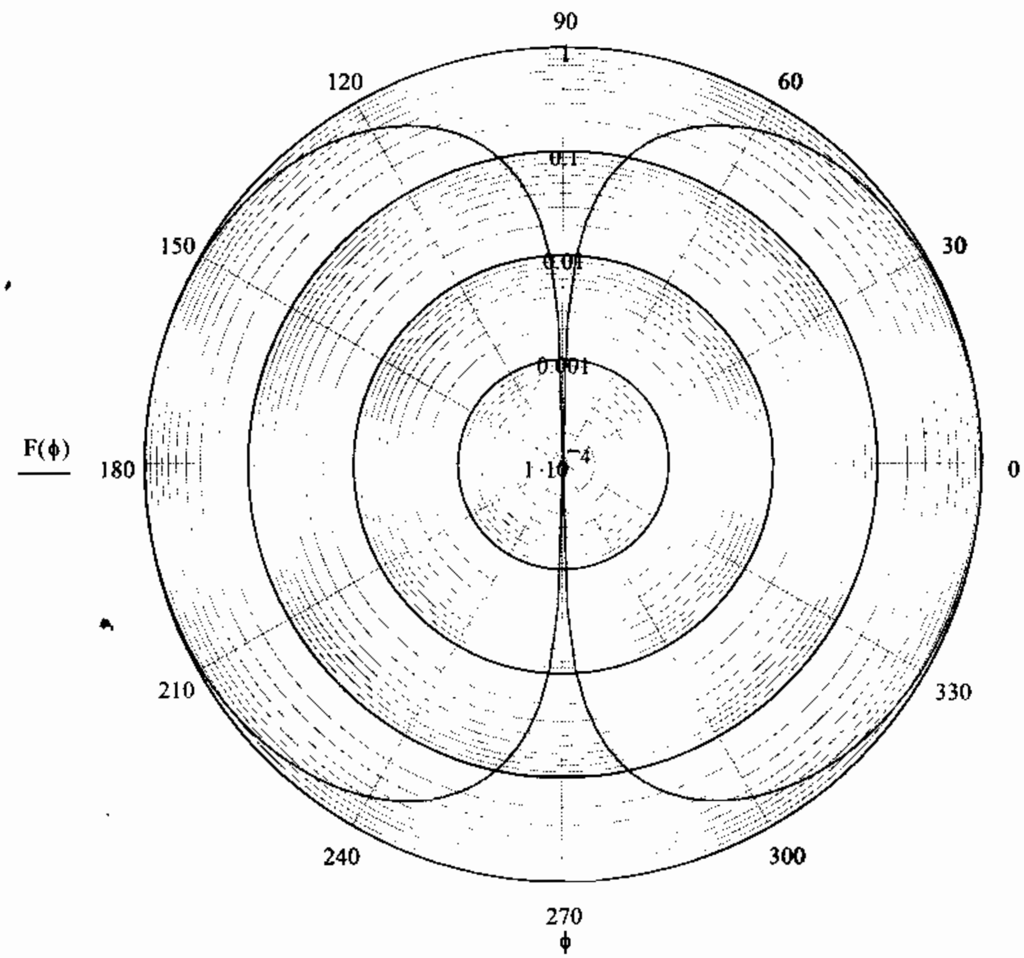
[Plot attached]

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2$; $d := \frac{1}{2}$; $\psi := \pi$; $j := \sqrt{-1}$; $\phi := 0, \frac{\pi}{100} .. 2\cdot\pi$ $n := 0, 1 .. N-1$; $F(\phi) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\phi)-\psi)}\right|\right]^2$

Where "N" is the number of array elements, "ψ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.

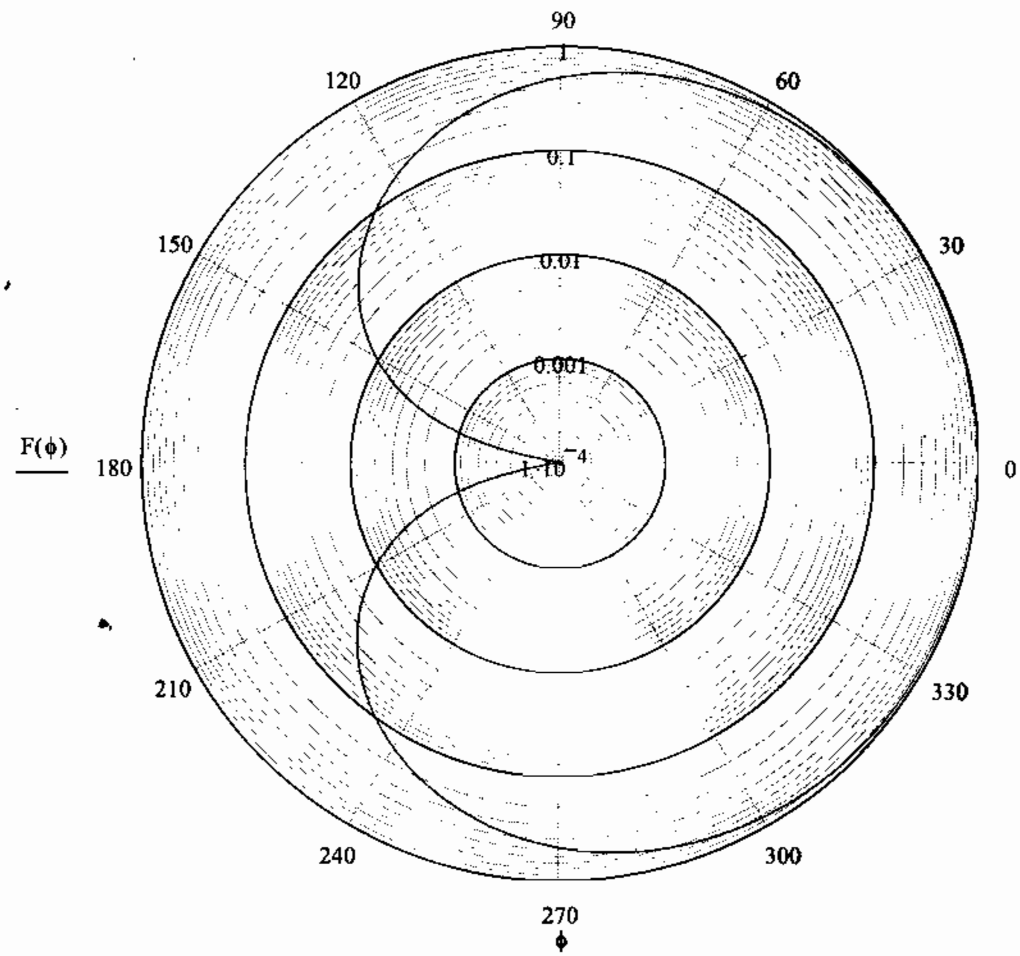


ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$$N := 2; \quad d := \frac{1}{4}; \quad \psi := \frac{\pi}{2}; \qquad j := \sqrt{-1}; \quad \phi := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\phi) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\phi)-\psi)}\right|\right]^2$$

Where "N" is the number of array elements, "ψ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.

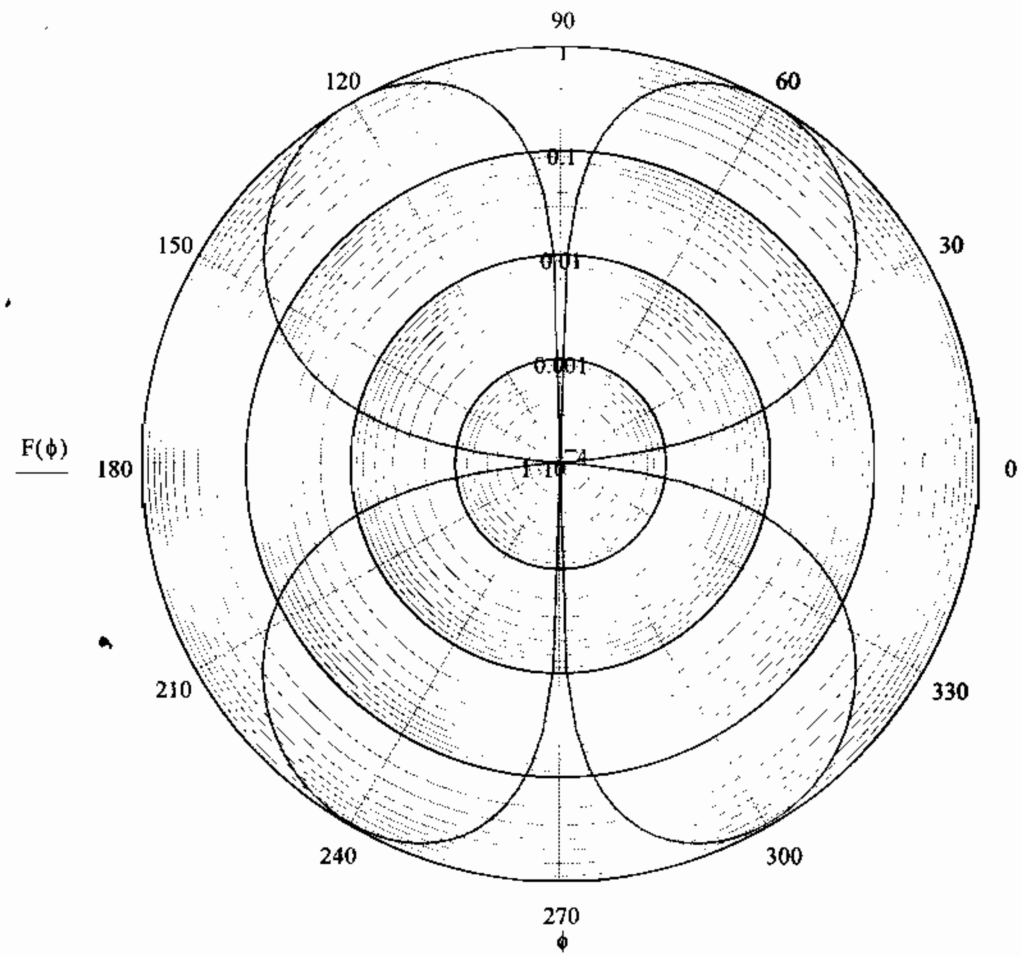


ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2$; $d := 1$; $\psi := \pi$; $j := \sqrt{-1}$; $\phi := 0, \frac{\pi}{100} .. 2\cdot\pi$ $n := 0, 1 .. N-1$; $F(\phi) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\phi)-\psi)}\right|\right]^2$

Where "N" is the number of array elements, "ψ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



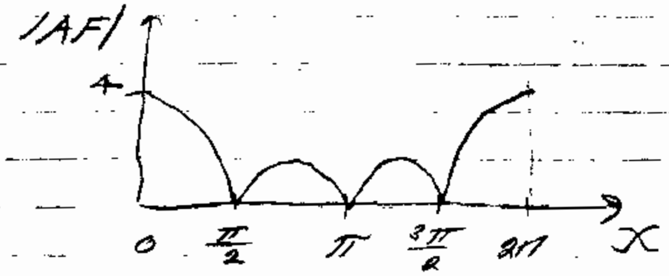
EE 434 Homework 10

9.11 4 element, $\phi_{o_{max}} = 45°$, no 2^{nd} main lobe

$|AF| = \dfrac{\sin\left(\frac{4\chi}{2}\right)}{\sin\left(\frac{\chi}{2}\right)}$ where $\chi = 2\pi \frac{d}{\lambda}\cos\phi - \psi$

maximum @ $\chi = 0$ ∴ $0 = 2\pi\frac{d}{\lambda}\cos 45° - \psi$ or $\boxed{\psi = \sqrt{2}\pi\frac{d}{\lambda}}$

$|AF|$ zero @ $\frac{4\chi}{2} = n\pi$ or $\chi = \frac{n\pi}{2}$



[for no part of 2^{nd} main lobe $|\chi| < \frac{3\pi}{2}$]

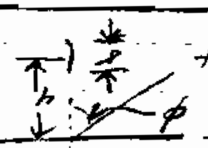
in the visible range $0 \le \phi \le \pi$ $-\beta d - \psi \le \chi \le \beta d - \psi$

∴ $-\beta d - \psi = -\frac{3\pi}{2}$ or $2\pi\frac{d}{\lambda} + \sqrt{2}\pi\frac{d}{\lambda} = \frac{3\pi}{2}$

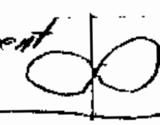
{This gives $\frac{d}{\lambda} = 0.439$ and from above $\psi = 1.95$} ←

see attached plot

9.19 short dipole $\ell = \frac{\lambda}{20}$



$f(\theta,\phi) = f_{element}$



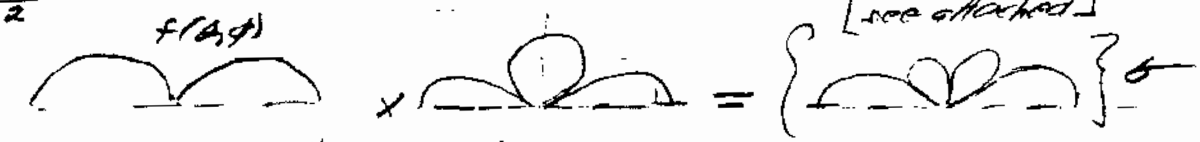
a) $h = \frac{\lambda}{8}$ ∴ this looks like a 2 element array with $d = \lambda, \psi = 0$ [image]

$\boxed{f(\theta\phi) = 0 \text{ @ } \phi = 0}$

$|AF| = \dfrac{\sin\chi}{\sin\frac{\chi}{2}}$; zeros @ $\chi = n\pi = \frac{2\pi}{\lambda}\cdot\lambda\cos\phi$; $\cos\phi = \frac{n}{2} \Rightarrow \boxed{\phi_{zero} = 60°}$

[see attached]

∴ total pattern



b) $h = 2\lambda$ (2 element array with $d = 4\lambda, \psi = 0$), zeros @ $\chi = n\pi = \frac{2\pi}{\lambda}4\lambda\cos\phi$

∴ zeros @ $\cos\phi = \frac{n}{8}$; {$\phi_{zero} = 82.8°, 68°, 51.3°, 28.9°$ plus same points in 2^{nd} quadrant

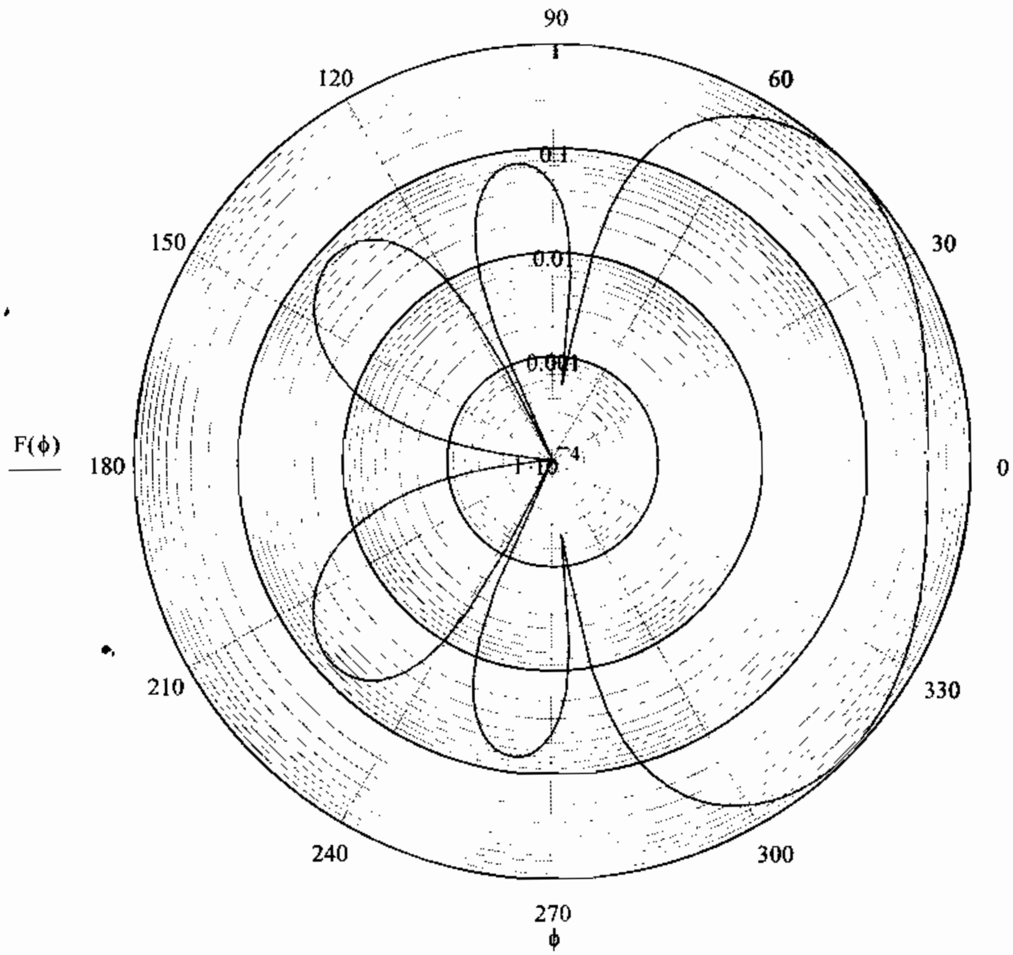
element zero still @ $\phi = 0°$ in plane of paper [plot attached]

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 4;\quad d := 0.439 \qquad \psi := 1.95 \qquad j := \sqrt{-1};\quad \phi := 0, \frac{\pi}{100} .. 2\cdot\pi \qquad n := 0, 1 .. N-1;\quad F(\phi) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\phi)-\psi)}\right|\right]^2$

Where "N" is the number of array elements, "ψ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



9.11

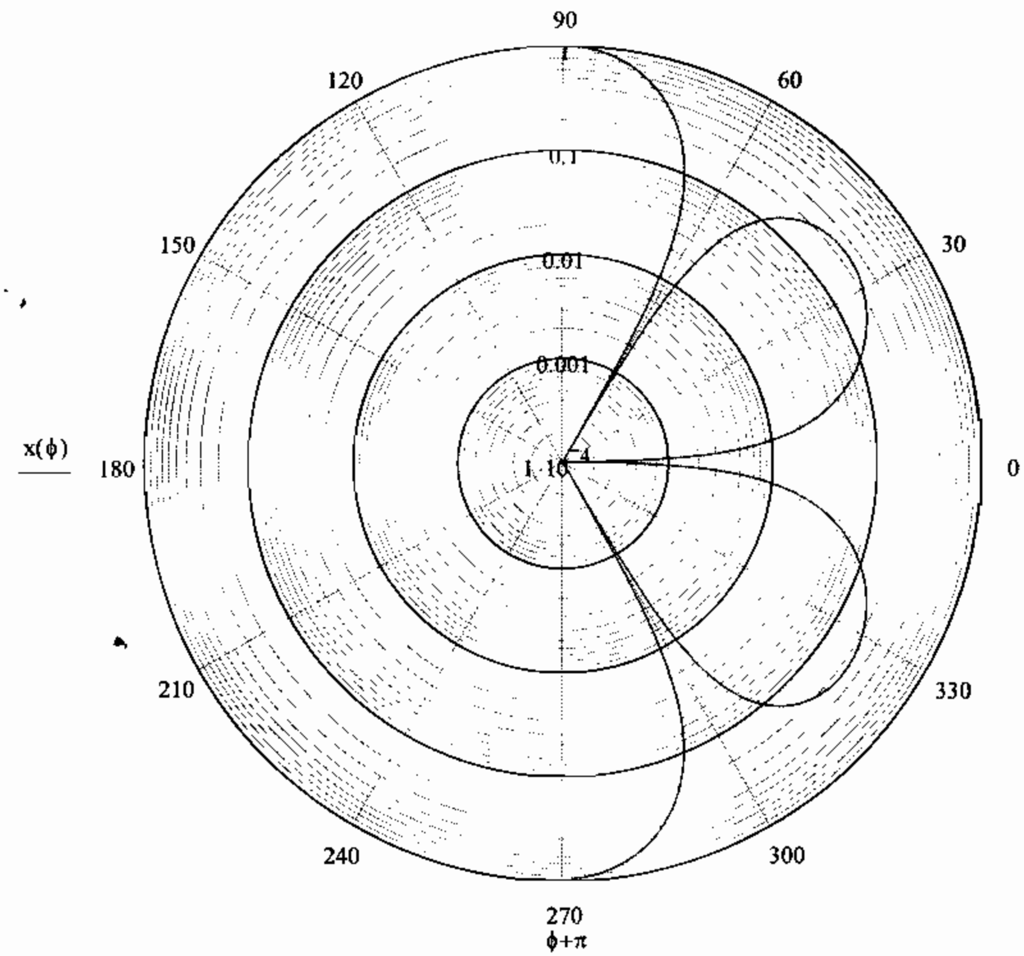
ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2$; $d := 1$; $\psi := 0$ $j := \sqrt{-1}$ $\phi := \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{1000} .. \frac{\pi \cdot 3}{2}$ $n := 0, 1 .. N - 1$ $F(\phi) := \left[\frac{1}{N} \cdot \left|\sum_n e^{-j \cdot n \cdot (2 \cdot \pi \cdot d \cdot \cos(\phi) - \psi)}\right|\right]^2$

$x(\phi) := F(\phi) \cdot (\sin(\phi))^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



9/19a

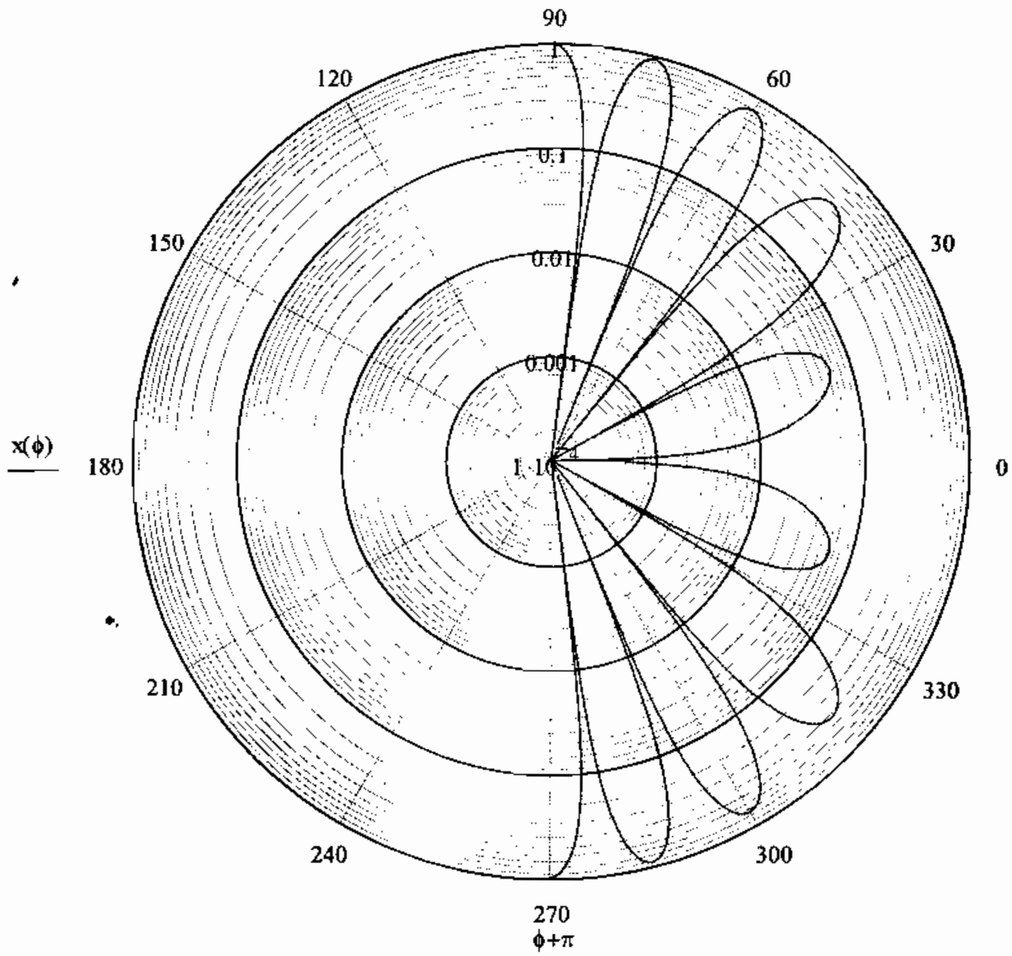
ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2;\quad d := 4;\qquad \psi := 0\quad j := \sqrt{-1}\quad \phi := \frac{\pi}{2}, \frac{\pi}{2} + \frac{\pi}{1000} .. \frac{\pi \cdot 3}{2}\qquad n := 0, 1 .. N - 1\quad F(\phi) := \left[\frac{1}{N} \cdot \left|\sum_{n} e^{-j \cdot n \cdot (2 \cdot \pi \cdot d \cdot \cos(\phi) - \psi)}\right|\right]^2$

$x(\phi) := F(\phi) \cdot (\sin(\phi))^2$

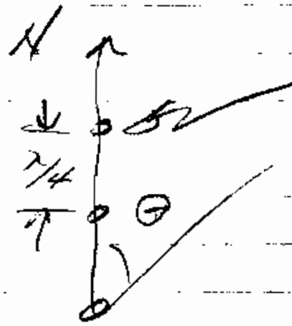
Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



9.19b

EE 434 Homework 11

Broadcast Antenna Array



λ/4 monopoles

→ East (maximum here!)

Zeroes @ $\theta = 26°$ and $154°$

$\boxed{AF = (z - z_1)(z - z_2)}$ where $\boxed{z = e^{jx}}$

$x = \beta d \cos\theta - \psi$ but for broadside array $\psi = 0$

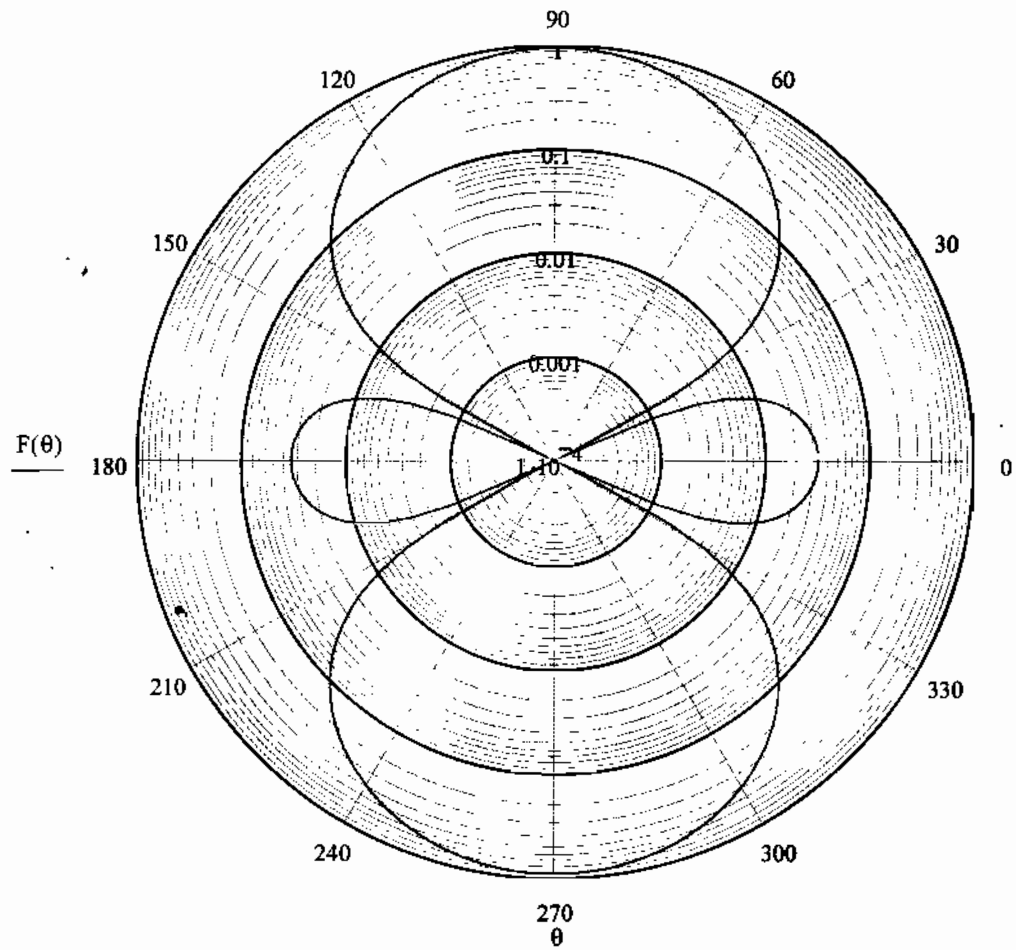
$\therefore\ x = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} \cos\theta = \frac{\pi}{2} \cos\theta$

$\boxed{z_1 = e^{j\frac{\pi}{2}\cos(26°)}}$; $\boxed{z_2 = e^{j\frac{\pi}{2}\cos(154°)}}$

[see attached pattern]

Broadcast Array Special Problem

$j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad z(\theta) := e^{j\cdot\frac{\pi}{2}\cdot\cos(\theta)} \quad z_1 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(26\cdot\frac{\pi}{180}\right)} \quad z_2 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(154\cdot\frac{\pi}{180}\right)} \quad F(\theta) := \left[\left|\frac{1}{\sqrt{3}}\cdot(z(\theta) - z_1)\cdot(z(\theta) - z_2)\right|\right]^2$



Broadcast Antenna – SEE previous analysis with plot of radiation pattern

9.5 2 m dipole, $f = 10^6$ Hz $\quad \lambda = \frac{3\times10^8}{10^6} = 300$ m

copper wire with radius = 1 mm = a $\quad$ very short dipole

$\sigma_{Cu} = 5.8\times10^7 \qquad \delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

a) $\eta = \frac{P_{rad}}{P_{rad}+P_{loss}} = \frac{R_{rad}}{R_{rad}+R_{loss}}$

(for constant current) $R_{rad} = \frac{2\pi\eta}{3}\left(\frac{\text{length}}{\lambda}\right)^2 = \frac{2\pi\, 120\pi}{3}\left(\frac{2}{300}\right)^2 = 0.0351\ \Omega$

$R_{loss} = \frac{\text{length}}{\sigma\, 2\pi a\, \delta} = \frac{l}{2a}\sqrt{\frac{\mu f}{\pi\sigma}} = \frac{2}{2\times10^{-3}}\sqrt{\frac{4\pi\times10^{-7}\times10^6}{\pi\, 5.8\times10^7}} = 0.83\times10$

so $\eta = \frac{0.0351}{0.0351+0.083} = 0.297 = \boxed{29.7\%}$

b) $G = \eta D = 1.5\times0.297 = \boxed{0.445 = -3.5\text{ dB}}$

c) $P_{rad} = 20 = \frac{1}{2} I^2 R_{rad} \quad \therefore I = \sqrt{\frac{40}{0.0351}} = \boxed{33.76\text{A}}$

$P_{transmitter} = \frac{P_{rad}}{0.297} = \frac{20}{0.297} = 67.34$ Watts

9.10

$\Gamma = \frac{73-60}{73+60} = 0.0977$

$\boxed{VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.2}$ ←

9.14 $\lambda/2$ dipole $D=1.64$ 100MHz ; $\lambda = \frac{3\times10^8}{10^8} = 3m$

$\therefore A_{eff} = \frac{\lambda^2}{4\pi}\times1.64 = \frac{9\times1.64}{4\pi} = \boxed{1.17\,m^2}$ ←

projected area of wire $= 10^{-2}\times1.5 = \boxed{0.015\,m^2}$ ←

9.16 1KW @ 50MHz $P_{transmitted}$

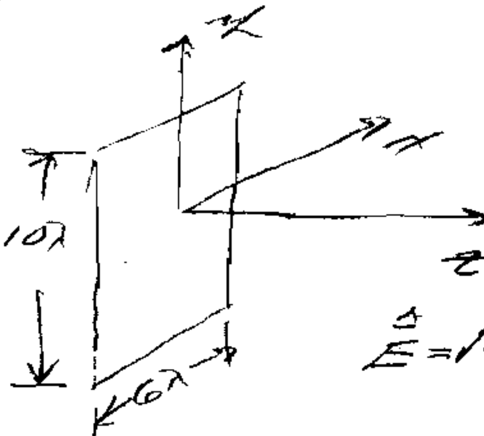
home antenna 13dB Gain = 19.95 power gain

$P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi R}\right)^2 = 10^3\times19.95\times1.64\left(\frac{6}{4\pi\times30\times10^3}\right)^2$

or $\boxed{P_r = 8.29\times10^{-6}\text{ Watts}}$ ←

Homework 12

Rectangular Aperature

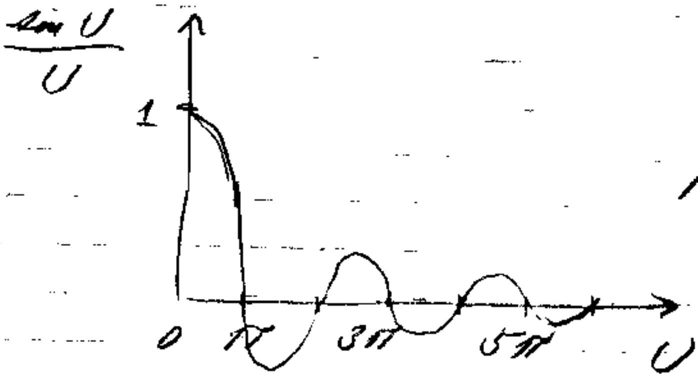


- $\phi = 0$ pattern (xz plane)

$$\hat{E} = j\frac{\beta E_m^+ ab(1+\cos\theta)e^{-j\beta r}}{4\pi r}\cdot\frac{\sin U}{U}\,\bar{a}_\theta$$

where $U = \frac{\beta a}{2}\sin\theta = \frac{2\pi 6\lambda}{\lambda 2}\sin\theta = \boxed{6\pi\sin\theta}$

visible range $0 \le \theta \le 90^\circ$ or $0 \le U \le 6\pi$



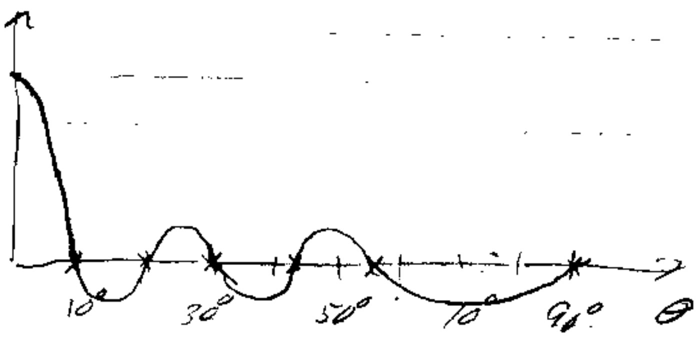
{zeros in pattern for $U = n\pi$}

pattern zeros ⇒

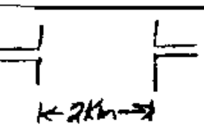
U	θ
π	9.6°
2π	19.5°
3π	30°
4π	42°
5π	56°
6π	90°

$$\boxed{\theta_{zero} = \sin^{-1}\frac{U}{6\pi}}$$

Radiation Pattern $\phi = 0$



9-17



$f = 150\,MHz$ ∴ $\lambda = 2m$; $B = 3\,MHz$; $T_{sys} = 600^\circ K$

$S/N = 20dB$

so $N = kTB = 1.38\times10^{-23}\times 6\times10^{2}\times 3\times10^{6} = 24.84\times10^{-15}$

$$\text{so } P_t = \frac{24.84\times10^{-13}}{1.64^2\left(\frac{\lambda}{4\pi R}\right)^2} = \frac{24.84\times10^{-13}\times16\pi^2\times4\times10^{6}}{1.64^2\times4} = \boxed{1.45\times10^{-4}\text{ watts}}$$

$\lambda/2$ dipoles