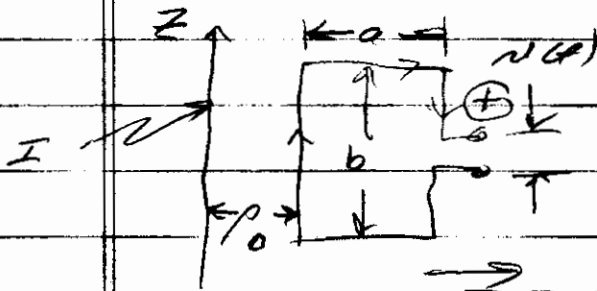


5-21 (Jabuk)

$$\oint \vec{H} \cdot d\vec{l} = I \Rightarrow 2\pi r H_\phi = I$$



$$\therefore B_\phi = \frac{\mu I}{2\pi r}$$

$$\oint \vec{B} \cdot d\vec{s} = \int_{\phi=0}^{2\pi} \int_{z=0}^b \frac{\mu I}{2\pi r} r dz d\phi$$

$$\vec{u} = \rho \vec{a}_\rho$$

$$\phi_m = \frac{\mu I b}{2\pi} \ln\left(\frac{\rho+a}{\rho}\right)$$

but $\rho = \rho_0 + \rho_1 t$ so $\phi_m = \frac{\mu I b}{2\pi} \ln\left(\frac{\rho_0 + \rho_1 t + a}{\rho_0 + \rho_1 t}\right)$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{d\phi_m}{dt} = - \frac{\mu I b}{2\pi} \left(\frac{\rho_0 + \rho_1 t}{\rho_0 + \rho_1 t + a}\right) \left[\frac{\rho_1}{\rho_0 + \rho_1 t} - \frac{(\rho_0 + \rho_1 t)\rho_1}{(\rho_0 + \rho_1 t)^2}\right]$$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{\mu I b}{2\pi} \left[\frac{\rho_1}{\rho_0 + \rho_1 t} - \frac{\rho_1}{\rho_0 + \rho_1 t}\right]$$

$$\oint \vec{E} \cdot d\vec{l} = - \frac{\mu I b \rho_1}{2\pi} \left[\frac{\rho_1 \rho_0 + \rho_1^2 t - \rho_1 \rho_0 - \rho_1^2 t}{(\rho_0 + \rho_1 t)(\rho_0 + \rho_1 t)}\right]$$

$$\oint \vec{E} \cdot d\vec{l} = \frac{\mu I b \rho_1^2 a}{(\rho_0 + \rho_1 t)(\rho_0 + \rho_1 t)}$$

also $\oint \vec{E} \cdot d\vec{l} = \oint \vec{u} \times \vec{B} \cdot d\vec{l} = \int \frac{\mu I}{2\pi(\rho_0 + \rho_1 t)} \vec{a}_\phi \cdot b \vec{a}_z$

$$= \int \frac{\mu I}{2\pi(\rho_0 + \rho_1 t)} \vec{a}_\phi \cdot b \vec{a}_z$$

$$\oint \vec{E} \cdot d\vec{l} = \frac{\mu I b \rho_1}{2\pi} \left[\frac{1}{\rho_0 + \rho_1 t} - \frac{1}{\rho_0 + \rho_1 t}\right] \text{ check!}$$

(Top terminal positive) $\vec{u} \times \vec{B}$ on inner path is up forcing positive to upper terminal!

$$\begin{aligned}
 P_{ave} &= \frac{1}{2} \overline{P_{ave}} \{ \vec{E} \times \vec{H} \} = \frac{1}{2} \overline{P_{ave}} \left\{ \overline{P_{ave}} \left[|\vec{E}_r| |\vec{H}_d| \sin \theta \cos \theta \left(-\frac{\beta_0}{r^2} + \frac{j}{r^3} \right) \right. \right. \\
 &\quad \left. \left. \times \left(\frac{j\beta_0}{r} + \frac{1}{r^2} \right) + \overline{P_{ave}} \left[|\vec{E}_d| |\vec{H}_d| \sin^2 \theta \left(\frac{2\beta_0}{r} + \frac{\beta_0}{r^2} - \frac{j}{r^3} \right) \left(-\frac{j\beta_0}{r} + \frac{1}{r^2} \right) \right] \right\} \\
 &= \frac{1}{2} \overline{P_{ave}} \left\{ \overline{P_{ave}} |\vec{E}_r| |\vec{H}_d| \sin \theta \cos \theta \left(\frac{j\beta_0}{r^3} - \frac{\beta_0}{r^4} + \frac{\beta_0}{r^4} + j \frac{1}{r^5} \right) \right. \\
 &\quad \left. + \overline{P_{ave}} |\vec{E}_d| |\vec{H}_d| \sin^2 \theta \left(\frac{\beta_0}{r^2} + \frac{j\beta_0}{r^3} - \frac{j\beta_0}{r^3} + \frac{\beta_0}{r^4} - \frac{\beta_0}{r^4} - \frac{j}{r^5} \right) \right\}
 \end{aligned}$$

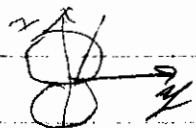
only real term

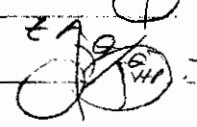
$$\therefore P_{ave} = \frac{1}{2} |\vec{E}_d| |\vec{H}_d| \sin^2 \theta \left(\frac{\beta_0}{r^2} \right) \overline{P_{ave}}$$

from 9.14 & 9.15 $|\vec{E}_d| = \frac{\eta I dl}{4\pi r^2}$, $|\vec{H}_d| = \frac{I dl}{4\pi r^2}$

$$\therefore P_{ave} = \frac{\beta_0 \eta I^2 dl^2}{2 \times 4^2 \pi^2 r^2 \sin^2 \theta} = \frac{\eta_0 (\beta_0 I dl \sin \theta)^2}{2 \times 4\pi r^2} \quad \text{which is equation 9.21}$$

9.3 $\vec{P} = K \frac{\sin \theta \cos \theta}{r^2} \hat{a}_r \quad W/m^2$

a) azimuthal pattern = $K \cos \theta$  $\therefore \cos(\frac{\phi_{HP}}{2}) = \frac{1}{2}$
 $\phi_{HP} = 60^\circ$; $\phi_{HP} = 120^\circ$

equation pattern = $K \sin \theta$  $\frac{1}{2} \cos \theta = \sin^{-1}(\frac{1}{2})$; $\theta_{0V} = 120^\circ$

b) ϕ_{0V} to first null = 180°
 θ_{0V} to first null = 180°

c) $P_{rad} = K \int_{\phi=-\pi/2}^{\pi/2} \int_{\theta=0}^{\pi} \frac{\sin \theta \cos \theta}{r^2} r^2 \sin \theta d\theta d\phi = K \int_{-\pi/2}^{\pi/2} \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_{\theta=0}^{\pi}$

$P_{rad} = 2K \times \frac{\pi}{2} = K\pi$; $\therefore D_{CSF} = \frac{K \sin \theta \cos \theta \times 4\pi r^2}{r^2 \times \pi} = 4 \sin \theta \cos \theta$

and Directivity = $D_{CSF, max} = 4$

(handout)

9.5 2 m linear antenna @ 1 MHz $\therefore \lambda = \frac{3 \times 10^8}{10^6} = 3 \times 10^2$
 (short dipole)! for Cu $\sigma = 5.8 \times 10^7$

so $\delta = \frac{1}{\sqrt{\pi f \mu_0 \sigma}} = 0.066 \times 10^{-3}$ wire radius is 2mm so current is not uniform across wire

$$a) R_{loss} \approx \frac{l}{\sigma \pi a \delta} = \frac{l}{2a} \sqrt{\frac{\mu_0}{\pi \sigma}} = 0.83 \times 10^{-1} \Omega$$

$$R_{rad} = \frac{2\pi^2}{3} \left(\frac{l}{\lambda}\right)^2 = \frac{2\pi^2 \times 100 \pi}{3} \left(\frac{2}{300}\right)^2 = 0.0351 \Omega$$

$$\eta_{eff} = \frac{R_{rad}}{R_{rad} + R_{loss}} = \frac{0.0351}{0.0351 + 0.083} = 0.297 = 29.7\%$$

$$b) G = \eta_{eff} D = 1.5 \times 0.297 = 0.445 = 3.5 \text{ dB}$$

$$c) P_{rad} = \frac{1}{2} I^2 R_{rad} = \frac{1}{2} I^2 \times 0.0351 = 20 \text{ Watts}$$

$$\text{so } I = \sqrt{\frac{2 \times 20}{0.0351}} = 33.76 \text{ A}$$

$$P_{transmitter} = \frac{P_{rad}}{\eta_{eff}} = \frac{20}{0.297} = 67.34 \text{ Watts}$$

9.14 $\lambda/2$ dipole, 100 MHz $\Rightarrow \lambda = \frac{3 \times 10^8}{10^8} = 3 \text{ m}$

$$A_{eff} = \frac{\lambda^2}{4\pi} D = \frac{9 \times 1.64}{4\pi} = 1.17 \text{ m}^2$$

$$\text{projected area} = 1.5 \times 10^{-2} = 0.015 \text{ m}^2$$

9.16 $\lambda/2$ dipole, 50 MHz $\Rightarrow \lambda = \frac{3 \times 10^8}{5 \times 10^7} = 6 \text{ m}$

$$G = 13.48 = 19.95 \text{ power ratio}$$

$$\text{so } P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi r}\right)^2 = 10 \times 19.95 \times 1.64 \left(\frac{6}{4\pi \times 30 \times 10^3}\right)^2$$

$$P_r = 8.29 \times 10^{-6} \text{ Watts}$$

Q.18 $\left(\xleftarrow{2 \times 10^4} \text{20km} \xrightarrow{\quad} \right)$

$$G_t = 20 \text{ dB} = 100$$

$$G_r = 23 \text{ dB} = 199.5$$

$$P_t = 10 \text{ W}, f = 6 \text{ GHz} \quad \tau = \frac{3 \times 10^8}{6 \times 10^9} = 0.05 \text{ m}$$

$$a) \quad Q_t = \frac{P_t}{4\pi r^2} \quad \text{or} \quad P_t = \frac{P_r A_t}{4\pi r^2}$$

$$\therefore P_r = \frac{10 \times 100}{4\pi \times 4 \times 10^8} = \frac{10^3}{16\pi \times 10^8} = 0.0199 \times 10^{-5} \text{ watts/m}$$

$$b) \quad P_r = P_t A_r = 1.99 \times 10^{-7} \times \frac{\lambda^2}{4\pi} = 1.99 \times 10^{-7} \times 199.5 \times \frac{25 \times 10^{-6}}{4\pi}$$

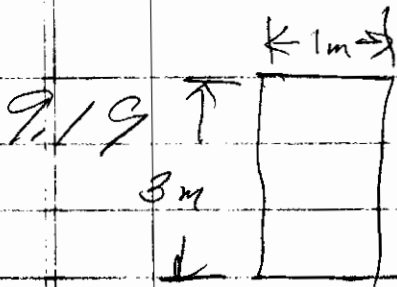
$$P_r = 789.59 \times 10^{-11} = 7.896 \times 10^{-9} \text{ Watts}$$

$$\text{ck } P_r = P_t G_r G_t \left(\frac{\lambda}{4\pi r} \right)^2 = 10 \times 199.5 \times 100 \left(\frac{0.05 \times 10^{-2}}{4\pi \times 2 \times 10^4} \right)^2$$

$$= 7.896 \times 10^{-9}$$

$$P_r = P_t A_r = P_t G_t \left(\frac{\lambda}{4\pi r} \right)^2 \left(\frac{\lambda}{4\pi} \right)^2 G_r$$

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r} \right)^2$$



$f = 10 \text{ GHz}$ so $\lambda = \frac{3 \times 10^8}{10^{10}} = 3 \times 10^{-2} \text{ m}$

a) $\theta_{HP \text{ vertical}} \approx \frac{3 \times 10^{-2}}{3} = 10^{-2} = 0.57^\circ$

$\theta_{HP \text{ horizontal}} \approx \frac{3 \times 10^{-2}}{1} = 3 \times 10^{-2} = 1.72^\circ$

b) $D = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi \times 3}{9 \times 10^{-4}} = 4.19 \times 10^4 = 62.2 \text{ dB}$

9.20 $\theta_{HP} = 1.5^\circ$, $f = 20 \text{ GHz}$ so $\lambda = \frac{3 \times 10^8}{2 \times 10^{10}} = 1.5 \times 10^{-2} \text{ m}$

$\theta_{HP} = 0.02618 \text{ radians}$

a) $D = \frac{4\pi}{\lambda^2} A_{eff}$ and $\theta_{HP} \approx \frac{\lambda}{\text{dia}}$ so $\text{dia} = \frac{\lambda}{\theta_{HP}} = 0.57 \text{ m}$

$A_{eff} = A_{actual} = \frac{\pi \text{dia}^2}{4} = 0.2578 \text{ m}^2$

and $D = \frac{4\pi \times 0.2578}{2.25 \times 10^{-4}} = 1.44 \times 10^4 = 41.58 \text{ dB}$

b) $D = K A_{eff}$ so $D_{new} \Rightarrow 2 D_{old}$

$\theta_{HP} \approx \frac{\lambda}{\text{dia}}$ so $\theta_{HP \text{ new}} \Rightarrow \frac{\theta_{HP \text{ old}}}{\sqrt{2}}$

c) $f \Rightarrow 2f$ so $\lambda_{new} = \frac{\lambda_{old}}{2}$

so $\theta_{HP \text{ new}} = \frac{1}{2} \theta_{HP \text{ old}}$; $D_{new} = 4 D_{old}$

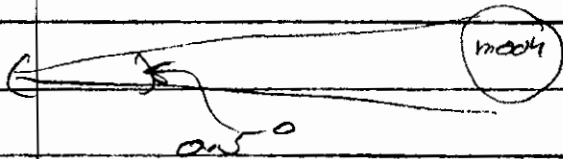
9.21 94 GHz $\lambda = \frac{3 \times 10^8}{94 \times 10^9} = 0.0319 \times 10^{-1} \text{ m}$

a) $\theta_{HP \text{ azimuthal}} = \frac{\lambda}{1} = 3.19 \times 10^{-3} \text{ radians} = 0.18^\circ$

$\theta_{HP \text{ elevation}} = \frac{\lambda}{0.1} = 3.19 \times 10^{-2} \text{ radians} = 1.8^\circ$

d) 300 m $\alpha = 300 \times 3.19 \times 10^{-3} = 0.957 \text{ m}$

9.22 100m parabolic dish, $f = 10 \text{ GHz}$



$$\theta_{\text{HP antenna}} \approx \frac{\lambda}{\text{dia}} = \frac{3 \times 10^8}{10^{10} \times 10^2} = 3 \times 10^{-4} \text{ radians}$$

distance to moon is $2.389 \times 10^5 \text{ miles} = 3.844 \times 10^5 \text{ km}$

$$\theta_{\text{HP antenna}} = 3 \times 10^{-4} \times \frac{180}{\pi} = 17.188 \times 10^{-4} \text{ degrees}$$

$$\frac{\text{diameter of area illuminated by antenna}}{\text{dia of moon}} = \frac{\theta_{\text{HP ant}} \times \text{distance}}{0.5 \times \frac{\pi}{180} \times \text{distance}}$$

$$= \frac{0.017188}{0.5} = 3.438 \times 10^{-2}$$

area goes as dia squared

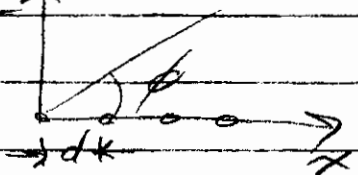
so ratio of area illuminated to total area is:

$$(3.438 \times 10^{-2})^2 = 1.18 \times 10^{-3} = 0.118 \times 10^{-2}$$

(0.118% of moon is illuminated) ←

from text

9.9 $\lambda/2$



a) $d = \frac{\lambda}{2}$, broadside so $\psi = 0$

$\chi = \beta d \cos \phi$

$$|AF| = \sum_{n=0}^3 e^{jn\beta d \cos \phi} = \sum_{n=0}^3 e^{jn \frac{2\pi d}{\lambda} \cos \phi}$$

[see attached plot]

b) $d = \lambda/4$, $\psi = \frac{\pi}{2}$

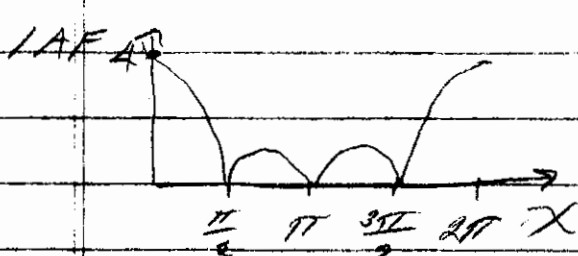
$$|AF| = \sum_{n=0}^3 e^{jn \left(\frac{\beta \pi d}{\lambda} \cos \phi - \frac{\pi}{2} \right)} = \sum_{n=0}^3 e^{jn \left(\frac{\pi}{2} \cos \phi - \frac{\pi}{2} \right)}$$

[see attached plot]

9.11 4 element uniform array, maximum @ $\phi = 45^\circ$
 (no part of 2nd main lobe present)

$$|AF| = \frac{\sin \frac{4\chi}{2}}{\sin \frac{\chi}{2}} \quad \chi = \frac{2\pi d}{\lambda} \cos \phi - \psi$$

maximum when $\chi = 0$ so $\psi = \frac{2\pi d}{\lambda} \cos 45^\circ = \frac{2\pi d}{\lambda} \frac{1}{\sqrt{2}} = \sqrt{2} \frac{\pi d}{\lambda}$



for $N=4$, zero of χ
 @ $\chi = \frac{4\pi}{4} = \pi$

for no part of 2nd main lobe in pattern

$$\beta d + \psi \leq \frac{3\pi}{2}, \quad \beta d \leq \frac{3\pi}{2} - \sqrt{2} \frac{\pi d}{\lambda}$$

$$\frac{2\pi d}{\lambda} + \sqrt{2} \frac{\pi d}{\lambda} \leq \frac{3\pi}{2} \quad \text{or} \quad \frac{d}{\lambda} \leq \frac{3/2}{2 + \sqrt{2}} = \frac{3}{4 + 2\sqrt{2}} = 0.439$$

so $\psi = \sqrt{2} \frac{\pi d}{\lambda} = 1.9516$ [see attached plot]

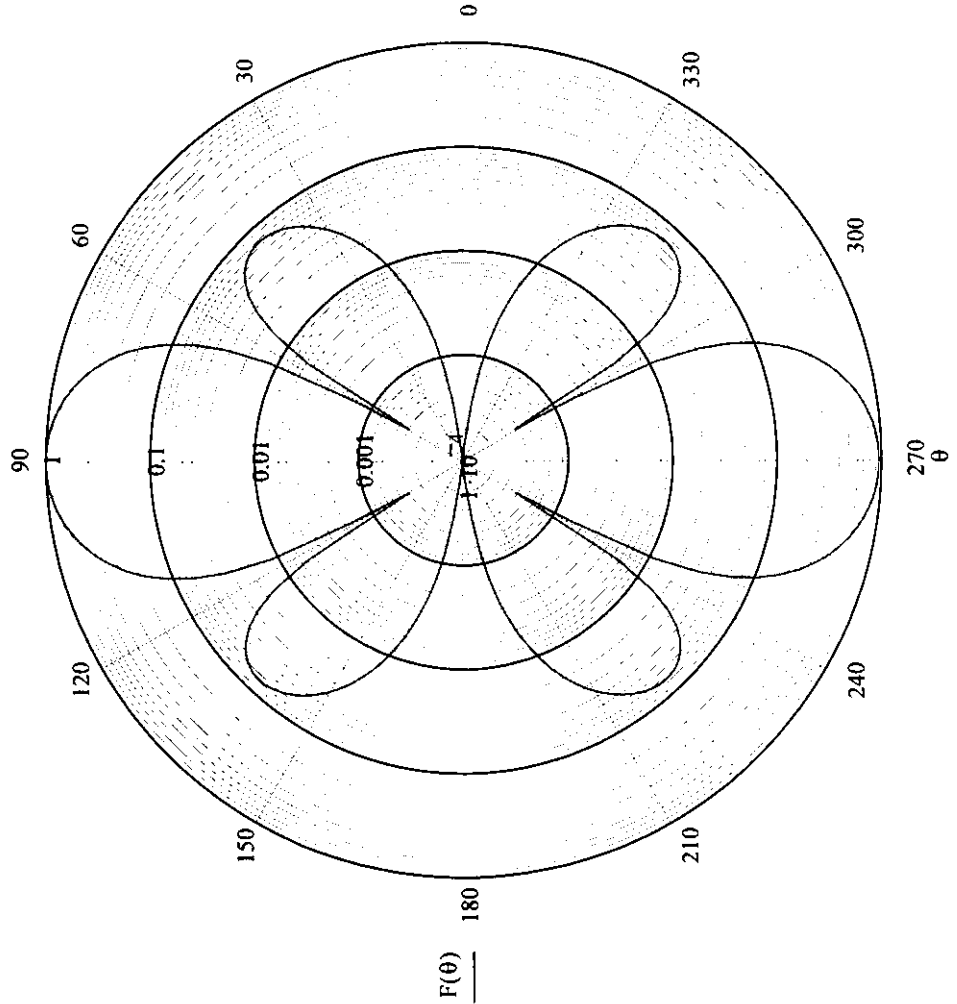
9.9a)

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$$N := 4 \quad d := \frac{1}{2} ; \quad \phi := 0 ; \quad j := \sqrt{-1} ; \quad \theta := 0, \frac{\pi}{100} .. 2 \cdot \pi \quad n := 0, 1 .. N - 1 ; \quad F(\theta) := \left[\frac{1}{N} \cdot \left| \sum_n e^{-j \cdot n \cdot (2 \cdot \pi \cdot d \cdot \cos(\theta) - \phi)} \right| \right]^2$$

Where "N" is the number of array elements, "φ" the progressive phase shift (element to element), and "d" the element spacing in wavelengths.



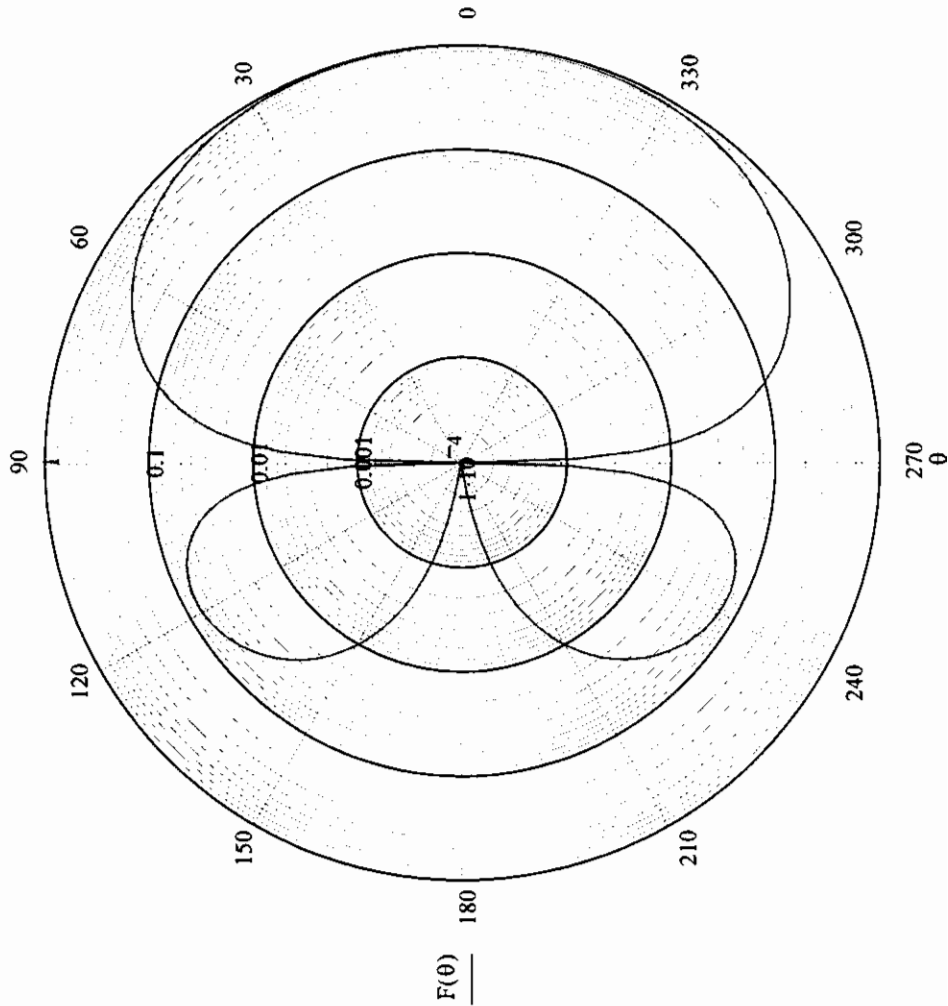
9,9 b)

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$$N := 4 \quad d := \frac{1}{4} ; \quad \phi := \frac{\pi}{2} ; \quad j := \sqrt{-1} ; \quad \theta := 0, \frac{\pi}{100} \dots 2 \cdot \pi \quad n := 0, 1 \dots N - 1 ; \quad F(\theta) := \left[\frac{1}{N} \cdot \sum_n e^{-j \cdot n \cdot (2 \cdot \pi \cdot d \cdot \cos(\theta) - \phi)} \right]^2$$

Where "N" is the number of array elements, "φ" the progressive phase shift (element to element), and "d" the element spacing in wavelengths.



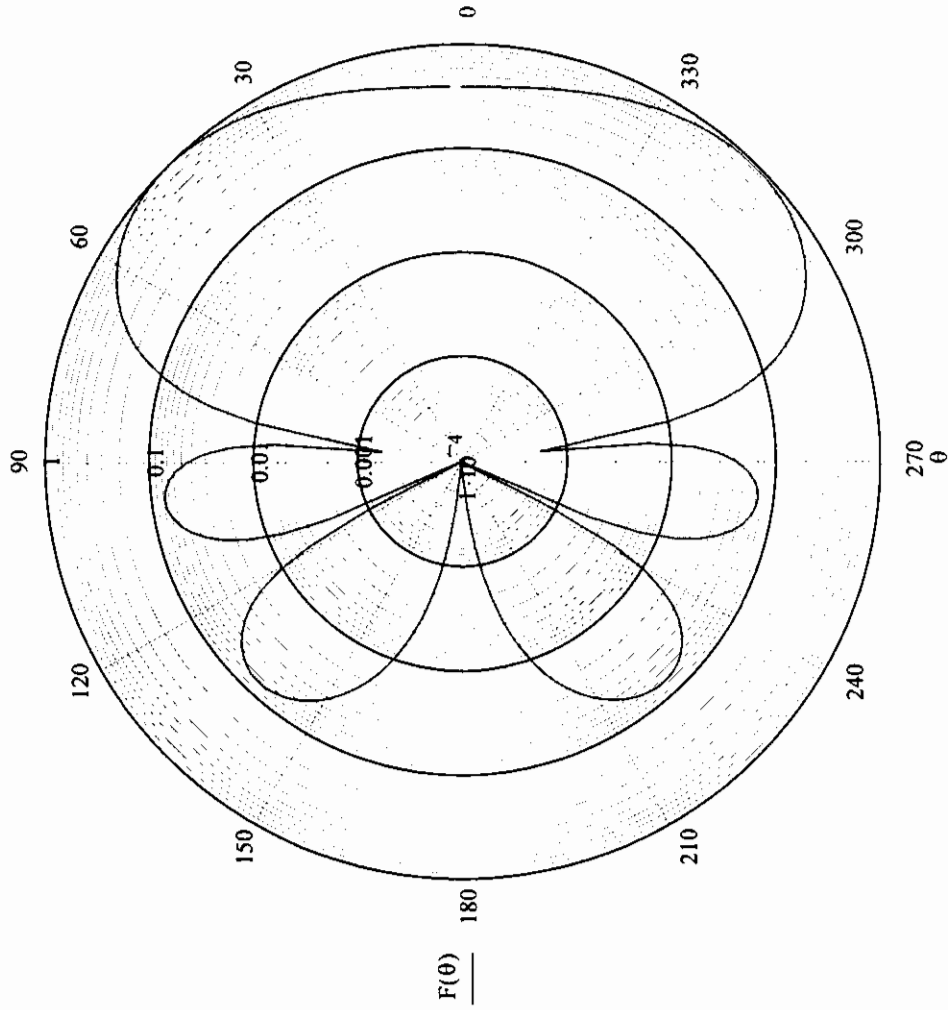
9.11

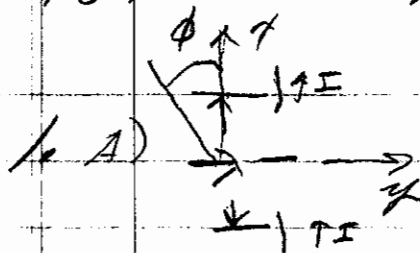
ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$$N := 4 \quad d := 0.439 \quad \phi := 1.9516 \quad j := \sqrt{-1} \quad \theta := 0, \frac{\pi}{100} \dots 2\pi \quad n := 0, 1 \dots N - 1; \quad F(\theta) := \left[\frac{1}{N} \cdot \sum_n e^{-j \cdot n \cdot (2 \cdot \pi \cdot d \cdot \cos(\theta) - \phi)} \right]^2$$

Where "N" is the number of array elements, " ϕ " the progressive phase shift (element to element), and "d" the element spacing in wavelengths.





This is a 2 element array with $\psi = 0, d = \lambda$

ψ s. AF zeros @ $X = n\pi = 2\pi \cos \phi \Rightarrow \cos \phi = \frac{n}{2}$

or zeros @ $\phi = \cos^{-1} \frac{1}{2} = 60^\circ$

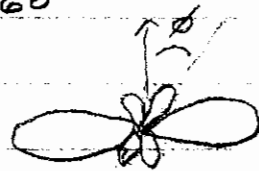
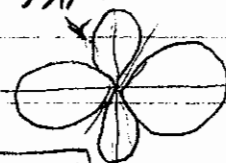
factor

$|AF|$

total pattern



x



zeros @

$$\phi = \pm 60^\circ, 0^\circ, 180^\circ, \pm 120^\circ$$

B) $\psi = 0, d = 4\lambda$ zeros @ $X = n\pi = \frac{2\pi \times 4\lambda \cos \phi}{\lambda}$

or zeros @ $\cos \phi = \frac{n}{4}$

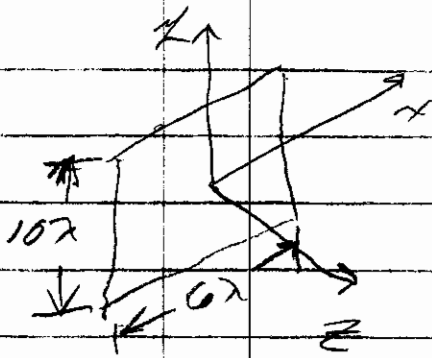
since $X = n\pi$ only n odd produces zeros

i.e. $X = 0, 2\pi, 4\pi \dots$ are maxima

\therefore zeros @ $\phi = \pm 82.8^\circ, \pm 67.9^\circ, \pm 51.32^\circ, \pm 28.95^\circ$

plus $7/2$ dipole zeros @ $\phi = 0^\circ$

There will also be zeros that are symmetrical to the above ... reflected about the $y-z$ plane



$\phi = 0$ pattern (i.e. xz plane)

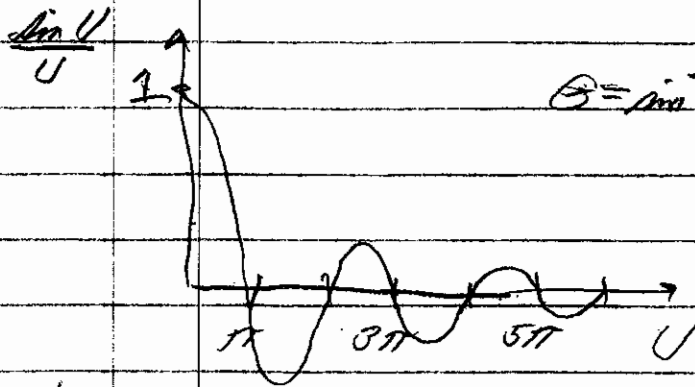
$$\vec{E} = \frac{j\beta E_m^+ ab (1 + \cos\theta) e^{-j\beta r}}{4\pi r} \frac{\sin(\frac{\beta a}{2} \sin\theta)}{\frac{\beta a}{2} \sin\theta} \frac{\sin(\frac{\beta b}{2} \sin\theta)}{\frac{\beta b}{2} \sin\theta}$$

i.e. $U = \frac{\beta a}{2} \sin\theta$

(aperture is 6λ wide in this plane)

so $\frac{\beta a}{2} = \frac{2\pi a}{\lambda} = \frac{2\pi \cdot 6\lambda}{\lambda} = 6\pi$ (which is the visible range) of U

→ i.e. $0 < U < 6\pi$ for $0 < \theta < \frac{\pi}{2}$

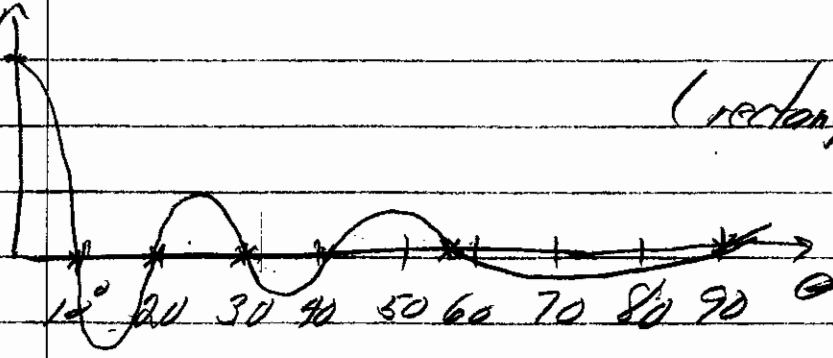


zeros of $\frac{\sin U}{U}$

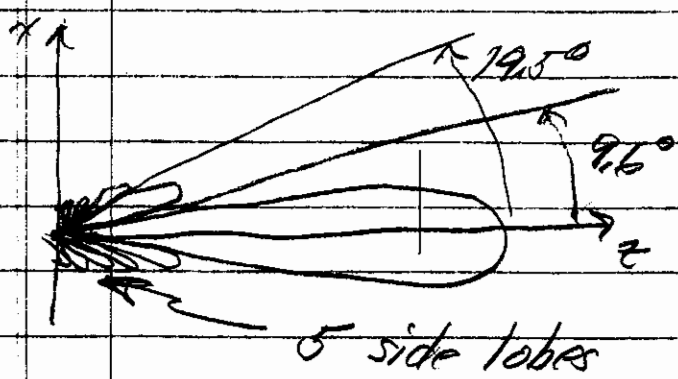
$$\theta = \sin^{-1}\left(\frac{U}{6\pi}\right)$$

U	θ
π	9.59°
2π	19.5°
3π	30.0°
4π	41.8°
5π	56.4°
6π	90°

Radiation pattern ($\phi = 0$)



(rectangular plot)

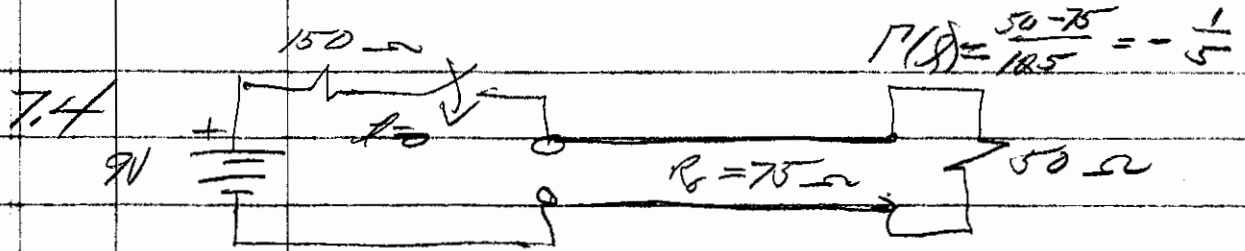


from table in hand out

$$\theta \approx 50^\circ = 9.33^\circ$$

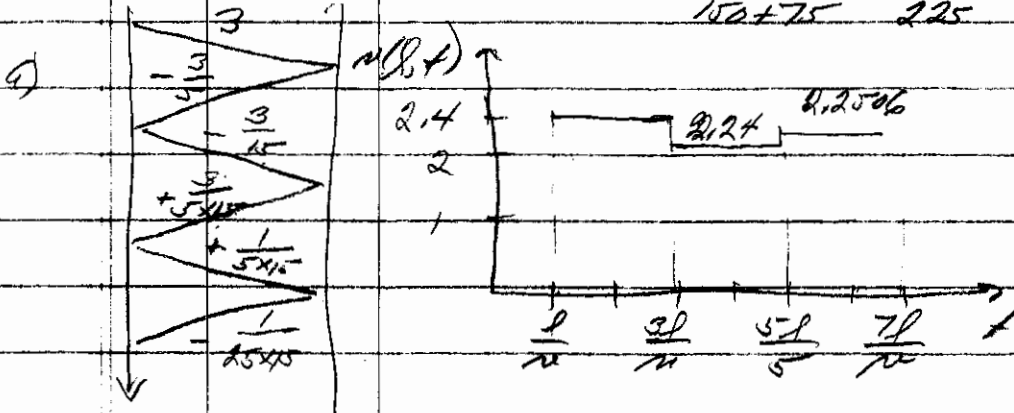
HP

first side lobe is down 13.2 dB

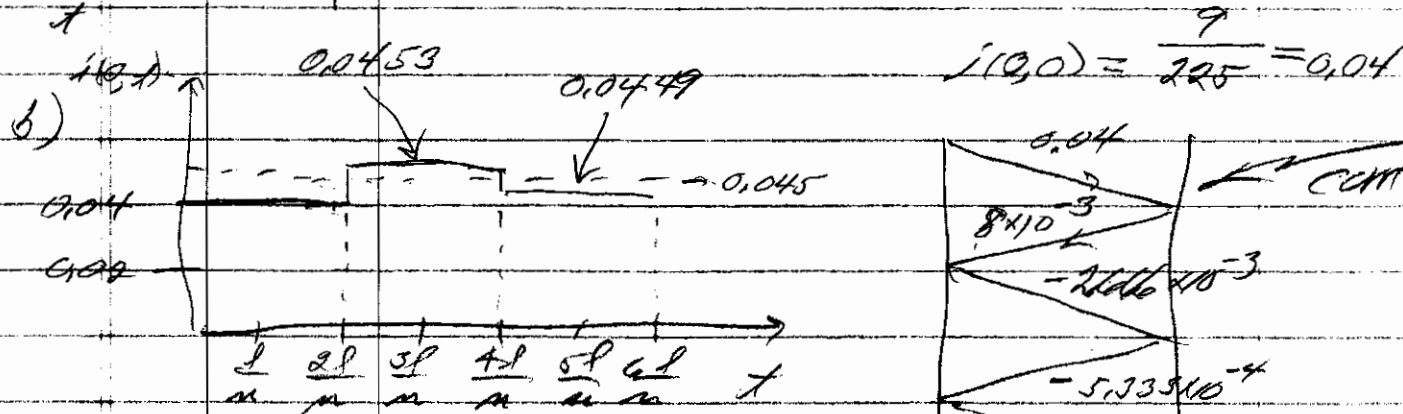


$$r(t) = \frac{50 - 75}{125} = -\frac{1}{5}$$

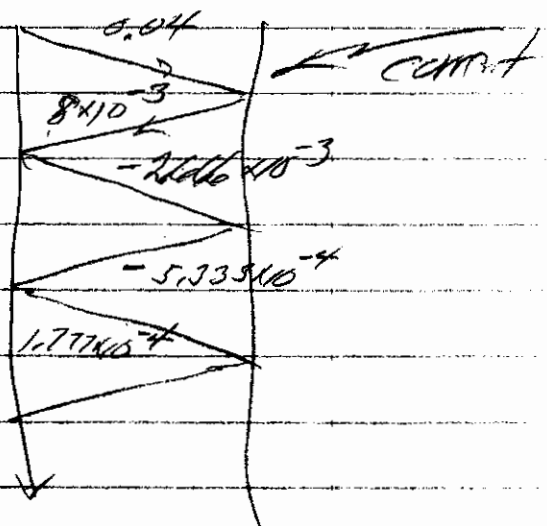
$$r(t) = \frac{150 - 75}{150 + 75} = \frac{75}{225} = \frac{1}{3}$$



$$n(t)_{\text{final}} = 9 \frac{50}{200} = 2.25$$

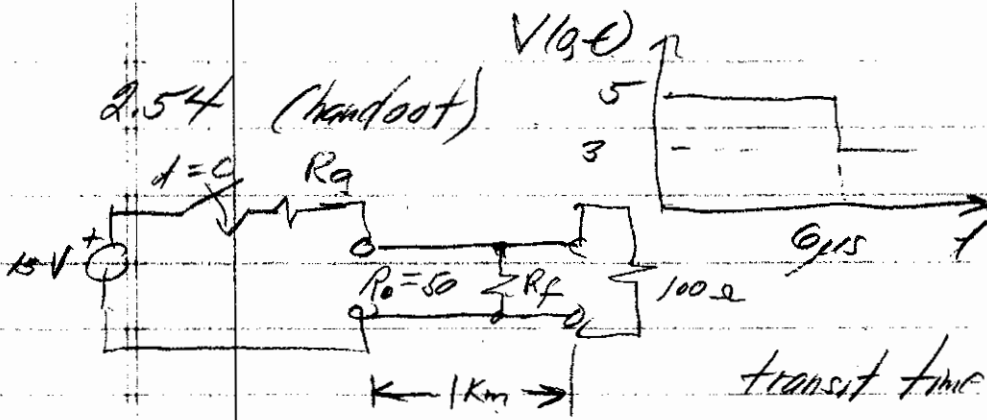


$$i(0,0) = \frac{9}{225} = 0.04$$



$$i(t)_{\text{final}} = \frac{9}{200} = 0.045$$

c) $n_{\text{final}} = 2.25$
 $i_{\text{final}} = 0.045$ } ← see above



$$\text{transit time} = \frac{l}{v} = \frac{10^3}{10^8} = 10^{-5} = 10 \mu\text{sec}$$

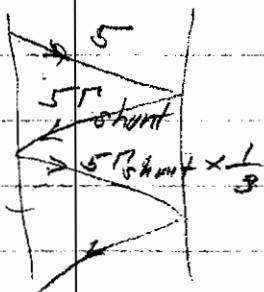
$$v = 10^8 \text{ m/sec}$$

Since the transit is 10 μsec the reflection reducing the voltage by 2 volts is due to a discontinuity on the line before the load resistor

a) $15 \frac{50}{50 + R_g} = 5$; $15 \times 50 = 250 + 5R_g$ or $R_g = \frac{750 - 250}{5} = 100 \Omega$
 or $\Gamma(0) = \frac{100 - 50}{150} = \frac{1}{3}$

b) wave reflected from load would not be seen for two transit times (20 μsec)

c) Resistance at short is $\frac{R_L 50}{50 + R_L} = R_{eq}$



To obtain above waveform $+5(1 + \Gamma_s + \frac{\Gamma_s}{3}) = 3$
 $5(1 + \frac{\Gamma_s}{3}) = \frac{3}{5} - 1$; $\Gamma_s = \frac{-2/5}{4/3} = -\frac{3}{10}$

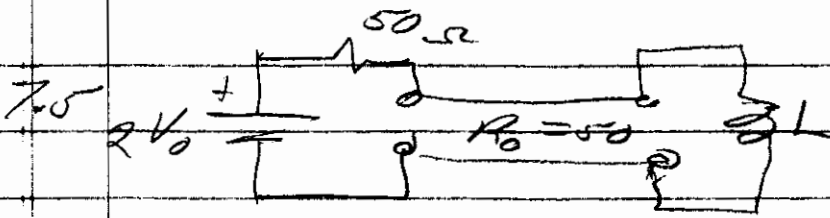
let $\Gamma_s = \frac{R_{eq} - 50}{R_{eq} + 50} = -\frac{3}{10}$; $-\frac{3}{10}(R_{eq} + 50) = R_{eq} - 50$

$$R_{eq}(1 + \frac{3}{10}) = 50 - 15$$
; $R_{eq} = \frac{35}{13/10} = \frac{350}{13} \approx 26.92 \Omega$

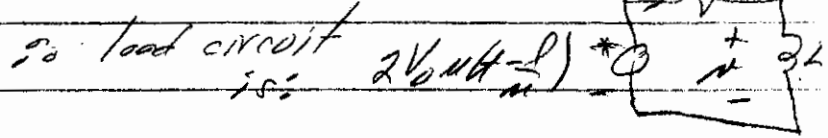
$$50 R_L = R_{eq}(50 + R_L)$$
; $R_L(50 - R_{eq}) = 50 R_{eq}$

$$R_L = \frac{50 R_{eq}}{50 - R_{eq}} = 58.33 \Omega$$

Homework 8



$i_{load}^+ = V_0 \left(1 - \frac{1}{n}\right)$



so $2V_0 - iR_0 - L \frac{di}{dt} = 0$ where $t' = t - \frac{l}{u}$

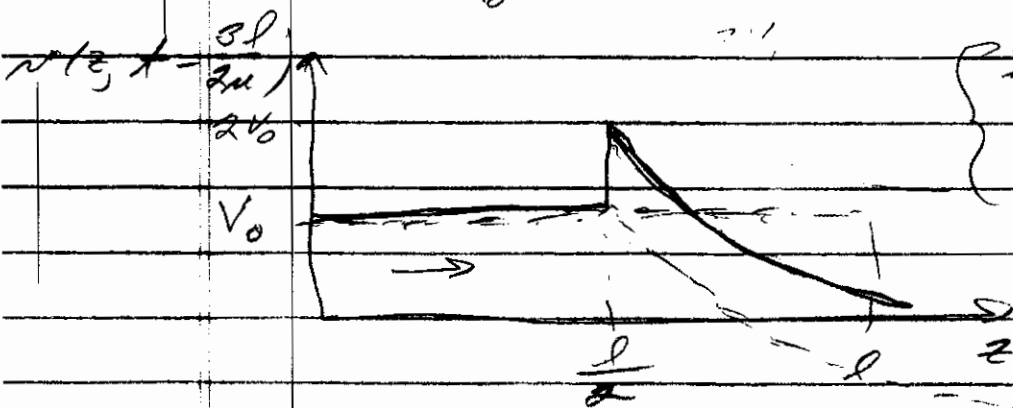
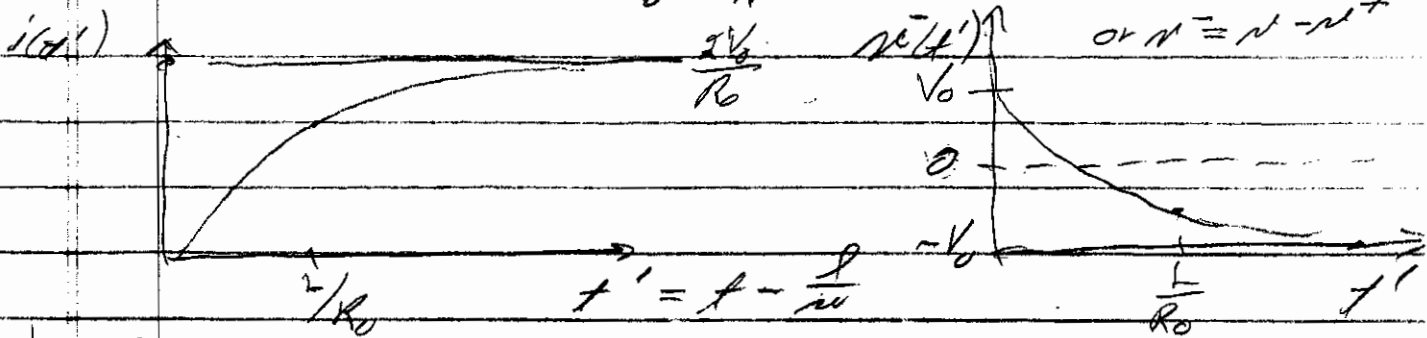
$i = K_0 e^{st}$ so $R_0 K_0 e^{st} + L s K_0 e^{st} = 0$
 $s = -\frac{R_0}{L}$

particular $= K' \Rightarrow 2V_0 - K'R_0 = 0$ or $K' = \frac{2V_0}{R_0}$

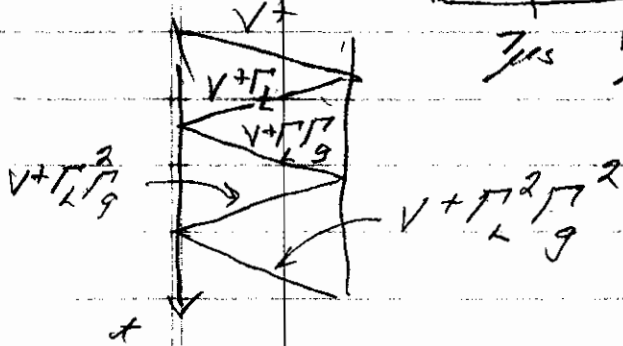
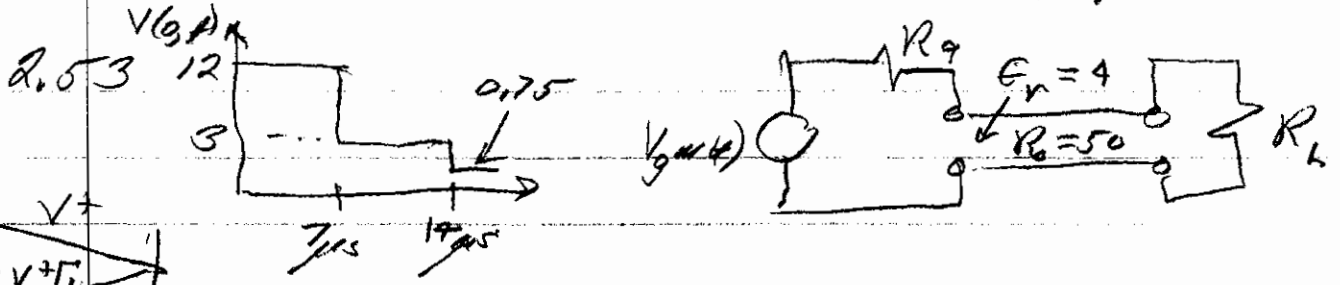
$i = i_p + i_h = \frac{2V_0}{R_0} + K e^{-\frac{R_0 t'}{L}}$ but $i(0) = 0$

so $i = \frac{2V_0}{R_0} \left(1 - e^{-\frac{R_0 t'}{L}}\right) u(t')$

and $v = L \frac{di}{dt} = \frac{2V_0}{R_0} \times \frac{R_0}{L} \left(1 - e^{-\frac{R_0 t'}{L}}\right) = v^+ + v^-$
 or $v^- = v - v^+$



Initially inductor looks like an "open" ... +1 reflection
 finally inductor looks like a short



$$R_g = 50 \frac{1 + \Gamma_g}{1 - \Gamma_g} \quad \left\{ \begin{array}{l} \Gamma_L = \frac{R_L - 50}{R_L + 50} \\ \Gamma_g = \frac{R_g - 50}{R_g + 50} \end{array} \right.$$

$$12 = V_g \frac{50}{50 + R_g} = V^+ \quad \leftarrow$$

$$\begin{array}{l} 1) \quad V^+ (\Gamma_L + \Gamma_L \Gamma_g) = -9 \\ 2) \quad V^+ (\Gamma_L^2 \Gamma_g + \Gamma_L^2 \Gamma_g^2) = -\frac{9}{4} \end{array} \quad \left\{ \begin{array}{l} \Gamma_L (1 + \Gamma_g) = -\frac{9}{12} = -\frac{3}{4} \\ \Gamma_L^3 \Gamma_g (1 + \Gamma_g) = -\frac{3 \cdot 9}{4 \cdot 12} = -\frac{3}{16} \end{array} \right.$$

$$\text{from 1) } \Gamma_L = -\frac{9}{4} \cdot \frac{1}{1 + \Gamma_g} \Rightarrow \frac{9}{16} \cdot \frac{1}{(1 + \Gamma_g)^2} \cdot \Gamma_g (1 + \Gamma_g) = -\frac{3}{16}$$

$$\text{or } \frac{\Gamma_g}{1 + \Gamma_g} = -\frac{1}{3} \quad ; \quad -\frac{1}{3} (1 + \Gamma_g) = \Gamma_g$$

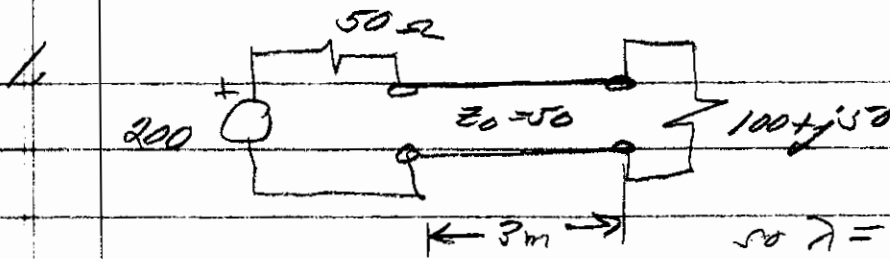
$$\Gamma_g (1 + \frac{1}{3}) = -\frac{1}{3} \quad ; \quad \Gamma_g = -\frac{\frac{1}{3}}{\frac{4}{3}} = -\frac{1}{4} \quad \leftarrow$$

$$R_g = 50 \frac{1 + \Gamma_g}{1 - \Gamma_g} = 50 \frac{1 - \frac{1}{4}}{1 + \frac{1}{4}} = 50 \frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5} \times 50 = 30 \Omega \quad \leftarrow$$

$$\therefore V_g = 12 \left(\frac{50 + 30}{50} \right) = 19.2 \text{ Volts} \quad \leftarrow$$

$$\text{transit time} = \frac{7}{2} \mu\text{s} \quad \mu = \frac{3 \times 10^8}{2}$$

$$\therefore \text{line length} = \frac{3 \times 10^8}{2} \times \frac{7}{2} \times 10^{-6} = 525 \text{ m} \quad \leftarrow$$



$\epsilon_r = 2.25$
 $f = 5 \times 10^7$

$50 \lambda = \frac{3 \times 10^8}{\sqrt{2.25} \cdot 5 \times 10^7} = 4 \text{ m}$

∴ line length is $\frac{3}{4} \lambda$

a) $\Gamma_{load} = \frac{100 + j50 - 50}{100 + j50 + 50} = \frac{50(1 + j)}{50(3 + j)} = \frac{\sqrt{2} e^{j45^\circ}}{\sqrt{10} e^{j18.43^\circ}} = 0.447 e^{j26.56^\circ}$

$\Gamma_{input} = \Gamma_{load} e^{-j2 \times \frac{3\pi}{4}} = 0.447 e^{-j3\pi} e^{j26.56^\circ} = 0.447 e^{j153.4^\circ} = -0.4 - j0.2$

b) $Z_{in} = Z_0 \frac{1 + \Gamma_{in}}{1 - \Gamma_{in}} = 50 \frac{0.6 - j0.2}{1.4 + j0.2} = 50 \frac{0.78 e^{-j18.43^\circ}}{1.414 e^{j8.13^\circ}} = 28.37 \Omega e^{-j26.58^\circ}$

$50 I_{in} = \frac{200}{50 + Z_{in}} = \frac{200}{50 + 28.013 - j10.018} = \frac{200}{70.725 e^{-j8.115^\circ}} = 2.828 e^{j8.115^\circ}$

$P_{ave, in} = \frac{1}{2} \text{Re} \{ I_{in} \hat{V}_{in} \} = \frac{1}{2} |I_{in}|^2 R_{in} = 80 \text{ Watts}$

$\frac{I_L}{I_{in}} = e^{-j\frac{3\pi}{4}} \frac{1 - \Gamma_{load}}{1 - \Gamma_{in}} = e^{-j\frac{3\pi}{4}} \frac{1 - 0.4 - j0.2}{1 + 0.4 + j0.2}$

$I_{load} = 2.828 e^{j8.115^\circ} \times j \times \frac{0.6 - j0.2}{1.4 + j0.2} = 2.828 e^{j8.115^\circ} \times 0.6324 e^{-j18.4^\circ} = 1.796 e^{-j10.285^\circ}$

$I_{load} = 1.764 e^{-j11.57^\circ}$

$P_{ave, load} = \frac{1}{2} |I_{load}|^2 \times 100 = 80 \text{ Watts}$

2. same problem with $Z_{load} = 50$

a) $\Gamma_{load} = 0 = \Gamma_{in}$

b) $Z_{in} = Z_0 = 50 \quad \angle 0^\circ \quad I_{in} = \frac{250}{100} = 2A$

$P_{avg, in} = \frac{1}{2} 2^2 \times 50 = 100 \text{ WCHS}$

c) $I_L = I_{in} \angle^{+90^\circ} \left\{ \frac{1 - \Gamma_{load}}{1 - \Gamma_{in}} \right\} = 2 \angle^{+90^\circ}$

$P_{avg, load} = \frac{1}{2} \times 2^2 \times 50 = 100 \text{ Watts}$

3. distortionless line $RC = LG$ or $\frac{R}{L} = \frac{G}{C}$

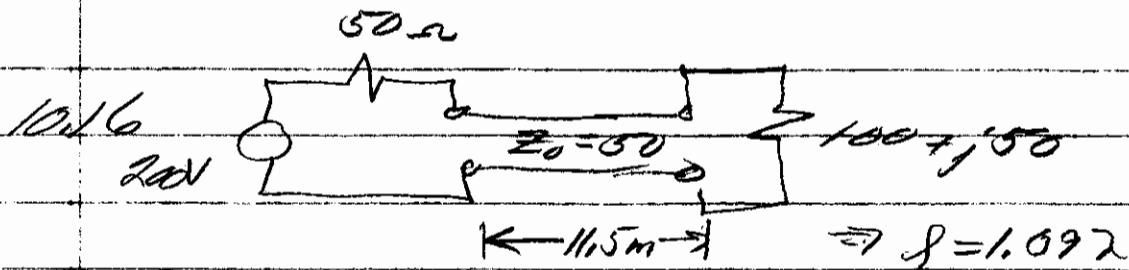
$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{LC} \sqrt{\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)} = \sqrt{LC} \left(\frac{R}{L} + j\omega\right)$

so $\alpha = \frac{R}{L} \sqrt{LC} = R \sqrt{\frac{C}{L}} = R \sqrt{\frac{G}{R}} = \sqrt{RG}$

$\beta = \omega \sqrt{LC}$

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{L}{C}} \sqrt{\frac{\frac{R}{L} + j\omega}{\frac{G}{C} + j\omega}} = \sqrt{\frac{L}{C}}$

EE 434 Homework 10



$$Z_{L, \text{norm}} = 2 + j$$

From chart $\Gamma_{\text{load}} = 0.455 \angle 26.5^\circ$

rotate 1.092 toward generator $[0.2125 + 0.09 = 0.3025]$

$$Z_{in, \text{norm}} = 1.6 - j1.13 \Rightarrow Z_{in} = 80 - j56.5$$

$$\Gamma_{in} = 0.455 \angle -38^\circ$$

10.17 Same as 10.16 with $\alpha = 0.00197 \text{ Np/m}$; $l = 11.5m$

$$e^{-2\alpha l} = e^{-2 \times 1.97 \times 10^{-3} \times 11.5} = 0.956$$

@ input $|\Gamma| = 0.956 \times 0.455 = 0.435$

rotation not changed

from chart $Z_{in, \text{norm}} = 1.59 - j1.03$; $Z_{in} = 79.5 - j51.5$

10.21 $Z_L = 36 + j20$; $\therefore Z_{L, \text{norm}} = 0.72 + j0.4$

From chart $Y_{L, \text{norm}} = 1.06 - j0.58$

$$\frac{1}{Z_{L, \text{norm}}} = \frac{1}{0.72 + j0.4} = \frac{1}{0.82365 \angle 29.05^\circ}$$

$$= 1.06 - j0.59$$

QED

$$10.23 \quad Y_{in2} = G_2 + jB_2 \quad ; \quad Y_{in3} = G_3 + jB_3 \quad \vec{I} = \vec{V} Y$$

$$P_{ave} = \frac{1}{2} \text{Re} \{ \vec{V} \vec{I}^* \} = \frac{1}{2} \text{Re} \{ \vec{V} \vec{V}^* Y^* \} = \frac{1}{2} \text{Re} \{ |\vec{V}|^2 (G_2 + jB_2)^* \}$$

$$\begin{aligned} \frac{P_{ave2}}{P_{ave3}} &= \frac{G_2}{G_3} & \left. \begin{aligned} G_2 &= 0,0366 = \frac{1,83}{50} \\ G_3 &= 0,012 \end{aligned} \right\} \leftarrow \end{aligned}$$

$$\frac{1}{2} |\vec{V}|^2 (G_2 + G_3) = 8,5 = \frac{1}{2} |\vec{V}|^2 G_2 \left(1 + \frac{G_3}{G_2} \right) = P_{ave2} \left(1 + \frac{G_3}{G_2} \right)$$

$$\therefore P_{ave2} = \frac{8,5}{1 + \frac{G_3}{G_2}} = \boxed{6,4 \text{ Watts}} \quad \leftarrow$$

$$\text{and } P_{ave3} = 8,5 - 6,4 = \boxed{2,1 \text{ Watts}} \quad \leftarrow$$

IMPEDANCE OR ADMITTANCE COORDINATES

Problem 10.16
and 10.17
Load

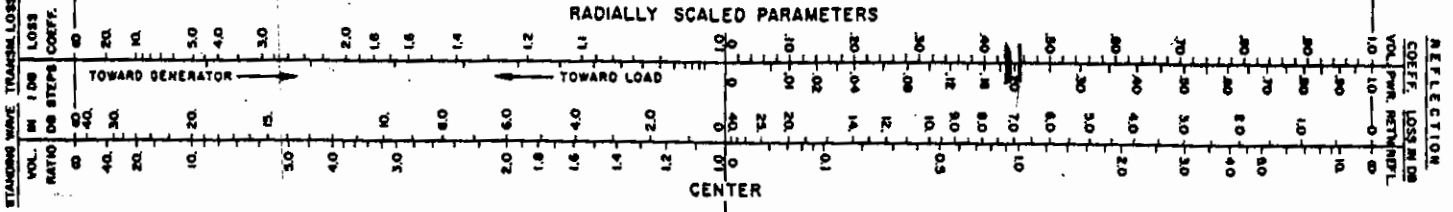
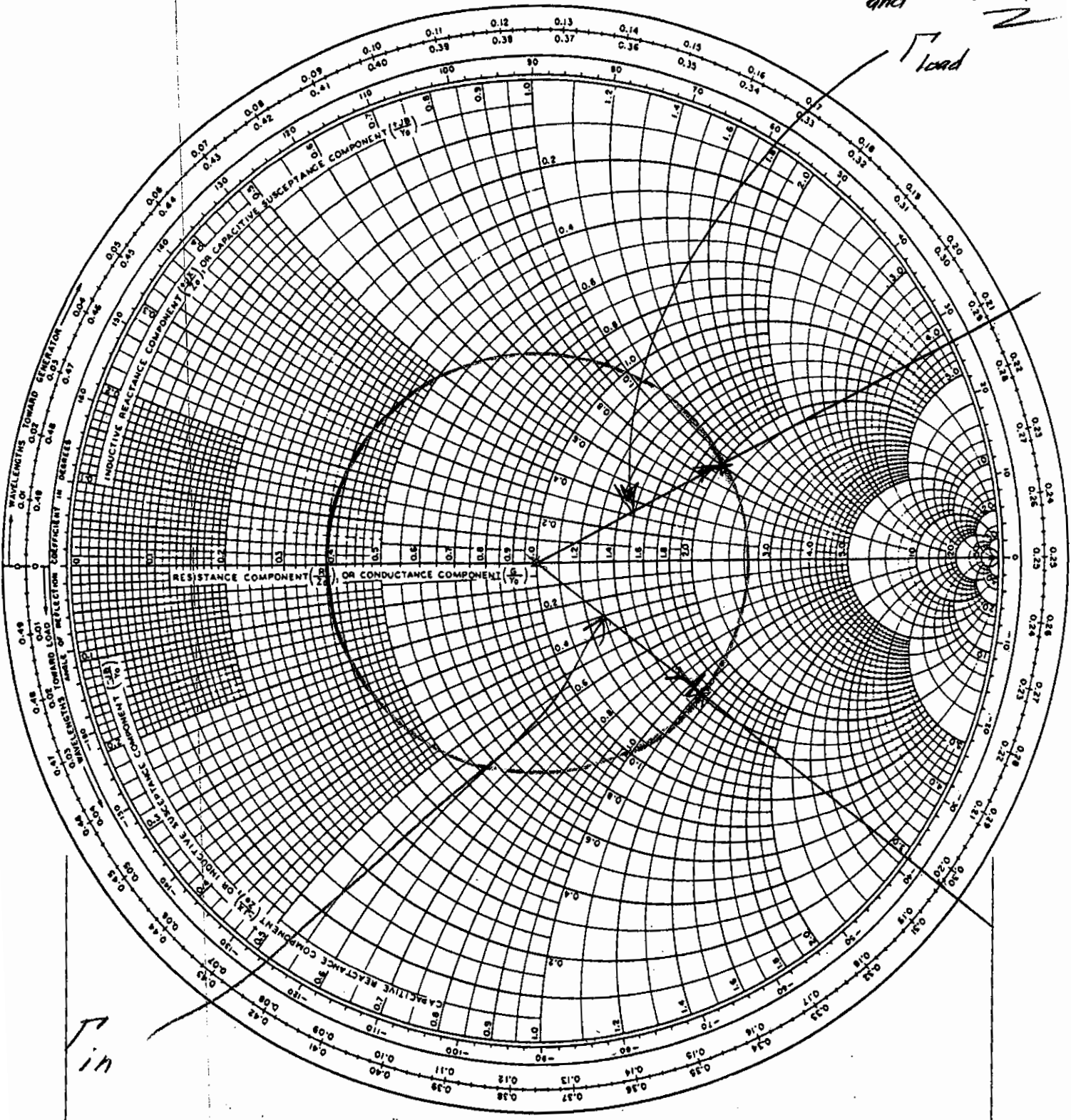


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

