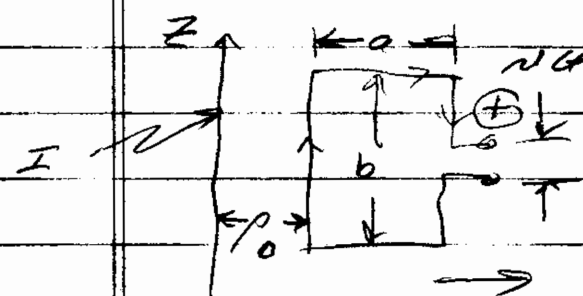
EE 434 Homework 1

5-21 (Johnk)



$\int \overline{H}\cdot\overline{dl} = I \Rightarrow 2\pi\rho H_\phi = I$

$\therefore B_\phi = \frac{\mu I}{2\pi\rho}$

$\therefore \psi_m = \int \overline{B}\cdot\overline{ds} = \int_{\rho=\rho}^{\rho+a}\int_{z=0}^{b} \frac{\mu I}{2\pi\rho}\, dz\, d\rho$

$\overrightarrow{v} = v_\rho \overline{a}_\rho$

$\psi_m = \frac{\mu I b}{2\pi} \ln\left(\frac{\rho+a}{\rho}\right)$

but $\rho = \rho_0 + v_\rho t$ so $\psi_m = \frac{\mu I b}{2\pi}\ln\left(\frac{\rho_0+v_\rho t+a}{\rho_0+v_\rho t}\right)$

$\oint \overline{E}\cdot\overline{dl} = -\frac{d\psi_m}{dt} = -\frac{\mu I b}{2\pi}\left(\frac{\rho_0+v_\rho t}{\rho_0+a+v_\rho t}\right)\left[\frac{v_\rho}{\rho_0+v_\rho t} - \frac{(\rho_0+a+v_\rho t)v_\rho}{(\rho_0+v_\rho t)^2}\right]$

$\oint \overline{E}\cdot\overline{dl} = -\frac{\mu I b}{2\pi}\left[\frac{v_\rho}{\rho_0+a+v_\rho t} - \frac{v_\rho}{\rho_0+v_\rho t}\right]$

$\oint \overline{E}\cdot\overline{dl} = -\frac{\mu I b v_\rho}{2\pi}\left[\frac{\cancel{\rho_0}+\cancel{v_\rho t} - \cancel{\rho_0} - a - \cancel{v_\rho t}}{(\rho_0+a+v_\rho t)(\rho_0+v_\rho t)}\right]$

$$\oint \overline{E}\cdot\overline{dl} = \frac{\mu I b v_\rho^2 a}{(\rho_0+a+v_\rho t)(\rho_0+v_\rho t)}$$

also $\oint \overline{E}\cdot\overline{dl} = \oint \overline{v}\times\overline{B}\cdot\overline{dl} = v_\rho \overline{a}_\rho \times \frac{\mu I\, \overline{a}_\phi}{2\pi(\rho_0+v_\rho t)}\cdot b\overline{a}_z$

$- v_\rho \overline{a}_\rho \times \frac{\mu I\, \overline{a}_\phi}{2\pi(\rho_0+a+v_\rho t)}\cdot b\overline{a}_z$

or $\oint \overline{E}\cdot\overline{dl} = \frac{\mu I b v_\rho}{2\pi}\left[\frac{1}{\rho_0+v_\rho t} - \frac{1}{\rho_0+a+v_\rho t}\right]$ check!

(Top terminal positive!) $\overline{u}\times\overline{B}$ on inner path is up forcing positive to upper terminal!

EE 434 Homework 2

9.2 $\overline{\mathcal{P}}_{ave} = \frac{1}{2}\mathrm{Re}\{\hat{E}\times\hat{H}^*\} = \frac{1}{2}\mathrm{Re}\Big\{\bar{a}_\theta\Big[|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(-j\frac{\beta_o}{r^2}+\frac{1}{r^3}\right)$

$\times\left(j\frac{\beta_o}{r}+\frac{1}{r^2}\right)\Big] + \bar{a}_r\Big[|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(j\frac{\beta_o^2}{r}+\frac{\beta_o}{r^2}-j\frac{1}{r^3}\right)\left(-j\frac{\beta_o}{r}+\frac{1}{r^2}\right)\Big]\Big\}$

$= \frac{1}{2}\mathrm{Re}\Big\{\bar{a}_\theta|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(j\frac{\beta_o^2}{r^3}-\frac{\beta_o}{r^4}+\frac{\beta_o}{r^4}+j\frac{1}{r^5}\right)$

$+\bar{a}_r|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta_o^3}{r^2}+\cancel{j\frac{\beta_o^2}{r^3}}-\cancel{j\frac{\beta_o^2}{r^3}}+\cancel{\frac{\beta_o}{r^4}}-\cancel{\frac{\beta_o}{r^4}}-\frac{j}{r^5}\right)\Big\}$

only real term

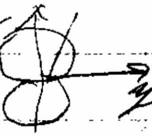
$\therefore\ \mathcal{P}_{ave} = \frac{1}{2}|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta_o^3}{r^2}\right)\bar{a}_r$

from 9.14 & 9.15 $|\hat{E}_\theta| = \frac{\eta_o I dl}{4\pi\beta_o}$, $|\hat{H}_\phi| = \frac{I dl}{4\pi}$

$\therefore \mathcal{P}_{ave} = \frac{\beta_o^{3}\eta_o I^2 dl^2}{2\times 4^2\pi^2\beta_o r^2}\sin^2\theta\,\bar{a}_r = \boxed{\frac{\eta_o}{2}\left(\frac{\beta_o I dl\sin\theta}{4\pi r}\right)^2\bar{a}_r}$ which is equation 9.21

9.3 $\overline{\mathcal{P}} = K\frac{\sin\theta\cos\phi}{r^2}\bar{a}_r$ W/m²

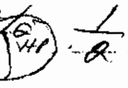
a) azimuthal pattern $= K\cos\phi$



$\therefore \cos\left(\frac{\phi_{HP}}{2}\right) = \frac{1}{2}$

$\frac{\phi_{HP}}{2} = 60°$; $\boxed{\phi_{HP} = 120°}$

elevation pattern $= K\sin\theta$



$\frac{1}{2}\theta_{BW} = 90° - \sin^{-1}\left(\frac{1}{2}\right)$; $\boxed{\theta_{BW} = 120°}$

b) ϕ_{BW} to first null $= 180°$

θ_{BW} to first null $= 180°$

c) $P_{rad} = K\int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}}\int_{\theta=0}^{\pi}\frac{\sin\theta\cos\phi}{r^2}r^2\sin\theta\,d\theta\,d\phi = K\sin\phi\Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left[\frac{\theta}{2}-\frac{\sin 2\theta}{4}\right]_0^{\pi}$

$P_{rad} = 2K\times\frac{\pi}{2} = K\pi$; $\therefore \boxed{D(\theta,\phi) = \frac{K\sin\theta\cos\phi\times 4\pi r^2}{r^2 K\pi} = 4\sin\theta\cos\phi}$

and Directivity $= \boxed{D(\theta,\phi)_{max} = 4}$

EE 434 Homework 3

(handout)

9.5 2 m linear antenna @ 1 MHz $\therefore \lambda = \frac{3\times10^8}{10^6} = 3\times10^2$

(short dipole)! for Cu $\sigma = 5.8\times10^7$

so $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}} = 0.066\times10^{-3}$ wire radius is 2mm so current is not uniform across wire

a) $R_{loss} \cong \frac{\ell}{\sigma 2\pi a \delta} = \frac{\ell}{2a}\sqrt{\frac{\mu f}{\pi \sigma}} = 0.83\times10^{-1}\Omega$

$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{\ell}{\lambda}\right)^2 = \frac{2\pi\times120\pi}{3}\left(\frac{2}{300}\right)^2 = 0.0351\,\Omega$

$\eta_{eff} = \frac{R_{rad}}{R_{rad}+R_{loss}} = \frac{0.0351}{0.0351+0.083} = 0.297 = \boxed{29.7\%}$

b) $G = \eta_{eff} D = 1.5\times0.297 = 0.445 = \boxed{-3.5\text{ dB}}$

c) $P_{rad} = \frac{1}{2}I^2 R_{rad} = \frac{1}{2}I^2\times0.0351 = 20\text{ Watts}$

so $I = \sqrt{\frac{2\times20}{0.0351}} = \boxed{33.76\text{ A}}$

$P_{transmitter} = \frac{P_{rad}}{\eta_{eff}} = \frac{20}{0.297} = 67.34\text{ Watts}$

9.14 $\lambda/2$ dipole; 100 MHz $\Rightarrow \lambda = \frac{3\times10^8}{10^8} = 3\text{ m}$

$A_{eff} = \frac{\lambda^2}{4\pi}D = \frac{9\times1.64}{4\pi} = \boxed{1.17\text{ m}^2}$

projected area $= \boxed{1.5\times10^{-2} = 0.015\text{ m}^2}$

9.16 $\lambda/2$ dipole, 50 MHz $\Rightarrow \lambda = \frac{3\times10^8}{5\times10^7} = 6\text{ m}$

$\boxed{G = 13\text{dB} = 19.95}$ ~~power ratio~~

so $P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi r}\right)^2 = 10^3\times19.95\times1.64\left(\frac{6}{4\pi\times30\times10^3}\right)^2$

$P_r = \boxed{8.29\times10^{-6}\text{ Watts}}$

P.18 $\left(\xleftrightarrow[20\,Km]{2\times10^4} \right)$

$G_t = 20dB = 100$ $\quad G_r = 23dB = 199.5$

$P_t = 10W$, $f = 6GHz$ $\therefore$ $\lambda = \frac{3\times10^8}{6\times10^9} = \boxed{0.05m}$

a) $\mathcal{P}_t = \frac{\cancel{\mathcal{P}_t}}{P_t/4\pi r^2}$ or $\mathcal{P}_t = \frac{P_t D_t}{4\pi r^2}$

$\therefore$ $\mathcal{P}_t = \frac{10\times100}{4\pi\times4\times10^8} = \frac{10^3}{16\pi\times10^8} = \underline{\underline{0.0199\times10^{-5}\ watts/m^2}}$

b) $P_r = \mathcal{P}_t A_r = 1.99\times10^{-7}\times G_r\frac{\lambda^2}{4\pi} = 1.99\times10^{-7}\times199.5\times\frac{25\times10^{-4}}{4\pi}$

$P_r = 789.59\times10^{-11} = \boxed{7.896\times10^{-9}\ Watts}$

ck $P_r = P_t G_r G_t \left(\frac{\lambda}{4\pi r}\right)^2 = 10\times199.5\times100\left(\frac{5\times10^{-2}}{\cancel{4\pi\times2\times10^4}}\right)^2$ $\quad 0.03938\times10^{-12}$

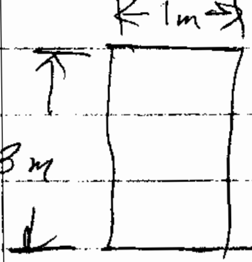
$= 7.896\times10^{-9}$

$P_r = \mathcal{P}_t A_r = P_t G_t\left(\frac{1}{4\pi r^2}\right)\left(\frac{\lambda^2}{4\pi}\right)G_r$

$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2$

EE 434 Homework 4

9.19



$f = 10\ GHz$ so $\lambda = \frac{3\times10^{8}}{10^{10}} = 3\times10^{-2}$ m

a) $\theta_{HP_{vertical}} \cong \frac{3\times10^{-2}}{3} = \boxed{10^{-2} \text{ radians} = 0.57^\circ}$

$\theta_{HP_{horizontal}} \cong \frac{3\times10^{-2}}{1} = \boxed{3\times10^{-2} \text{ rad} = 1.72^\circ}$

b) $D = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi\times 3}{9\times10^{-4}} = \boxed{4.19\times10^{4} = 6.22\ dB}$

9.20 $\theta_{HP} = 1.5^\circ$; $f = 20\ GHz$ so $\lambda = \frac{3\times10^{8}}{2\times10^{10}} = 1.5\times10$

$\theta_{HP} = 0.02618$ radians

a) $D = \frac{4\pi}{\lambda^2} A_{eff}$ and $\theta_{HP} \cong \frac{\lambda}{dia}$ so $dia = \frac{\lambda}{\theta_{HP}} = 0.57\ m$

$A_{eff} = A_{actual} = \frac{\pi\, dia^2}{4} = 0.2578\ m^2$

and $D = \frac{4\pi\times0.2578}{2.25\times10^{-4}} = \boxed{1.44\times10^{4} = 41.58\ dB}$

b) $D = K A_{eff}$ $\therefore$ $\boxed{D_{new} \Rightarrow 2 D_{orig}}$

$\theta_{HP} \cong \frac{\lambda}{dia}$ so $\boxed{\theta_{HP_{new}} \Rightarrow \frac{\theta_{HP_{old}}}{\sqrt{2}}}$

c) $f \Rightarrow 2f$ so $\lambda_{new} = \frac{\lambda_{old}}{2}$

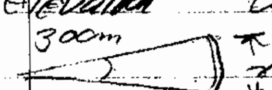
so $\boxed{\theta_{HP_{new}} = \frac{1}{2}\theta_{HP_{old}}}$; $\boxed{D_{new} = 4 D_{old}}$

9.21 94 GHz $\lambda = \frac{3\times10^{8}}{94\times10^{9}} = 0.0319\times10^{-1}$ m

a) $\theta_{HP_{azimuthal}} = \frac{\lambda}{1} = 3.19\times10^{-3}$ radians $\boxed{= 0.18^\circ}$

? $\theta_{HP_{elevation}} = \frac{\lambda}{0.1} = 3.19\times10^{-2}$ radians $\boxed{= 1.8^\circ}$

d)



$x = 300\times3.19\times10^{-3} = 0.957\ m$

EE 434 Homework 4 (page 2)

9.22 100 m parabolic dish, $f = 10\,GHz$

$$\theta_{HP_{antenna}} \cong \frac{\lambda}{dia} = \frac{3\times10^{8}}{10^{10}\times10^{2}} = 3\times10^{-4} \text{ radians}$$

distance to moon is 2.389×10^{5} miles $= 3.844\times10^{5}$ km

$$\theta_{HP_{antenna}} = 3\times10^{-4}\times\frac{180}{\pi} = 171.88\times10^{-4} \text{ degrees}$$

diameter of area illuminated by antenna / dia of moon $= \dfrac{\theta_{HP_{ant}} \cancel{\times \text{distance}}}{0.5\times\frac{\pi}{180}\times \cancel{\text{distance}}}$

$$= \frac{0.017188}{0.5} = 3.438\times10^{-2}$$

area goes as dia squared

so ratio of ~~area~~ illuminated to total area is:

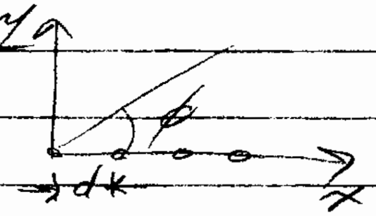
$$(3.438\times10^{-2})^{2} = 1.18\times10^{-3} = 0.118\times10^{-2}$$

$$\boxed{0.118\% \text{ of moon is illuminated}}$$

EE 434 Homework 5

from text

9.9



a) $d = \frac{\lambda}{2}$, Broadside so $\psi = 0$

$\chi = \beta d \cos\phi$

$$|AF| = \sum_{n=0}^{3} e^{jn\beta d\cos\phi} = \sum_{n=0}^{3} e^{jn\frac{2\pi d}{\lambda}\cos\phi}$$

[see attached plot]

b) $d = \lambda/4$, $\psi = \frac{\pi}{2}$

$$|AF| = \sum_{n=0}^{3} e^{jn\left(\frac{2\pi d}{\lambda}\cos\phi - \frac{\pi}{2}\right)} = \sum_{n=0}^{3} e^{jn\left(\frac{\pi}{2}\cos\phi - \frac{\pi}{2}\right)}$$

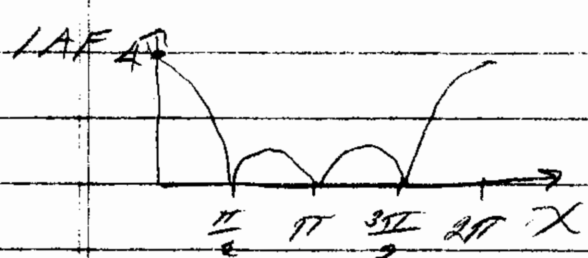
[see attached plot]

9.11 4 element uniform array, maximum @ $\phi = 45^\circ$

(no part of 2nd main lobe present

$$|AF| = \frac{\sin\frac{4\chi}{2}}{\sin\frac{\chi}{2}} \qquad \chi = \frac{2\pi d}{\lambda}\cos\phi - \psi$$

maximum when $\chi = 0$ $\therefore$ $\psi = \frac{2\pi d}{\lambda}\cos 45^\circ = \frac{2\pi d}{\lambda\sqrt{2}} = \sqrt{2}\frac{\pi d}{\lambda}$



for $N=4$, ~~zeros~~ of χ

@ $\chi = \frac{n 2\pi}{4} = n\frac{\pi}{2}$

for no part of 2nd main lobe in pattern

$\beta d + \psi \le \frac{3\pi}{2}$, $\beta d \le \frac{3\pi}{2} - \sqrt{2}\frac{\pi d}{\lambda}$

$\frac{2\pi d}{\lambda} + \sqrt{2}\pi\frac{d}{\lambda} \le \frac{3\pi}{2}$ or $\frac{d}{\lambda} \le \frac{3\frac{\pi}{2}}{2\pi + \sqrt{2}\pi} = \frac{3}{4+2\sqrt{2}} = \boxed{0.439}$

so $\psi = \sqrt{2}\pi\frac{d}{\lambda} = 1.9516$ ← [see attached plot]

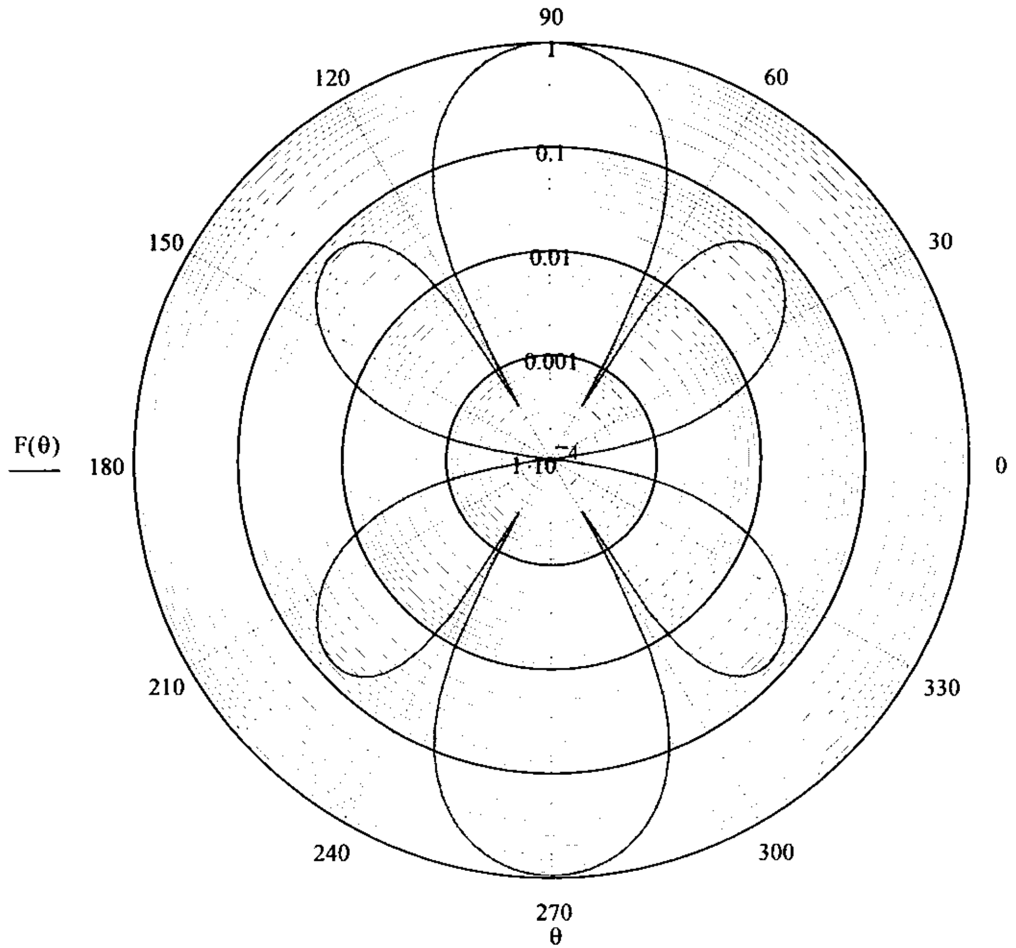
9.9a)

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 4 \quad d := \frac{1}{2}; \quad \phi := 0 \;; \qquad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



9,9 b)

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 4 \quad d := \frac{1}{4}; \quad \phi := \frac{\pi}{2}; \qquad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

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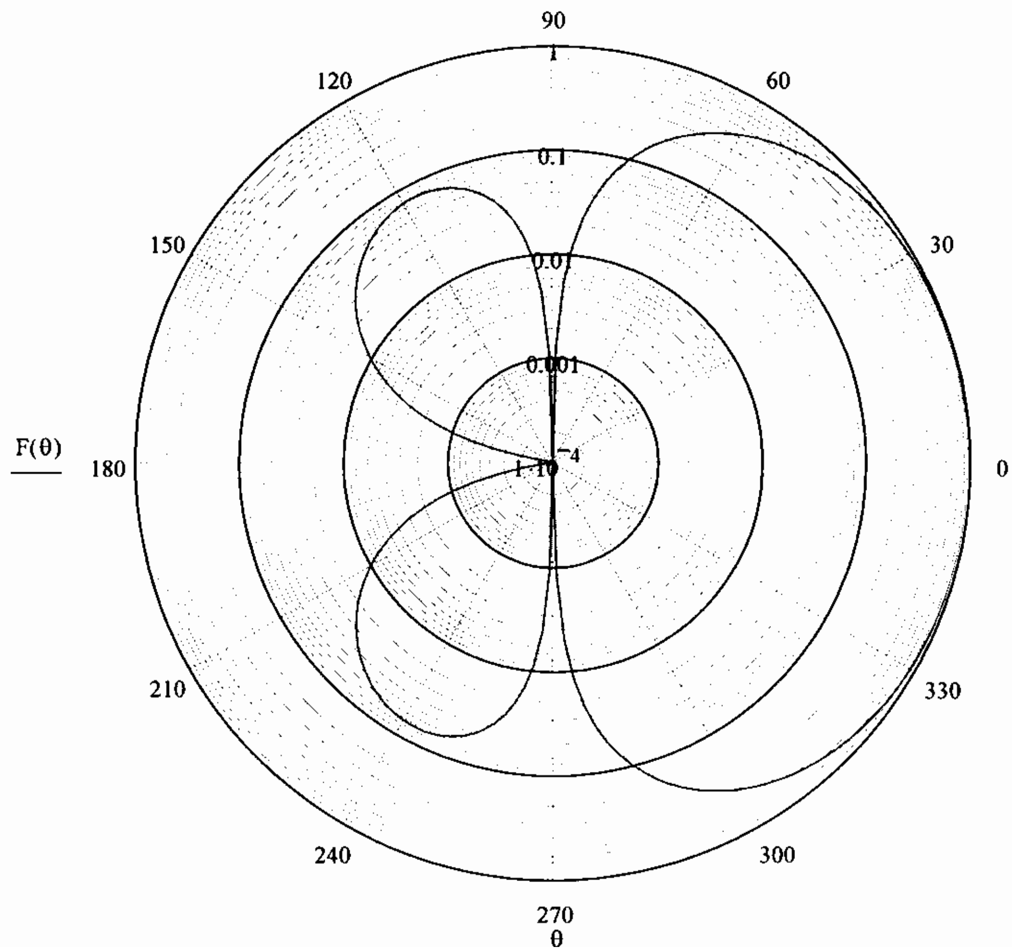


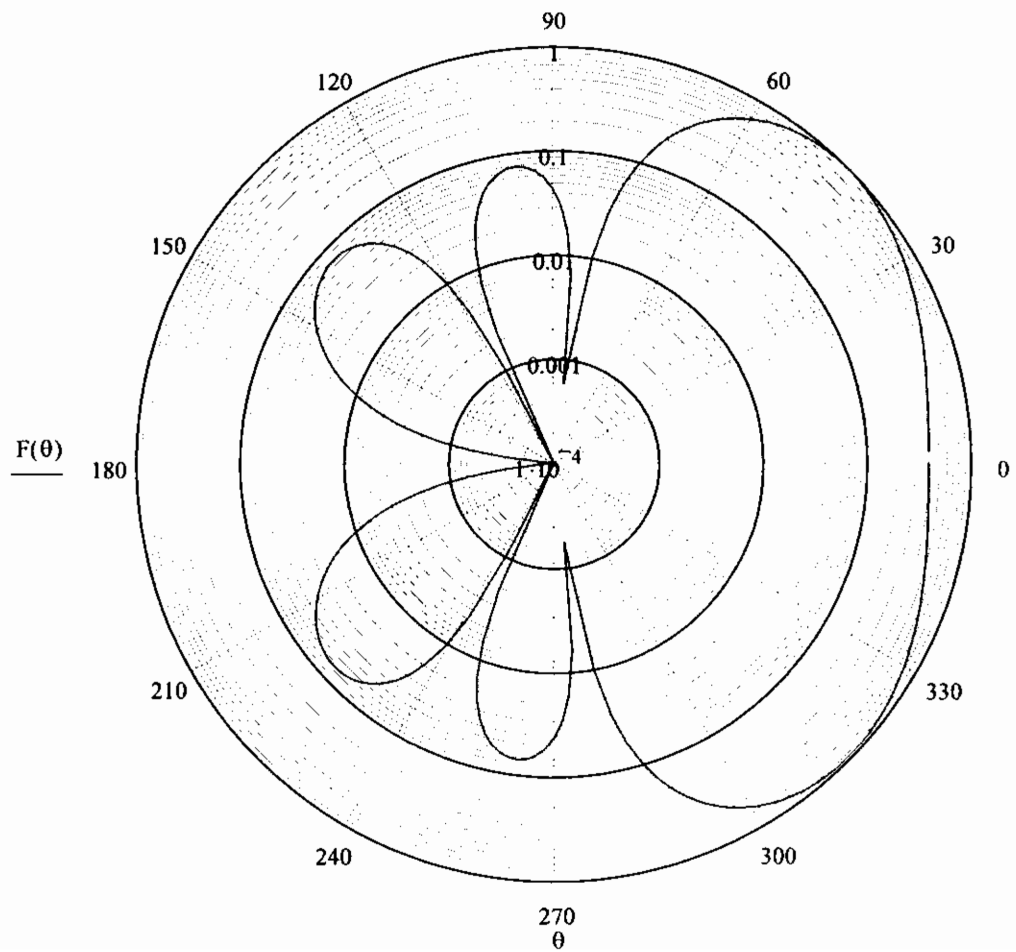
ANTENNA ARRAY RADIATION PATTERNS

9/11

The radiation pattern of a linear antenna array is given by the following:

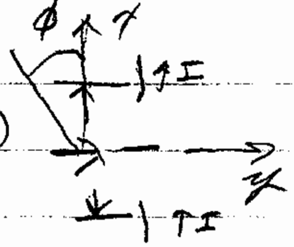
$N := 4 \quad d := 0.439 \quad \phi := 1.9516 \ ; \quad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



EE 434 Homework 6

1. A)

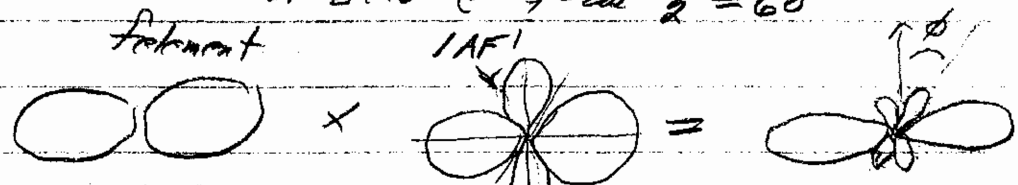


This is a 2 element array with $\psi = 0, d = \lambda$

$\therefore$ AF zeros @ $X = n\pi = 2\pi\cos\phi \ni \cos\phi = \frac{n}{2}$

or zero @ $\phi = \cos^{-1}\frac{1}{2} = 60°$

total pattern



zeros @ $\boxed{\phi = \pm 60°, 0°, 180°, \pm 120°}$

B.) $\psi = 0$, $d = 4\lambda$ zeros @ $X = n\pi = \frac{2\pi \times 4\lambda}{\lambda}\cos\phi$

or zeros @ $\cos\phi = \frac{n}{8}$

since $X_{zero} = n\pi$ only n odd produces zeros

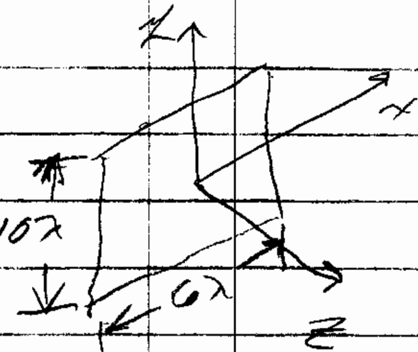
i.e. $X = 0, 2\pi, 4\pi \cdots$ are maxima

$\therefore$ zeros @ $\phi = \pm 82.8°, \pm 67.98°, \pm 51.32°, \pm 28.95°$

plus $\lambda/2$ dipole zeros @ $\phi = 0°$

There will also be zeros that are symmetrical to the above ... reflected about the yz plane

EE 434 Homework 6 (page 2)



$\phi=0$ pattern (i.e. xz plane)

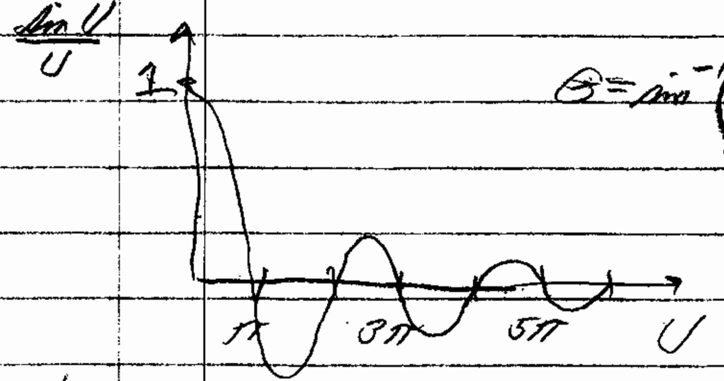
$$\hat{E} = \frac{j\beta E_m^+ ab(1+\cos\theta)\, e^{-j\beta r}}{4\pi r} \cdot \frac{\sin\left(\frac{\beta a}{2}\sin\theta\right)}{\frac{\beta a}{2}\sin\theta}\, \bar{a}_\theta$$

i.e. $U = \frac{\beta a}{2}\sin\theta$

(aperture is 6λ wide in this plane)

so $\frac{\beta a}{2} = \frac{2\pi a}{2\lambda} = \frac{2\pi 6\lambda}{2\lambda} = 6\pi$ (which is the visible range of U)

i.e. $0 < U < 6\pi$ for $0 < \theta < \frac{\pi}{2}$

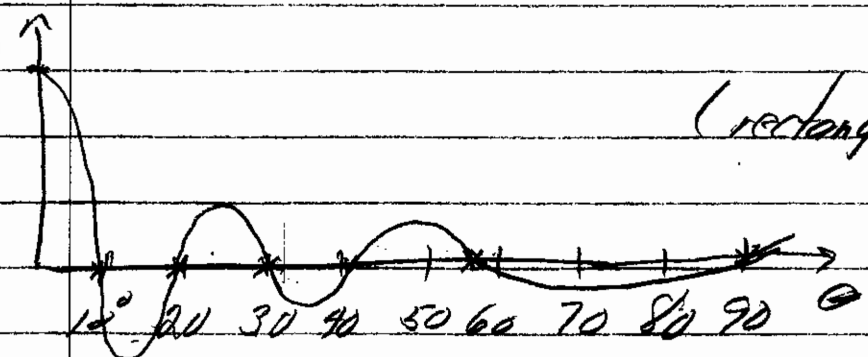


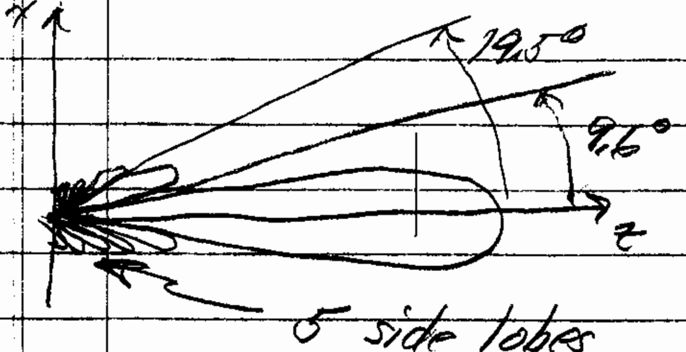
zeros of $\frac{\sin U}{U}$

U	θ
π	9.59°
2π	19.5°
3π	30.0°
4π	41.8°
5π	56.4°
6π	90°

Radiation pattern ($\phi=0$)

(rectangular plot)





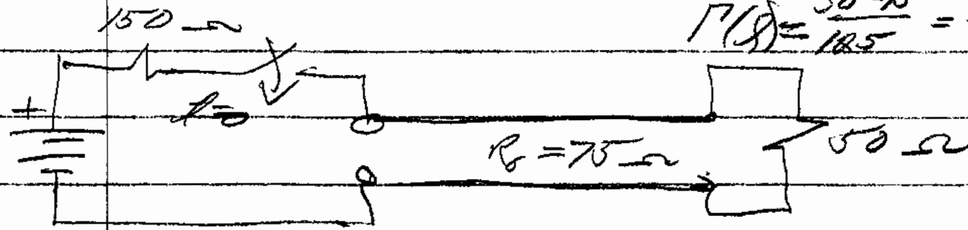
from table in hand out

$\theta_{HP} \cong \frac{50°}{6} = \boxed{9.33°}$

$\boxed{\text{first side lobe is down 13.2dB}}$

EE 434 Homework 7

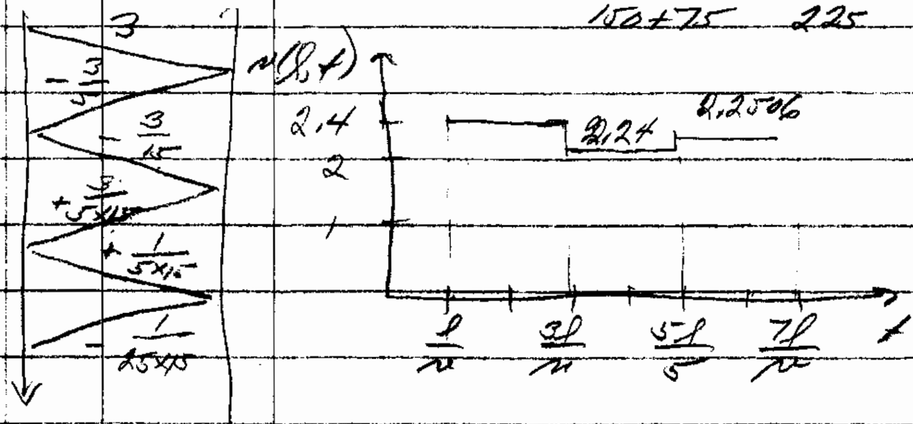
7.4



$\Gamma(l) = \frac{50-75}{125} = -\frac{1}{5}$

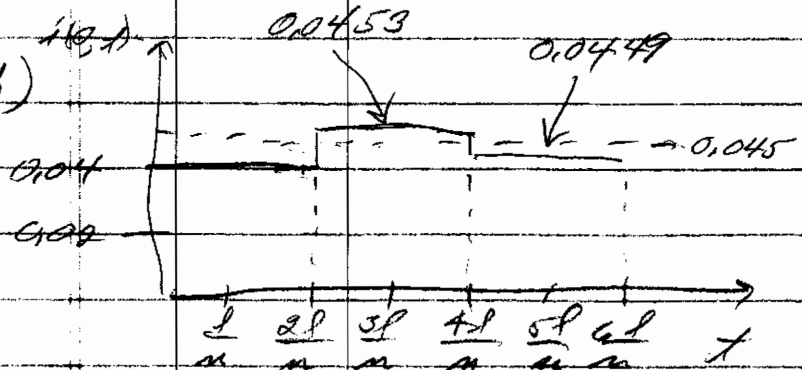
$\Gamma(0) = \frac{150-75}{150+75} = \frac{75}{225} = \frac{1}{3}$

a)

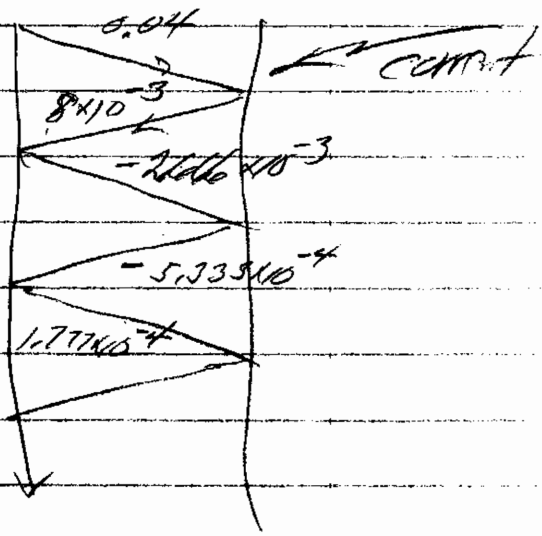


$$\boxed{v(l)_{final} = 9\,\frac{50}{200} = 2.25}$$

b)



$i(0,0) = \frac{9}{225} = 0.04$

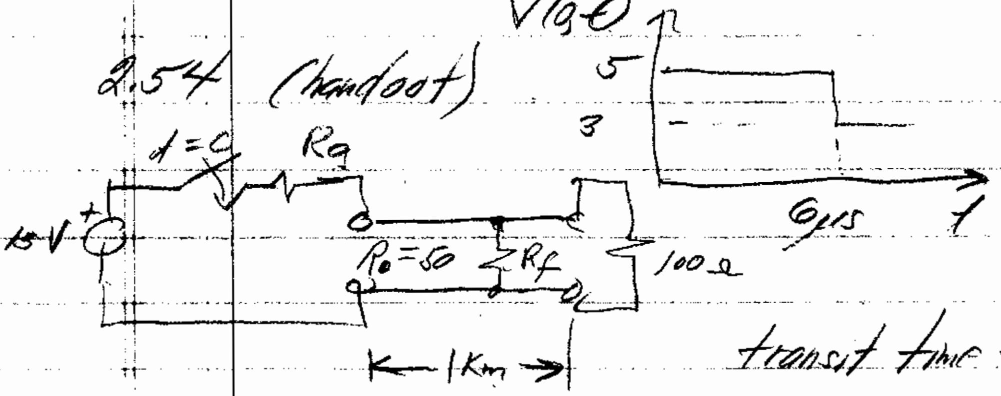


$$\boxed{i(0)_{final} = \frac{9}{200} = 0.045}$$

c) $v_{final} = 2.25$
$i_{final} = 0.045$ ← see above

EE 434 Homework 7 (page 2)

2.54 (handout)



transit time $= \frac{l}{v} = \frac{10^3}{10^8} = 10^{-5} = 10\mu sec$

$v = 10^8 \, m/sec$

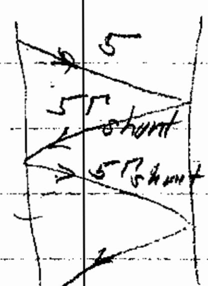
Since the transit is 10μsec the reflection reducing the voltage by 2 volts is due to a discontinuity on the line before the load resistor

a) $15 \frac{50}{50+R_g} = 5$; $15 \times 50 = 250 + 5R_g$ or $\boxed{R_g = \frac{750-250}{5} = 100\,\Omega}$

so $\Gamma(0) = \frac{100-50}{150} = \frac{1}{3}$

b) wave reflected from load would not be seen for two transit times (20μsec)

c) Resistance at shunt is $\frac{R_f \, 50}{50+R_f} = R_{eq}$



, To obtain above waveform $+5(1+\Gamma_s + \frac{\Gamma_s}{3}) = 3$

so $\Gamma_s(1+\frac{1}{3}) = \frac{3}{5} - 1$; $\Gamma_s = \frac{-2/5}{4/3} = -\frac{3}{10}$

but $\Gamma_s = \frac{R_{eq}-50}{R_{eq}+50} = -\frac{3}{10}$; $-\frac{3}{10}(R_{eq}+50) = R_{eq} - 50$

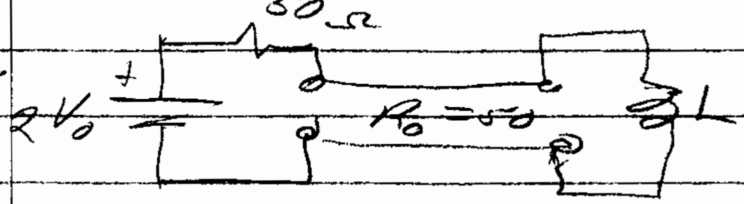
$R_{eq}(1+\frac{3}{10}) = 50 - 15$; $R_{eq} = \frac{35}{13/10} = \frac{350}{13} \cong 26.923$

$\therefore 50R_f = R_{eq}(50+R_f)$; $R_f(50-R_{eq}) = 50R_{eq}$

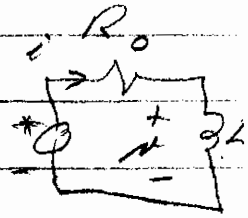
$R_f = \frac{50R_{eq}}{50-R_{eq}} = \boxed{58.33\,\Omega}$ ⟵

EE 434 Homework 8

7.5



$v^+_{load} = V_0 (t - \frac{l}{v})$ ∴ load circuit is: $2V_0 u(t-\frac{l}{u})$



so $2V_0 - iR_0 - L\frac{di}{dt'} = 0$ where $t' = t - \frac{l}{u}$

$i_{homo} = Ke^{st}$ ∴ $R_0 K e^{st} + L s K e^{st} = 0$

$s = -\frac{R_0}{L}$

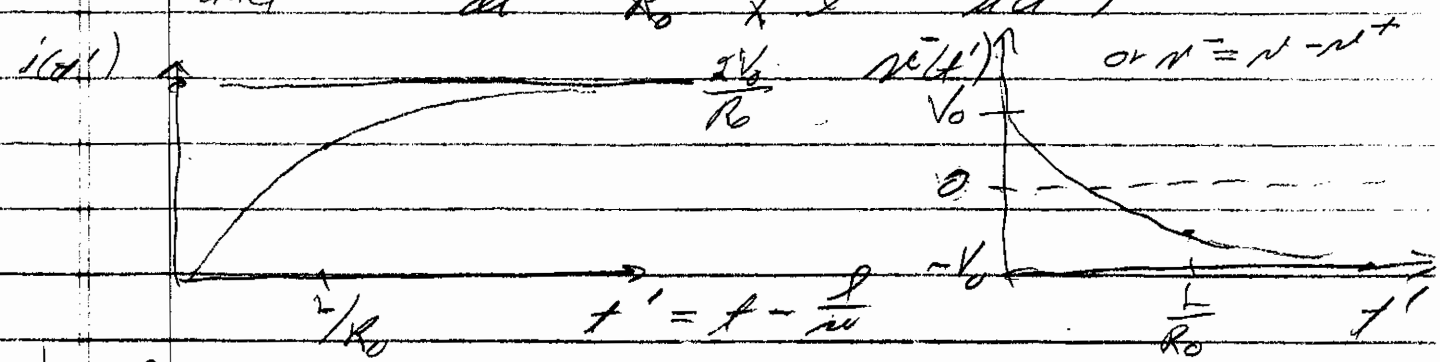
$i_{particular} = K' \Rightarrow 2V_0 - K'R_0 = 0$ or $K' = \frac{2V_0}{R_0}$

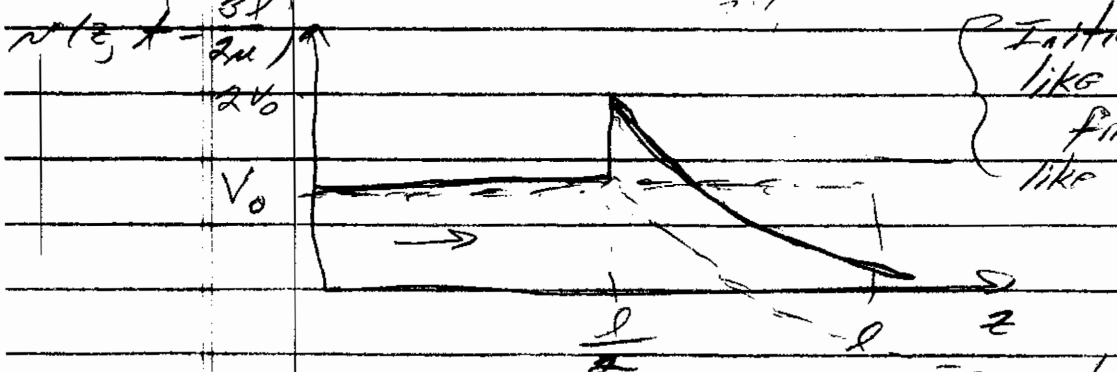
$i = i_p + i_h = \frac{2V_0}{R_0} + Ke^{-\frac{R_0 t}{L}}$ but $i(0) = 0$

so $\boxed{i = \frac{2V_0}{R_0}\left(1 - e^{-\frac{R_0 t'}{L}}\right) u(t')}$

and $v = L\frac{di}{dt'} = \frac{2V_0 L}{R_0} \cdot \frac{R_0}{L} e^{-\frac{R_0 t'}{L}} u(t') = v^+ + v^-$

or $v^- = v - v^+$

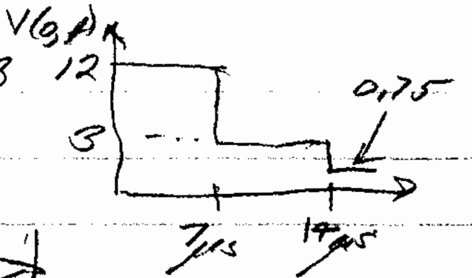


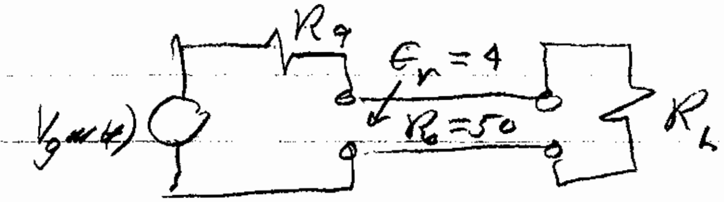


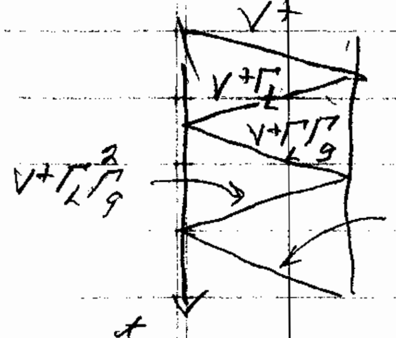
~~Initialy inductor looks~~ like an "open" ∴ +1 reflection
finaly inductor looks like a short

EE 434 Homework 8 (page 2)

2.53







$$R_g = 50\,\frac{1+\Gamma_g}{1-\Gamma_g} \Leftarrow \left\{ \begin{array}{l} \Gamma_L = \dfrac{R_L-50}{R_L+50} \\ \Gamma_g = \dfrac{R_g-50}{R_g+50} \end{array} \right\}$$

$$\boxed{12 = V_g\,\frac{50}{50+R_g} = V^+}$$

1) $V^+(\Gamma_L + \Gamma_L\Gamma_g) = -9$

2) $V^+(\Gamma_L^2\Gamma_g + \Gamma_L^2\Gamma_g^2) = -\frac{9}{4}$

$$\left\{ \begin{array}{l} \Gamma_L(1+\Gamma_g) = -\dfrac{9}{12} = -\dfrac{3}{4} \\ \Gamma_L^2\Gamma_g(1+\Gamma_g) = -\dfrac{9}{4\times 12} = -\dfrac{3}{16} \end{array} \right.$$

from 1) $\Gamma_L = -\frac{3}{4}\cdot\frac{1}{1+\Gamma_g} \Rightarrow \frac{9}{16}\cdot\frac{1}{(1+\Gamma_g)^{\cancel{2}}}\,\Gamma_g\cdot\cancel{(1+\Gamma_g)} = -\frac{3}{16}$

or $\frac{\Gamma_g}{1+\Gamma_g} = -\frac{1}{3}$; $-\frac{1}{3}(1+\Gamma_g) = \Gamma_g$

$\Gamma_g\left(1+\frac{1}{3}\right) = -\frac{1}{3}$; $\boxed{\Gamma_g = -\frac{\frac{1}{3}}{\frac{4}{3}} = -\frac{1}{4}}$

$$R_g = 50\,\frac{1+\Gamma_g}{1-\Gamma_g} = 50\,\frac{1-\frac{1}{4}}{1+\frac{1}{4}} = 50\,\frac{\frac{3}{4}}{\frac{5}{4}} = \frac{3}{5}\times 50 = \boxed{30\,\Omega}$$

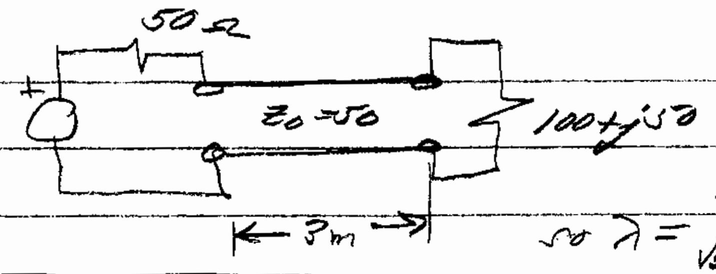
$$\therefore V_g = 12\left(\frac{50+30}{50}\right) = \boxed{19.2\text{ Volts}}$$

transit time $= \frac{7}{2}\,\mu\text{sec}$ $\quad u = \frac{3\times10^8}{2}$

$$\therefore \text{line length} = \frac{3\times10^8}{2}\times\frac{7}{2}\times10^{-6} = \boxed{525\text{ m}}$$

EE 434 Homework 9

1.



$\epsilon_r = 2.25$

$f = 5\times10^7$

so $\lambda = \dfrac{3\times10^8}{\sqrt{2.25}\,5\times10^7} = 4\text{m}$

∴ $\boxed{\text{line length is } \frac{3}{4}\lambda}$

a) $\Gamma_{load} = \dfrac{100+j50-50}{150+j50} = \dfrac{50(1+j)}{50(3+j)} = \dfrac{\sqrt{2}\,e^{j45^\circ}}{\sqrt{10}\,e^{j18.43^\circ}} = 0.447\,e^{j26.56^\circ}$

$\Gamma_{input} = \Gamma_{load}\,e^{-j2\times\frac{2\pi}{\lambda}\times\frac{3\lambda}{4}} = 0.447\,e^{-j3\pi}\,e^{j26.56^\circ} = 0.447\,e^{-j153.4^\circ}$

$= -0.4 - j0.2$

b) $Z_{in} = Z_0\dfrac{1+\Gamma_{in}}{1+\Gamma_{in}} = 50\,\dfrac{0.6-j0.2}{1.4+j0.2} = 50\,\dfrac{0.72\,e^{-j18.43^\circ}}{1.414\,e^{j8.13^\circ}} = 22.37\,e^{-j26.58^\circ}$

so $\hat{I}_{in} = \dfrac{200}{50+Z_{in}} = \dfrac{200}{50+20.013-j10.018} = \dfrac{200}{70.725\,e^{-j8.13}} = 2.828\,e^{j8.1}$

$P_{ave_{in}} = \frac{1}{2}\,Re\{\hat{I}_{in}\hat{V}^*_{in}\} = \frac{1}{2}|\hat{I}_{in}|^2 R_{in} = \boxed{80\text{ Watts}}$

$\dfrac{I_L}{I_{in}} = e^{-j\frac{2\pi}{\lambda}\cdot\frac{3}{4}\lambda}\left\{\dfrac{1-\Gamma_{load}}{1-\Gamma_{in}}\right\} = e^{-j\frac{3\pi}{2}}\left\{\dfrac{1-0.4-j0.2}{1+0.4+j0.2}\right\}$

$I_{load} = 2.828\,e^{j8.13}\times j\times\left\{\dfrac{0.6-j0.2}{1.4+j0.2}\right\} = 2.828\,e^{j8.13}\times e^{j90^\circ}\times\dfrac{0.6324\,e^{-j18.4}}{1.414\,e^{j8.13}}$

$\boxed{I_{load} = 1.2648\,e^{j71.57^\circ}}$

$P_{ave_{load}} = \frac{1}{2}|\hat{I}_{load}|^2\times100 = \boxed{80\text{ Watts}}$

2. same problem with $Z_{load} = 50$

a) $\Gamma_{load} = 0 = \Gamma_{in}$

b) $Z_{in} = Z_0 = 50$ $\therefore$ $\boxed{\hat{I}_{in} = \frac{200}{100} = 2A}$

$P_{ave_{in}} = \frac{1}{2} 2^2 \times 50 = \boxed{100 \text{ watts}}$

c) $I_L = I_{in} e^{-j90°} \left\{ \frac{1-\Gamma_{load}}{1-\Gamma_{in}} \right\} = \boxed{2 e^{-j90°}}$

$P_{ave_{load}} = \frac{1}{2} \times 2^2 \times 50 = \boxed{100 \text{ watts}}$

---

3. distortionless line $RC = LG$ or $\frac{R}{L} = \frac{G}{C}$

$\gamma = \sqrt{(R+j\omega L)(G+j\omega C)} = \sqrt{LC} \sqrt{\left(\frac{R}{L}+j\omega\right)\left(\frac{G}{C}+j\omega\right)} = \sqrt{LC} \left(\frac{R}{L}+j\omega\right)$

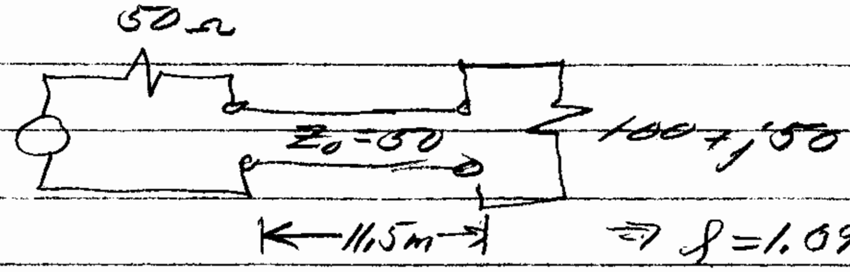
so $\boxed{\alpha = \frac{R}{L}\sqrt{LC} = R\sqrt{\frac{C}{L}} = R\sqrt{\frac{G}{R}} = \sqrt{RG}}$

$\beta = \omega\sqrt{LC}$

$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \boxed{\sqrt{\frac{L}{C}} \sqrt{\frac{\frac{R}{L}+j\omega}{\frac{G}{C}+j\omega}} = \sqrt{\frac{L}{C}}}$

EE 434 Homework 10

10.16



$\Rightarrow \ell = 1.092\lambda$

$Z_{L_{norm}} = 2 + j$

From Chart $\Gamma_{load} = 0.455 e^{j26.5°}$ ←

rotate 1.092λ toward generator $[0.2125 + 0.09 = 0.3025]$

$Z_{in_{norm}} = 1.6 - j1.13 \Rightarrow \boxed{Z_{in} = 80 - j56.5}$ ←

$\Gamma_{in} = 0.455 e^{-j38°}$ ←

10.17 Same as 10.16 with $\alpha = 0.00197$ Np/m ; $\ell = 11.5$ m

$e^{-2\alpha\ell} = e^{-2 \times 1.97 \times 10^{-3} \times 11.5} = \boxed{0.956}$ ←

$\therefore$ @ input $|\Gamma| = 0.956 \times 0.455 = 0.435$ ←

rotation not changed

from chart $Z_{in_{norm}} = 1.59 - j1.03$; $Z_{in} = 79.5 - j51.5$

10.21 $Z_L = 36 + j20$; $\therefore Z_{L_{norm}} = 0.72 + j0.4$

from chart $\boxed{Y_{L_{norm}} = 1.06 - j0.58}$ ←

$$\frac{1}{Z_{L_{norm}}} = \frac{1}{0.72 + j0.4} = \frac{1}{0.82365 e^{j29.05°}}$$

$= 1.06 - j0.59$ ←

$\underline{\underline{\ }}$ QED

EE 434 Homework 10 (page 2)

10.23 $Y_{in_2} = G_2 + jB_2$; $Y_{in_3} = G_3 + jB_3$ $\hat{I} = \hat{V}Y$

$P_{ave} = \frac{1}{2}Re\{\hat{V}\hat{I}^*\} = \frac{1}{2}Re\{\hat{V}\hat{V}^*Y^*\} = \frac{1}{2}Re\{|\hat{V}|^2(G - jB)\}$

$\therefore \boxed{\dfrac{P_{ave_2}}{P_{ave_3}} = \dfrac{G_2}{G_3}}$ $\left.\begin{array}{l} G_2 = 0.0366 = \frac{1.83}{50} \\ G_3 = 0.012 \end{array}\right\}$

$\frac{1}{2}|\hat{V}|^2(G_2 + G_3) = 8.5 = \frac{1}{2}|\hat{V}|^2 G_2\left(1 + \frac{G_3}{G_2}\right) = P_{ave_2}\left(1 + \frac{G_3}{G_2}\right)$

$\therefore P_{ave_2} = \dfrac{8.5}{1 + \frac{G_3}{G_2}} = \boxed{6.4 \text{ Watts}}$

and $P_{ave_3} = 8.5 - 6.4 = \boxed{2.1 \text{ Watts}}$

Problem 10.16
and 10.17

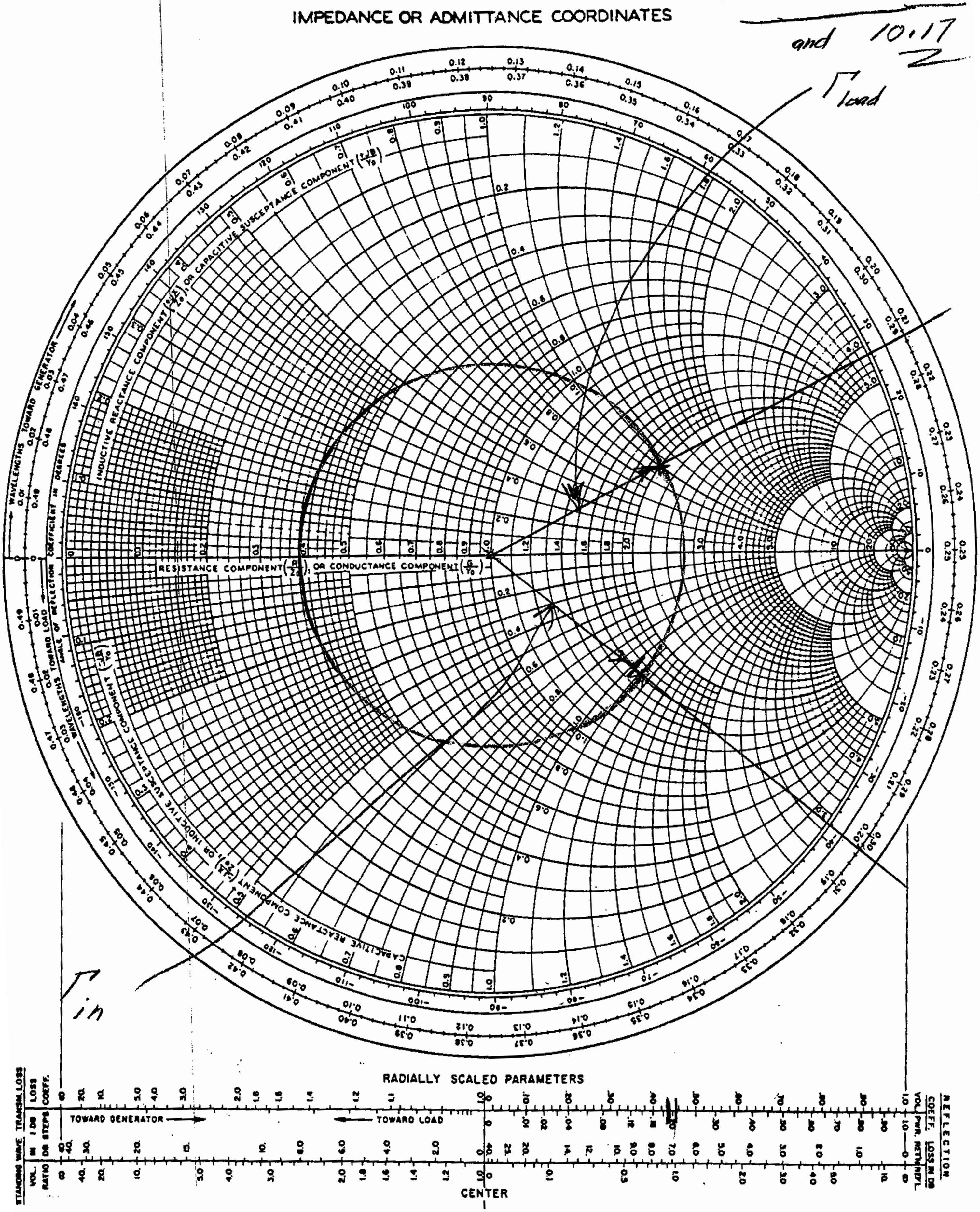
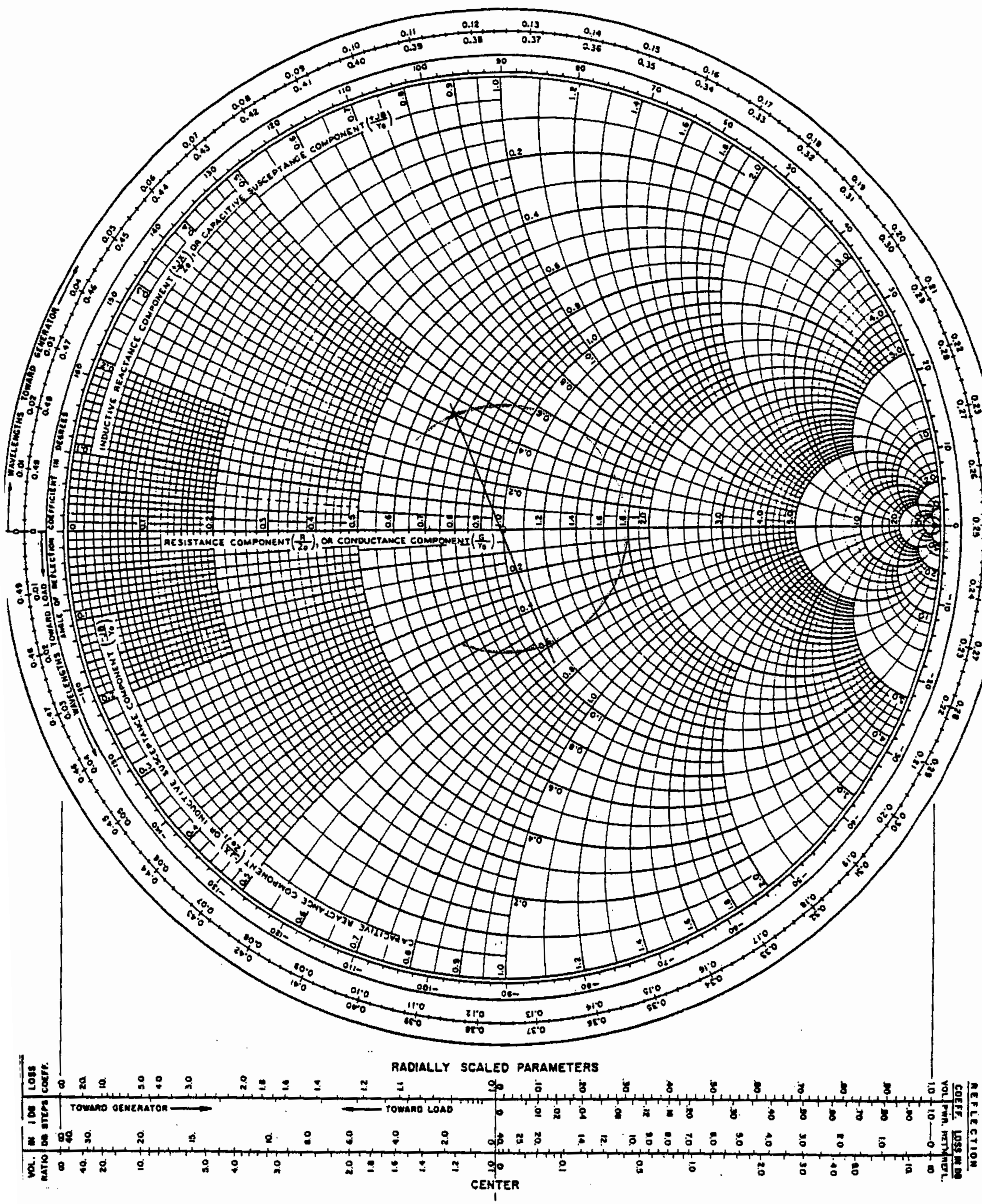


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

IMPEDANCE OR ADMITTANCE COORDINATES

Problem 10.21



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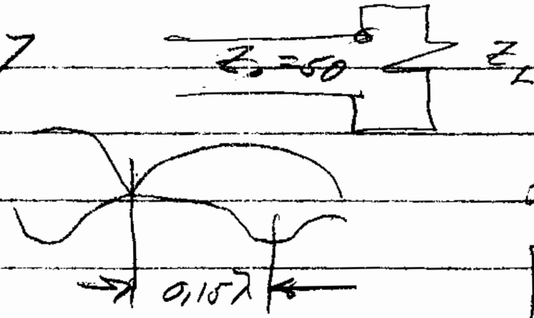
EE 434 Homework 11

10.27

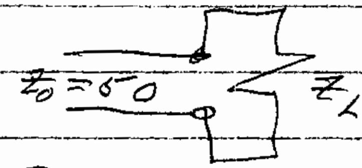


VSWR = 4.0

from chart $Z_{L_{norm}} = 0.67 + j1.14$

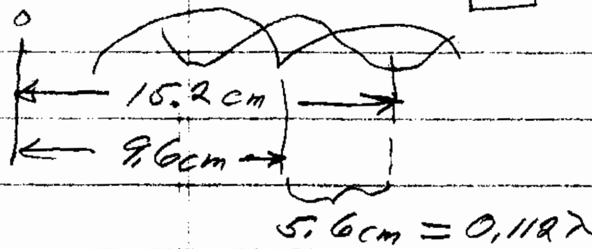
$\boxed{\therefore Z_L = 33.5 + j57}$

10.28



$f = 600\text{MHz}$; VSWR = 3.5

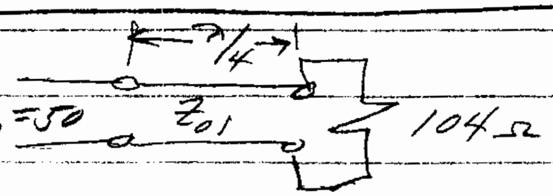
$\lambda = \frac{3\times10^8}{6\times10^8} = 0.5\text{m}$



from chart $Z_{L_{norm}} = 0.475 + j0.73$

$\therefore \boxed{Z_L = 23.75 + j36.5}$

10.36

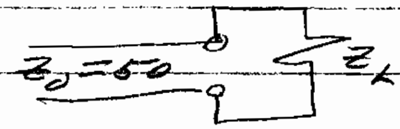


$f = 500\text{MHz}$, $\epsilon_r = 2.26$

$\boxed{Z_{01} = \sqrt{50\times104} = 72.1\,\Omega}$

$\lambda = \frac{3\times10^8}{\sqrt{2.26}\times5\times10^8} = 0.4\text{m}$ $\therefore$ $\boxed{\lambda/4 = 0.1\text{m}}$

10.39



$f = 500\text{MHz}$; $\lambda = 60\text{cm}$

VSWR = 3.5

(see chart) place stub $(0.25 - 0.171)\lambda = 0.08\lambda = 4.8\text{cm}$

moving this distance (in either direction) from V_{min} will locate stub

a) ~~for stub toward load~~ $Y_{norm\,stub} = -j1.3$ so stub length is $0.355 - 0.25 = 0.105\lambda$ or 6.3cm

b) VSWR from generator to stub = 1 and from stub to load = 3.5

c) for stub toward generator $Y_{norm\,stub} = +j1.3$; stub length = 23.7cm

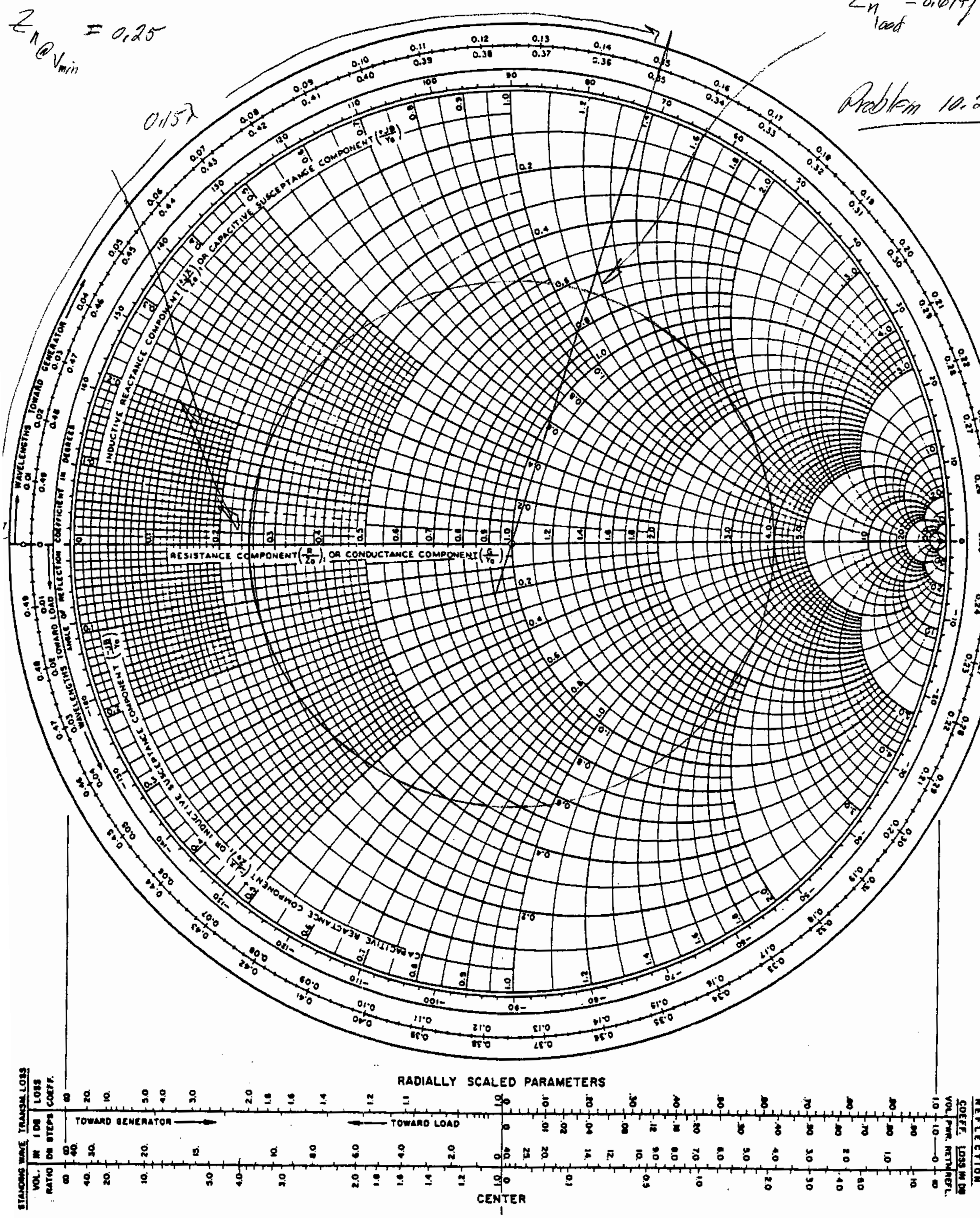
IMPEDANCE OR ADMITTANCE COORDINATES

$Z_{n\,@V_{min}} = 0.25$

$Z_{n\,load} = 0.67 + j1.14$

Problem 10.27

0.157λ



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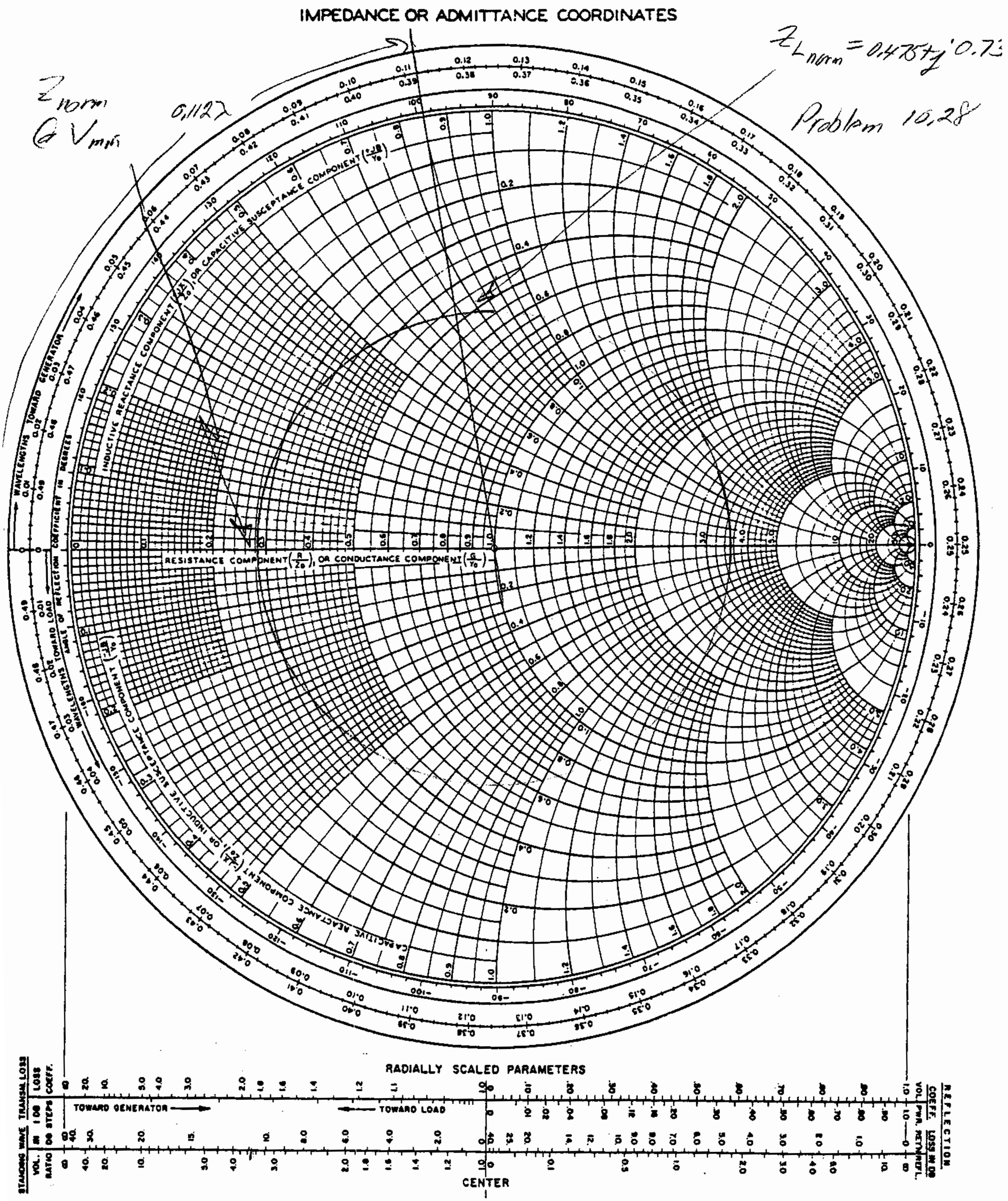

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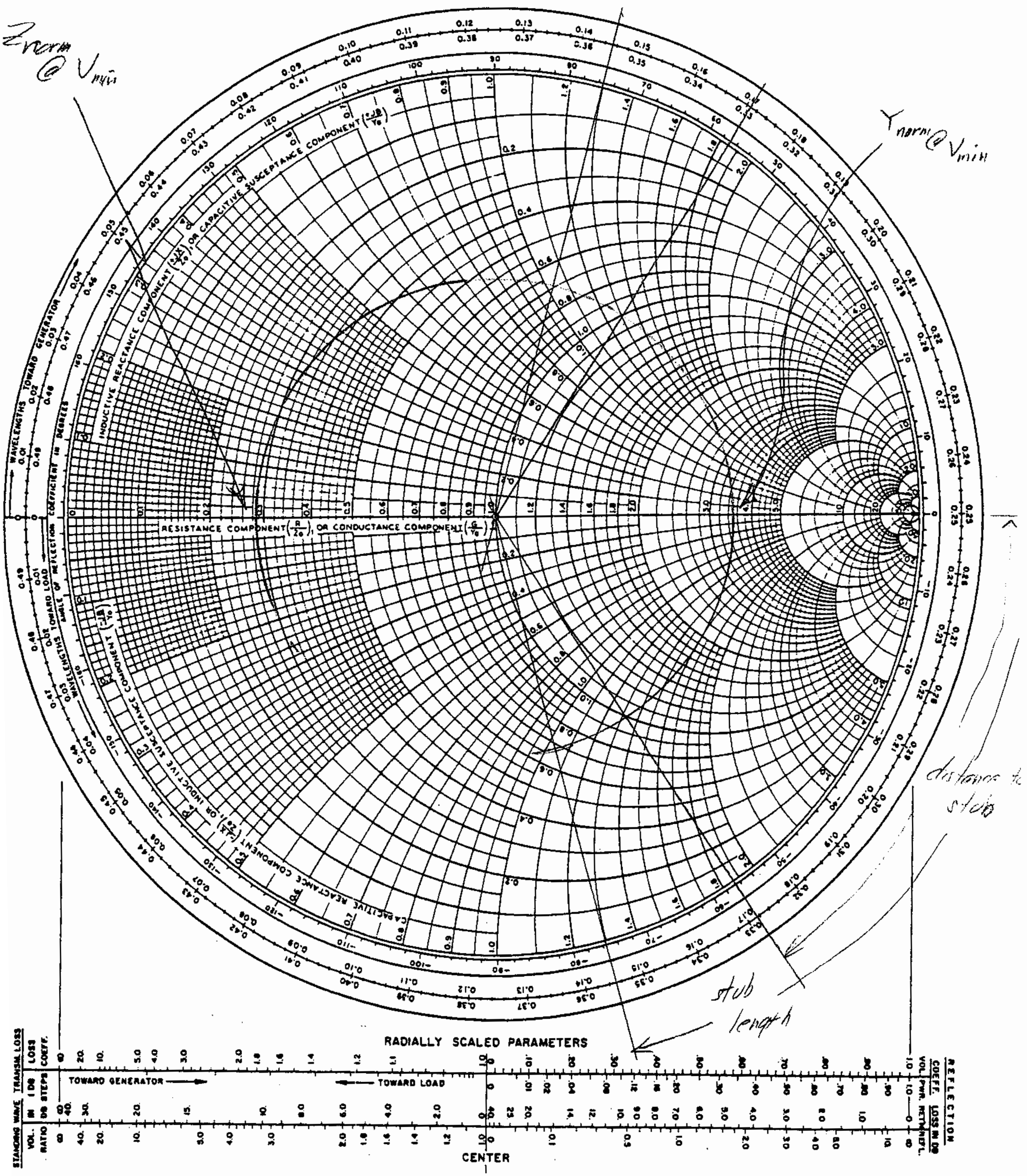
IMPEDANCE OR ADMITTANCE COORDINATES

Problem 10.39



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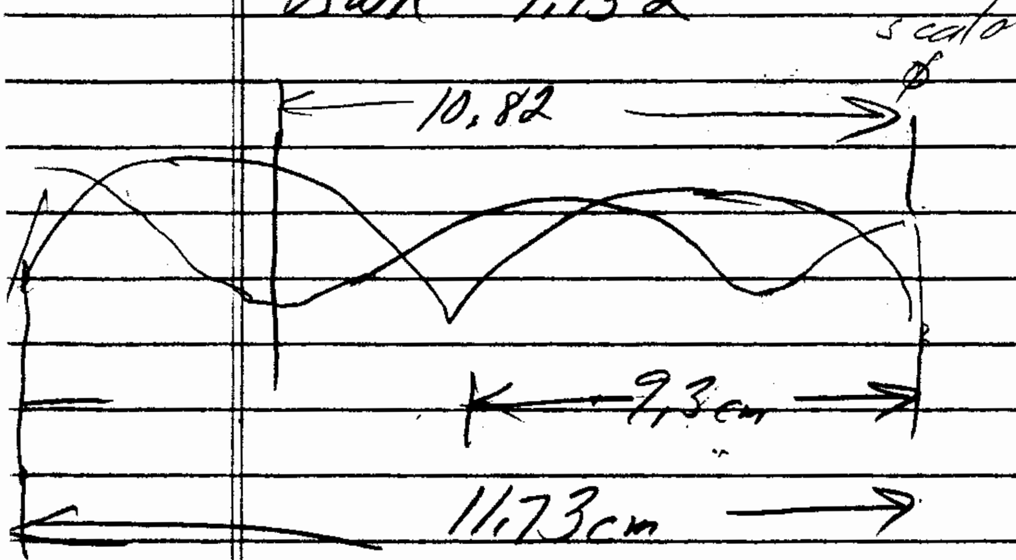
EE 434 Homework 12

9.1938 GHz (lab measurements)

VSWR = 1.152



or $\lambda = 4.86$cm

from above $\lambda/2 = (11.73 - 9.3)$cm $= 2.43$cm

move $(10.82 - 9.3)$cm toward load from V_{min} $= 1.52$cm

i.e. move $\frac{1.52}{2 \times 2.43} = \underline{\underline{0.3128\lambda}}$ toward load

from above $Z_{norm @ V_{min}} = \frac{1}{1.152} = 0.868$

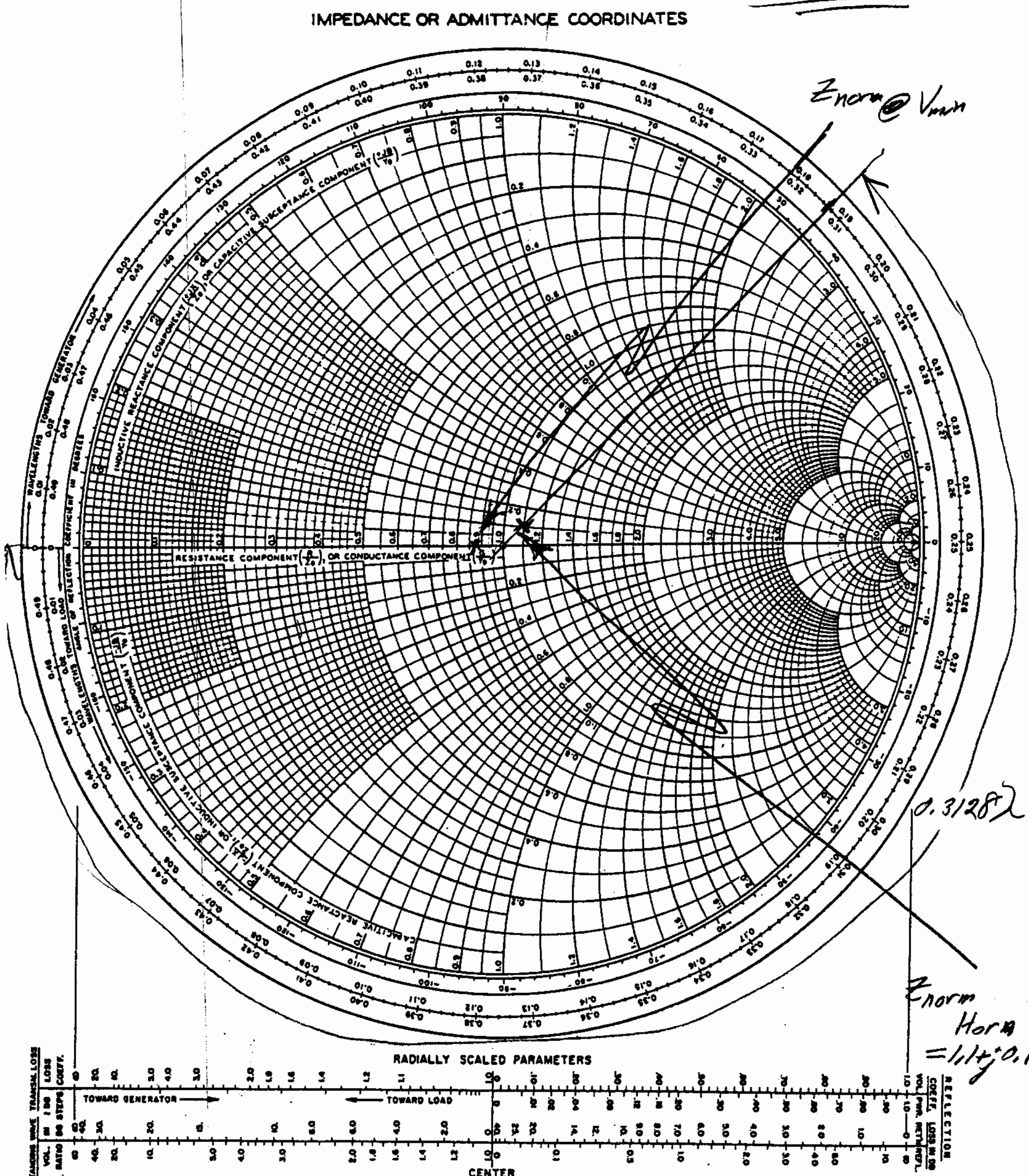
from chart $Z_{norm\ Horn} = 1.1 + j0.1$ ←

$\lambda_{calculated} = \lambda^0 \frac{1}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{3\times10^8}{9.1938\times10^9} \cdot \frac{1}{\sqrt{1-\left(\frac{6.557}{9.1938}\right)^2}}$

$\lambda_{calc} = 3.263\times10^{-2} \times 1.4266 = 4.655\times10^{-2}$m about 4% error? due to slot

$\eta_{10} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = 377 \times 1.4266 = 537.8\ \Omega$

lab demo



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EE 434 Homework 13

1. a) rectangular tunnel 15m by 6m

lowest $f_{c_{TE_{10}}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{15}\right)^2} = \frac{3\times10^8}{30} = 10^7$ Hertz ⟸

b) Vertical polarization

c) AM 535 → 1605 KHz ∴ will not propagate

FM 88 → 108 MHz so will propagate

8.11 TE_{10} mode $a = 2b$

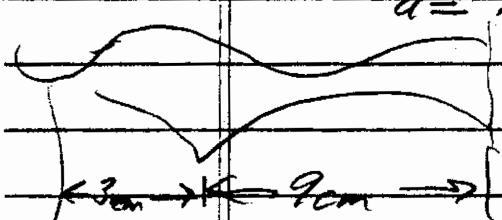
$f_c = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{3\times10^8}{2a}$ next lowest cutoff is twice this value

∴ want $1.5 f_c = 9\times10^9 = \frac{4.5\times10^8}{2a}$ or $a = \frac{4.5\times10^8}{18\times10^9} = 0.025m$

$\boxed{a = 2.5\,cm}$ ⟸

8.15 VSWR = 2.1 ; $\lambda/2 = 9cm$; ∴ z_{norm} @ $V_{min} = \frac{1}{VSWR} = 0.476$

$a = 7.6\times10^{-2}$



on smith chart move $\frac{3}{18}\lambda$ toward the load

This gives $z_{norm_{horn}} = \boxed{1.12 - j0.78}$ ⟸

$\eta_{TE_{10}} = \frac{\eta_0}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$ ∴ need f and f_c

$f_c = \frac{3\times10^8}{2\pi}\sqrt{\frac{\pi^2}{a^2}} = 1.5\times10^8\left(\frac{1}{a}\right) = \boxed{0.19737\times10^{10}}$

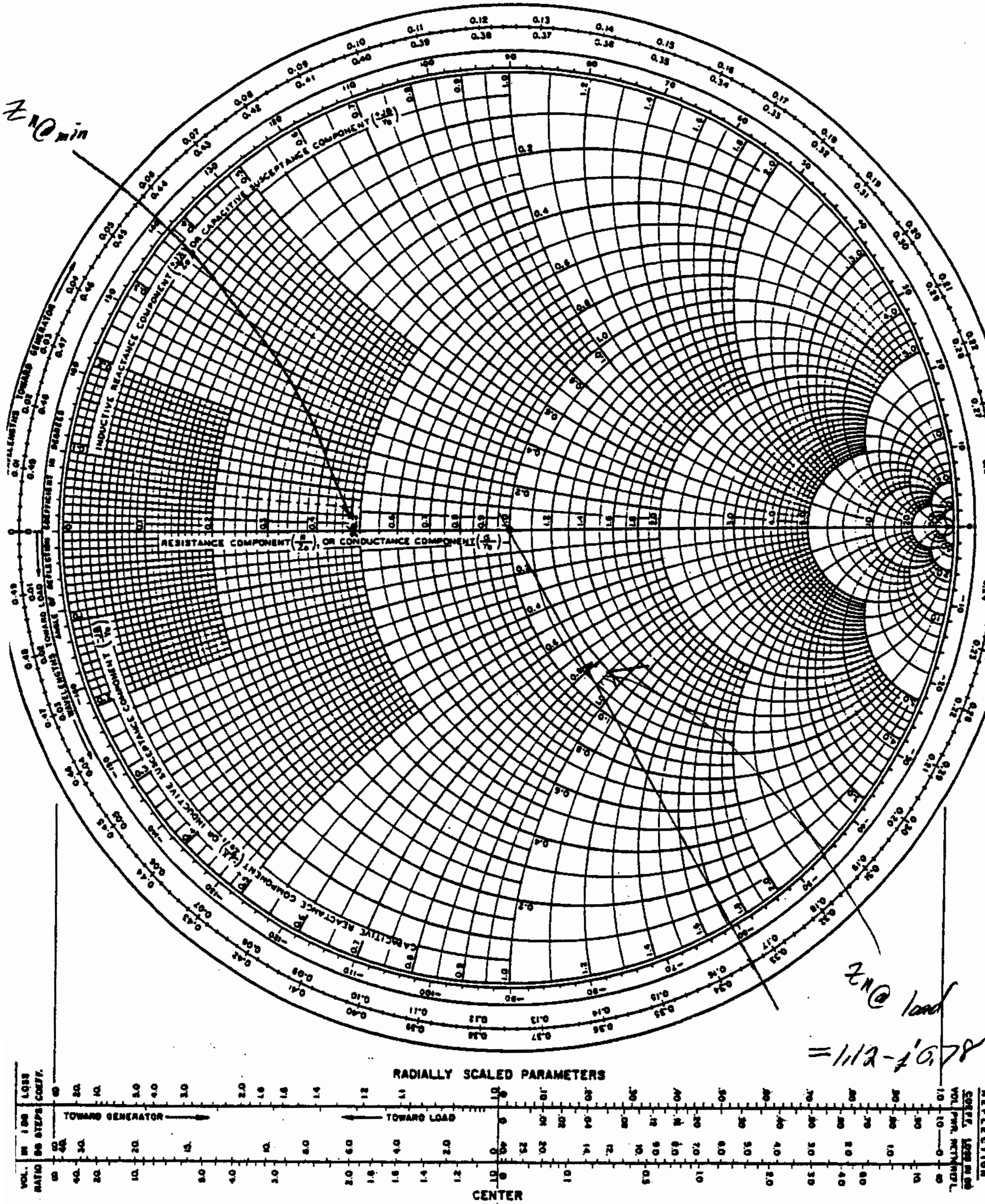
need f $\quad \lambda = \frac{2\pi}{\beta} = \frac{2\pi c}{\omega\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \frac{c}{\sqrt{f^2 - f_c^2}}$

$\sqrt{f^2 - f_c^2} = \frac{3\times10^8}{18\times10^{-2}} = 0.1666\times10^{10}$ giving $f = \sqrt{1.67^2 + 1.97^2}\times10^9$

$\boxed{f = 2.583\times10^9}$ giving $\eta_{TE_{10}} = \frac{377}{\sqrt{1-0.583}} = 584.4\ \Omega$

so $Z_{horn} = \eta_{TE_{10}}(1.12 - j0.78) = \boxed{654 - j455.8}$ ⟸

IMPEDANCE OR ADMITTANCE COORDINATES



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EE 434 Homework 13 (page 2)

8.18 $f = 1.3 f_c$ $E_{max} = 0.75 \times 2 \times 10^6$ V/m

rectangular waveguide $a = 2.3$ cm, $b = 1.2$ cm

from notes or book

$$\mathcal{P}_z = -E_y \times H_x = H_0^2 \frac{\omega\mu a^2 \beta_{10}}{\pi^2} \sin^2\left(\frac{\pi}{a}x\right) \rightarrow \frac{1}{2}\left[1 - \cos\left(\frac{2\pi x}{a}\right)\right]$$

so $$P_{ave} = \frac{1}{2} \mathrm{Re} \int_{y=0}^{b} \int_{x=0}^{a} \mathcal{P}_z \, dx\, dy = \frac{H_0^2 \omega\mu a^2 \beta_{10}}{\pi^2} \cdot b \cdot \frac{a}{2}$$

$$E_{max} = H_0 \frac{\omega\mu a}{\pi} \therefore P_{ave} = \frac{E_{max}^2 \beta_{10}\, ab}{4\omega\mu}$$

$$\text{or } P_{ave} = \frac{ab\, E_{max}^2}{4\omega\mu} \omega\sqrt{\mu\epsilon} \sqrt{1 - \left(\frac{1}{1.3}\right)^2}$$

$$P_{ave} = E_{max}^2 \frac{ab\sqrt{1 - \left(\frac{1}{1.3}\right)^2}}{4\eta_0} = E_{max}^2\, 1.169 \times 10^{-7}$$

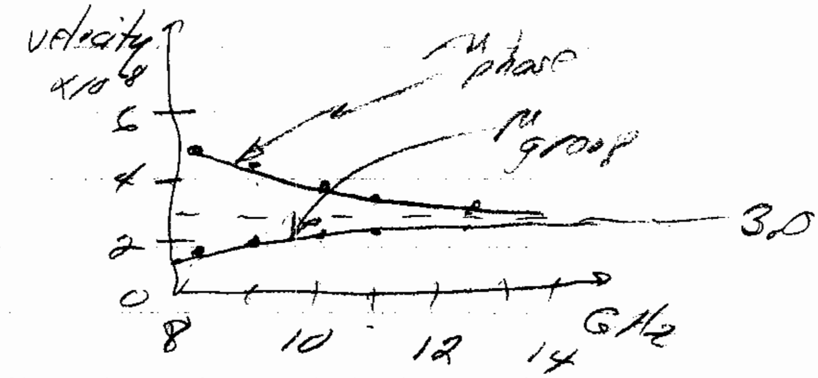
giving $P_{ave_{max}} = 2.25 \times 10^{12} \times 1.169 \times 10^{-7} = \boxed{0.263 \times 10^6 \text{ Watts}}$

EE 434 Homework 14

1. $\gamma = \sqrt{\omega_c^2\mu\epsilon - \omega^2\mu\epsilon} = \omega\sqrt{\mu\epsilon}\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$ ∴ $\beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$v_{ph} = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$; $v_{group} = \frac{\partial\omega}{\partial\beta} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$; $f_c = 6.557$ GHz

f	v_{ph}	v_{group}
8.2 GHz	5×10^8	1.8×10^8
9 GHz	4.38×10^8	2.05×10^8
10 GHz	3.97×10^8	2.26×10^8
11 GHz	3.73×10^8	2.41×10^8
12.4 GHz	3.53×10^8	2.54×10^8



2. a) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi\times3\times10^8}{2\pi\times10^{10}\underbrace{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}_{=0.755}} = 3.97\times10^{-2}\text{ m}$

b) must use $v_{group} = 3\times10^8\times0.755 = 2.26\times10^8$

∴ transit time $= \frac{100}{2.26\times10^8} = \boxed{0.44\ \mu sec}$

c) Change dielectric so that $f_c < 6$ GHz

3. $f_c = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2+\left(\frac{n\pi}{b}\right)^2}$ $f_{c_{TE_{10}}} = 6.557$ (E field vertical)

for horizontal E field TE_{01} mode has the lowest cutoff frequency. Since $b < 2a$ cutoff frequency for TE_{01} mode is $> 2f_{c_{TE_{10}}}$ (i.e. > 13 GHz)

Therefore horizontal portion of the circularly polarized wave will not propagate ∴ Output would be linearly polarized

4. TE_{11} $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p'_{11}}{a}\right) = \frac{3\times10^8}{2\pi}\left(\frac{1.841}{2.54\times10^{-2}}\right) = \boxed{3.46\text{ GHz}}$

TM_{01} $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p_{01}}{a}\right) = \frac{3\times10^8}{2\pi}\left(\frac{2.405}{2.54\times10^{-2}}\right) = \boxed{4.52\text{ GHz}}$

TE_{01} $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p'_{01}}{a}\right) = \frac{3\times10^8}{2\pi}\left(\frac{3.832}{2.54\times10^{-2}}\right) = \boxed{7.2\text{ GHz}}$