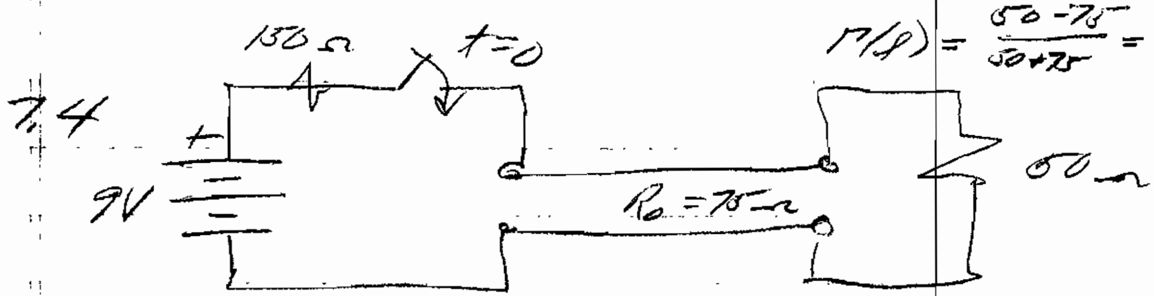
EE 434 Homework 1

2.4

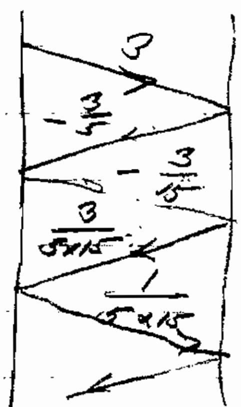


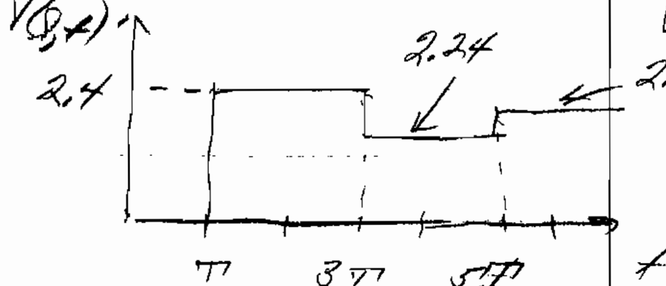
$$\Gamma(\ell) = \frac{50-75}{50+75} = -\frac{1}{5}$$

$$\Gamma(0) = \frac{150-75}{150+75} = \frac{1}{3}$$

[T = transit time $\frac{\ell}{u}$]

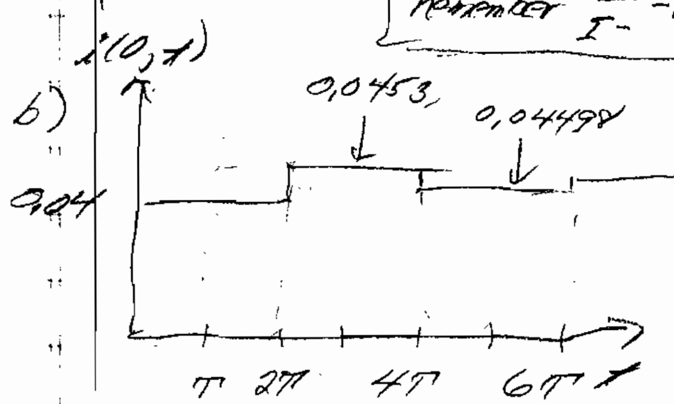
a)



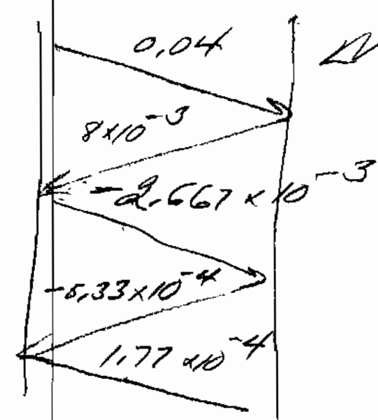


$$\boxed{\text{remember } \frac{V^+}{I^+} = R_0 = -\frac{V^-}{I^-}}$$

b)



current

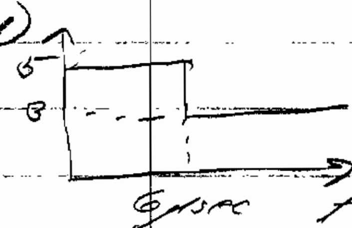


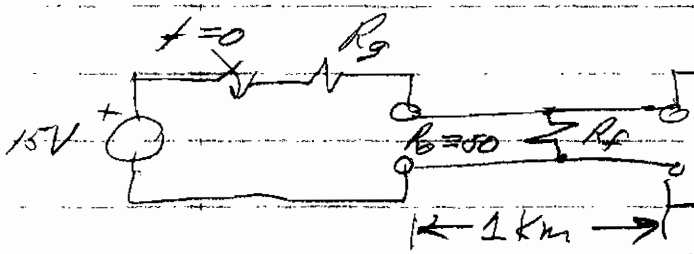
c) $v_{final} = 9\,\frac{50}{200} = \boxed{2.25 \text{ volts}}$

$i_{final} = \frac{v_{final}}{50} = \boxed{0.045 \text{ Amperes}}$

EE 434 Homework 1 (page 2)

2.54 from handout





$[u = 10^8 \text{ m/sec}]$

Transit time $= \frac{1\times10^3}{10^8} = 10^{-5} = \boxed{10 \mu sec}$

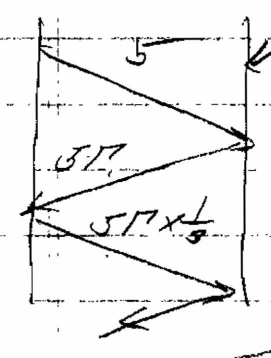
Since the transit time is 10μsec the reduction in V(0,t) at 6μsec is due to reflection from R_f ~~not the 100 Ω load~~

a) $5 = 15 \frac{R_0}{R_0+R_g}$ or $\frac{1}{3} = \frac{50}{50+R_g}$; $\boxed{R_g = 100\,\Omega}$

b) Reflection from load will not be seen for 2T or 20μsec.

c) At R_f the initial positive going wave sees a resistance of $\frac{R_f 50}{R_f+50} = R_{eq}$

$\Gamma(z=0) = \frac{100-50}{150} = \frac{1}{3}$



To obtain waveform given

$3 = 5(1+\Gamma+\frac{\Gamma}{3})$

$\therefore \Gamma_{@R_f} = \frac{3-5}{5+\frac{5}{3}} = \frac{3(-2)}{20} = \boxed{-\frac{3}{10}}$

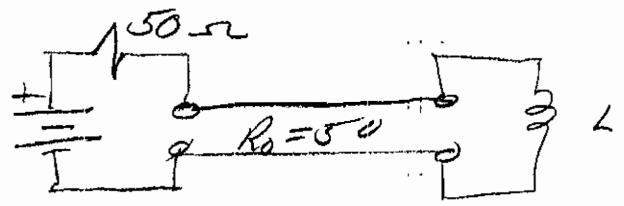
but $\Gamma_{@R_f} = \frac{R_{eq}-50}{R_{eq}+50}$; $R_{eq} = \frac{50-15}{1+\frac{3}{10}} = \frac{350}{13}\,\Omega = \underline{26.923}$

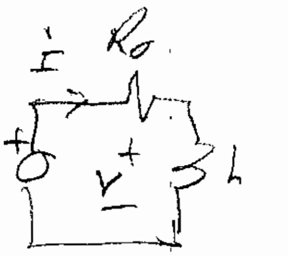
from above, $R_f 50 = R_{eq}(R_f+50)$

$\boxed{R_f = 58.33\,\Omega}$

EE 434 Homework 2

7.5





$V^+_{load} = V_0(t - \frac{l}{u})$ ∴ load circuit is: $2V_0 u(t - \frac{l}{u})$

so $2V_0 - IR_0 - L\frac{dI}{dt'} = 0$ where $t' = t - \frac{l}{u}$

$I_{homogeneous} = Ke^{bt}$ ∴ $R_0 K e^{bt} + L b K e^{bt} = 0 \Rightarrow b = -\frac{R_0}{L}$

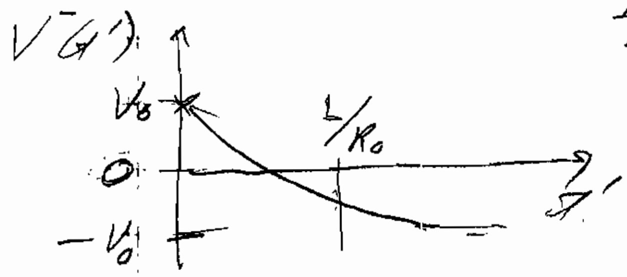
$I_{particular} = K' \Rightarrow 2V_0 - K'R_0 = 0$ or $K' = \frac{2V_0}{R_0}$

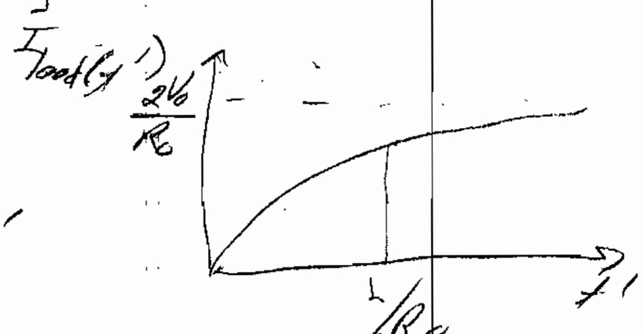
$I = I_h + I_p = \frac{2V_0}{R_0} + Ke^{-\frac{R_0 t'}{L}}$ but $I(0) = 0$

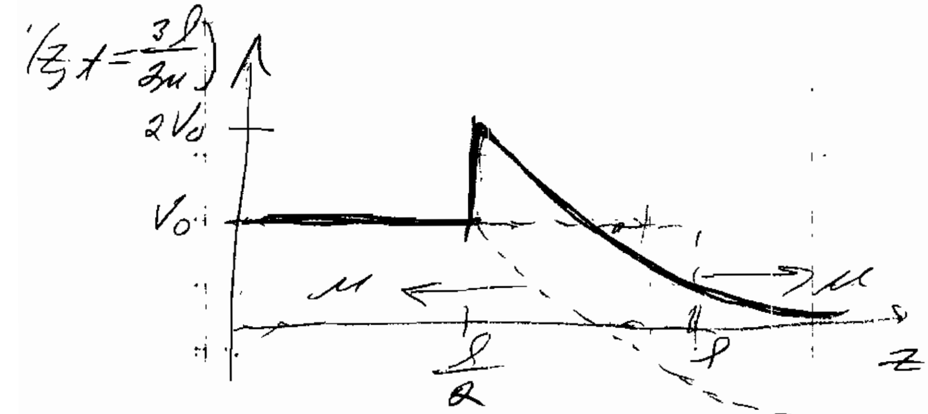
∴ $I = \frac{2V_0}{R_0}\left(1 - e^{-\frac{R_0 t'}{L}}\right)u(t')$

$V = L\frac{dI}{dt'} = 2V_0 e^{-\frac{R_0 t'}{L}} u(t') = V^+_{load} + V^-_{load}$

so $V^-_{load} = \left[2V_0 e^{-\frac{R_0 t'}{L}} - V_0\right]u(t')$



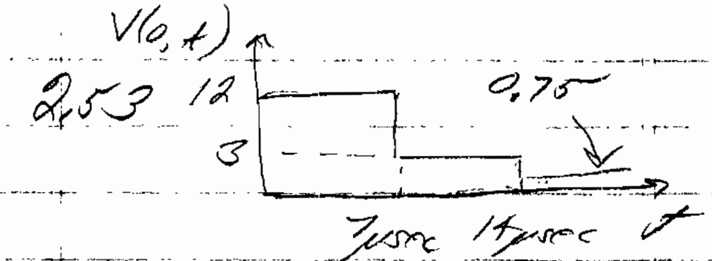


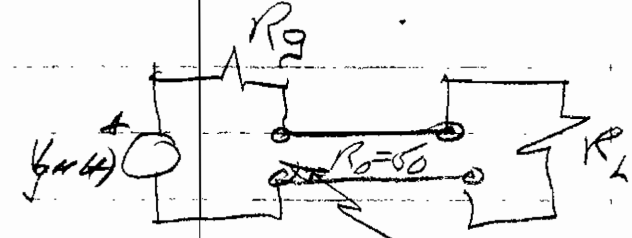


Initially inductor looks like an open circuit $\Gamma = +1$
After transients die out inductor looks like a short circuit

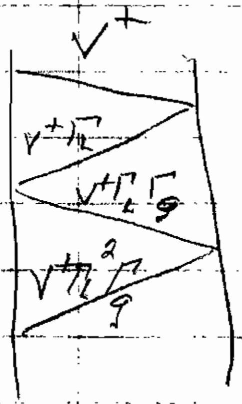
EE 434 Homework 2 (page 2)

2.53





$\epsilon_r = 4$



where $\boxed{V^+ = V_g \dfrac{50}{50+R_g} = 12}$

$\Gamma_L = \dfrac{R_L - 50}{R_L + 50}$

$\Gamma_g = \dfrac{R_g - 50}{R_g + 50}$

From diagram we see that:

1) $V^+(\Gamma_L + \Gamma_L \Gamma_g) = -9$

2) $V^+(\Gamma_L^2 \Gamma_g + \Gamma_L^2 \Gamma_g^2) = -\dfrac{9}{4}$

$\Gamma_L(1+\Gamma_g) = -\dfrac{9}{12} = -\dfrac{3}{4}$

$\Gamma_L^2 \Gamma_g (1+\Gamma_g) = -\dfrac{9}{4 \times 12} = -\dfrac{3}{16}$

from 1) $\Gamma_L = -\dfrac{3}{4} \cdot \dfrac{1}{1+\Gamma_g}$ substituting into 2) $\dfrac{9}{16} \dfrac{1}{[1+\Gamma_g]^2} \Gamma_g \cancel{(1+\Gamma_g)} = -\dfrac{3}{16}$

or $\dfrac{\Gamma_g}{1+\Gamma_g} = -\dfrac{1}{3}$ giving $\boxed{\Gamma_g = -\dfrac{1}{4}}$

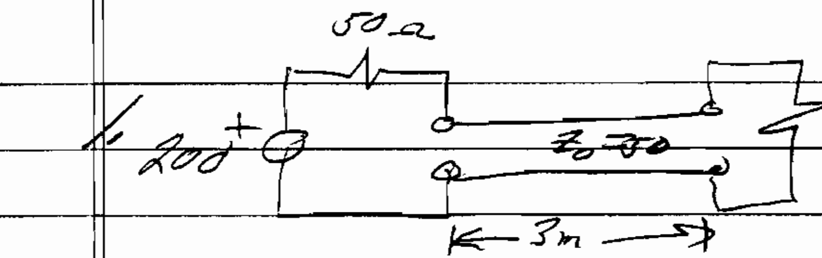
but $\Gamma_g = \dfrac{R_g - 50}{R_g + 50} \Rightarrow \boxed{R_g = 50 \dfrac{1+\Gamma_g}{1-\Gamma_g} = 50 \dfrac{\frac{3}{4}}{\frac{5}{4}} = 30\ \Omega}$

$\therefore \boxed{V_g = 12 \dfrac{50+30}{50} = 19.2 \text{ Volts}}$

Transit time $= \dfrac{7}{2}$ μsec and for $\epsilon_r = 4$ $u = \dfrac{3 \times 10^8}{2}$

so $\boxed{\text{line length} = \dfrac{3 \times 10^8}{2} \times \dfrac{7}{2} \times 10^{-6} = 525\text{m}}$

EE 434 Homework 3



$\left\{ \begin{array}{l} \epsilon_r = 2.25 \\ f = 5\times10^7 \end{array} \right.$

$\lambda = \frac{3\times10^8}{\sqrt{2.25}\;5\times10^7} = 4m$

$\boxed{\therefore \text{ line length is } \frac{3}{4}\lambda}$

a) $\Gamma_{load} = \frac{100+j50-50}{150+j50} = \frac{50(1+j)}{50(3+j)} = \frac{\sqrt{2}\,e^{j45°}}{\sqrt{10}\,e^{j18.43°}} = 0.447\,e^{j26.56°}$

$\Gamma_{input} = \Gamma_{load}\, e^{-j2\times\frac{2\pi}{\lambda}\times\frac{3\lambda}{4}} = 0.447\,e^{-j3\pi}\,e^{j26.56°} = 0.447\,e^{-j153.4°}$

$(-0.4-j0.2)$

b) $Z_{in} = Z_0\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50\,\frac{0.6-j0.2}{1.4+j0.2} = 50\,\frac{0.632\,e^{-j18.43}}{1.414\,e^{j8.13}} = 22.377\,e^{-j26.56°}$

so $I_{in} = \frac{200}{50+20.013-j10.012} = \boxed{2.828\,e^{j8.13°}}$

and $P_{ave_{in}} = \frac{1}{2}Re\{\hat{I}_{in}\hat{V}_{in}^*\} = \frac{1}{2}|\hat{I}_{in}|^2 R_{in} = \boxed{80 \text{ Watts}}$

$\frac{\hat{I}_{load}}{\hat{I}_{in}} = e^{-j\frac{2\pi}{\lambda}\cdot\frac{3}{4}\lambda}\left\{\frac{1-\Gamma_{load}}{1-\Gamma_{in}}\right\} = e^{-j\frac{3\pi}{2}}\left\{\frac{1-0.4-j0.2}{1+0.4+j0.2}\right\}$

or $\hat{I}_{load} = 2.828\,e^{j8.13}\times j\times\left\{\frac{0.6-j0.2}{1.4+j0.2}\right\} = 2.828\,e^{j8.13}\times e^{j90°}\times\frac{0.6324\,e^{-j18.43}}{1.414\,e^{j8.13}}$

$\boxed{\hat{I}_{load} = 1.2648\,e^{j71.57°}}$

and $P_{ave_{load}} = \frac{1}{2}|\hat{I}_{load}|^2\times100 = 80 \text{ Watts}$

2. a) $\Gamma_{load} = \Gamma_{in} = 0$

b) $Z_{in} = Z_0 = 50\,\Omega$ $\therefore$ $\boxed{\hat{I}_{in} = \frac{200}{100} = 2A}$

$P_{avg_{in}} = \frac{1}{2} \times 2^2 \times 50 \boxed{= 100 \text{ Watts}}$

c) $\hat{I}_{load} = \hat{I}_{in} e^{j90^\circ} \left\{ \frac{1 - \Gamma_{load}}{1 - \Gamma_{in}} \right\} \boxed{= 2 e^{j90^\circ}}$

and $P_{avg_{load}} = \frac{1}{2} \times 2^2 \times 50 = 100 \text{ Watts}$

3. Distortionless Line $RC = LG$ or $\frac{R}{L} = \frac{G}{C}$

$$\gamma = \sqrt{(R + j\omega L)(G + j\omega C)} = \sqrt{LC}\sqrt{\left(\frac{R}{L} + j\omega\right)\left(\frac{G}{C} + j\omega\right)} = \sqrt{LC}\left(\frac{R}{L} + j\omega\right)$$

so $\boxed{\alpha = \frac{R}{L}\sqrt{LC} = R\sqrt{\frac{C}{L}} = R\sqrt{\frac{G}{R}} = \sqrt{RG}}$

from above

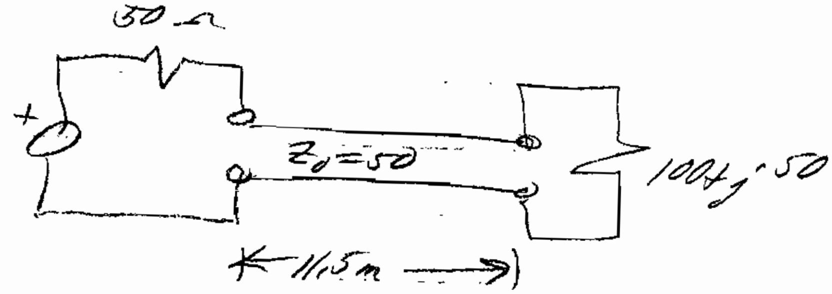
$\beta = \omega\sqrt{LC}$

$$\boxed{Z_0 = \sqrt{\frac{R + j\omega L}{G + j\omega C}} = \sqrt{\frac{L}{C}}\sqrt{\frac{\frac{R}{L} + j\omega}{\frac{G}{C} + j\omega}} = \sqrt{\frac{L}{C}}}$$

EE 434

Homework 4

10.16



(1.092)

$Z_{L_{norm}} = 2 + j$ ⟹ From Chart $\Gamma_{load} = 0.455\, e^{j26.5^\circ}$ ←

rotating 1.092 toward generator [0.2125 + 0.09 = 0.3025]

gives $Z_{in_{norm}} = 1.6 - j1.13 \Rightarrow \boxed{Z_{in} = 80 - j56.5}$ ←

and $\boxed{\Gamma_{in} = 0.455\, e^{-j38^\circ}}$ ←

10.17 Same as 10.16 with $\alpha = 0.00197$ Np/m

$e^{-2\alpha l} = e^{-2 \times 1.97 \times 10^{-3} \times 11.5} = 0.956$ ←

$\therefore$ @ input $|\Gamma| = 0.956 \times 0.455 = 0.435$ ←

(rotation is not changed) so $Z_{in_{norm}} = 1.59 - j1.03$; $\boxed{Z_{in} = 79.5 - j51.5}$ ←

10.21 $Z_{norm_{load}} = \frac{36 + j20}{50} = 0.72 + j0.4$

From chart $Y_{L_{norm}} = 1.06 - j0.58$ ←

$\frac{1}{Z_{L_{norm}}} = \frac{1}{0.72 + j0.4} = \frac{1}{0.8237\, e^{j29.05^\circ}}$

$Z_{L_{norm}} = 1.06 - j0.59$!? ←

Problem 10.16

and 10.17

IMPEDANCE OR ADMITTANCE COORDINATES

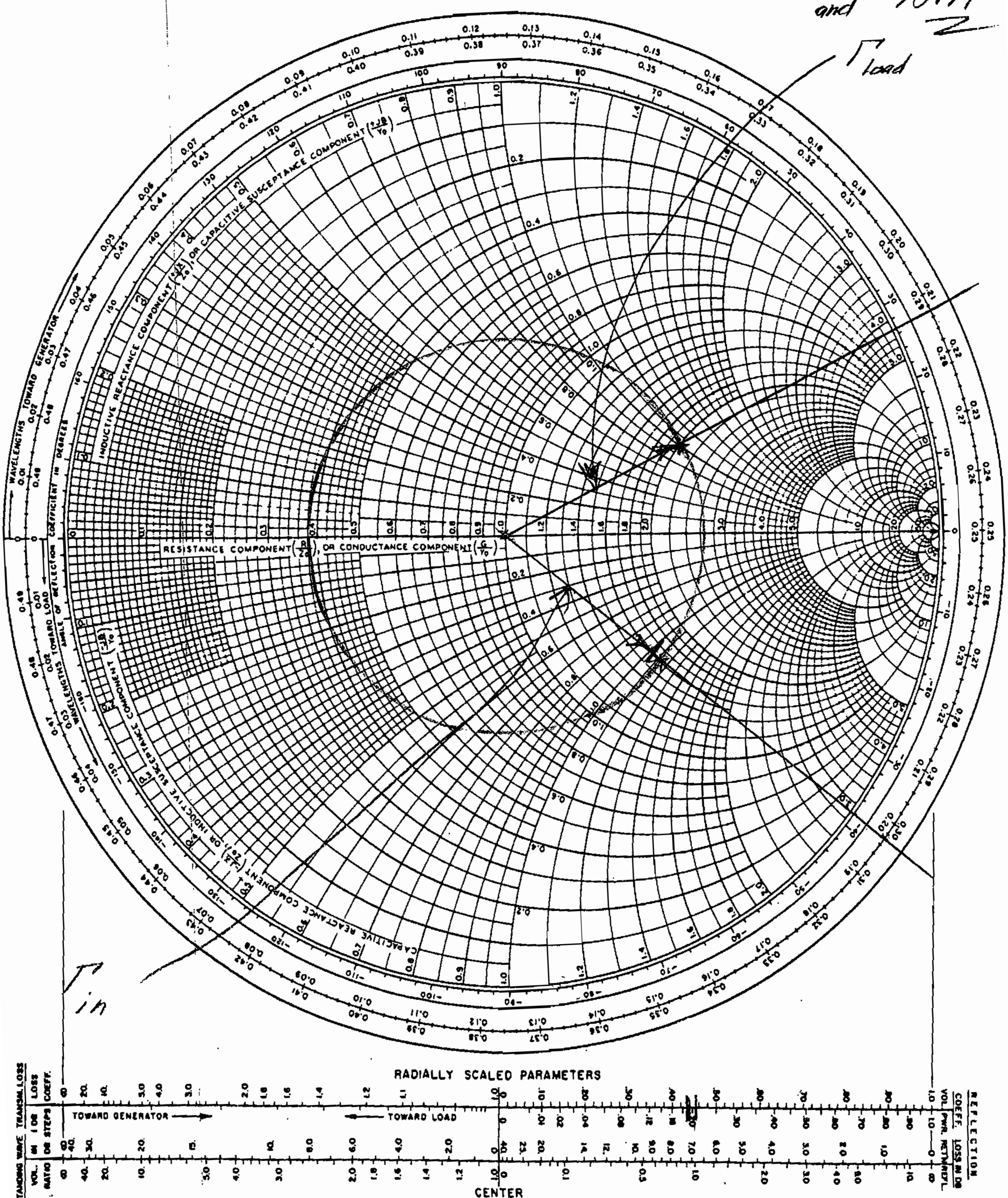


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

IMPEDANCE OR ADMITTANCE COORDINATES

Problem 10.21

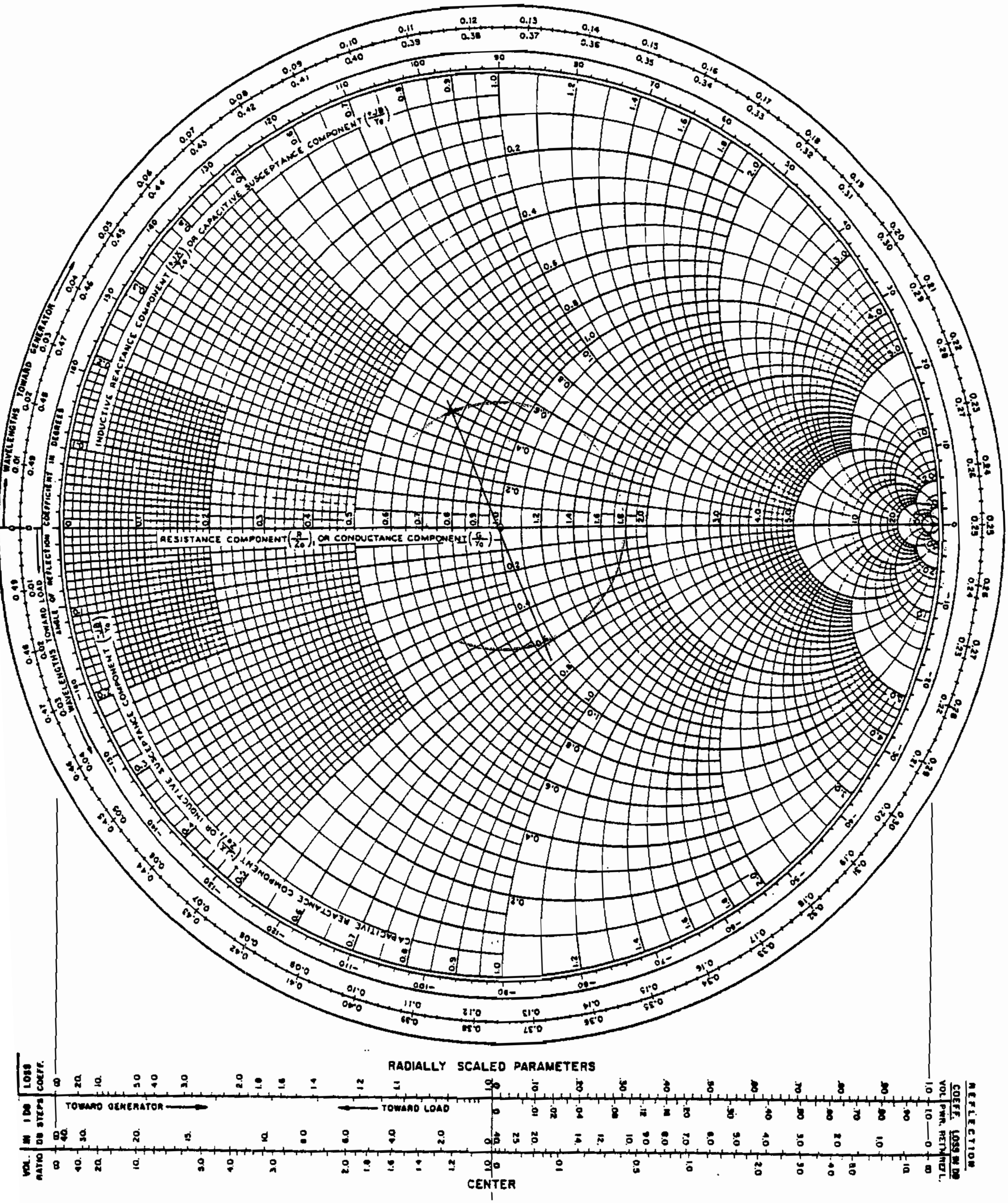


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

10.23 $Y_{in_2} = G_2 + jB_2$; $Y_{in_3} = G_3 + jB_3$; $\hat{I} = \hat{V} Y$

$$P_{ave} = \tfrac{1}{2}\mathrm{Re}\{\hat{V}\hat{I}^*\} = \tfrac{1}{2}\mathrm{Re}\{\hat{V}\hat{V}^* Y^*\} = \tfrac{1}{2}\mathrm{Re}\{|\hat{V}|^2 (G+jB)\}$$

or $P_{ave} = \dfrac{G|\hat{V}|^2}{2}$; $|\hat{V}|$ the same for both lines

$$\therefore \frac{P_{ave_2}}{P_{ave_3}} = \frac{G_2}{G_3} \quad \text{where} \begin{cases} G_2 = \frac{1.83}{50} = 0.0366 \\ G_3 = ? \end{cases}$$

$$z_{L_{3_{norm}}} = \frac{36 + j20}{50} = 0.72 + j0.4$$

from chart $Y_{3_{in\,norm}} = 0.6 - j0.23$ or $\boxed{Y_{3_{in}} = 0.012 - j0.0046}$

so $\boxed{G_3 \text{ for above is } 0.012}$

from Example $Y_1(0)_{norm} = 0.30 + j0.47$; from Smith chart $Z_1(0)_{norm} = 0.97 - j1.57$

$$\therefore \hat{I}_{in} = \frac{100}{100 + 70(0.97 - j1.57)} = \frac{100}{167.9 - j109.9} = \frac{100}{200.67 e^{-j33.2^\circ}}$$

$67.9 - j109.9$

so average power from source $= \tfrac{1}{2}|\hat{I}_{in}|^2 67.9 = 8.43$ Watts

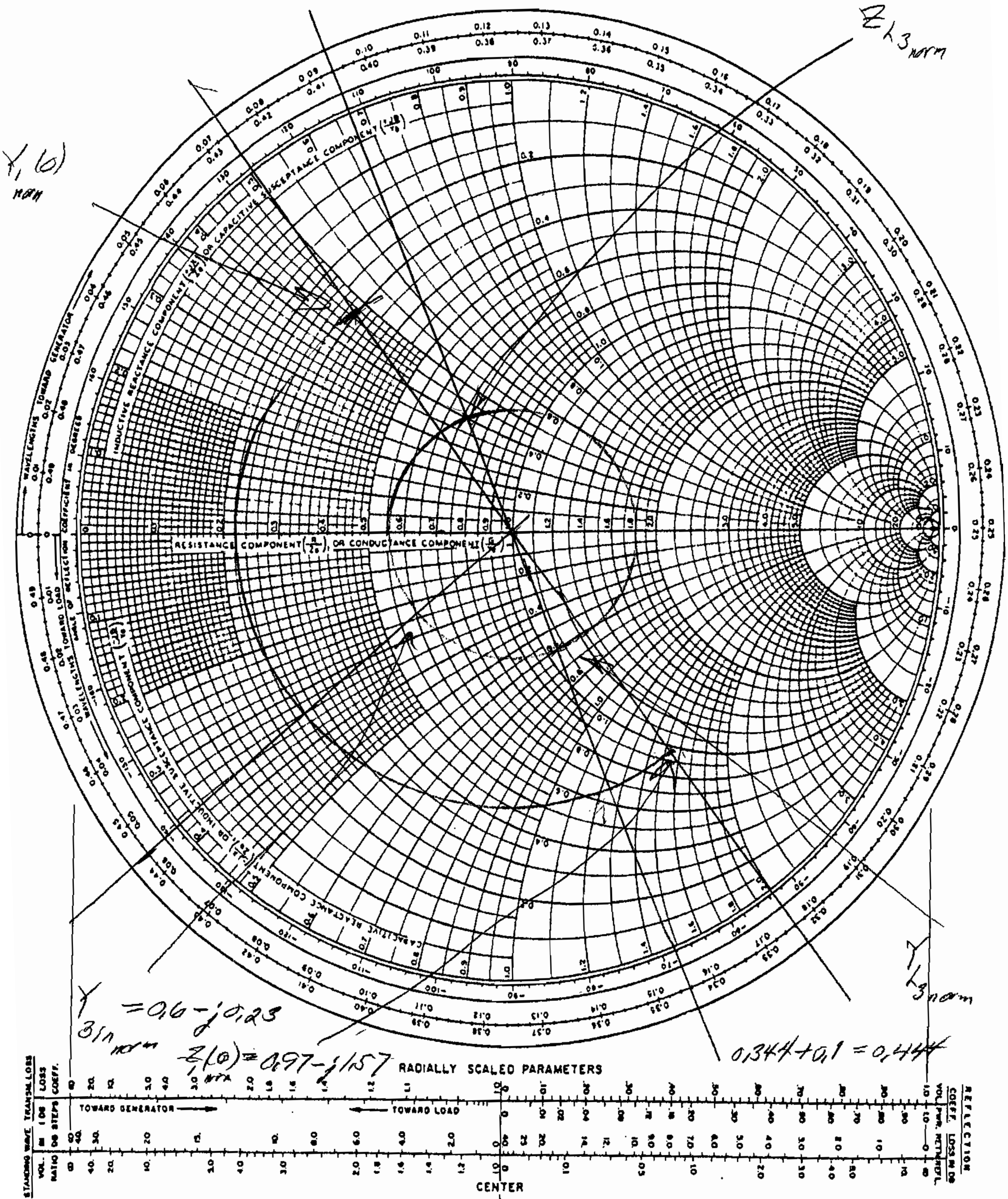
$$\tfrac{1}{2}|\hat{V}_{BB}|^2 (G_2 + G_3) = 8.43 = \tfrac{1}{2}|\hat{V}_{BB}|^2 G_2\left(1 + \frac{G_3}{G_2}\right) = P_{ave_2}\left(1 + \frac{G_3}{G_2}\right)$$

$$\therefore P_{ave_2} = \frac{8.43}{1 + \frac{G_3}{G_2}} = \frac{8.43}{1 + \frac{0.012}{0.0366}} = \boxed{6.34 \text{ Watts}}$$

and $P_{ave_1} = 8.43 - 6.34 = \boxed{2.08 \text{ Watts}}$

Problem 10.23

IMPEDANCE OR ADMITTANCE COORDINATES



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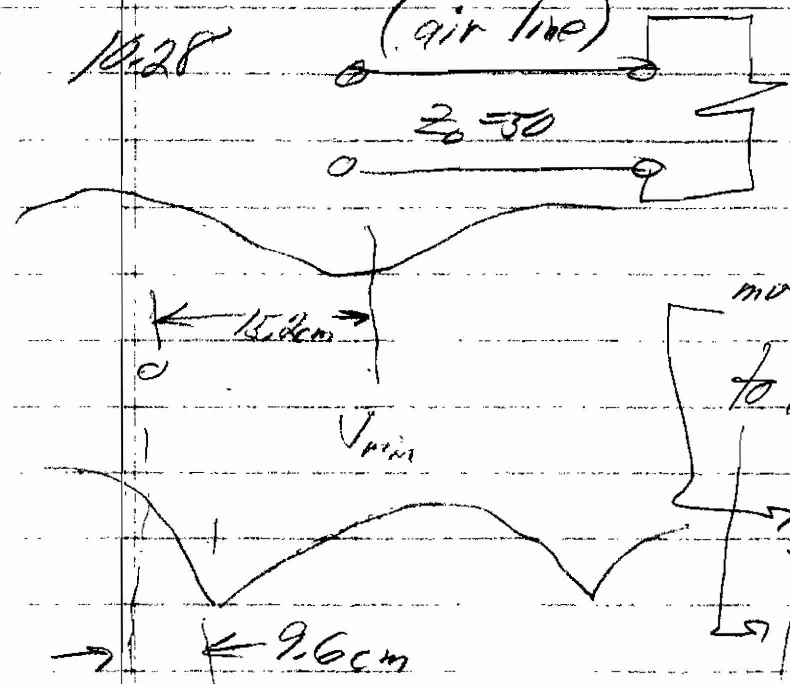
EE434 Homework 5

10.28



$Z_{n @ V_{min}} = \frac{1}{VSWR} = 0.2857$

$VSWR = 3.5$

$f = 600$ MHz

$\lambda = \frac{3\times10^8}{6\times10^8} = 0.5\,m = 50\,cm$

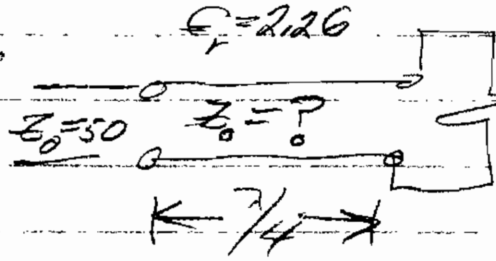
must move $(15.2 - 9.6)_{cm}$ from $Z_{norm @ V_{min}}$

to find $Z_{L_{norm}} = 0.48 + j0.73$

$\boxed{5.6\,cm = 0.112\lambda}$ toward generator

$\boxed{Z_L = 24 + j\,36.5}$

10.36



$f = 500$ MHz

$\lambda = \frac{3\times10^8}{\sqrt{2.26}\times5\times10^8}$

or $\lambda \cong 0.4\,m$

from above $\boxed{\lambda/4 = 0.1\,m}$

Z_0 of $\lambda/4$ transformer $= \sqrt{50\times104} = \boxed{72.1\ \Omega}$

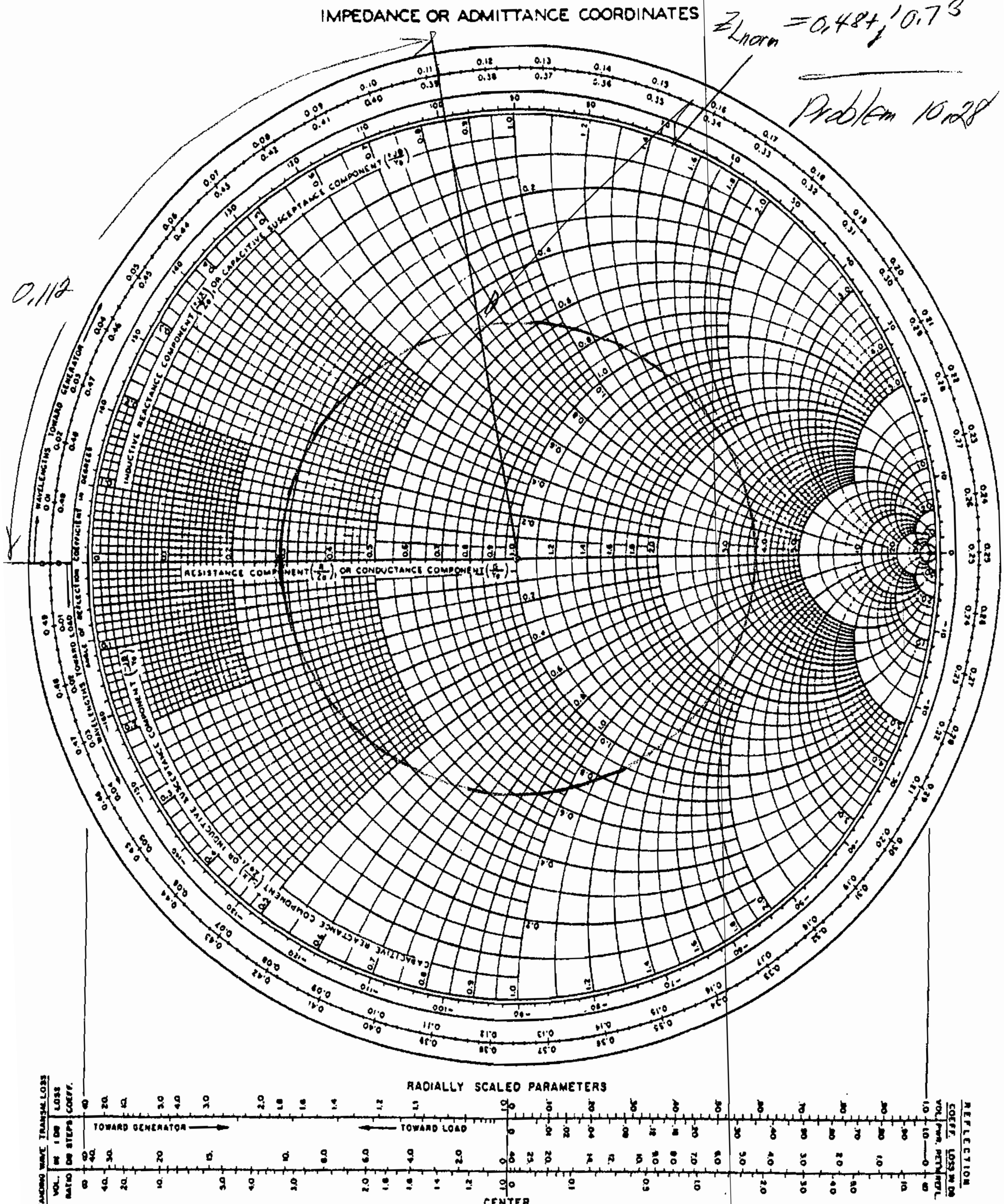


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 434 Homework 6

1. a) rectangular tunnel 15m by 6m

lowest $f_{c_{TE_{10}}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{15}\right)^2} = \frac{3\times10^8}{30} = \boxed{10^7 \text{ Hertz}}$

b) Vertical polarization

c) AM 535 → 1605 KHz ∴ will not propagate

FM 88 → 108 MHz so will propagate

8.11 TE_{10} mode $a = 2b$

$f_c = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{3\times10^8}{2a}$ next lowest cutoff frequency is twice this

want $1.5 f_c = 9\times10^9 = \frac{4.5\times10^8}{2a}$ or $a = \frac{4.5\times10^8}{18\times10^9} = 0.025$ m

or $\boxed{a = 2.5 \text{ cm}}$

8.18 $f = 1.3 f_c$ $E_{max} = 0.75\times2\times10^6$ V/m

rectangular waveguide $a = 2.3$ cm, $b = 1.2$ cm

$\mathcal{P}_z = -\hat{E}_y \times \hat{H}_x^* = H_0^2 \frac{\omega\mu a^2 \beta_{10}}{\pi^2} \sin^2\left(\frac{\pi x}{a}\right)$

$P_{ave} = \frac{1}{2}\text{Re}\int_{y=0}^{b}\int_{x=0}^{a} \mathcal{P}_z \, dx\, dy = \frac{H_0^2 \omega\mu a^2 \beta_{10}}{\pi^2} \cdot b \cdot \frac{a}{2}$

$E_{max} = H_0 \frac{\omega\mu a}{\pi}$ ∴ $P_{ave} = \frac{E_{max}^2 \beta_{10} ab}{4\omega\mu}$

or $P_{ave} = \frac{ab E_{max}^2}{4\omega\mu}\,\omega\sqrt{\mu\epsilon}\,\sqrt{1-\left(\frac{1}{1.3}\right)^2}$

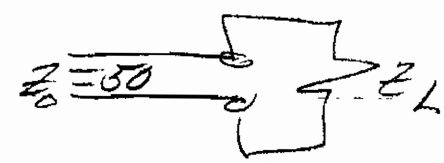
$P_{ave} = E_{max}^2 \frac{ab\sqrt{1-\left(\frac{1}{1.3}\right)^2}}{4\eta_0} = E_{max}^2\, 1.169\times10^{-7}$

giving $P_{ave_{max}} = 2.25\times10^{12} \times 1.169\times10^{-7} = \boxed{0.263\times10^6 \text{ Watts}}$

2

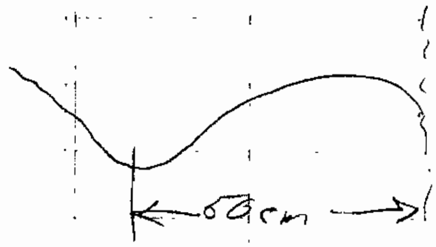
EE 434 Homework 6 (page 2)

10.39.



$f = 500\,MHz$; $\lambda = 60\,cm$
$VSWR = 3.5$

(see Smith chart)



(place stub $(0.25 - 0.17)\lambda = 0.08\lambda = 4.8\,cm$ from V_{min})

The stub can be placed 4.8cm in either direction from V_{min} because these positions are at $Y_{norm} = 1 \pm j$?

a) For stub toward the load from V_{min} $Y_{norm,stub} = -j1.3$
so stub length is $\boxed{(0.355 - 0.25)\lambda = 0.105\lambda = 6.3\,cm}$

b) VSWR from generator to stub = $\underline{1}$ and from stub to load VSWR = 3.5
(VSWR on stub = ∞)

c) For stub toward generator from V_{min} $Y_{norm,stub} = +j1.3$
$\left\{ \begin{array}{l} \text{and stub length is } 23.7\,cm \\ (0.25 + 0.145)\lambda = 0.395\lambda = 23.7\,cm \end{array} \right\}$

Problem 10.39

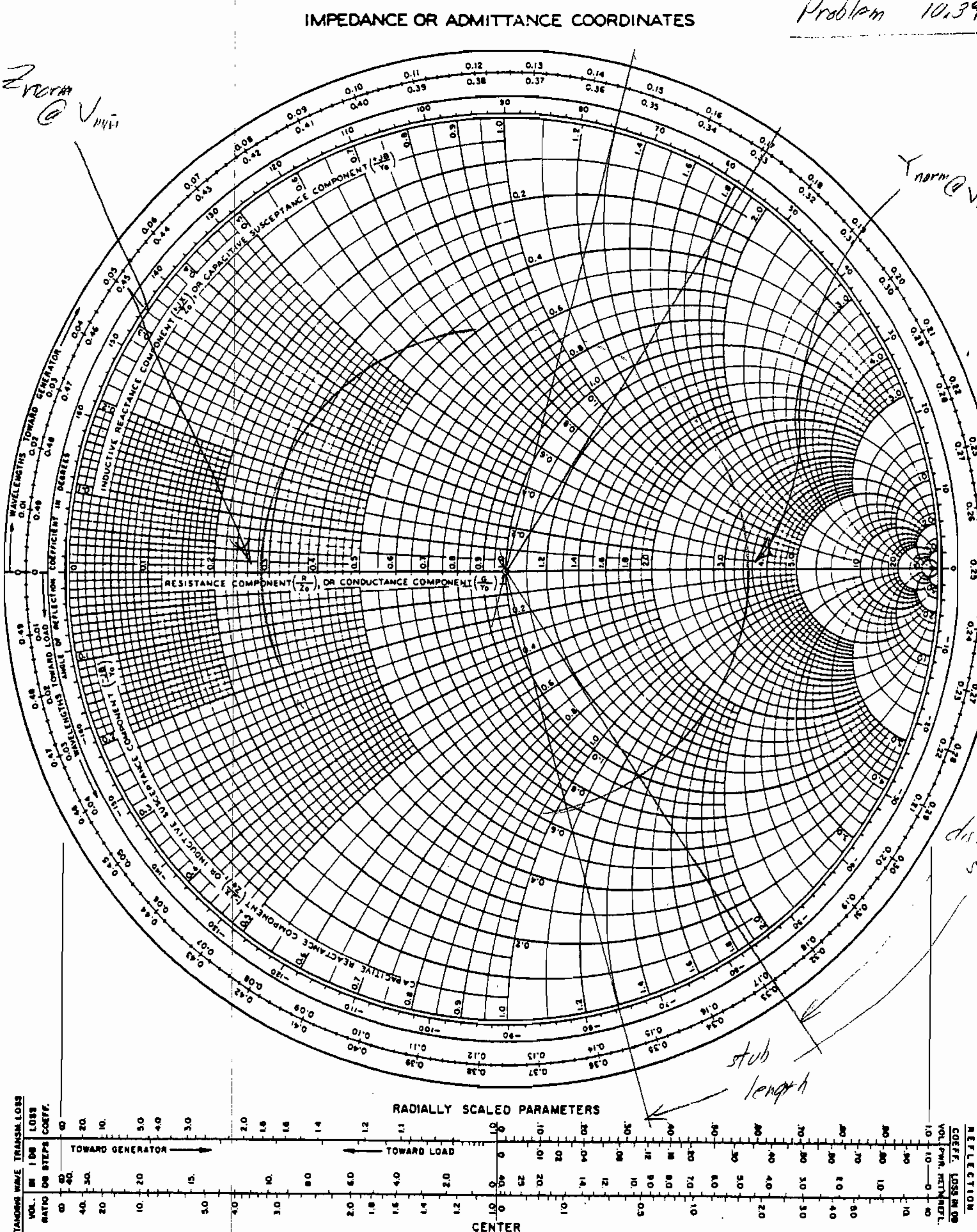


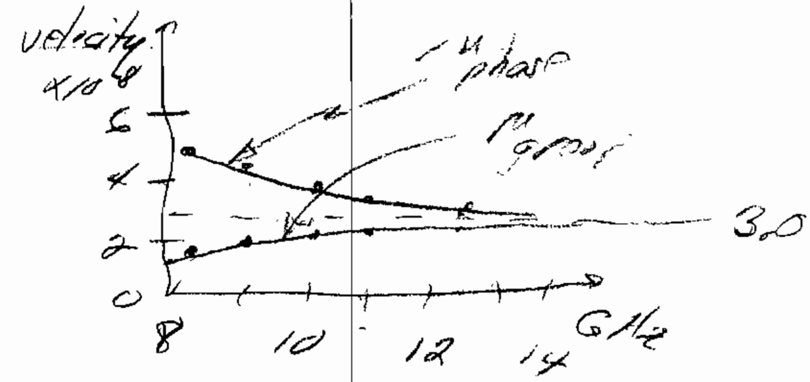
Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 434 Homework 7

1. $\gamma = \sqrt{\omega_c^2\mu\epsilon - \omega^2\mu\epsilon} = \omega\sqrt{\mu\epsilon}\sqrt{\left(\frac{f_c}{f}\right)^2 - 1}$ $\therefore \beta = \omega\sqrt{\mu\epsilon}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$v_{ph} = \frac{\omega}{\beta} = \frac{c}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}$; $v_{group} = \frac{\partial\omega}{\partial\beta} = c\sqrt{1-\left(\frac{f_c}{f}\right)^2}$; $f_c = 6.557$ GHz

f	v_{ph}	v_{group}
8.2 GHz	5×10^8	1.8×10^8
9 GHz	4.38×10^8	2.05×10^8
10 GHz	3.97×10^8	2.26×10^8
11 GHz	3.73×10^8	2.41×10^8
12.4 GHz	3.53×10^8	2.54×10^8



2. a) $\lambda = \frac{2\pi}{\beta} = \frac{2\pi\times3\times10^8}{2\pi\times10^{10}\underbrace{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}_{=0.755}} = 3.97\times10^{-2}$ m

b) must use $v_{group} = 3\times10^8\times0.755 = 2.26\times10^8$

$\therefore$ transit time $= \frac{100}{2.26\times10^8} = \boxed{0.44\,\mu\text{sec}}$

c) Change dielectric so that $f_c < 6$ GHz

3. $f_c = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2}$ $f_{c_{TE_{10}}} = 6.557$ (E field vertical,

for horizontal E field TE_{01} mode has the lowest cutoff frequency. Since $b < 2a$ cutoff frequency for TE_{01} mode is $> 2f_{c_{TE_{10}}}$ (i.e. >13 GHz)

Therefore horizontal portion of the circularly polarized wave will not propagate ∴ Output would be linearly polarized

4. TE_{11} $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p'_{11}}{a}\right) = \frac{3\times10^8}{2\pi}\left(\frac{1.841}{2.54\times10^{-2}}\right) = \boxed{3.46\text{ GHz}}$

TM_{01} $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p_{01}}{a}\right) = \frac{3\times10^8}{2\pi}\left(\frac{2.405}{2.54\times10^{-2}}\right) = \boxed{4.52\text{ GHz}}$

TE_{01} $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p'_{01}}{a}\right) = \frac{3\times10^8}{2\pi}\left(\frac{3.832}{2.54\times10^{-2}}\right) = \boxed{7.2\text{ GHz}}$

EE 434 Homework 8

9.1 $dl = 0.1m$, $\hat{I} = 1$, $f = 10MHz$, $\therefore \lambda = \frac{3\times10^8}{10^7} = 30m$

a) $\theta = \frac{\pi}{2}$

$\theta = \frac{\pi}{2}$ $\quad \frac{d\hat{E}}{e^{-j\beta r}} = \frac{I\,dz}{4\pi}\left\{\frac{120\pi}{r^2} + j\left(\frac{\omega\mu}{r} - \frac{1}{\omega\epsilon r^3}\right)\right\}\bar{a}_\theta$; $\frac{d\hat{H}}{e^{-j\beta r}} = \frac{I\,dz}{4\pi}\left(\frac{1}{r^2} + j\frac{\beta}{r}\right)\bar{a}_\phi$

i) $r = 1m$, $d\hat{H} = e^{-j\beta}\frac{0.1}{4\pi}(1 + j\beta)\bar{a}_\phi$; $\beta = \frac{2\pi\times10^7}{3\times10^8} = 2.094\times10^{-1}$

or $d\hat{H} = 7.958\times10^{-3}\, e^{-j0.2094}\,(1.022\, e^{j11.83°})\,\bar{a}_\phi$

$\boxed{d\hat{H} = 8.133\times10^{-3}\, e^{-j0.167°}\,\bar{a}_\phi}$ ←

ii) $r = 5m$, $d\hat{H} = 7.958\times10^{-3}\, e^{-j59.99°}\underbrace{(0.04 + j0.042)}_{0.058\, e^{j46.4°}}\bar{a}_\phi$

$\boxed{d\hat{H} = 0.4616\times10^{-3}\, e^{-j13.59°}\,\bar{a}_\phi}$ ←

iii) $r = 10m$ $\quad d\hat{H} = 7.958\times10^{-3}\, e^{-j119.98°}\underbrace{(0.01 + j0.02094)}_{0.0232\, e^{j64.47°}}\bar{a}_\phi$

$\boxed{d\hat{H} = 0.1846\times10^{-3}\, e^{-j55.51°}\,\bar{a}_\phi}$ ←

b) our far zone criterion was $r \gg \frac{\lambda}{2\pi}$

$\lambda/2\pi = \frac{30}{2\pi} = 4.77m$

$\therefore$ $r = 1m$ is in the near zone
$r = 5m$ is transitional
and $r = 10m$ is marginal far zone ←

c) see above

$$9.2\quad \overline{P}_{ave} = \tfrac{1}{2}\mathrm{Re}\{\hat{\overline{E}}\times\hat{\overline{H}}^*\} = \tfrac{1}{2}\mathrm{Re}\{\overline{a}_\theta|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(-\frac{\beta}{r^2}+\frac{j}{r^3}\right)$$

$$\times\left(-\frac{j\beta}{r}+\frac{1}{r^2}\right)+\overline{a}_r\left[|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{j\beta}{r}+\frac{\beta}{r^2}-\frac{j}{r^3}\right)\left(\frac{-j\beta}{r}+\frac{1}{r^2}\right)\right]\}$$

$$=\tfrac{1}{2}\mathrm{Re}\{\overline{a}_\theta|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(\frac{j\beta}{r^3}-\frac{\beta}{r^4}+\cancel{\frac{\beta}{r^4}}+\frac{j}{r^5}\right)$$

$$+\overline{a}_r|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta^3}{r^2}+\cancel{\frac{j\beta^2}{r^3}}-\frac{j\beta^2}{r^3}+\frac{\beta}{r^4}-\frac{\beta}{r^4}-\frac{j}{r^5}\right)\}$$

only real term!

$$\therefore\ \overline{P}_{ave}=\tfrac{1}{2}|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta^3}{r^2}\right)\overline{a}_r$$

from 9.14 and 9.15 $|\hat{E}_\theta| = \dfrac{\eta I\,d\ell}{4\pi\beta}$, $|\hat{H}_\phi| = \dfrac{I\,d\ell}{4\pi}$

$$\therefore\ \overline{P}_{ave}=\frac{\eta}{2}\left(\frac{\beta I\,d\ell\sin\theta}{4\pi r}\right)^2\overline{a}_r \quad \text{which is equation 9.21}$$

9.5 (handout) 2 m linear antenna @ 1 MHz

$\lambda = \dfrac{3\times10^8}{10^6} = 3\times10^2$ $\therefore$ a short dipole

for Cu $\sigma = 5.8\times10^7$ giving $\delta = \dfrac{1}{\sqrt{\pi f \mu_0 \sigma}} = 0.066\times10^{-3}$ m

because $\delta <<$ radius of wire $R_{loss} = \dfrac{\ell}{2\pi a\sigma\delta} = 0.83\times10^{-1}\ \Omega$

$R_{rad} = \dfrac{2\pi\eta}{3}\left(\dfrac{\ell}{\lambda}\right)^2 = 0.0351\ \Omega$

$\eta_{eff} = \dfrac{R_{rad}}{R_{rad}+R_{loss}} = \dfrac{0.0351}{0.0351+0.083} = 0.297;$ $\boxed{29.7\%}$

b) $G = \eta_{eff}D = 1.5\times0.297 = \boxed{0.445 \text{ or } -3.5\text{ dB}}$

c) $P_{rad} = \frac{1}{2}I^2R_{rad} = 20$ watts ; $\boxed{I = 33.76\text{ A}}$

$P_{\text{from transmitter}} = \dfrac{P_{rad}}{\eta_{eff}} = \dfrac{20}{0.297} = \boxed{67.34 \text{ Watts}}$

EE 434 Homework 8 (page 3)

9-14 (handout) $\lambda/2$ dipole; 100 MHz $\Rightarrow \lambda = \frac{3\times10^8}{10^8} = 3m$

$$A_{eff} = \frac{\lambda^2}{4\pi} D = \frac{9 \times 1.64}{4\pi} = \boxed{1.17 m^2}$$

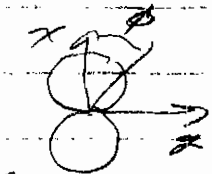
projected area $= 1.5\times10^{-2} = \boxed{0.015 m^2}$

EE 434

Homework 9

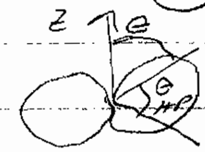
9.3 $\vec{P} = K \frac{\sin\theta \cos\phi}{r^2} \hat{a}_r$ for $-\frac{\pi}{2} \le \phi \le \frac{\pi}{2}, 0 \le \theta \le \pi$

a) azimuthal pattern $= K\cos\phi$



$\cos\left(\frac{\phi_{HP}}{2}\right) = \frac{1}{2}$ or $\boxed{\phi_{HP} = 120^\circ}$

elevation pattern $= K\sin\theta$



$\frac{1}{2}\theta_{HP} = 90^\circ - \sin^{-1}\left(\frac{1}{2}\right)$; $\boxed{\theta_{HP} = 120^\circ}$

b) ϕ_{BW} to first null $= 180^\circ$
θ_{BW} to first null $= 180^\circ$

c) $P_{rad} = K\int_{\phi=-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_{\theta=0}^{\pi} \frac{\sin\theta\cos\phi}{r^2} r^2 \sin\theta \, d\theta \, d\phi = K \sin\phi \Big|_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4}\right]_0^{\pi}$

$P_{rad} = 2K\frac{\pi}{2} = K\pi$; $\therefore D(\theta,\phi) = \frac{K\sin\theta\cos\phi \times 4\pi r^2}{r^2 K\pi} = \boxed{4\sin\theta\cos\phi}$

$\boxed{\text{and } D = D(\theta,\phi)_{max} = 4}$

9.16 $\lambda/2$ dipole, 50MHz $\Rightarrow \lambda = \frac{3\times10^8}{5\times10^7} = 6m$

$\boxed{G = 13dB = 19.95}$ power ratio

so $P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi r}\right)^2 = 10^3 \times 19.95 \times 1.64 \left(\frac{6}{4\pi \times 30\times10^3}\right)^2$

$\boxed{P_r = 8.29\times10^{-6} \text{ Watts}}$

EE434 Homework 10

9.18

$\overset{2\times10^4}{\longleftarrow 20\,\text{Km} \longrightarrow}$

$G_t = 20\,dB = 100$ $\quad G_r = 23\,dB = 199.5$

$P_t = 10W,\ f = 6\,GHz \ \therefore\ \lambda = \frac{3\times10^8}{6\times10^9} = \boxed{0.05\,m}$

a) $\quad \mathcal{O}_t = \frac{\cancel{\mathcal{O}_t}}{P_t/4\pi r^2} \quad$ or $\quad \mathcal{P}_t = \frac{P_t D_t}{4\pi r^2}$

$\therefore\ \mathcal{P}_t = \frac{10\times100}{4\pi\times4\times10^8} = \frac{10^3}{16\pi\times10^8} = 0.0199\times10^{-5}$ watts/m

b) $P_r = \mathcal{P}_t A_r = 1.99\times10^{-7}\times G_r\frac{\lambda^2}{4\pi} = 1.99\times10^{-7}\times199.5\times\frac{25\times10^{-4}}{4\pi}$

$$P_r = 789.59\times10^{-11} = \boxed{7.896\times10^{-9}\ \text{Watts}}$$

---

ck $\quad P_r = P_t G_r G_t \left(\frac{\lambda}{4\pi r}\right)^2 = 10\times199.5\times100\left(\frac{5\times10^{-2}}{4\pi\times2\times10^4}\right)^2$ $\quad 0.03958\times10^{-12}$

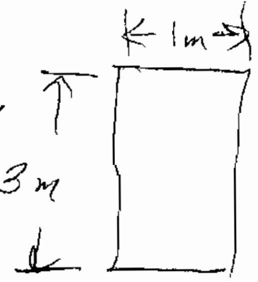
$= 7.896\times10^{-9}$

$\rightarrow\ P_r = \mathcal{P}_t A_r = P_t G_t\left(\frac{1}{4\pi r^2}\right)\left(\frac{\lambda^2}{4\pi}\right) G_r$

$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi r}\right)^2$

EE 434 Homework 10 (page 2)

9.19



$f = 10\ GHz$ so $\lambda = \frac{3\times10^{8}}{10^{10}} = 3\times10^{-2}\ m$

a) $\theta_{HP_{vertical}} \cong \frac{3\times10^{-2}}{3} = \boxed{10^{-2}\ \text{radians} = 0.57^\circ}$

$\theta_{HP_{horizontal}} \cong \frac{3\times10^{-2}}{1} = \boxed{3\times10^{-2}\ \text{rad} = 1.72^\circ}$

b) $D = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi\times3}{9\times10^{-4}} = \boxed{4.19\times10^{4} = 46.22\ dB}$

9.20 $\theta_{HP} = 1.5^\circ$; $f = 20\ GHz$ so $\lambda = \frac{3\times10^{8}}{2\times10^{10}} = 1.5\times10^{-2}$

$\to \theta_{HP} = 0.02618$ radians

a) $D = \frac{4\pi}{\lambda^2} A_{eff}$ and $\theta_{HP} \cong \frac{\lambda}{dia}$ so $dia = \frac{\lambda}{\theta_{HP}} = 0.57\ m$

$A_{eff} = A_{actual} = \frac{\pi\, dia^2}{4} = 0.2578\ m^2$

and $D = \frac{4\pi\times0.2578}{2.25\times10^{-4}} = \boxed{1.44\times10^{4} = 41.58\ dB}$

b) $D = K A_{eff}$ ∴ $\boxed{D_{new} \Rightarrow 2 D_{org}}$

$\theta_{HP} \cong \frac{\lambda}{dia}$ so $\boxed{\theta_{HP_{new}} \Rightarrow \frac{\theta_{HP_{old}}}{\sqrt{2}}}$

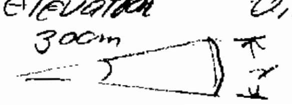
c) $f \Rightarrow 2f$ so $\lambda_{new} = \frac{\lambda_{old}}{2}$

so $\boxed{\theta_{HP_{new}} = \frac{1}{2}\theta_{HP_{old}}}$; $\boxed{D_{new} = 4 D_{old}}$

9.21 94 GHz $\lambda = \frac{3\times10^{8}}{94\times10^{9}} = 0.0319\times10^{-1}\ m$

a) $\theta_{HP_{azimuthal}} = \frac{\lambda}{1} = 3.19\times10^{-3}$ radians $\boxed{= 0.18^\circ}$

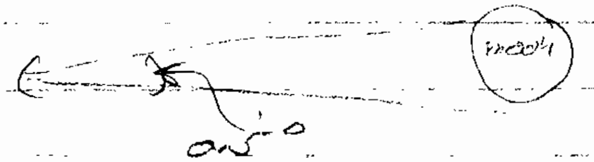
? $\theta_{HP_{elevation}} = \frac{\lambda}{0.1} = 3.19\times10^{-2}$ radians $\boxed{= 1.8^\circ}$

b) 

$x = 300\times3.19\times10^{-3} = 0.957\ m$

EE 434 Homework 10 (page 3)

9.22 100 m parabolic dish, $f = 10$ GHz



$$\theta_{HP_{antenna}} \cong \frac{\lambda}{dia} = \frac{3\times10^{8}}{10^{10}\times10^{2}} = 3\times10^{-4} \text{ radians}$$

distance to moon is 2.389×10^{5} miles $= 3.844\times10^{5}$ Km

$$\theta_{HP_{antenna}} = 3\times10^{-4}\times\frac{180}{\pi} = 171.88\times10^{-4} \text{ degrees}$$

$$\text{diameter of area illuminated by antenna} \Big/ \text{dia of moon} = \frac{\theta_{HP_{ant}} \cancel{\times \text{distance}}}{0.5\times\frac{\pi}{180}\times\cancel{\text{distance}}}$$

$$= \frac{0.017188}{0.5} = 3.438\times10^{-2}$$

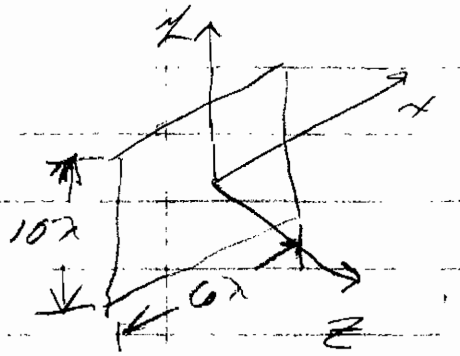
area goes as dia squared

so ratio of area illuminated to total area is:

$$(3.438\times10^{-2})^{2} = 1.18\times10^{-3} = 0.118\times10^{-2}$$

(0.118% of moon is illuminated)

EE 434 Homework 11



$\phi = 0$ pattern (i.e. xz plane)

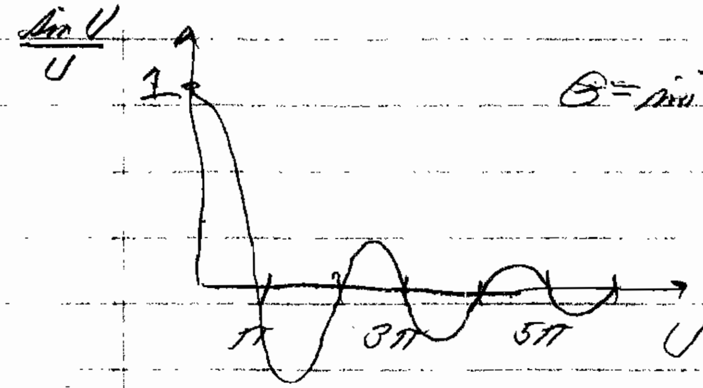
$$\hat{E} = \frac{j\beta E_m^+ ab(1+\cos\theta)e^{-j\beta r}}{4\pi r} \cdot \frac{\sin\left(\frac{\beta a}{2}\sin\theta\right)}{\frac{\beta a}{2}\sin\theta}\,\hat{a}_\theta$$

i.e. $U = \frac{\beta a}{2}\sin\theta$

(aperture is 6λ wide in this plane)

so $\frac{\beta a}{2} = \frac{2\pi a}{2\lambda} = \frac{2\pi 6\lambda}{2\lambda} = 6\pi$ (which is the visible range of U)

i.e. $0 < U < 6\pi$ for $0 < \theta < \frac{\pi}{2}$

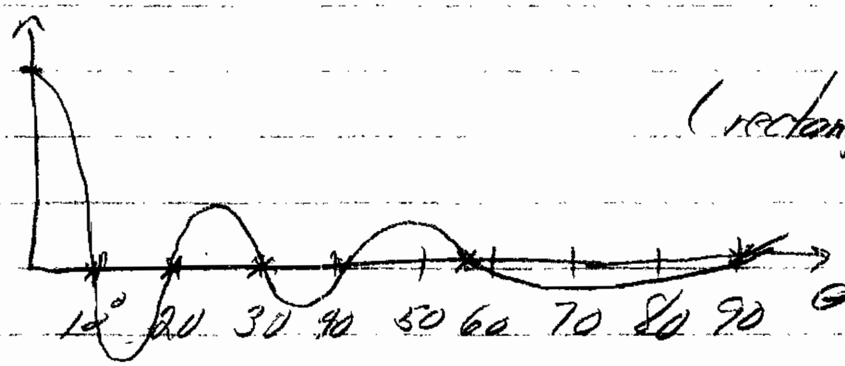


$\theta = \sin^{-1}\left(\frac{U}{6\pi}\right)$

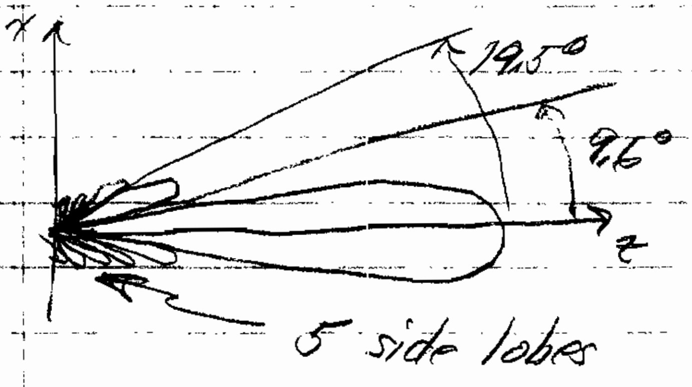
zeros of $\frac{\sin U}{U}$

U	θ
π	$9.59°$
2π	$19.5°$
3π	$30.0°$
4π	$41.8°$
5π	$56.4°$
6π	$90°$

Radiation pattern ($\phi=0$)



(rectangular plot)

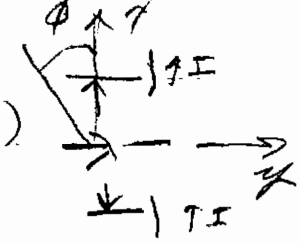


from table in hand out

$\theta_{HP} \cong \frac{50°}{6} = \boxed{8.33°}$

first side lobe is down 13.2 dB

EE 434 Homework 12

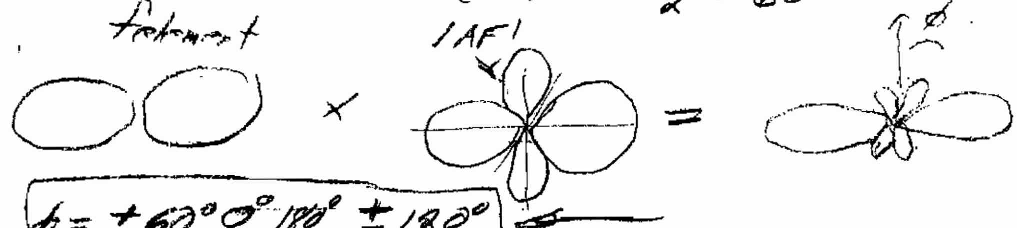


1. A) This is a 2 element array with $\psi = 0, d = \lambda$

So AF zeros @ $X = n\pi = 2\pi \cos\phi \Rightarrow \cos\phi = \frac{n}{2}$

or zero @ $\phi = \cos^{-1}\frac{1}{2} = 60°$

total pattern



zeros @ $\boxed{\phi = \pm 60°, 0°, 180°, \pm 120°}$

B) $\psi = 0$, $d = 4\lambda$ zeros @ $X = n\pi = \frac{2\pi \times 4\lambda}{\lambda}\cos\phi$

or zeros @ $\cos\phi = \frac{n}{8}$

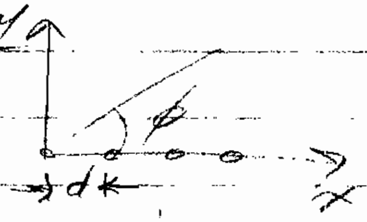
since $X_{zero} = n\pi$ only n odd produces zeroes

i.e. $X = 0, 2\pi, 4\pi \cdots$ are maxima

or zeros @ $\phi = \pm 82.8°, \pm 67.98°, \pm 51.32°, \pm 28.95°$

plus $\lambda/2$ dipole zeros @ $\phi = 0°$

There will also be zeros that are symmetrical to the above ... reflected about the yz plane from text

29 

a) $d = \frac{\lambda}{2}$. Broadside so $\psi = 0$

$x = \beta d \cos\phi$

$$|AF| = \sum_{n=0}^{3} e^{jn\beta d\cos\phi} = \sum_{n=0}^{3} e^{jn\frac{2\pi d}{\lambda}\cos\phi}$$

[see attached plot]

b) $d = \lambda/4$, $\psi = \frac{\pi}{2}$

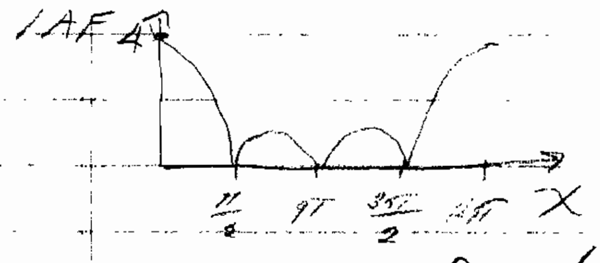
$$|AF| = \sum_{n=0}^{3} e^{jn\left(\frac{2\pi d}{\lambda}\cos\phi - \frac{\pi}{2}\right)} = \sum_{n=0}^{3} e^{jn\left(\frac{\pi}{2}\cos\phi - \frac{\pi}{2}\right)}$$

[see attached plot]

9.11 4 element uniform array, maximum @ $\phi = 45^\circ$ (no part of 2^{nd} main lobe present

$$|AF| = \frac{\sin\frac{4x}{2}}{\sin\frac{x}{2}} \qquad x = \frac{2\pi d}{\lambda}\cos\phi - \psi$$

maximum when $x = 0$ $\therefore$ $\psi = \frac{2\pi d}{\lambda}\cos 45^\circ = \frac{2\pi d}{\lambda\sqrt{2}} = \sqrt{2}\frac{\pi d}{\lambda}$



for $N = 4$; zeros of x

@ $x = \frac{n2\pi}{4} = n\frac{\pi}{2}$

for no part of 2^{nd} main lobe in pattern

$\beta d + \psi \le \frac{3\pi}{2}$, $\beta d \le \frac{3\pi}{2} - \sqrt{2}\frac{\pi d}{\lambda}$

$\frac{2\pi d}{\lambda} + \sqrt{2}\pi\frac{d}{\lambda} \le \frac{3\pi}{2}$ or $\frac{d}{\lambda} \le \frac{\frac{3\pi}{2}}{2\pi + \sqrt{2}\pi} = \frac{3}{4 + 2\sqrt{2}} = \boxed{0.439}$

so $\psi = \sqrt{2}\pi\frac{d}{\lambda} = 1.9516$ [see attached plot]

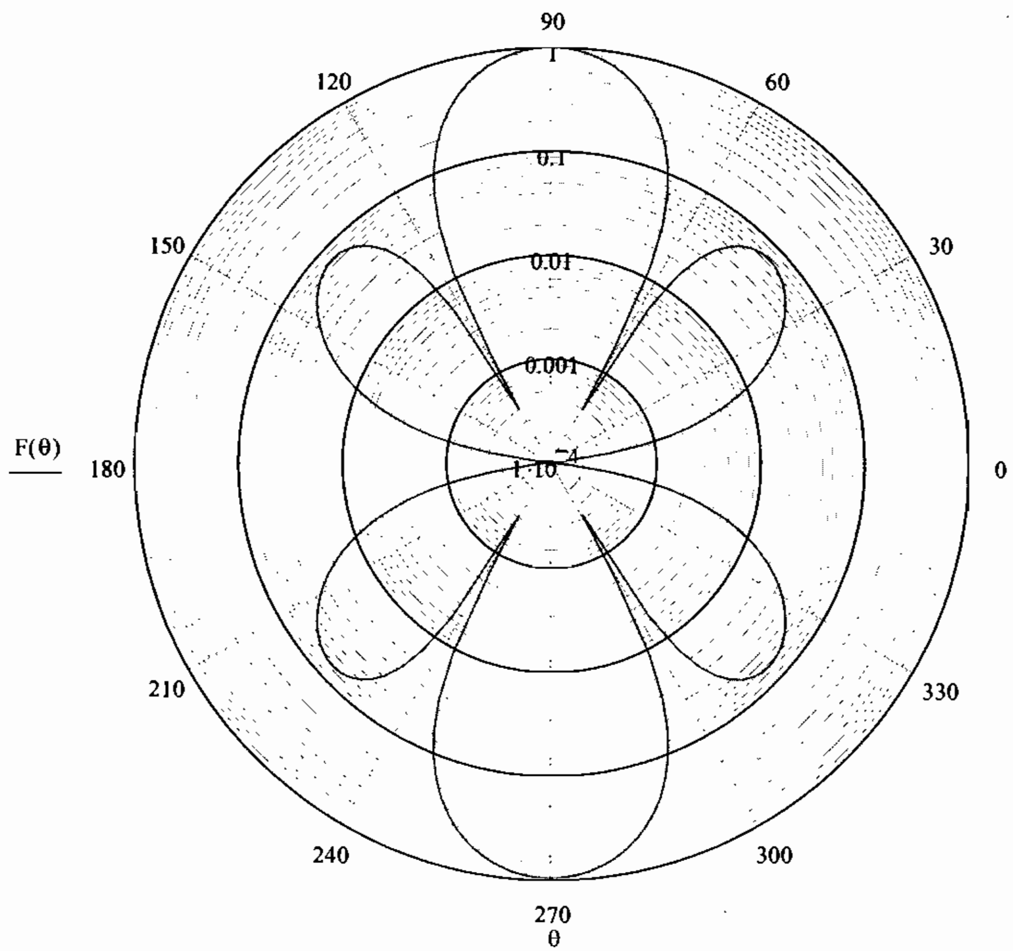
9.9a)

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 4 \quad d := \frac{1}{2}; \quad \phi := 0 \;; \qquad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



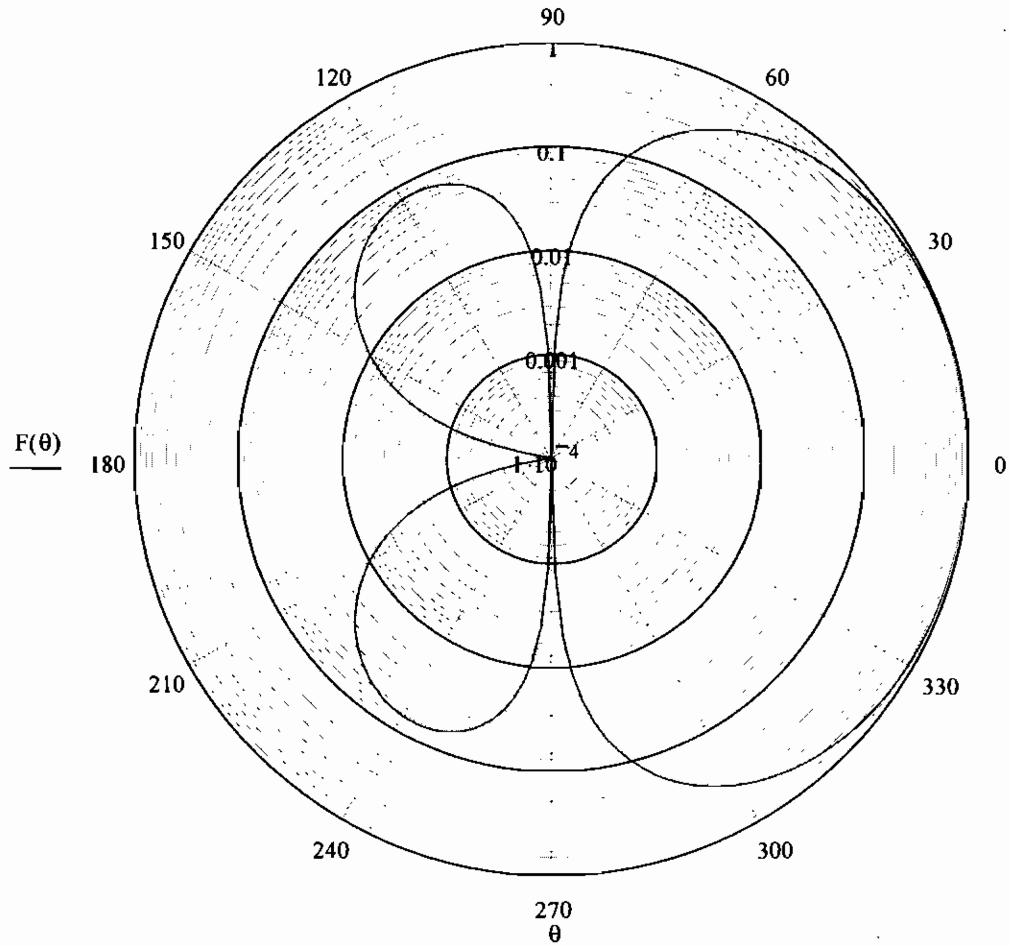
9.9 b)

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 4 \quad d := \frac{1}{4}; \quad \phi := \frac{\pi}{2};$ $\quad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



ANTENNA ARRAY RADIATION PATTERNS

9,11

The radiation pattern of a linear antenna array is given by the following:

$N := 4 \quad d := 0.439 \quad \phi := 1.9516\ ; \qquad j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1; \quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.