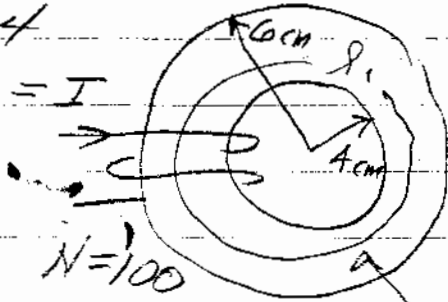
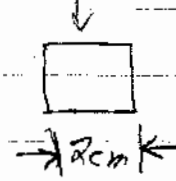


5-4

$0.1A = I$



cross section



$\mu_r = 10^3$

$R = \frac{2\pi r_1 \times 10^{-2}}{10^3 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}}$

$R = 0.0625 \times 10^7$

$\psi = \frac{NI}{A} = \frac{\mu_{rel} \mu_0 NI}{A} = \mu_{rel} \mu_0 \frac{NI}{A}$

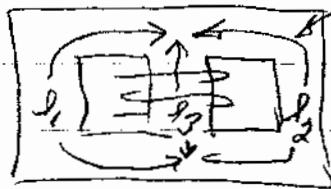
$\psi = \frac{NI}{R}$

or $H_{ave} = \frac{\psi}{\mu A} = \frac{NI}{\mu AR} = \frac{NI \times A}{\mu A R} = \frac{NI}{R} = \frac{10^{-1} \times 10^2}{8\pi \times 5 \times 10^{-2}} = 31.8$

b) $\oint H \cdot dl = NI = Hl$ or $H = \frac{NI}{l}$ as above

With a gap we don't have a path with constant A so we can not take H outside the line integral and calculate its value

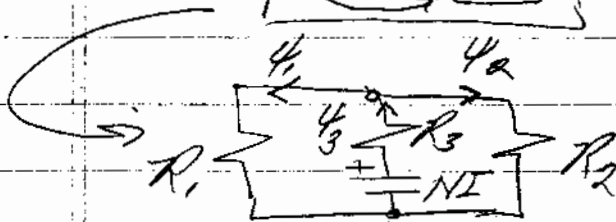
5-5



$\mu_r = 10^4, I = 0.1A, N = 80$

$l_3 = 4cm, l_1 = l_2 = 12cm$

$A = 8cm^2$



$\psi_1 = \psi_2 = \psi_3$

$\psi_3 = \frac{NI}{R_3 + \frac{R_1}{2}} = \frac{8}{\frac{4 \times 10^{-2}}{4\pi \times 10^{-7} \times 10^4 \times 2 \times 10^{-4}} + \frac{6 \times 10^{-2}}{4\pi \times 10^{-7} \times 10^4 \times 2 \times 10^{-4}}}$

$\psi_3 = 201.6 \times 10^{-6} = 0.202m \text{ Webers} = 2\psi_{3,2}$

$B_{ave3} = \frac{\psi_3}{A} = \frac{0.202 \times 10^{-3}}{4 \times 10^{-4}} \approx 1 \text{ Weber/m}^2$

$H_{ave3} = 2H_{ave1} = 2H_{ave2} = \frac{B_{ave3}}{\mu_0 \mu_r} \approx 80$

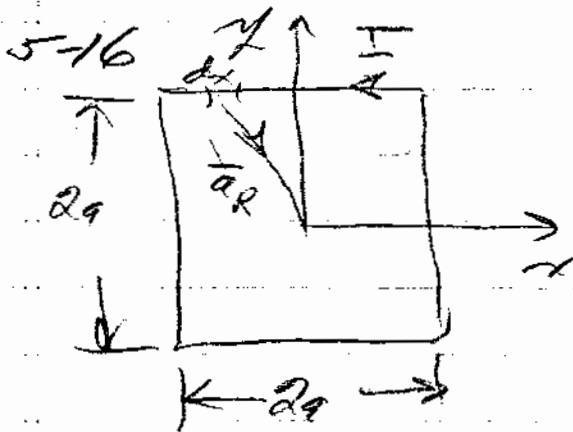
EE 434 Homework 1 (page 2)

5-5 (continued) Have, $I_1 + I_2 + I_3 = nI$?

$$\rightarrow 40 \times 0.12 + 80 \times 0.04 = 8$$

$$4.8 + 3.2 = 8$$

$$\boxed{8 = 8} !$$



for top of square we have:

$$dB = \frac{\mu I dx \bar{a}_x \times \bar{a}_R}{4\pi (x^2 + a^2)}$$

$$\bar{a}_R = \frac{-x\bar{a}_x - a\bar{a}_y}{\sqrt{x^2 + a^2}}$$

$$\therefore dB = \frac{\mu I dx (-a\bar{a}_y)}{4\pi (x^2 + a^2)^{3/2}}$$

$$B_{z\text{top}} = \int_{-a}^a \frac{-\mu I dx a}{4\pi (x^2 + a^2)^{3/2}} = -\frac{\mu I a}{4\pi} \left\{ \frac{x}{a^2 \sqrt{x^2 + a^2}} \right\}_{-a}^a$$

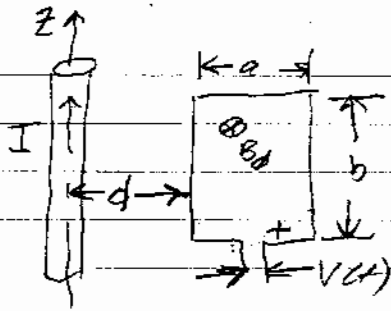
$$B_{z\text{top}} = \frac{-\mu I}{4\pi} \left\{ \frac{-a}{a\sqrt{a^2 + a^2}} - \frac{a}{a\sqrt{a^2 + a^2}} \right\}$$

$$B_{z\text{top}} = \frac{\mu I}{2\pi a \sqrt{2}} = \frac{\mu I \sqrt{2}}{4\pi a}$$

from symmetry all sides contribute the same

$$\therefore B_{z\text{total}} = \frac{\sqrt{2} \mu I}{\pi a}$$

5-19



$I(t) = I_m \sin \omega t$; $\oint \vec{H} \cdot d\vec{l} = I$

a) $H_\phi = \frac{I}{2\pi r} = \frac{I_m \sin \omega t}{2\pi r}$

clockwise integration

b) $\oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \left\{ \int_{\rho=d}^{d+a} \int_{z=0}^b \frac{\mu_0 I_m \sin \omega t}{2\pi \rho} dz d\rho \right\}$

$\oint \vec{E} \cdot d\vec{l} = - \frac{d}{dt} \left\{ \frac{\mu_0 I_m \sin \omega t}{2\pi} b \ln \frac{d+a}{d} \right\} = - \frac{\mu_0 I_m b \ln \frac{d+a}{d}}{2\pi} \omega \cos \omega t$

(right hand terminal positive)

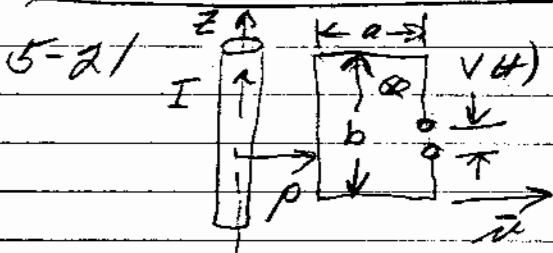
(see A page 282 of text)

For $I_m = 10A$, $f = 20kHz$, $d = 4 \times 10^{-3}m$, $a = b = 0.1m$

$V(t) = - \frac{2\pi \times 10 \times 10^{-7} \times 0.1 \times 10 \times 10^4}{2\pi} \ln \left[1 + \frac{0.1}{4 \times 10^{-3}} \right] \cos(2\pi \times 2 \times 10^4 t)$

or $V(t) = - 81.9 \cos(4\pi \times 10^4 t) mV$

c) 2 times above result with opposite polarity

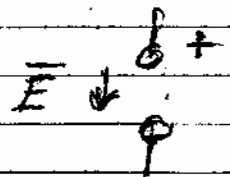


$\oint \vec{E} \cdot d\vec{l} = \oint \vec{v} \times \vec{B} \cdot d\vec{l}$

$\vec{B} = \frac{\mu_0 I}{2\pi r} \hat{\phi}$ (integrate clockwise)

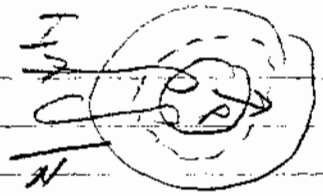
$\oint \vec{E} \cdot d\vec{l} = \int \vec{v} \times \vec{B} \cdot d\vec{l} = \int_0^b v \hat{y} \times \frac{\mu_0 I}{2\pi \rho} \hat{\phi} \cdot dz \hat{z} + \int_0^b v \hat{y} \times \frac{\mu_0 I}{2\pi(\rho+a)} \hat{\phi} \cdot dz \hat{z}$

$\oint \vec{E} \cdot d\vec{l} = \frac{\mu_0 I}{2\pi} \left\{ \frac{b}{\rho} - \frac{b}{\rho+a} \right\} a$



(top terminal positive)

$$5-23 \quad U_m = \frac{1}{2} \int_{\text{vol}} \vec{B} \cdot \vec{H} d\text{vol}$$



$$a) \int \vec{H} \cdot d\vec{l} = NI \quad \therefore \quad H_\phi = \frac{NI}{2\pi r}$$

$$\text{so } U_m = \frac{1}{2} \int_{z=0}^d \int_{\rho=0}^b \int_{\phi=0}^{2\pi} \frac{\mu N^2 I^2}{4\pi^2 \rho^2} \rho d\rho d\phi dz$$

$$U_m = \frac{\mu N^2 I^2 d}{2 \times 4\pi^2} \ln\left(\frac{b}{a}\right) = \frac{1}{2} LI^2$$

$$\therefore L = \frac{2U_m}{I^2} = \frac{\mu N^2 d \ln\left(\frac{b}{a}\right)}{4\pi^2}$$

$$\text{or } L = \frac{\mu d N^2}{2\pi^2} \ln\left(\frac{b}{a}\right)$$

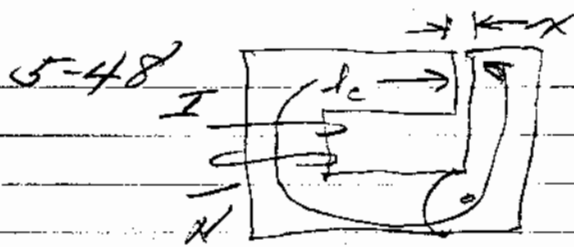
$$b) \quad I = 2, \quad \mu_r = 4 \times 10^3, \quad a = 10^{-2}, \quad b = 3 \times 10^{-2}, \quad d = 2 \times 10^{-2}$$

$$N = 150$$

$$U_m = 79.1 \times 10^{-2} \text{ J} \quad \leftarrow$$

$$L = \frac{2U_m}{I^2} = 0.395 \text{ H} \quad \leftarrow$$

c) if $\mu_r = f(I)$ or $f(H)$ which would be true for ferromagnetic material!



$$a) \phi_m = \frac{NI}{R_{\text{core}} + R_{\text{gap}}}$$

$$b) L = \frac{\lambda}{I} = \frac{N\phi_m}{I} = \frac{N^2}{R_{\text{total}}} \leftarrow$$

$$\infty U_m = \frac{1}{2} L I^2 = \frac{1}{2} \frac{N^2 I^2}{R_{\text{total}}} \leftarrow$$

$$\text{in general } F_x = \frac{\partial U_m}{\partial x} = \frac{I^2}{2} \frac{\partial L}{\partial x} = - \frac{\phi_m^2}{2} \frac{\partial R}{\partial x} \leftarrow$$

$$\left\{ \begin{array}{l} \text{for } l_c = 0.12, A_c = 4 \times 10^{-4}, \tau = 1.5 \times 10^{-3}, I = 1.25 \text{ A}, N = 200 \\ \mu = 10^5 \mu_0 \end{array} \right.$$

$$R_{\text{total}} = \frac{0.12}{10^5 \times 4\pi \times 10^{-7} \times 4 \times 10^{-4}} + \frac{\tau}{4\pi \times 10^{-7} \times 4 \times 10^{-4}} = 2.387 \times 10^3 + 19.89 \times 10^8$$

$$\text{for the above } R_{\text{total}} = 2.387 \times 10^3 + 19.89 \times 10^8 \times 1.5 \times 10^{-3}$$

$$\infty \phi_m = \frac{200 \times 1.25}{29.84 \times 10^5} = 8.37 \times 10^{-5} \text{ Webers } \leftarrow$$

$$\text{so } F_x = - \frac{1}{2} (8.37 \times 10^{-5})^2 \times 19.89 \times 10^8 = -6.97 \text{ Newtons } \leftarrow$$

$$B_{\text{ave}} = \frac{\phi_m}{A} = 0.21 \text{ W/m}^2 \leftarrow$$

$$H_{\text{ave gap}} = \frac{B_{\text{ave}}}{\mu_0} = 1.67 \times 10^5$$

$$H_{\text{ave iron}} = \frac{0.21}{1.4 \times 10^{-7} \times 10^5} = 1.67$$

$$L = \frac{N^2}{R_{\text{total}}} = \frac{4 \times 10^4}{29.84 \times 10^6} = 13 \text{ mH}$$

$$U_m = \frac{1}{2} I^2 L = 1.02 \times 10^{-2} \text{ J}$$

$$\text{if } \tau = 0.75 \text{ mm } R_{\text{new}} = \frac{R_{\text{old}}}{2}; \phi_{\text{new}} = 2\phi_{\text{old}} \text{ so } F_{x \text{ new}} = 4F_x \leftarrow$$

3,48 (continued) if $n \rightarrow 0$ $\frac{dR}{dr}$ is the same

$$\text{but } R = R_{\text{ind}} = 2.387 \times 10^9$$

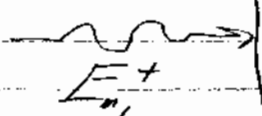
$$\text{so } \rho_{\text{in}} = \frac{2.5 \times 10^9}{2.387 \times 10^9} = 1.047 \times 10^{-1}$$

$$F_x = -\frac{1}{2} (1.047 \times 10^{-1})^2 \times 1.99 \times 10^9 = \boxed{-1.09 \times 10^7 \text{ N}}$$

6-5 (air)

$$E = \gamma_r \epsilon_0$$

$$\frac{E_{m2}^+}{E_{m1}^+} = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\eta_2 \frac{1}{\eta_2}}{\eta_1 + \eta_2 \frac{1}{\eta_2}}$$



$$\text{so } \boxed{\frac{E_{m2}^+}{E_{m1}^+} = \frac{2}{1 + \sqrt{\epsilon_r}}}$$

$$\frac{E_{m1}^-}{E_{m1}^+} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \boxed{\frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}}}$$

b) $E_{m1}^+ = 100$, $\epsilon_r = 2.25$, $\sqrt{\epsilon_r} = 1.5$

$$\text{so } E_{m2}^+ = 100 \frac{2}{2.5} = \boxed{80}$$

$$E_{m1}^- = 100 \frac{-0.5}{2.5} = \boxed{-20}$$

c) $\epsilon_r = 81$; $\sqrt{\epsilon_r} = 9$

$$\text{so } E_{m2}^+ = \frac{100 \times 2}{10} = \boxed{20 \frac{\text{V}}{\text{m}}}$$

$$E_{m1}^- = \frac{100 \times (-8)}{10} = \boxed{-80 \frac{\text{V}}{\text{m}}}$$

6-17 $\gamma_1, \eta_1 \mid \begin{matrix} \gamma_2, \epsilon_2 \\ \gamma_2 = 0 \end{matrix} \mid \gamma_3, \eta_3$ $\gamma_2 = j/\lambda_2$
 $z=0$ $z=d = \lambda/4$ $e^{j\beta d} = e^{j\pi/2} = j$
 $\hookrightarrow = e^{j\frac{\pi}{2}} \frac{\gamma_2}{\eta_2} = j$

\therefore from 6-99a $Z(0) = \eta_2 \frac{j(\eta_3 + \eta_2) + j(\eta_2 - \eta_3)}{j(\eta_3 + \eta_2) + j(\eta_2 - \eta_3)} = \frac{2\eta_2^2}{2\eta_3} = \frac{\eta_2^2}{\eta_3}$ ←

6-18 $\epsilon_{r3} = 2.56$ in the above problem we need $Z(0) = \eta_1$

$\therefore \eta_2 = \sqrt{\eta_1 \eta_3}$ assuming $\mu_1 = \mu_2 = \mu_3 = \mu_0$

this becomes $\frac{1}{\epsilon_2} = \frac{1}{\sqrt{\epsilon_1 \epsilon_3}}$; $\epsilon_{r2} = \sqrt{\epsilon_{r1} \epsilon_{r3}}$

$\epsilon_{r2} = \sqrt{1 \cdot 2.56} = 1.6$ ←

reciprocal because we obtain the same value of η_2 if η_1 and η_3 are interchanged?

6-20 problem 6-17 with $d = \frac{\lambda}{2}$; $e^{j\beta d} = e^{j\frac{2\pi}{\lambda} \frac{\lambda}{2}} = -1$

$\therefore Z_0 = \eta_2 \frac{-\eta_3 + \eta_2 - \eta_3 + \eta_2}{-\eta_3 - \eta_2 + \eta_3 - \eta_2} = \frac{\eta_2(-2\eta_3)}{-2\eta_2} = \eta_3$ ←

same result whenever $e^{j\frac{2\pi}{\lambda} d} = -1$

or $d = n \frac{\lambda}{2} \Rightarrow e^{j\frac{2\pi}{\lambda} \frac{n\lambda}{2}} = e^{jn\pi}$

$\rightarrow n$ odd

$$6-35 \quad \textcircled{1} \\ \mu_0, \epsilon_0$$

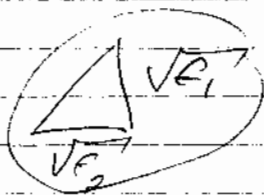
$$\textcircled{2} \\ \mu_0, 3\epsilon_0$$

$$\theta_i^\circ = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\text{so from } \textcircled{1} \rightarrow \textcircled{2} \quad \theta_i^\circ = \tan^{-1} \sqrt{3} = 60^\circ$$

$$\text{from } \textcircled{2} \rightarrow \textcircled{1} \quad \theta_i^\circ = \tan^{-1} \sqrt{\frac{1}{3}} = 30^\circ$$

must be // polarized } there is no Brewster angle?
 for \perp polarization }



$$\Rightarrow \tan^{-1} \sqrt{\frac{\epsilon_1}{\epsilon_2}} = 90^\circ - \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

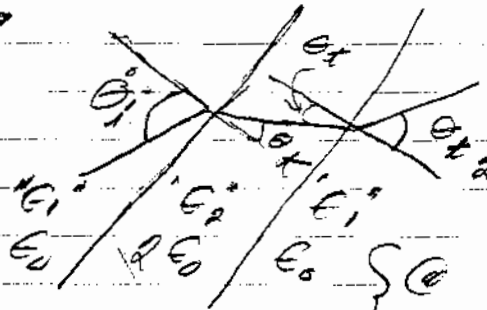
so we see that θ_i° 's from opposite directions are always complements

6-36 μ_0, ϵ_0 $\mu_0, 3\epsilon_0$ 

$$\theta_{\text{critical}} = \sin^{-1} \sqrt{\frac{1}{3}} = 35.26^\circ$$

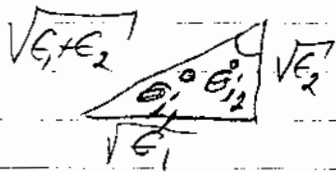
polarization not a factor

6-37



$$\theta_i^0 = \tan^{-1} \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\frac{\sin \theta_t}{\sin \theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$



$$\left. \begin{array}{l} \text{first interface} \\ \sin \theta_t = \sin \theta_i \sqrt{\frac{\epsilon_1}{\epsilon_2}} \end{array} \right\} \quad (1)$$

$\left. \begin{array}{l} \text{this } \theta_t \text{ is the angle of incidence} \\ \text{at the second interface} \end{array} \right\}$

$$\therefore \frac{\sin \theta_{t2}}{\sin \theta_1 \sqrt{\frac{\epsilon_1}{\epsilon_2}}} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\text{or } \boxed{\sin \theta_{t2} = \sin \theta_1} \quad \boxed{\theta_{t2} = \theta_1} \quad \leftarrow$$

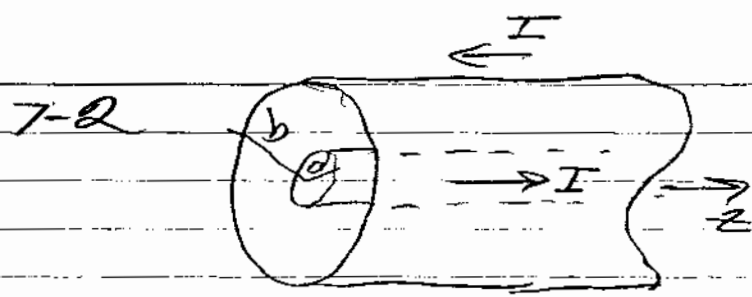
$$\text{Must show that } \theta_t = \theta_{t2}^0 = \tan^{-1} \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$

From (1) and the triangle shown above!

$$\sin \theta_t = \sin \theta_1 \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1 \epsilon_2}} \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sqrt{\frac{\epsilon_1}{\epsilon_1 \epsilon_2}} \quad \leftarrow$$

$$\text{or } \sin \theta_t = \sin \theta_{t2} \Rightarrow \theta_t = \theta_{t2}$$

So there is no reflection from the 2nd surface



in outer conductor

$$E_z = -\frac{J_z}{\sigma}$$

$$J_z = \frac{I}{\pi c^2 - \pi b^2}$$

$$H_\phi = \frac{I}{2\pi\rho} \leftarrow \text{for } a < \rho < b \text{ (found many times)}$$

$$\overline{E} \times \overline{H} \Big|_{\rho=b} = \frac{J_z}{\sigma} \cdot \frac{I}{2\pi b} \hat{\phi} = \frac{I^2}{2\pi^2 b \sigma (c^2 - b^2)} \hat{\phi}$$

in general $R = \frac{L}{\sigma A}$

$$P_{\text{into outer conductor}} / \text{length } L = \int_{z=0}^L \int_{\phi=0}^{2\pi} \frac{I^2}{2\pi^2 b \sigma (c^2 - b^2)} \cdot b \, d\phi \, dz = \frac{I^2 L}{\pi \sigma (c^2 - b^2)} = I^2 R$$

$$7-13 \quad \hat{E}_T = \hat{E}_m e^{-\alpha z} e^{j\beta z} [1 + \Gamma] ; \quad \hat{H}_T = \frac{\hat{E}_m}{\eta} e^{-\alpha z} e^{j\beta z} [1 - \Gamma]$$

$$P_{\text{ave}} = \frac{1}{2} \text{Re} \{ \hat{E} \times \hat{H}^* \} = \frac{1}{2} \text{Re} \left\{ \frac{|\hat{E}_m|^2}{\eta} (1 + \Gamma)(1 - \Gamma^*) \right\} \hat{a}_z$$

$$P_{\text{ave}} = \frac{|\hat{E}_m|^2}{2\eta} \text{Re} \{ 1 + \Gamma - \Gamma\Gamma^* - |\Gamma|^2 \} \hat{a}_z$$

$\Gamma\Gamma^*$ is imaginary

$$\therefore P_{\text{ave}} = \frac{|\hat{E}_m|^2}{2\eta} \{ 1 - |\Gamma|^2 \} \hat{a}_z = P_{\text{ave}}^+ + P_{\text{ave}}^-$$

$$P_{\text{ave}} \text{ through area } A = \overline{P_{\text{ave}}} A = P_{\text{ave}}^+ + P_{\text{ave}}^-$$

$$\therefore \frac{|P_{\text{ave}}^-|}{|P_{\text{ave}}^+|} = |\Gamma|^2$$

$\text{return loss} = 10 \log_{10} |\Gamma|^2$

$$a) \quad 7-17 \quad P_{avg} = \frac{1}{2} \text{Re} \left\{ \frac{|\hat{E}_m|^2 e^{-2\alpha z}}{\eta} e^{j\theta} [1+\Gamma][1-\Gamma] \right\}$$

$$P_{avg} = \frac{|\hat{E}_m|^2 e^{-2\alpha z}}{2\eta} \text{Re} \left\{ \cos\theta + j\sin\theta [1+\Gamma + \Gamma^* - \Gamma - \Gamma^*] \right\}$$

$$\Rightarrow P_{avg} = \frac{|\hat{E}_m|^2 e^{-2\alpha z}}{2\eta} \left\{ \cos\theta [1-|\Gamma|^2] - \sin\theta [2\Gamma_i] \right\} \quad (1)$$

$$b) \quad \text{For } \Gamma = 0 \text{ this becomes } P_{avg} = \frac{|\hat{E}_m|^2 e^{-2\alpha z}}{2\eta} \cos\theta$$

and for a lossless region $\alpha=0, \theta=0$

$$\text{so } P_{avg} = \frac{|\hat{E}_m|^2}{2\eta} \quad (\text{Eq. 7-59 in text})$$

$$c) \quad P_{avg} = \frac{1}{2} \text{Re} \left\{ \frac{\hat{E}_m^+ e^{-\alpha z} e^{j\beta z} + \hat{E}_m^- e^{-\alpha z} e^{j\beta z}}{\eta} \right\} = \frac{|\hat{E}_m|^2 e^{-2\alpha z} \cos\theta}{2\eta}$$

positive wave only

$$P_{avg} = \frac{1}{2} \text{Re} \left\{ \frac{\hat{E}_m^+ e^{-\alpha z} e^{j\beta z} + \hat{E}_m^* e^{-\alpha z} e^{j\beta z} (-\Gamma^*)}{\eta e^{j\theta}} \right\} = \frac{|\hat{E}_m|^2 e^{-2\alpha z}}{2\eta} |\Gamma|^2$$

negative wave only

Summing the above two results would miss the $-\sin\theta [2\Gamma_i]$ term in equation (1) above!

$$7-21 \quad P_{\text{ave sun @ earth}} = 1340 \text{ W/m}^2$$

assuming a single frequency wave we have:

$$P_{\text{ave}} = \frac{|\hat{E}_m|^2}{2\eta} \quad \text{or} \quad |\hat{E}_m| = \sqrt{2 \times 377 \times 1340} = \boxed{1 \times 10^3 \text{ V/m}} \leftarrow$$

$$\text{and } |\hat{H}_m| = \frac{|\hat{E}_m|}{\eta} = \boxed{2.65 \text{ A/m}} \leftarrow$$

$$P_{\text{ave total from sun}} = 4\pi (1.48 \times 10^{11})^2 \times 1.34 \times 10^3 = \boxed{36.88 \times 10^{25} \text{ Watts}} \leftarrow$$

P_{max} for x -band waveguide @ 10 GHz

$$\hat{E}_y = E_{y\text{max}} \sin \frac{\pi}{a} x \quad ; \quad \hat{H}_x = -E_{y\text{max}} \sqrt{\frac{\epsilon'}{\mu}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \sin \frac{\pi}{a} x$$

$$\bar{P}_{\text{ave}} = \frac{1}{2} \text{Re} \{ \hat{E} \times \hat{H}^* \} = \frac{1}{2} \text{Re} \left\{ \frac{1}{2} E_{y\text{max}}^2 \sin^2 \left(\frac{\pi}{a} x \right) \sqrt{\frac{\epsilon'}{\mu}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \right\}$$

$$P_{\text{ave}} = \frac{1}{2} E_{y\text{max}}^2 \sqrt{\frac{\epsilon'}{\mu}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \int_{y=0}^b \int_{x=0}^a \underbrace{\sin^2 \left(\frac{\pi}{a} x \right)}_{\frac{1}{2} [1 - \cos \left(\frac{2\pi}{a} x \right)]} dx dy$$

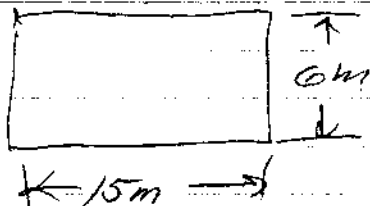
$$P_{\text{ave}} = \frac{1}{2} E_{y\text{max}}^2 \sqrt{\frac{\epsilon'}{\mu}} \sqrt{1 - \left(\frac{f_c}{f}\right)^2} \frac{ba}{2} \quad \text{(Equation 8-90)}$$

$$\text{or } P_{\text{ave}} = 1.16 \times 10^{-7} E_{y\text{max}}^2$$

for air filled waveguide $E_{\text{max}} \approx 3 \times 10^6 \text{ V/m}$

$$\therefore P_{\text{ave max}} = 1.16 \times 10^{-7} \times 9 \times 10^{12} = \boxed{1.04 \times 10^6 \text{ Watts}} \leftarrow$$

8-27

TE₁₀ mode

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}} \frac{1}{15} = \frac{3 \times 10^8}{30} = 10 \text{ MHz}$$

Vertical polarization

AM will not propagate

535 → 1605 KHz

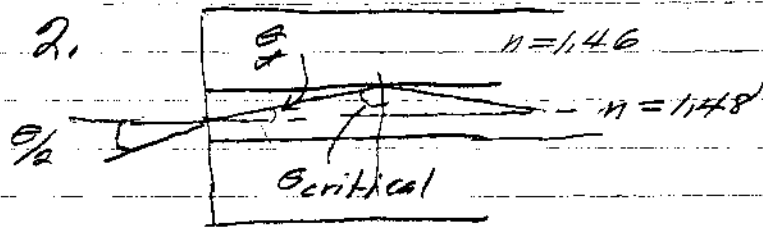
FM will propagate

88 → 108 MHz

1. 1" radius circular waveguide

TE₁₁ cutoff frequency $f_c = \frac{3 \times 10^8 \times 10^2}{2\pi \times 2.54} P_{1,1}$

$$f_c = \frac{3 \times 10^{10}}{2\pi \times 2.54} \times 1.841 = 3.46 \times 10^9 \text{ Hz} \quad \leftarrow$$



$$\theta_c = \sin^{-1}\left(\frac{1.46}{1.48}\right) = 80.57^\circ$$

$$\therefore \theta_f = 90 - \theta_c = 9.43^\circ$$

$$\sin \theta_f = \sqrt{\frac{\epsilon_1}{\epsilon_2}} \sin\left(\frac{\theta}{2}\right); \quad \sin \frac{\theta}{2} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} \sin \theta_f = 1.48 \sin 9.43^\circ = 0.2425$$

$$\text{so } \theta = 2 \sin^{-1}(0.2425) = 28^\circ \quad \leftarrow$$