EE 434 Spring 2007
Assignment 1

1. A variable air capacitor, using a rotating, multiplate rotor, provides a linear capacitance variation from 30 to 500pF as the rotor rotates from 0 to 260 degrees.

   Determine the electrostatic torque on this rotor at any arbitrary angle setting, when 5000 volts are applied to the capacitor.

   $dU = \boldsymbol{T} \cdot \boldsymbol{d\theta}$ therefore $T_1 = \overset{+}{-}\dfrac{\partial U_e}{\partial \theta_1}$ etc.

$$T = +\frac{\partial U_e}{\partial \theta}$$

$$U_e = \frac{1}{2} C V^2$$

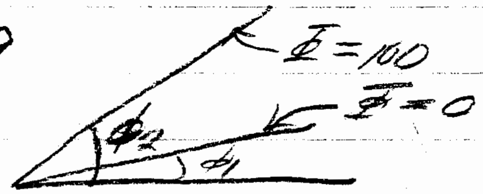
$$\therefore T = +\frac{V^2}{2}\frac{\partial C}{\partial \theta} \doteq +\frac{V^2}{2} \cdot \frac{\Delta C}{\Delta \theta} = +\frac{25 \times 10^6}{2} \cdot \frac{470 \times 10^{-12}}{260 \times \frac{\pi}{180}}$$

$$\left[T = 1294.7 \times 10^{-6} = 1.295 \times 10^{-3} \text{ n-m} \right.$$

in direction to increase C $\Big]$

EE 434 Homework 2

4.9



$\nabla^2 \Phi = 0 = \frac{1}{\rho^2}\frac{\partial^2 \Phi}{\partial \phi^2}$

a) $\therefore \; \Phi = A\phi + B$

so $0 = A\phi_1 + B$ and $100 = A\phi_2 + B$

$B = -A\phi_1$ so $100 = A\phi_2 - A\phi_1$

$A = \frac{100}{\phi_2 - \phi_1}$; $B = -\frac{100\phi_1}{\phi_2 - \phi_1}$

$$\boxed{\Phi = \frac{100}{\phi_2 - \phi_1}(\phi - \phi_1)}$$

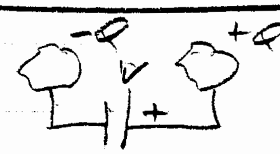
b) $\overline{E} = -\nabla \Phi = -\frac{1}{\rho}\frac{\partial \Phi}{\partial \phi}\overline{a}_\phi = \boxed{-\frac{1}{\rho}\cdot\frac{100}{\phi_2 - \phi_1}\overline{a}_\phi}$

c) $\rho_s(\phi = \phi_1) = D(\phi = \phi_1) = -\frac{\epsilon\, 100}{\rho(\phi_2 - \phi_1)}$

$\rho_s(\phi = \phi_2) = -D(\phi = \phi_2) = \frac{\epsilon\, 100}{\rho(\phi_2 - \phi_1)}$

4.8 a) $W = \frac{1}{2}\int \rho_v \Phi \, dv$

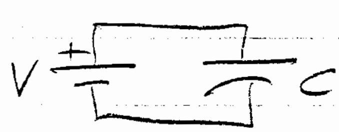
$$\boxed{W = \frac{1}{2} Q V}$$



charge only on surface of plates!

b) $C = \frac{Q}{V}$ $\therefore$ $W = \frac{1}{2} Q \frac{Q}{C} = \frac{1}{2}\frac{Q^2}{C}$

c)



$Q = CV$ initially

after battery is removed $Q =$ constant

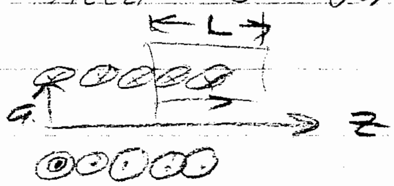
$C = \frac{\epsilon A}{d}$ if $d' \to 3d$ $C' = \frac{C}{3}$ $\therefore$ from b) $\boxed{W' = 3W}$

EE 434 Homework 2 (page 2)

4.25 a) $\bar{B} = \nabla \times \bar{A}$

$$\int \bar{B}\cdot d\bar{s} = \int \nabla \times \bar{A}\cdot d\bar{s} = \oint \bar{A}\cdot d\bar{l} \quad \longleftarrow$$

b) Need $\bar{B}$ for solenoid



$\bar{B} = 0$ for $\rho > a$

$\oint \bar{H}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{s}$

or $H_z \cancel{l} = NI\cancel{l}$ or $B_z = \mu NI$ for $\rho > a$

for $\rho > a$ $\quad A_\phi 2\cancel{\pi}\rho = \cancel{\pi} a^2 \mu NI$

$$\text{or } A_\phi = \frac{\mu a^2 NI}{2\rho} \quad \longleftarrow$$

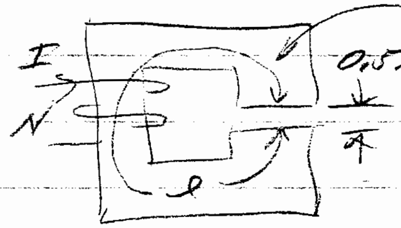
for $\rho < a$ $\quad A_\phi 2\cancel{\pi}\cancel{\rho} = \cancel{\pi}\rho^{\cancel{2}}\mu NI$

$$\text{or } A_\phi = \frac{\rho \mu NI}{2} \quad \longleftarrow$$

EE 434 Homework 3

$B = \mu H$

4.27



$A_{eff} = 1\,cm^2$ $\quad l = 40\,cm$

[Want $B = 1\ W/m^2$ in air gap]

μ_{iron} (not saturated) $= \dfrac{\Delta B}{\Delta H} = \dfrac{1.3}{0.025} = 52$

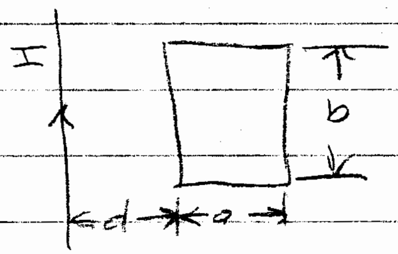
$$R_{gap} = \frac{5\times10^{-4}}{4\pi\times10^{-7}\times10^{-4}} = 0.398\times10^{7}$$

$$R_{iron} = \frac{0.4}{52\times10^{-4}} = 7.69\times10^{192}$$

$$\psi_m = B_{gap}\times10^{-4} = 10^{-4} = \frac{NI}{R_g + R_{iron}}$$

$$\therefore \left[NI = 10^{-4}(0.398\times10^{7} + 769) = \underline{398 \text{ Ampere turns}} \right]$$

4.30



$$L_{12} = \frac{\psi_{12}}{I}$$

$$\psi_{12} = \int_{\rho=d}^{d+a}\int_{z=0}^{b} \frac{\mu_0 I}{2\pi\rho}\, dz\, d\rho$$

$$\therefore L_{12} = \frac{\mu_0 b}{2\pi}\ln\left(\frac{d+a}{d}\right)$$

4.31 $\quad \bar{H} = NI\,\bar{a}_z$ for $\rho < a$; $\bar{H} = 0$ for $\rho > a$

N is turns/unit length

$$W_m = \frac{1}{2}\int \bar{B}\cdot\bar{H}\,dv = \frac{1}{2}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a}\int_{z=0}^{L} \mu N^2 I^2 \rho\, d\phi\, d\rho\, dz$$

$$\boxed{W_m/L = \frac{\mu N^2 I^2 L\, 2\pi a^2}{4\cdot 2}} = \frac{\mu (NI)^2 \pi a^2 L}{2}$$

Energy stored in length L with gap of length $l = \dfrac{(NI)^2 \pi a^2}{2}\left[\mu_0\mu_r(L-l) + \mu_0 l\right]$

$$\boxed{F_l = \frac{\partial W_m}{\partial l} = \frac{(NI)^2 \pi a^2}{2}(\mu_0 - \mu_0\mu_r) = \frac{\mu_0 (NI)^2 \pi a^2}{2}(1-\mu_r)}$$

EE 434 Homework 4

5.3 $\eta_1 = 80$ (μ_o) | $\sigma = \infty$ $f = 150\,MHz$

$\leftarrow 2m \rightarrow$

a) $\bar{E} = 0$ [$\lambda/2$ in front of conducting plane]

$\eta_1 = \sqrt{\frac{\mu_o}{\epsilon}}$ $\epsilon = \frac{4\pi \times 10^{-7}}{80^2} = 1.96 \times 10^{-10}$

$$\text{velocity} = \frac{1}{\sqrt{\mu_o \epsilon}} = \frac{1}{\sqrt{4\pi \times 10^{-7} \times 1.96 \times 10^{-10}}} = 0.637 \times 10^8 \, m/sec$$

$\beta = 14.8$

$$\text{so } \boxed{\lambda/2 = \frac{0.637 \times 10^8}{2 \times 1.5 \times 10^8} = 0.212\,m}$$

b) zero H @ $\boxed{\lambda/4 = 0.106\,m}$

c) $\hat{E}^i = 100 \angle 0°$; $\boxed{|\hat{H}(z=0)| = 2\,\frac{100}{\eta} = 2.5\ A/m}$

$$|\hat{H}|_{z=-2} = \left|2\,\frac{100}{80} \cos\beta_1 z\right|_{z=-2} = \left|2.5 \cos\underbrace{\left(-2 \times \frac{2\pi}{0.424}\right)}_{-0.206}\right| = \boxed{+0.515\ A/m}$$

5.5 20k Hz

a) $\boxed{\lambda/2 \text{ in air} = \frac{3 \times 10^8}{2 \times 10^4 \times 2} = 0.75 \times 10^4}$

$\lambda/2$ in sea water = ?

$$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}} \left[\sqrt{1 + \left(\frac{\sigma}{\omega\epsilon}\right)^2} + 1\right]^{\frac{1}{2}}$$

$$\frac{\sigma}{\omega\epsilon} = \frac{\cancel{4} \times 36\pi}{\cancel{4\pi} \times 2 \times 10^4 \times 81 \times 10^{-9}} = 0.444 \times 10^5$$

$$\therefore \beta \approx \frac{2\pi \times 2 \times 10^4 \sqrt{81}}{3 \times 10^8 \sqrt{2}} \sqrt{4.44 \times 10^4} = 56.198 \times 10^{-2}$$

$$\text{so } \boxed{\lambda_w/2 = \frac{2\pi}{\beta\,2} = 5.59\,m}$$

b) $E_{incident} = E_0$

$$\hat{T} = \frac{2\hat{\eta}_2}{\hat{\eta}_1 + \hat{\eta}_2} \; ; \; \hat{\eta}_2 = \sqrt{\frac{\mu}{\epsilon}}\sqrt{\frac{\omega\epsilon}{\sigma}}\, e^{j\frac{1}{2}\tan^{-1}\left(\frac{\sigma}{\omega\epsilon}\right)}$$

$$\hat{\eta}_2 = \frac{377}{\sqrt{81}} \times 4.74\times10^{-3}\, e^{j45^\circ} = 0.198\, e^{j45^\circ}$$

$$\text{so } \hat{T} = \frac{0.396\, e^{j45^\circ}}{377 + 0.198\, e^{j45^\circ}} \cong 1.05\times10^{-3}\, e^{j45^\circ}$$

$$\therefore \; E^t_{@surface} = 1.05\times10^{-3} E_0$$

$$\text{so } 1.05\times10^{-3} E_0\, e^{-0.561918 z} = 10^{-4} E_0$$

$$-0.561918 z = \ln\frac{10^{-4}}{1.05\times10^{-3}} = -2.35$$

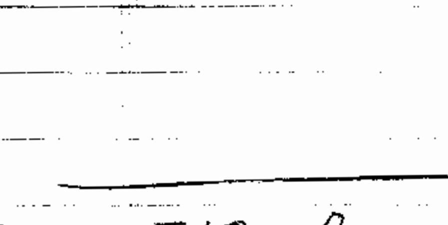
$$\boxed{\therefore \; z = 4.18\,\text{m}}$$

5.10 $f = 10\,\text{GHz}$; $\epsilon_r = 4.9$, μ_0, $\sigma = 0$ for radome

a) radome thickness $= \lambda/2 = \frac{3\times10^8}{\sqrt{4.9}\times10^{10}\times2} \cong \boxed{0.678\times10^{-2}\,\text{m}}$

b) $\hat{E}^+_{m_1} = 100\, e^{j0^\circ}$; $f = 9\,\text{GHz}$ $\quad \lambda/2_{new} = \frac{10}{9}\lambda_{old} = 7.53\times10^{-3}\,\text{m}$

$\therefore$ radome thickness $= 0.45\,\lambda_{new}$



$$\eta_0 = 377 = Z(d)$$

$$\eta_{radome} \cong \frac{377}{\sqrt{4.9}} = 170.31\,\Omega$$

$$\therefore \; z_{norm}(d^-) = \frac{377}{170.31} = 2.214 .$$

from chart $\Gamma(d^-) = 0.383\, e^{j0^\circ}$

from chart $z_{norm}(0) = 1.61 + j0.82$ ←

and $\Gamma(0^+) = 0.383\, e^{j36^\circ}$

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from above $Z(0) = 170.31\, z_{norm}(0) = \boxed{274.2 + j139.65}$

$\therefore\ z_{norm}(0^-) = \frac{Z(0)}{377} = 0.727 + j0.370$

so from chart $\boxed{\Gamma(0^-) = 0.27\, e^{j115^\circ}}$

$\hat{E}_{total}(0^-) = 100\,[1 + \Gamma(0^-)] = E_{m_2}^+\,[1 + \Gamma(0^+)]$

so $E_{m_2}^+ = \frac{100\,[1 + \Gamma(0^-)]}{1 + \Gamma(0^+)}$ ✓

$\hat{E}_{total}(d^-) = \hat{E}_{total}(d^+) = \hat{E}_3(d^+) = E_{m_2}^+\,[1 + \Gamma(d^-)]\,e^{-j\beta d}$

$\therefore\ \hat{E}_3(d^+) = 100\,\frac{1 + \Gamma(0^-)}{1 + \Gamma(0^+)} \cdot [1 + \Gamma(d^-)]\,e^{-j\beta d}$

$\hat{E}_3(d) = 100\,\frac{1 + 0.27e^{j115^\circ}}{1 + 0.383\,e^{j36^\circ}}\,[1 + 0.383]\,\underbrace{e^{-j\frac{2\pi}{\lambda}\cdot 0.452\lambda}}_{\text{phase only ... interested in magnitude}}$

$\therefore\ |\hat{E}_3(d)| = \left|138.3\,\frac{1 + [-0.114 + j0.24]}{1 + [0.31 + j0.225]}\right| = \left|138.3\,\frac{0.886 + j0.24}{1.31 + j0.225}\right|$

$|\hat{E}_3(d)| = \left|138.3\,\frac{0.918\,e^{j15.15^\circ}}{1.33\,e^{j9.74^\circ}}\right| = \boxed{95.46\ \text{V/m}}$ ←

$|E_{m_2}^+| = 69.02$ V/m

answer in book

IMPEDANCE OR ADMITTANCE COORDINATES

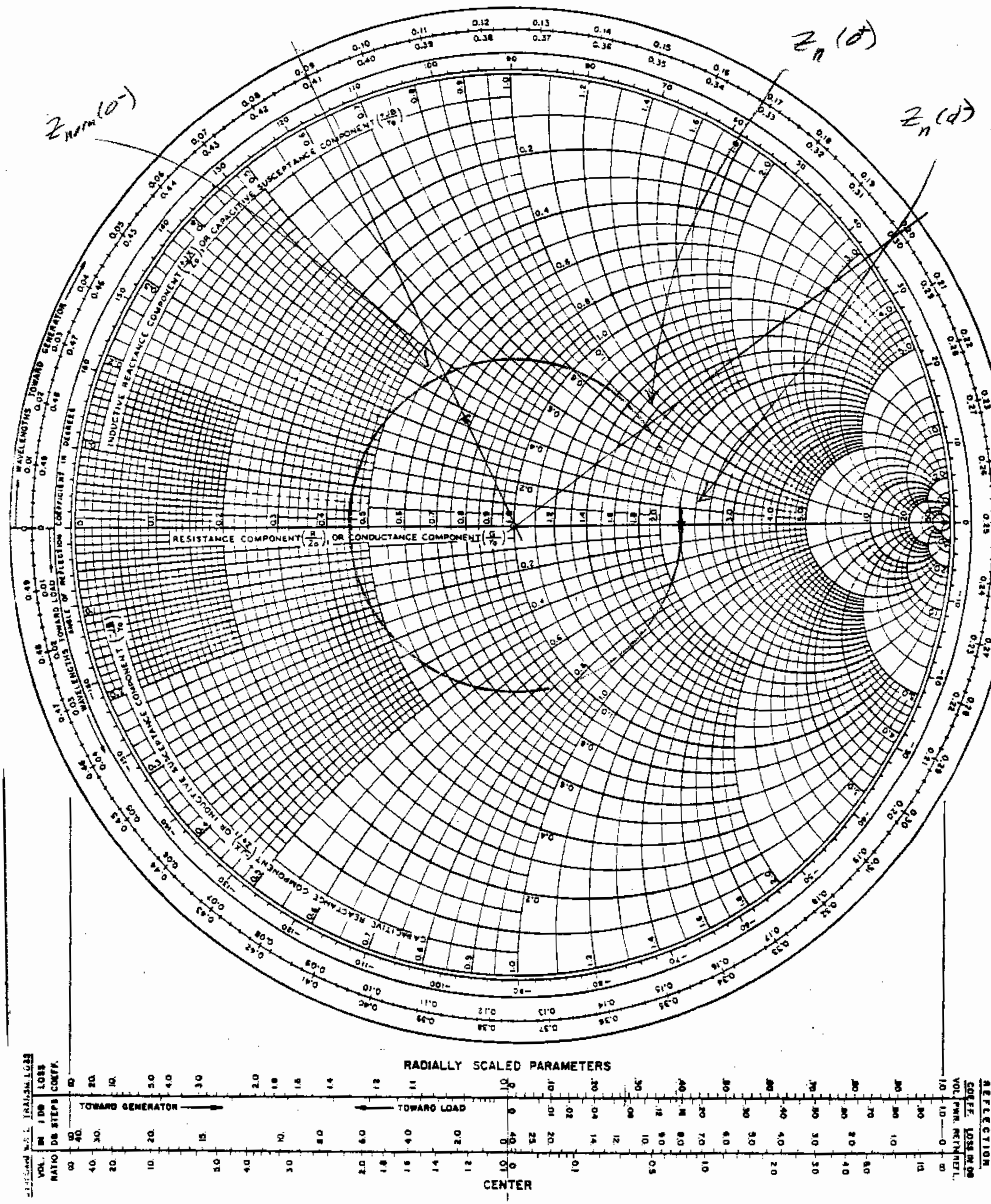


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 434 Homework 5

6.2 $\hat{\bar{E}} = [2\bar{a}_x + 0.5\bar{a}_y + 5\bar{a}_z]\, e^{-j(\beta_x x + 1.8y - 2.1z)}$

$\beta\bar{n}\cdot\bar{r} = \beta_x x + 1.8y - 2.1z$ so $\beta\bar{n} = \beta_x\bar{a}_x + 1.8\bar{a}_y - 2.1\bar{a}_z$

$\bar{n}$ and $\bar{E}$ are normal vector so $\bar{E}\cdot\beta\bar{n} = 0$

or $2\beta_x + 0.5\times 1.8 + 5(-2.1) = 0$

$\beta_x = 4.8$ ✓

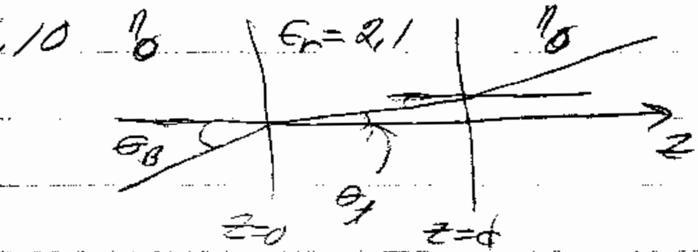
so $\beta = \sqrt{4.8^2 + 1.8^2 + 2.1^2} = 5.54$ ✓

a) $\bar{n} = \frac{\bar{\beta}}{|\beta|} = \frac{4.8\bar{a}_x + 1.8\bar{a}_y - 2.1\bar{a}_z}{5.54} = \underline{0.866\bar{a}_x + 0.325\bar{a}_y - 0.38\bar{a}_z}$

b) $\boxed{\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{5.54} = 0.88\ m}$ ←

c) $\hat{\bar{H}} = \frac{\bar{n}\times\hat{\bar{E}}}{\eta_0} = \frac{1}{\eta_0}[0.433\bar{a}_z - 4.33\bar{a}_y - 0.65\bar{a}_z + 1.655\bar{a}_x - 0.76\bar{a}_y + 0.19\bar{a}_x]\, e^{-j\beta\cdot r}$

$\boxed{\bar{H} = \frac{1}{\eta_0}[1.845\bar{a}_x - 5.09\bar{a}_y - 0.217\bar{a}_z]\, e^{-j\beta\cdot r}}$ ←

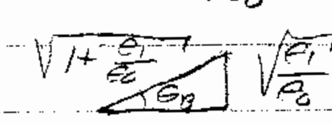
6.10 

Snell's Law

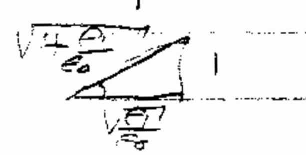
$$\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\therefore \sin\theta_t = \sqrt{\frac{\epsilon_0}{\epsilon_1}}\sin\theta_i = \sin\theta_i$ @ 2nd boundary — but $\theta_i = \theta_B$ or $\tan\theta_B = \sqrt{\frac{\epsilon_1}{\epsilon_0}}$

so $\sin\theta_i$ @ 2nd boundary $= \sqrt{\frac{\epsilon_0}{\epsilon_1}}\sin\theta_B = \sqrt{\frac{\epsilon_0}{\epsilon_1}}\left(\frac{\sqrt{\epsilon_1/\epsilon_0}}{\sqrt{1+\epsilon_1/\epsilon_0}}\right)$

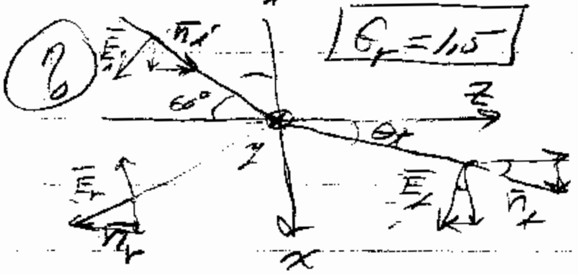


$\boxed{\theta_i\ @\ 2\text{nd boundary} = \sin^{-1}\frac{1}{\sqrt{1+\epsilon_1/\epsilon_0}} = \tan^{-1}\sqrt{\frac{\epsilon_0}{\epsilon_1}} = \theta_{B@z=d}}$



6.17 $\overline{E}_i = E_m \left(\frac{\sqrt{3}}{2}\overline{a}_x - \frac{1}{2}\overline{a}_z\right)\cos\left(6\pi\times10^9 t - 10\pi(x+\sqrt{3}z)\right)$

(b)



$\boxed{\epsilon_r = 1.5}$

$\theta_i = 60^\circ$

$\boxed{\parallel \text{polarization}}$

$\eta_2 = \frac{\eta_0}{\sqrt{1.5}}$

$\sin\theta_t = \sin\theta_i \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \sin 60^\circ \frac{1}{\sqrt{1.5}} = 0.707$ $\boxed{\text{or } \theta_t = 45^\circ}$ ←

$$\Gamma_\parallel = \frac{\cos\theta_i - \frac{1}{\sqrt{1.5}}\cos\theta_t}{\cos\theta_i + \frac{1}{\sqrt{1.5}}\cos\theta_t} = \frac{0.5-0.577}{0.5+0.577} = \boxed{-0.0715} \leftarrow$$

so $\overline{E}_r = \Gamma_\parallel E_m \left(-\frac{\sqrt{3}}{2}\overline{a}_x - \frac{1}{2}\overline{a}_z\right)\cos\left(6\pi\times10^9 t - 10\pi(x-\sqrt{3}z)\right)$

$\boxed{\text{or } \overline{E}_r = E_m (0.0619\overline{a}_x + 0.0369\overline{a}_z)\cos\left(6\pi\times10^9 t - 10\pi(x-\sqrt{3}z)\right)} \leftarrow$

$\overline{\beta}_i \cdot \overline{r}_i = 10\pi(x+\sqrt{3}z)$ $\therefore$ $\beta_1 = 10\pi\times2 = 20\pi$

$\boxed{\beta_2 = \beta_1\sqrt{1.5} = 24.5\pi}$ so $\overline{\beta}_2 = \sin\theta_t\,\beta_2\overline{a}_x + \cos\theta_t\,\beta_2\overline{a}_z$

$$\tau_\parallel = \frac{2\frac{1}{\sqrt{1.5}}\cos\theta_i}{1.077} = 0.758 \leftarrow$$

so $\overline{E}_t = 0.758 E_m \overbrace{|\overline{E}_i|}^{\wedge} (\cos\theta_t\,\overline{a}_x - \sin\theta_t\,\overline{a}_z)\cos\left(6\pi\times10^9 t - \beta_2(\sin\theta_t\, x + \cos\theta_t\, z)\right)$

$\overline{E}_t = 0.758 E_m (0.707\overline{a}_x - 0.707\overline{a}_z)\cos\left(6\pi\times10^9 t - 24.5\pi(0.707x + 0.707z)\right)$

$$\boxed{\overline{E}_t = E_m (0.536\overline{a}_x - 0.536\overline{a}_z)\cos\left(6\pi\times10^9 t - 17.32\pi(x+z)\right)}$$

EE 434 Homework 6

Q1/19 $\frac{\sin\theta_i}{\sin\theta_t} = \sqrt{\frac{\epsilon_2}{\epsilon_1}} = \frac{n_2}{n_1}$

for glass $\boxed{n = A + \frac{B}{\lambda^2}}$ with $A = 1.5$, $B = 5\times10^{-15}$

$\sin\theta_t = \frac{n_1}{n_2}\sin\theta_i$ $\theta_i = 30^\circ$ so $\sin\theta_i = \frac{1}{2}$

$\sin\theta_t = \frac{1}{n}\sin\theta_i = \frac{1}{2n}$

$n = 1.5 + \frac{5\times10^{-15}}{\lambda^2}$

color	λ nm	n	θ_t
violet	400	1.5312	19.058°
blue	450	1.5247	19.14°
green	550	1.516	19.25°
yellow	600	1.5139	19.28°
orange	650	1.5118	19.31°
red	700	1.5102	19.33°

EE 434 Homework 7

8.6 $E_x = A\cos\left(\frac{\pi}{a}x\right)\sin\left(\frac{2\pi y}{b}\right)\sin\left(7\pi\times10^{10}t - \beta z\right)$

a) $\boxed{TE_{12} \text{ or } TM_{12} \text{ mode}}$

b) $f = \frac{7}{2}\times10^{10} = \boxed{35\,GHz}$

c) $\beta_{12} = \sqrt{\frac{49\pi^2\times10^{20}}{9\times10^{16}} - \left(\frac{\pi}{2.3\times10^{-2}}\right)^2 - \left(\frac{2\pi}{1.2\times10^{-2}}\right)^2} = \sqrt{53.73\times10^4 - 1.86\times10^4 - 27.4\times10^4}$

$$\boxed{\beta_{12} = 4.95\times10^2\ /m}$$

d) $f_c = \frac{3\times10^8}{2\pi}\sqrt{29.26\times10^4} = 25.8\times10^9 = \boxed{25.8\,GHz}$

$\eta_{TE} = \frac{120\pi}{\sqrt{1-\left(\frac{f_c}{f}\right)^2}} = \boxed{557.9\ \Omega}$

$\eta_{TM} = 120\pi\underbrace{\sqrt{1-\left(\frac{f_c}{f}\right)^2}}_{0.6757} = \boxed{254.7\ \Omega}$

8.11 TE_{10}, $\boxed{a = 2b}$; $1.5 f_{c_{10}} = 9\times10^9$

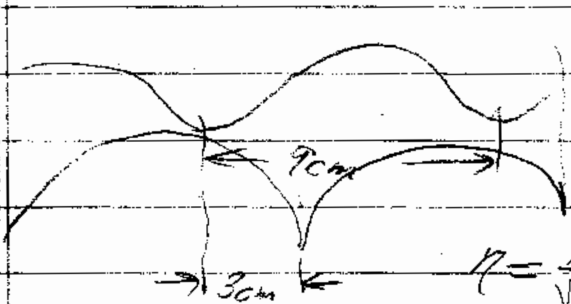
so $6\times10^9 = \frac{3\times10^8}{2\pi}\sqrt{\left(\frac{\pi}{a}\right)^2} = \frac{3\times10^8}{2a}$; $\boxed{a = 2.5\,cm}$

8.15 VSWR = 2.1 , $\lambda/2 = 9\,cm$; $f_c = \frac{3\times10^8}{2\pi}\sqrt{\frac{\pi^2}{0.076^2}} = 1.974\,GHz$

$\lambda = \frac{2\pi}{\beta} = 0.18$; $\beta = \frac{2\pi}{0.18} = \frac{2\pi f}{3\times10^8}\sqrt{1-\left(\frac{f_c}{f}\right)^2}$

$\sqrt{f^2 - f_c^2} = \frac{3\times10^8}{0.18} = 1.6\times10^9$

$f = \sqrt{(1.6\times10^9)^2 + (1.974\times10^9)^2} = 2.54\times10^9$



$\eta = \frac{120\pi}{\sqrt{1-0.604}} = 599\ \Omega$

$(\lambda/6)$ (from chart) $\boxed{\therefore Z_{load} = 671 - j467}$

IMPEDANCE OR ADMITTANCE COORDINATES

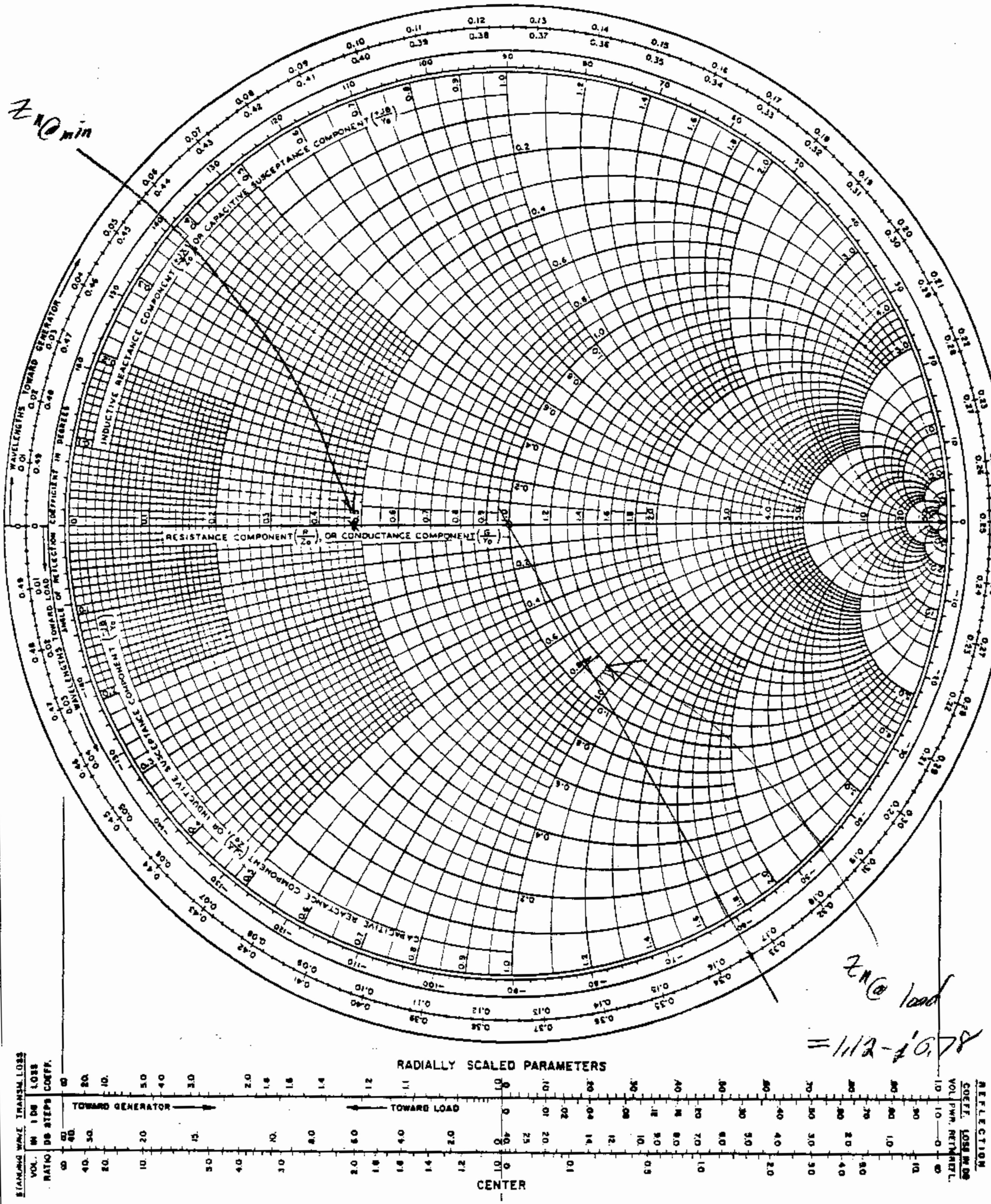


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 434 Homework 8

1. rectangular waveguide (tunnel) 15m by 6m

a) (lowest) $f_{c_{TE_{10}}} = \frac{c}{2\pi}\sqrt{\left(\frac{\pi}{15}\right)^2} = \frac{3\times10^8}{30} = \boxed{10^7 \text{ Hertz}}$

b) $\boxed{\text{Vertical polarization}}$

c) AM 535–1605 kHz will not propagate
FM 88–108 MHz will propagate

$f = 1.3 f_c$, $E_{o_{max}} = 0.75\times2\times10^6$ V/m

2. (8.18) rectangular waveguide $a = 2.3$ cm, $b = 1.2$ cm
TE$_{10}$ mode

$\therefore |\hat{E}_y|_{max} = \frac{\omega\mu a}{\pi}H_0 = E_0$; $P_{ave} = \frac{\omega\mu\beta_{10}a^3b}{(2\pi)^2}H_0^2 = \frac{\omega\mu\beta_{10}a^3b}{(2\pi)^2}\cdot\frac{E_0^2\pi^2}{\omega^2\mu^2a^2}$

or $P_{ave} = \frac{\beta_{10}ab}{4\omega\mu}E_0^2 = \frac{\omega\sqrt{\mu\epsilon}\overbrace{\sqrt{1-\left(\frac{1}{1.3}\right)^2}}^{0.639}ab}{4\omega\mu}E_0^2 = 0.159\sqrt{\frac{\epsilon}{\mu}}ab\,E_0^2$

so $P_{ave_{max}} = \frac{0.159\times2.3\times1.2\times10^{-4}}{377}\times2.25\times10^{12} = \boxed{2.6\times10^5 \text{ Watts}}$

---

3. TE$_{11}$ $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p'_{11}}{2.54\times10^{-2}}\right) = \frac{3\times10^8}{2\pi}\left(\frac{1.841}{2.54\times10^{-2}}\right) = 3.46$ GHz

TM$_{01}$ $f_c = \frac{3\times10^8}{2\pi}\left(\frac{p_{01}}{2.54\times10^{-2}}\right) = 4.5$ GHz (2.405)

TE$_{01}$ $f_c = \quad = 7.2$ GHz $p'_{01} = 3.832$

EE 434 Homework 9

9.1 $d\ell = 0.1\,m$; $\hat{I} = 1e^{j0^\circ}$; $f = 10\,MHz$ ∴ $\beta = \frac{2\pi f}{3\times10^8} = 2.09\times10^{-1}$

a) i) $\boxed{r = 1\,m}$

$$\hat{H}_\phi = \frac{10^{-1}}{4\pi} e^{-j0.209}\underbrace{(j0.209+1)}_{1.02e^{j11.829^\circ}} = 8.13\times10^{-3} e^{-j0.15^\circ}$$

$$\hat{E}_\theta = \frac{e^{-j90^\circ}\,\cancel{120\pi}\times10^{-1}\,30}{\cancel{4\pi}\,0.209}\underbrace{(0.9563+j0.209)}_{0.979e^{j12.328^\circ}} e^{-j11.975^\circ} = 14.05\,e^{-j89.6^\circ}$$

ii) $r = 5\,m$

$$H_\phi = \frac{10^{-1}}{4\pi} e^{-j59.875^\circ}\underbrace{(j0.0418+0.04)}_{0.0578e^{j46.26^\circ}} = 4.60\times10^{-4} e^{-j13.6^\circ}$$

$$E_\theta = \frac{e^{-j90^\circ}\,\cancel{120\pi}\times10^{-1}\,30}{\cancel{4\pi}\,0.209}\underbrace{(-7.362\times10^{-4}+j8.36\times10^{-3})}_{10^{-2}\times0.8392\,e^{j95.03}} e^{-j59.875^\circ} = 0.1204\,e^{-j54.8^\circ}$$

83.6×10^{-4}

iii) $r = 10\,m$

$$H_\phi = \frac{10^{-1}e^{-j119.75^\circ}}{4\pi}\underbrace{(j0.0209+0.01)}_{0.0213e^{j64.43^\circ}} = 1.83\times10^{-4} e^{-j55.3^\circ}$$

$$E_\theta = \frac{e^{-j90}\,\cancel{120\pi}\times10^{-1}\,3}{\cancel{4\pi}\,0.209}\underbrace{(-4.368\times10^{-3}+10^{-3}+j2.09\times10^{-3})}_{3.96\times10^{-3}e^{j148.18}} e^{-j119.75^\circ} = 5.68\times10^{-2} e^{-j61.57^\circ}$$

b) i) look @ H_ϕ ; $0.209 < 1$ ∴ near field dominates

ii) " ; $0.0418 \approx 0.04$ ∴ near and far field terms of same magnitude

iii) " ; $0.0209 > 0.01$ ∴ far field term larger

c) not quite true ... only a factor of 2 greater!

9.2 $P_{ave} = \frac{1}{2}Re\{\hat{\bar{E}} \times \hat{\bar{H}}^*\} = \frac{1}{2}Re\Big\{\bar{a}_\theta\Big[|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(-\frac{\beta_0}{r^2}+\frac{j}{r^3}\right)\cdot\left(-j\frac{\beta_0}{r}+\frac{1}{r^2}\right)\Big]$

$+\bar{a}_r\Big[|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{j\beta_0^2}{r}+\frac{\beta_0}{r^2}-\frac{j}{r^3}\right)\cdot\left(-j\frac{\beta_0}{r}+\frac{1}{r^2}\right)\Big]\Big\}$

$= \frac{1}{2}Re\Big\{\bar{a}_\theta|\hat{E}_r||\hat{H}_\phi|\sin\theta\cos\theta\left(j\frac{\beta_0^2}{r^3}-\cancel{\frac{\beta_0}{r^4}}+\cancel{\frac{\beta_0}{r^4}}+j\frac{1}{r^5}\right)$

$+\bar{a}_r|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta_0^3}{r^2}+\cancel{\frac{j\beta_0^2}{r^3}}-\cancel{\frac{j\beta_0^2}{r^3}}+\cancel{\frac{\beta_0}{r^4}}-\cancel{\frac{\beta_0}{r^4}}-\frac{j}{r^5}\right)\Big\}$

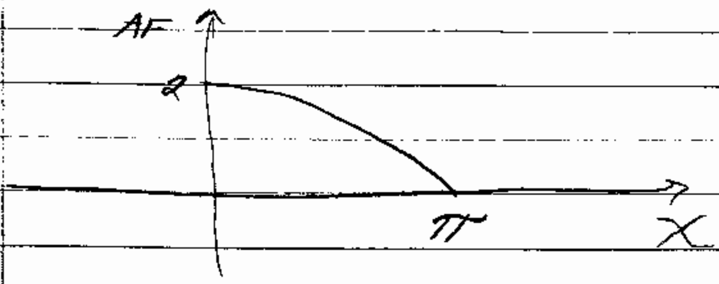
only non imaginary term!

so $P_{ave} = \frac{1}{2}|\hat{E}_\theta||\hat{H}_\phi|\sin^2\theta\left(\frac{\beta_0^3}{r^2}\right)$

EE 434 Homework 10

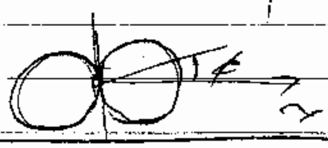
9.7 a) $\lambda/2$ $\psi = \pi$ 2 isotropic radiators

$X = \frac{2\pi\lambda}{2\lambda}\cos\phi - \pi = \pi(\cos\phi - 1)$; $|AF| = \frac{\sin X}{\sin\frac{X}{2}}$



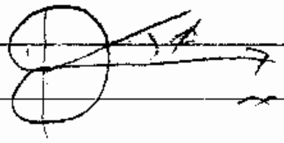
ϕ	X	$\lvert AF \rvert$
0	0	2
30°	-0.42	2
60°	-1.57	1.99
90°	$-\pi$	0

see attached radiation pattern



b) $d = \frac{\lambda}{4}$, $\psi = 90°$ $X = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4}\cos\phi - \frac{\pi}{2} = \frac{\pi}{2}\cos\phi - \frac{\pi}{2}$

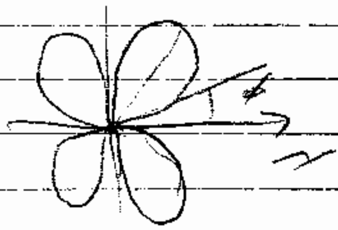
see attached



ϕ	X	$\lvert AF \rvert$
0	0	2
π	$-\pi$	0

c) $d = \lambda$ $\psi = 180°$

see attached

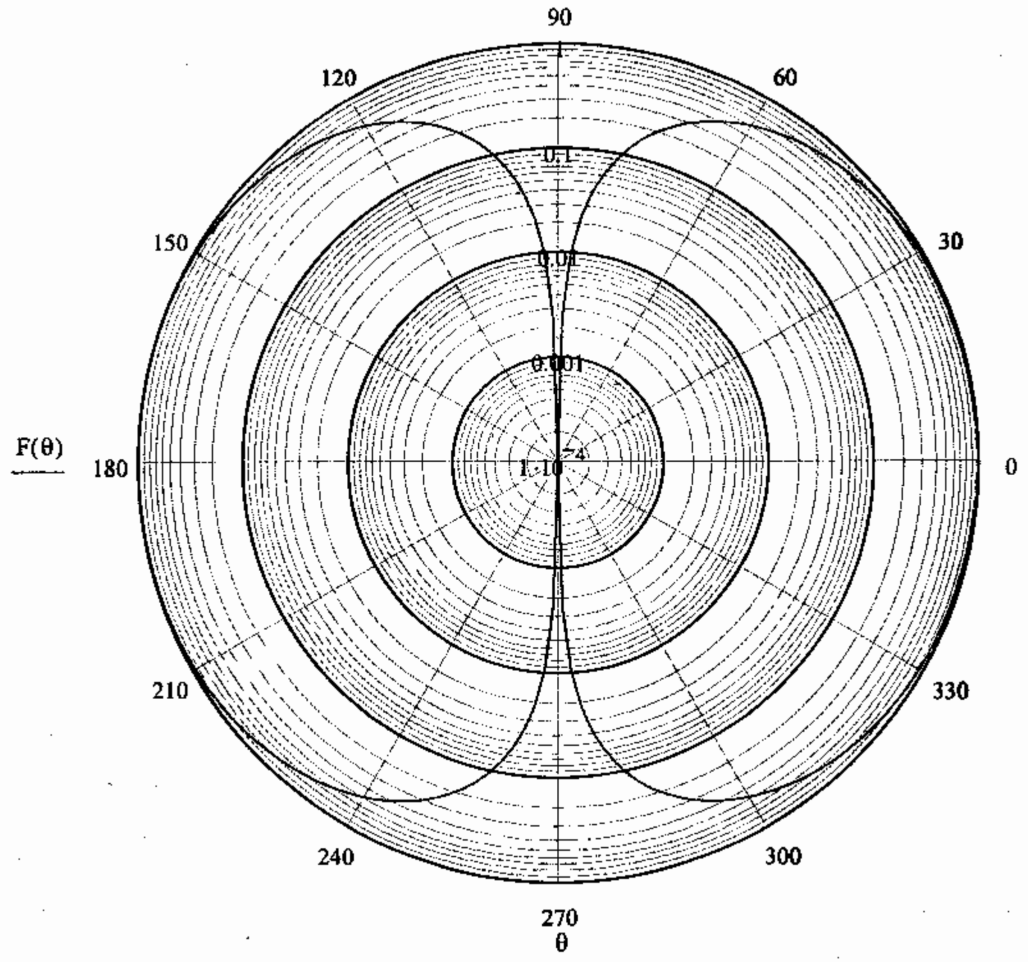


ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2\,;\quad d := \frac{1}{2};\quad \phi := \pi\ ;\qquad j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1;\quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.

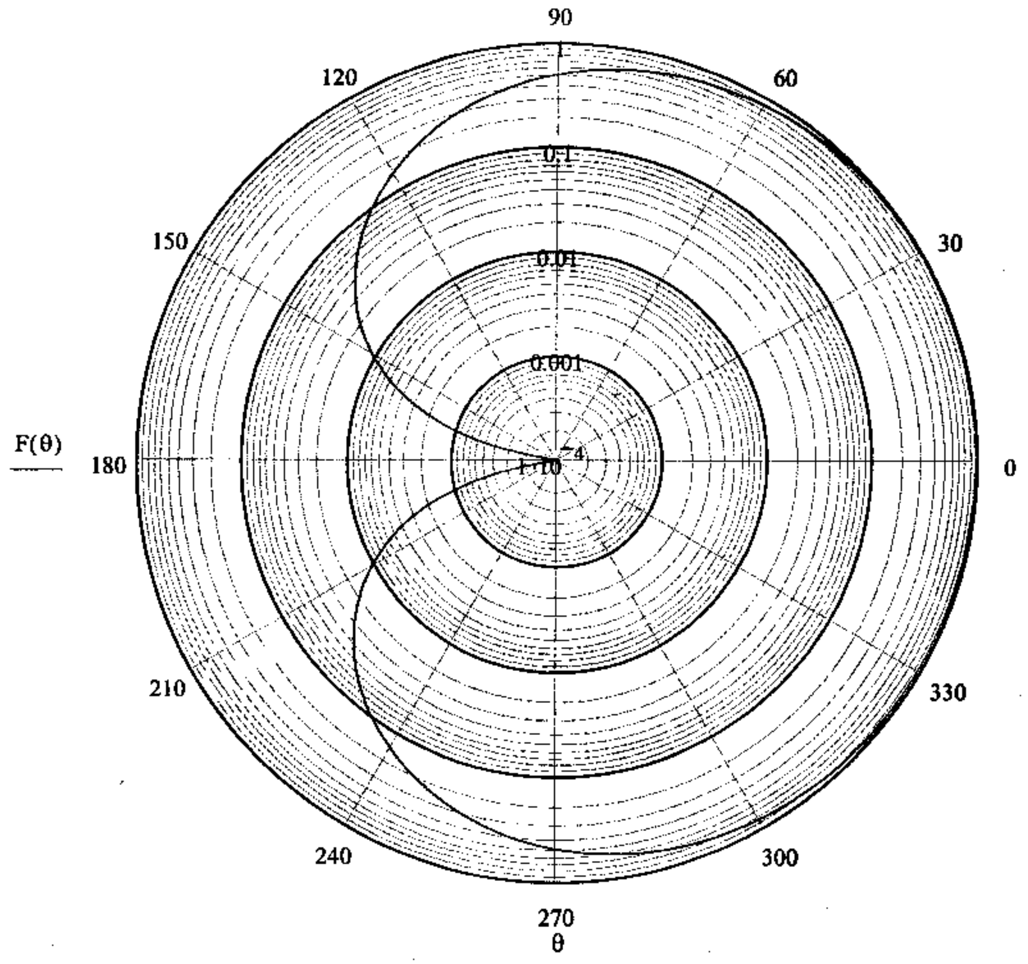


ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2\,;\quad d := \frac{1}{4};\quad \phi := \frac{\pi}{2}\,;\qquad j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1;\quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.

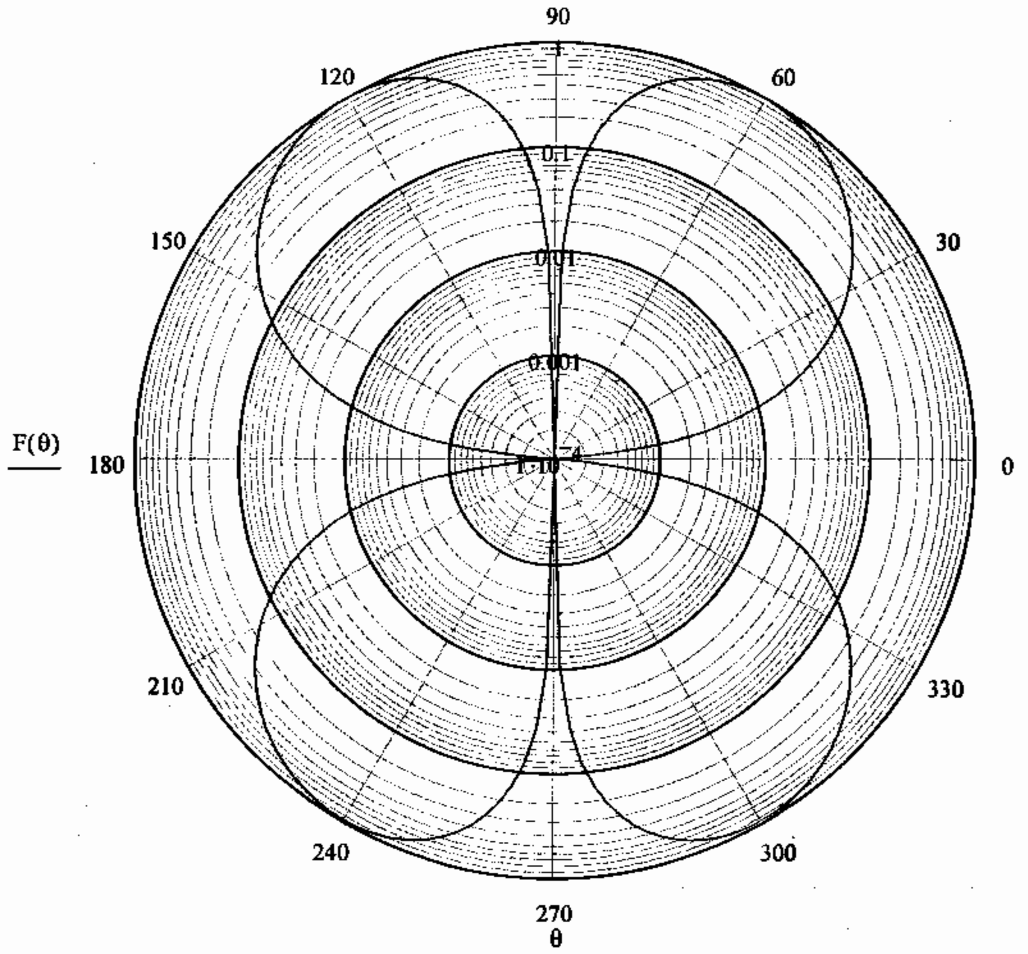


ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2;\quad d := 1;\quad \phi := \pi\;;\qquad j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad n := 0, 1 .. N-1;\quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



EE 434 Homework 11

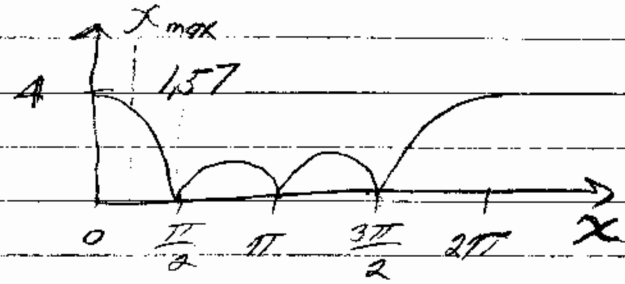
9.11 4 element, $\phi_0 = 45°$, no 2nd main lobe

$$|AF| = \frac{\sin\left(\frac{4\chi}{2}\right)}{\sin\left(\frac{\chi}{2}\right)} \qquad \chi = \beta d \cos\phi - \psi$$

4.44

$\chi = 0$ @ $\phi = 45°$ $\therefore$ $\boxed{\psi = \frac{2\pi d}{\lambda}\cos 45° = 4.4429\,\frac{d}{\lambda}}$

$|AF|$ zero @ $\frac{4\chi}{2} = n\pi$ or $\chi = n\frac{\pi}{2}$



[for no part of 2nd main lobe $|\chi| < \frac{3\pi}{2}$]

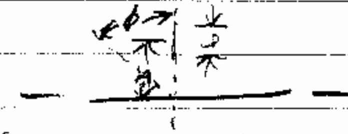
for $0 \le \phi \le \pi$ "visible range" $-\beta d - \psi \le \chi \le \beta d - \psi$

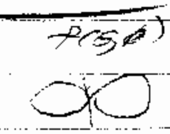
$\therefore$ $-\beta d - \psi = -\frac{3\pi}{2}$ or $+\beta d + 4.4429\frac{d}{\lambda} = \frac{3\pi}{2}$

so $\frac{d}{\lambda}(2\pi + 4.4429) = \frac{3\pi}{2}$; $\boxed{\frac{d}{\lambda} = 0.439 \quad \psi = 1.95}$

[see attached plot]

9.19 short dipole $\ell = \frac{\lambda}{20}$

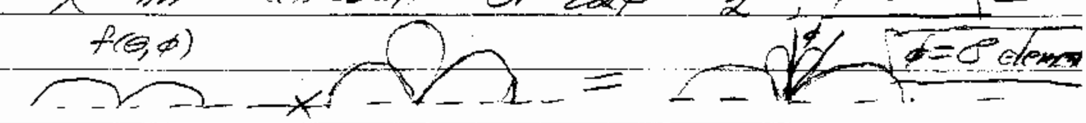




a) $h = \frac{\lambda}{2}$ 2 element "in phase" array with $d = \lambda$ ($\psi = 0$)

AF zeros @ $\chi = n\pi = 2\pi\cos\phi$ or $\cos\phi = \frac{n}{2}$; $\boxed{\phi = 60°}$ array

total pattern



b) ~~zeros @ $\chi = n\pi = \frac{n\pi}{4}$ that~~ $\therefore$ zeros @ $\cos = \frac{n}{8}$

$\phi_{zero} = 82.8°, 68°, 51.3°, 28.9°$ plus symmetrical points in other three quadrants!

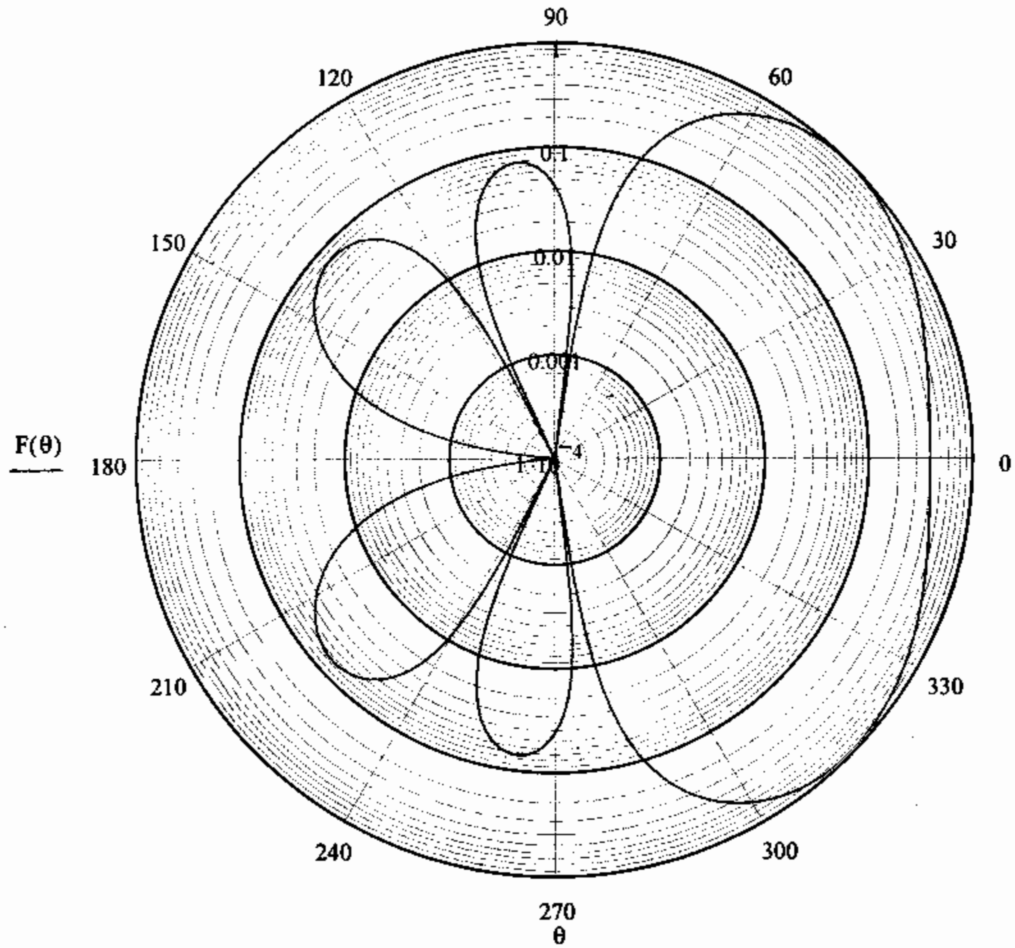
element zero @ $\phi = 0°$ in plane of paper!

ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 4\,;\quad d := 0.439 \quad \phi := 1.95 \qquad j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{200} .. 2\cdot\pi \quad n := 0, 1 .. N-1;\quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_{n} e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



9/11

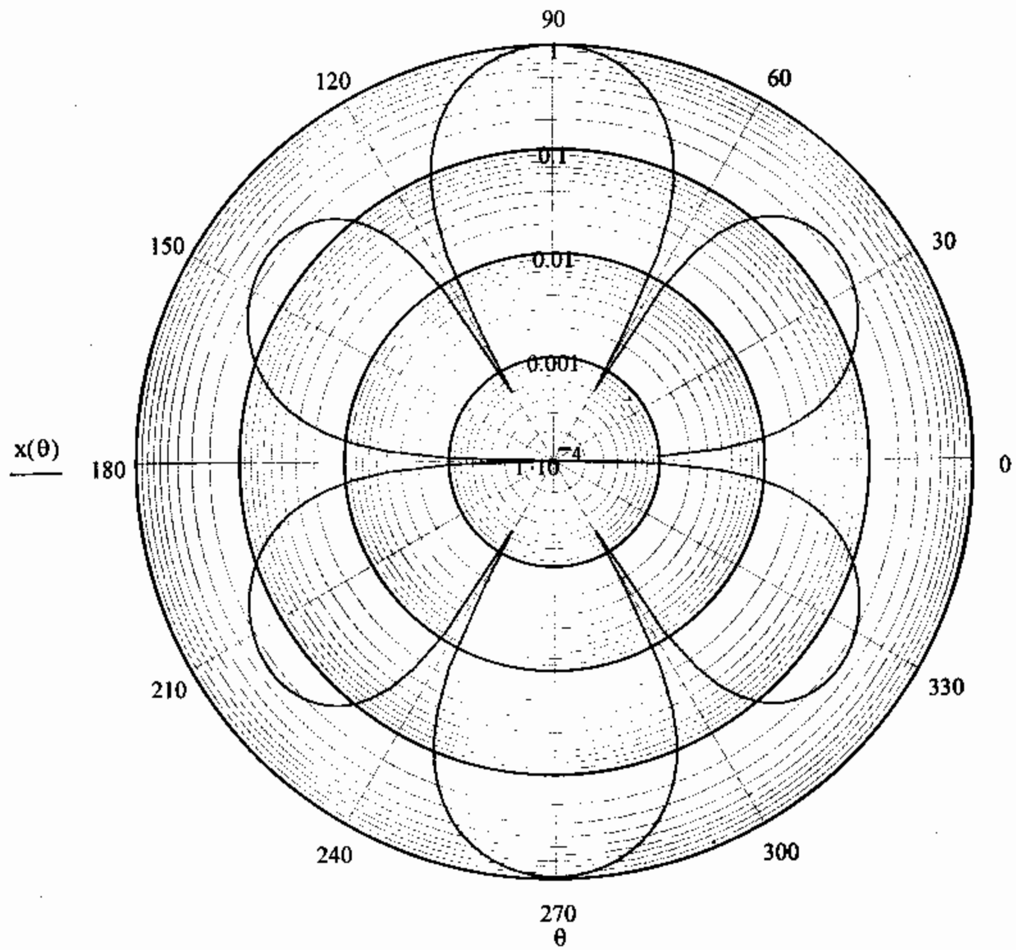
ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2;\quad d := 1;\qquad \phi := 0\qquad\qquad j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi\quad n := 0, 1 .. N-1;\quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

$x(\theta) := F(\theta)\cdot(\sin(\theta))^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in wavelengths.



9.19 a)

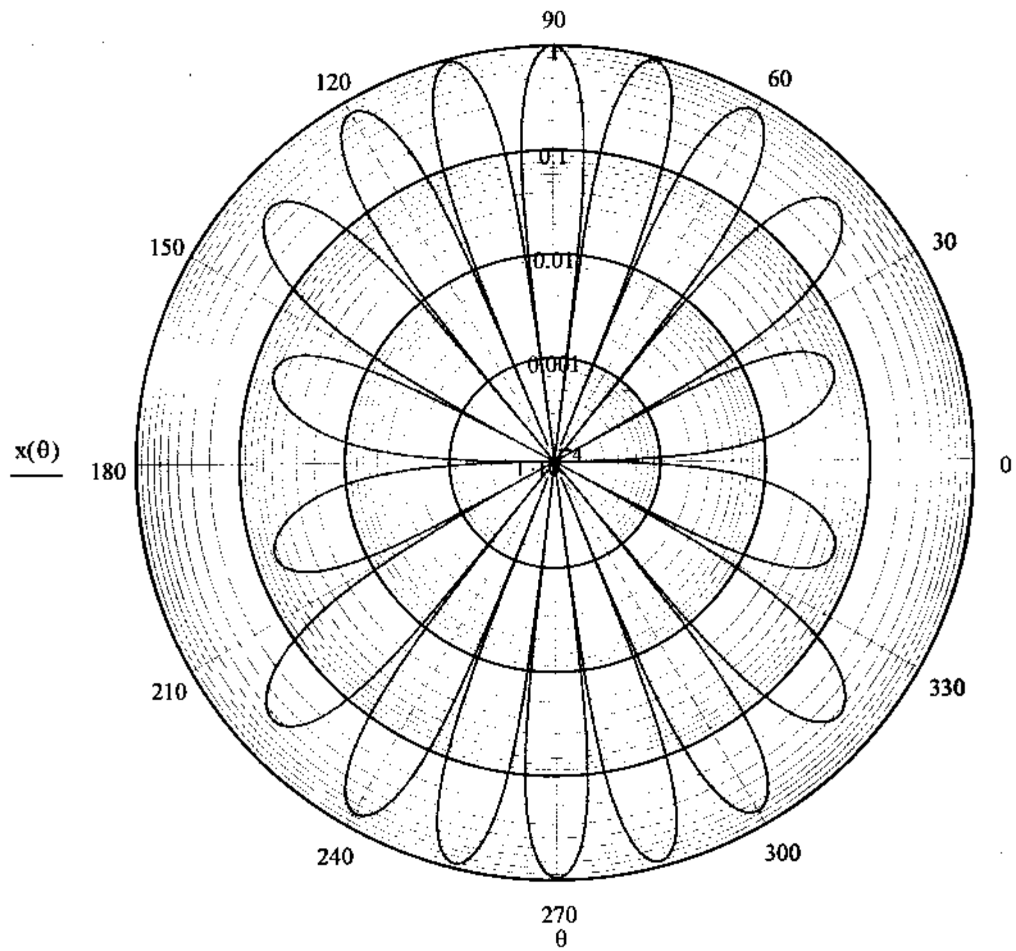
ANTENNA ARRAY RADIATION PATTERNS

The radiation pattern of a linear antenna array is given by the following:

$N := 2;\quad d := 4;\qquad \phi := 0 \qquad\qquad j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{1000} .. 2\cdot\pi \quad n := 0, 1 .. N-1;\quad F(\theta) := \left[\frac{1}{N}\cdot\left|\sum_n e^{-j\cdot n\cdot(2\cdot\pi\cdot d\cdot\cos(\theta)-\phi)}\right|\right]^2$

$x(\theta) := F(\theta)\cdot(\sin(\theta))^2$

Where "N" is the number of array elements, "ϕ" the progressive phase shift (element to element), and "d" the the element spacing in **wavelengths.**



9/19 b)

EE 434 Homework 12

9.5
9.10
9.14
9.16

Broadcast Antenna - see attached solution with plot of radiation pattern

9.5 2m dipole, $f = 10^6$ Hz $\lambda = \frac{3\times10^8}{10^6} = 300\,m$

copper wire with radius = 1mm = a

very short dipole

$\sigma_{Cu} = 5.8\times10^7$ $\delta = \frac{1}{\sqrt{\pi f \mu \sigma}}$

a) $\eta = \frac{P_{rad}}{P_{rad}+P_{loss}} = \frac{R_{rad}}{R_{rad}+R_{loss}}$

(for constant current) $R_{rad} = \frac{2\pi\eta}{3}\left(\frac{\text{length}}{\lambda}\right)^2 = \frac{2\pi\,120\pi}{3}\left(\frac{2}{300}\right)^2 = 0.0351\,\Omega$

$R_{loss} = \frac{\text{length}}{\sigma\,2\pi a\,\delta} = \frac{l}{2a}\sqrt{\frac{\mu f}{\pi\sigma}} = \frac{2}{2\times10^{-3}}\sqrt{\frac{4\pi\times10^{-7}\times10^6}{\pi\,5.8\times10^7}} = 0.83\times10^{-1}$

so $\eta = \frac{0.0351}{0.0351+0.083} = 0.297 = \boxed{29.7\%}$

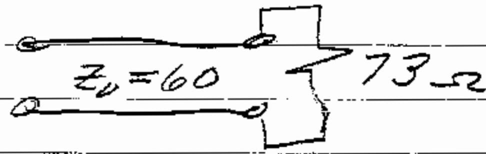
b) $G = \eta D = 1.5\times0.297 = \boxed{0.445 = -3.5\,dB}$

c) $P_{rad} = 20 = \frac{1}{2}I^2R_{rad}$ $\therefore I = \sqrt{\frac{40}{0.0351}} = \boxed{33.76\,A}$

$P_{transmitter} = \frac{P_{rad}}{0.297} = \frac{20}{0.297} = 67.34$ Watts

Homework 12 (page 2)

9.10



$$\Gamma = \frac{73-60}{73+60} = 0.0977$$

$$\boxed{VSWR = \frac{1+|\Gamma|}{1-|\Gamma|} = 1.2} \leftarrow$$

9.14 $\lambda/2$ dipole $D = 1.64$ 100MHz; $\lambda = \frac{3\times10^8}{10^8} = 3m$

$$\therefore A_{eff} = \frac{\lambda^2}{4\pi} \times 1.64 = \frac{9 \times 1.64}{4\pi} = \boxed{1.17\, m^2} \leftarrow$$

projected area of wire $= 10^{-2} \times 1.5 = \boxed{0.015\, m^2} \leftarrow$

9.16 1 kW @ 50MHz. $P_{transmitted}$

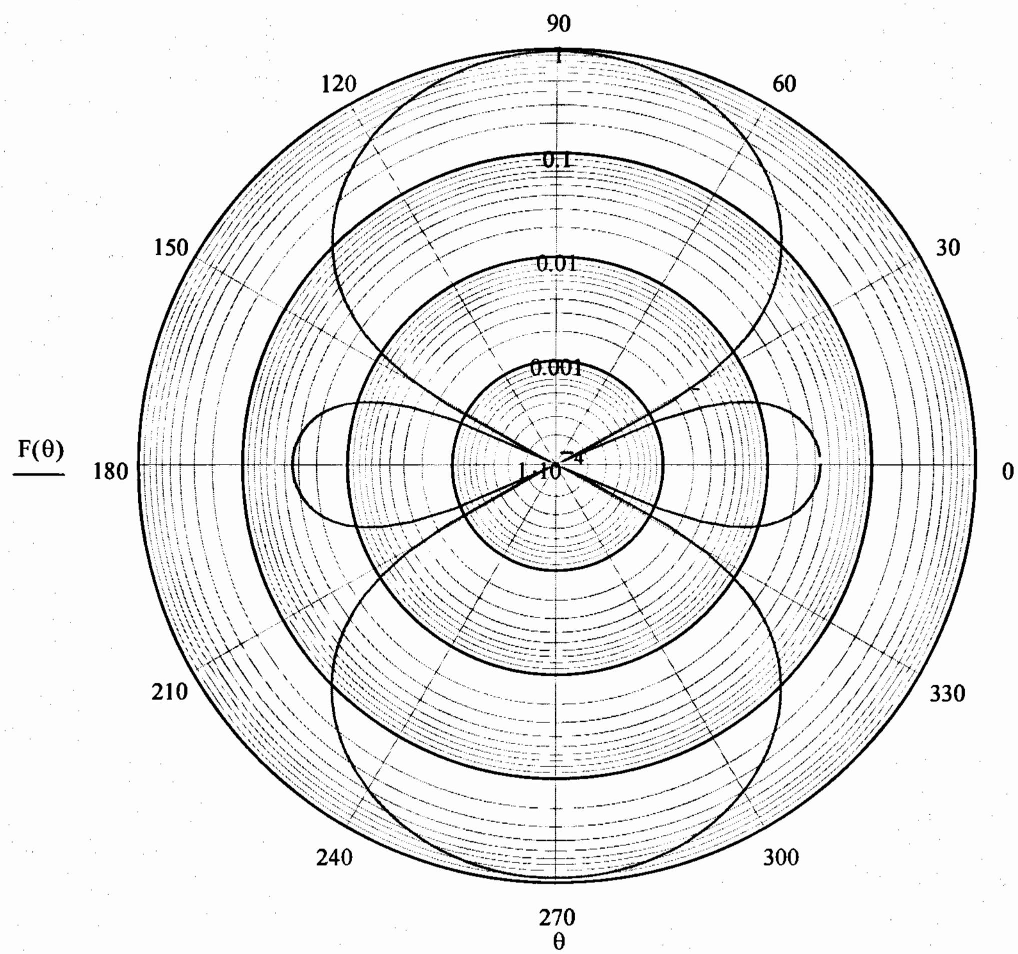
home antenna 13dB Gain = 19.95 power gain

$$P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi R}\right)^2 = 10^3 \times 19.95 \times 1.64 \left(\frac{6}{4\pi \times 30 \times 10^3}\right)^2$$

or $\boxed{P_r = 8.29 \times 10^{-6} \text{ Watts}} \leftarrow$

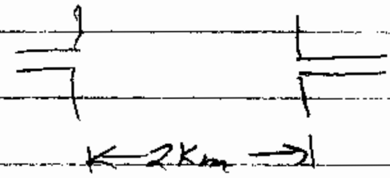
Broadcast Array Special Problem

$j := \sqrt{-1}; \quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad z(\theta) := e^{j\cdot\frac{\pi}{2}\cdot\cos(\theta)} \quad z_1 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(26\cdot\frac{\pi}{180}\right)} \quad z_2 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(154\cdot\frac{\pi}{180}\right)} \quad F(\theta) := \left[\left|\frac{1}{\sqrt{3}}\cdot(z(\theta) - z_1)\cdot(z(\theta) - z_2)\right|\right]^2$



EE 434 Homework 13

9.17



$f = 150\,MHz \therefore \boxed{\lambda = 2m}$

$Bandwidth = 3\,MHz$

$T_{sys} = 600°K$

$S/N = 20\,dB$

$N = kTB = 1.38\times10^{-23}\times 6\times10^{2}\times 3\times10^{6} = 24.84\times10^{-15}$

$$\text{so } P_t = \frac{24.84\times10^{-13}}{G_r G_t \left(\frac{\lambda}{4\pi R}\right)^2} = \frac{24.84\times10^{-13}\times 16\pi^2 \cancel{4}\times10^{6}}{1.64^2\times\cancel{4}} = 1458\times10^{-7} = \underline{1.45\times10^{-4}}$$

9.20 $\theta_{HP} = 1.5°$ @ $20\,GHz$ $\left(\lambda = \frac{3\times10^{8}}{2\times10^{10}} = 1.5\times10^{-2}\,m\right)$

a) $D = \frac{4\pi}{\lambda^2} Area = \frac{\cancel{4}\pi}{\lambda^2}\cdot\frac{\pi(dia)^2}{\cancel{4}} = \left(\frac{\pi\, dia}{\lambda}\right)^2 = \left(\frac{\pi}{\theta_{HP}}\right)^2$

$\therefore D = \left(\frac{\pi\, 180}{1.5\times\pi}\right)^2 = 14,400 = 41.58\,dB$

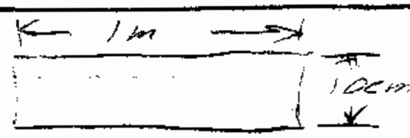
b) twice the area then $D = 2D_{original}$

$\theta_{HP} = \frac{\lambda}{dia} \quad \therefore \theta_{HP} = \frac{1}{\sqrt{2}}\theta_{HP\,original}$

c) frequency doubled $D = 4\cdot D_{original}$

$\theta_{HP} = \frac{1}{2}\theta_{HP\,original}$

9.21 94 GHz

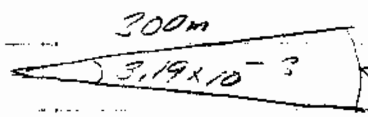


$\lambda = 3.19\times10^{-3}$

a) $\theta_{HP\,elevation} = \left(\frac{10^{-1}}{\lambda}\right)^{-1} = (31.33)^{-1} = 0.0319\,radians = \underline{1.83°}$

$\theta_{HP\,azimuth} = \frac{\lambda}{1} = 3.19\times10^{-3} = \underline{0.183°}$

b)



arc length $= 300\times3.19\times10^{-3} = \underline{0.957\,m}$

EE 434 Homework 13 (page 2)

9.22 100m dish ! 10GHz

$$\theta_{HP} = \frac{\lambda}{100} = \frac{3\times10^8}{10^{12}} = 3\times10^{-4} \text{ radians}$$

$(\text{diameter of coverage})^2$ is proportional to area covered

$$\therefore \ \% \text{ coverage} = \left(\frac{3\times10^{-4}/180}{0.5\times\pi}\right)^2 = \underline{\underline{1.18\times10^{-3}}}$$