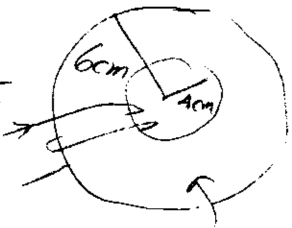
EE 434 Homework 1 5-16

5-4

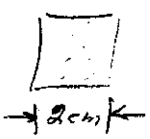
$\underline{0.1 = I}$

$\underline{n = 100}$



$\mu_r = 10^3$

cross section



a) $R = \dfrac{2\pi 5\times10^{-2}}{10^3\, 4\pi\times10^{-7}\times 4\times10^{-4}}$

$R = \dfrac{10^{-1}}{16\times10^{-8}} = 0.0625\times10^{7}$

$\therefore\ \psi_m = \dfrac{nI}{R} = B_{ave}A = \mu H_{ave} A$

or $H_{ave} = \dfrac{nI}{R\mu A} = \dfrac{n\, I \times A}{\ell\, \mu A} = \dfrac{nI}{\ell} = \dfrac{10}{2\pi 5\times10^{-2}}\ \boxed{= 0.318\times10^{2}}$

b) $\oint \overline{H}\cdot d\overline{\ell} = nI$ or $H_\phi = \dfrac{nI}{\ell}$ (the same as above)

with air gap don't have a path with constant $\overline{H}$ so cannot take $\overline{H}$ outside of the integral to find its value

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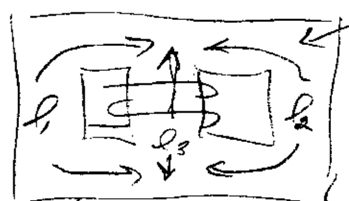
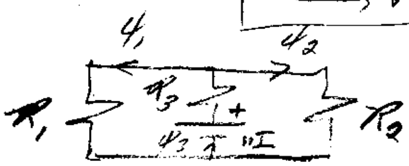
5-5



$\mu_r = 10^4$, $I = 0.1$, $n = 80$

$\ell_3 = 4cm$, $\ell_1 = \ell_2 = 12cm$

$A = 2cm^2 = 2\times10^{-4} m^2$



$\boxed{\psi_1 = \psi_2 = \dfrac{\psi_3}{2}}$

$\psi_3 = \dfrac{nI}{R_3 + \frac{R_1}{2}}$

$\psi_3 = \dfrac{8}{\dfrac{4\times10^{-2}}{4\pi\times10^{-7}\times10^{4}\times2\times10^{-4}} + \dfrac{6\times10^{-2}}{4\pi\times10^{-7}\times10^{4}\times2\times10^{-4}}} = \dfrac{8\times4\pi\times10^{-7}\times10^{4}\times2\times10^{-4}}{10^{-1}}$

$\boxed{\psi_3 = 201.06\times10^{-6} = 0.201 mW = 2\psi_{1,2}}$

$B_{ave_3} = \dfrac{\psi_3}{A} = \dfrac{0.201\times10^{-3}}{2\times10^{-4}} = 0.1\times10\ \boxed{\cong 1\ Weber/m^2}$

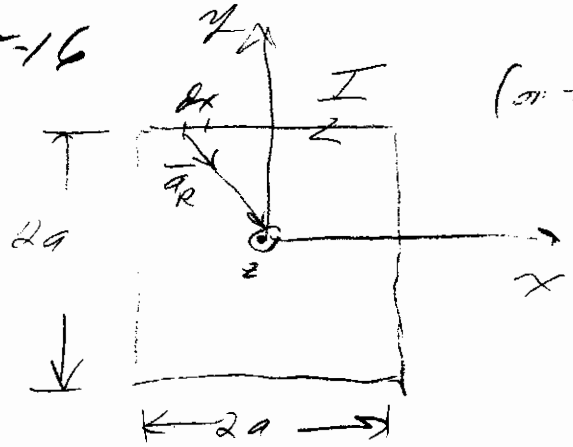
$B_{ave_1} = B_{ave_2} = \dfrac{\psi_1}{A} \cong \boxed{0.5\ Weber/m^2}$

$H_{ave_3} = 2H_{ave_1} = 2H_{ave_2} = \dfrac{B_{ave_3}}{\mu_0\mu_r} = \dfrac{1}{4\pi\times10^{-7}\times10^{4}}\ \boxed{\approx 80}$

$\left.\begin{array}{l}[H_{ave_1}\ell_1 + H_{ave_3}\ell_3 = nI]? \Rightarrow 40\times0.12 + 80\times0.04 = 8 \\ 4.8 + 3.2 = 8\end{array}\right\}$ QED

EE 434   Homework 1   (Solutions page 2)

5-16



(on top)

$$\overline{dB} = \frac{\mu I\, dx\, \bar a_x \times \bar a_R}{4\pi (x^2+a^2)}$$

$$\bar a_R = \frac{-x\bar a_x - a\bar a_y}{\sqrt{x^2+a^2}}$$

$$\therefore \ \overline{dB} = \frac{\mu I\, dx\,(-a\bar a_z)}{4\pi (x^2+a^2)^{3/2}}$$

$$B_{z_{top}} = \int_a^{-a} \frac{-\mu I a\, dx}{4\pi (x^2+a^2)^{3/2}} = \frac{-\mu I a}{4\pi}\left\{\frac{x}{a^2(x^2+a^2)^{1/2}}\right\}_a^{-a}$$

$$B_{z_{top}} = \frac{-\mu I}{4\pi}\left\{\frac{-a}{a\sqrt{2}\,a} - \frac{a}{a\sqrt{2}\,a}\right\} = \frac{2\mu I}{4\pi\sqrt{2}\,a}$$

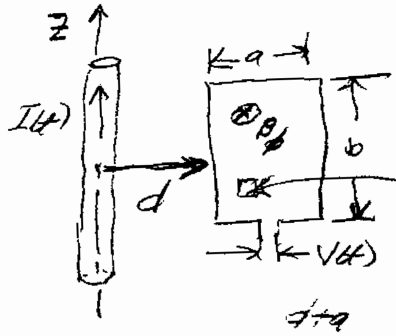
$$B_{z_{top}} = \frac{\mu I}{2\pi a\sqrt{2}} = \frac{\mu I\sqrt{2}}{4\pi a}$$

by symmetry other 3 sides contribute the same

$$\therefore \ B_{z\ @\ center} = 4B_{z_{top}} = \frac{\mu I\sqrt{2}}{\pi a}$$

EE 434 Homework 2

5.19



$I(t) = I_m \sin \omega t$

a) $H_\phi = \dfrac{I_m \sin \omega t}{2\pi\rho}$ from $\oint \bar{H} \cdot d\bar{l} = I$

$d\bar{S} = dz\, d\rho\, \bar{a}_\phi$

b) $\oint \bar{E} \cdot d\bar{l} = V(t) = -\dfrac{d}{dt} \displaystyle\int_{\rho=d}^{d+a} \int_{z=0}^{b} \dfrac{\mu_0 I_m \sin \omega t}{2\pi\rho}\, dz\, d\rho$

$V(t) = -\dfrac{d}{dt} \left\{ \dfrac{\mu_0 I_m \sin \omega t\, b}{2\pi} \ln\left(\dfrac{d+a}{d}\right) \right\} = -\dfrac{\mu_0 I_m \omega b \cos \omega t}{2\pi} \ln\left(\dfrac{d+a}{d}\right)$

(right hand terminal positive) $\boxed{\text{see A page 282}}$

or ... because $\bar{F}$ on positive charges would be in the z direction on both long sides of the loop but of greater magnitude on the left hand side driving a net positive charge to the right hand terminal.

for $I_m = 10A$, $f = 20kHz$, $d = 4 \times 10^{-3} m$, $a = b = 0.1 m$

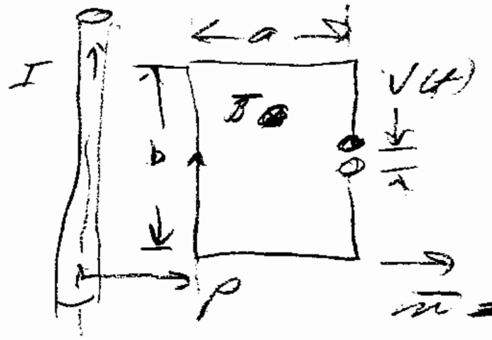
$V(t) = -\dfrac{4\pi \times 10^{-7} \times 10 \times \cancel{2\pi} \times 2 \times 10^{4} \times 10^{-1}}{\cancel{2\pi}} \ln\left(1 + \dfrac{0.1}{4 \times 10^{-3}}\right) \cos(2\pi \times 2 \times 10^{4} t)$

$V(t) = -81.87 \times 10^{-3} \cos(4\pi \times 10^4 t) = \boxed{-81.9 \cos(4\pi \times 10^4 t)\ mV}$

c) 2 times above with opposite polarity

EE 434 Homework 2 (solutions - page 2)

5.21



$V(t) = \oint \bar{E}\cdot d\bar{l} = \int \bar{u}\times\bar{B}\cdot d\bar{l}$

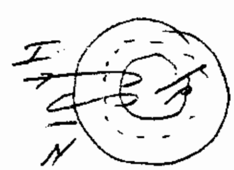
$\bar{B} = \frac{\mu_0 I}{2\pi\rho}\bar{a}_\phi$

$\bar{u} = \bar{\rho}\, u_\rho$

so $V(t) = \int_0^b \frac{\mu_0 I u}{2\pi\rho}\bar{a}_z\cdot dz\,\bar{a}_z + \int_b^0 \frac{\mu_0 I u}{2\pi(\rho+a)}dz = \underline{\underline{\frac{\mu_0 I u}{2\pi}\left\{\frac{b}{\rho} - \frac{b}{\rho+a}\right\}}}$

top terminal positive

5.23 $W_m = \frac{1}{2}\int_V \bar{B}\cdot\bar{H}\,dv$



$\oint \bar{H}\cdot d\bar{l} = NI$

$H_\phi = \frac{NI}{2\pi\rho}$

a)

so $W_m = \frac{1}{2}\int_{\rho=a}^{b}\int_{\phi=0}^{2\pi}\int_{z=0}^{d} \frac{\mu N^2 I^2}{4\pi^2\rho^2}\,dz\,\rho\,d\phi\,d\rho = \frac{\mu N^2 I^2\, 2\pi d}{2\times 4\pi^2}\ln\frac{b}{a}$

or $W_m = \frac{\mu d N^2 I^2}{4\pi}\ln\frac{b}{a} = \frac{1}{2}L I^2$

$\therefore \boxed{L = \frac{2W_m}{I^2} = \frac{\mu d N^2}{2\pi}\ln\frac{b}{a}}$ same as Example 5-17

b) $I = 2$, $\mu_r = 4\times10^3$, $a = 10^{-2}$, $b = 3\times10^{-2}$, $d = 2\times10^{-2}$, $N = 150$

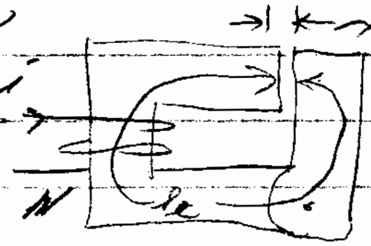
$W_m = \frac{4\times10^3\times\cancel{4\pi}\times10^{-7}\times2\times10^{-2}\times4\times1.5^2\times10^4}{\cancel{4\pi}} = 72\times10^{-2}\ln 3 = \boxed{79.1\times10^{-2}\text{ J}}$

so $L = \frac{2W_m}{I^2} = \frac{2\times79.1\times10^{-2}}{4} = \boxed{0.395\text{ H}}$

c) if $\mu_r = f(I)$ or $f(H)$ which would be true for ferromagnetic material?

EE 434 Homework 3

5.48



a) $\phi_m = \frac{NI}{R_{iron}+R_{gap}} = \boxed{\frac{NI}{R_{total}}}$

b) $L = \frac{\lambda}{I} = \frac{N\phi_m}{I} = \boxed{\frac{N^2}{R_{total}}}$

$\therefore U_m = \frac{1}{2}LI^2 = \frac{1}{2}\frac{N^2}{R}I^2$

found that $F_x = \frac{\partial U_m}{\partial x} = \frac{I^2}{2}\frac{\partial L}{\partial x} = \frac{I^2}{2}\frac{d}{dx}\left(\frac{N^2}{R}\right) = \frac{-N^2I^2}{2R^2}\frac{\partial R}{\partial x}$

or $F_x = -\frac{1}{2}\phi_m^2\frac{\partial R}{\partial x}$

so for $l_c = 0.12$, $A_c = 4\times10^{-4}$, $x = 1.5\times10^{-3}$, $I = 1.25A$, $N = 200$

and $\mu = 10^5\mu_0$

$$R_{total} = \frac{0.12}{10^5\times4\pi\times10^{-7}\times4\times10^{-4}} + \frac{\overset{10^{-3}}{x}}{4\pi\times10^{-7}\times4\times10^{-4}} = \boxed{2.387\times10^3 + \underset{\uparrow\ 1.99\times10^9x}{0.0298\times10^8}}$$

$$\phi_m = \frac{200\times1.25}{R_{total}} = \frac{2.5\times10^2}{2.98\times10^6} = \boxed{0.839\times10^{-4}\text{ Webers}}$$

$$\therefore F_x = -\frac{1}{2}(0.839\times10^{-4})^2\times1.99\times10^9 = \boxed{-7\text{ Newtons}}$$

$$B_{ave} = \frac{\phi_m}{A} = \frac{0.839\times10^{-4}}{4\times10^{-4}} = \boxed{0.21\ W/m^2}$$

$$H_{ave_{gap}} = \frac{B_{ave}}{\mu} = \frac{0.21}{4\pi\times10^{-7}} = \boxed{1.67\times10^5}$$

$$H_{ave_{iron}} = \frac{0.21}{4\pi\times10^{-7}\times10^5} = \boxed{1.67}$$

$$L = \frac{4\times10^4}{2.98\times10^6} = 1.31\times10^{-2} \boxed{\approx 13\text{ mH}}$$

$$U_m = \frac{1}{2}LI^2 = \frac{1}{2}\,1.31\times10^{-2}\times1.25^2 = \boxed{1.02\times10^{-2}\text{ J}}$$

If $x = 0.75$ mm $R_{new} = \frac{R_{old}}{2}$ so $\phi_{m_{new}} = 2\phi_{m_{old}}$ and $\boxed{F_{x_{new}} = 4F_{x_{old}}}$

5.48 $x = 0$ $\frac{\partial \mathcal{R}}{\partial x}$ is same as before but $\mathcal{R} = \mathcal{R}_{1mm} = 2.387 \times 10^{3}$

so $\psi_m = \frac{2.5 \times 10^{2}}{2.387 \times 10^{3}} = 1.047 \times 10^{-1}$

and $F_x = -\frac{1}{2}(1.047 \times 10^{-1})^2 \times 1.99 \times 10^{9} = \boxed{-1.09 \times 10^{7}\ N}$

6-5 air | $\epsilon = \epsilon_r \epsilon_0$

$\hat{E}^{+}_{m_1}$

$$\frac{\hat{E}^{+}_{m_2}}{\hat{E}^{+}_{m_1}} = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2\cancel{\eta_0}\frac{1}{\sqrt{\epsilon_r}}}{\cancel{\eta_0} + \cancel{\eta_0}\frac{1}{\sqrt{\epsilon_r}}}$$

a) so $\boxed{\frac{\hat{E}^{+}_{m_2}}{\hat{E}^{+}_{m_1}} = \frac{2}{1+\sqrt{\epsilon_r}}}$

$$\boxed{\frac{\hat{E}^{-}_{m_1}}{\hat{E}^{+}_{m_1}} = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{\frac{1}{\sqrt{\epsilon_r}} - 1}{\frac{1}{\sqrt{\epsilon_r}} + 1} = \frac{1 - \sqrt{\epsilon_r}}{1 + \sqrt{\epsilon_r}}}$$

b) $\hat{E}^{+}_{m_1} = 100$, $\epsilon_r = 2.25$ $\sqrt{\epsilon_r} = 1.5$

$\hat{E}^{+}_{m_2} = \frac{100 \times 2}{2.5} = \boxed{80}$

$\hat{E}^{-}_{m_1} = 100 \frac{-0.5}{2.5} = \boxed{-20}$

c) $\epsilon_r = 81$; $\sqrt{81} = 9$

so $\hat{E}^{+}_{m_2} = \frac{100 \times 2}{10} = \boxed{20\ \frac{V}{m}}$

$E^{-}_{m_1} = 100 \frac{-8}{10} = \boxed{-80\ \frac{V}{m}}$

EE 434 Homework 3 (solutions - page 3)

6-6 $v_{phase} = \frac{\omega}{\beta} = \frac{\omega}{\omega\sqrt{\mu\epsilon}} = \frac{c}{\sqrt{\epsilon_r}}$ ←

index of refraction $n = \sqrt{\epsilon_r}$ so $\boxed{n = \frac{c}{v_{ph}}}$ ←

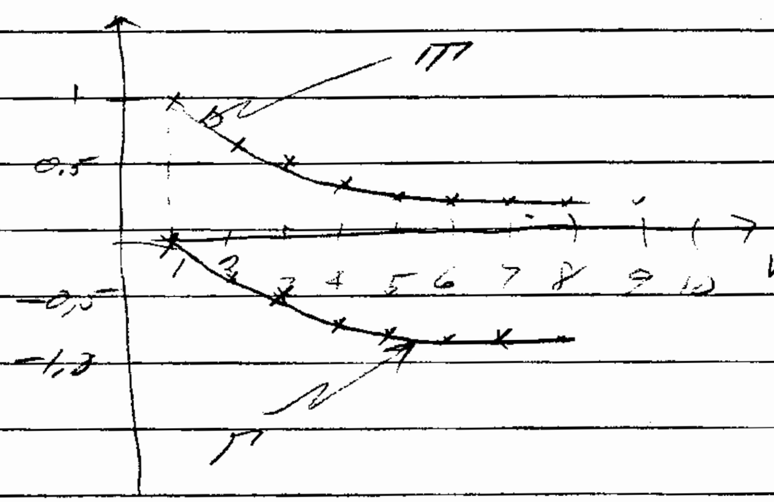
$n_{polyethylene} = \sqrt{2.25} = 1.5$ ←

$n_{water} = \sqrt{81} = 9$ ←

$\Gamma = \frac{1-n}{1+n}$; $T = \frac{2}{1+n}$

$\boxed{T - \Gamma = \frac{2-1+n}{1+n} = 1}$ ← {E field is continuous at boundary}

n	T	Γ
1	1	0
2	0.67	-0.33
3	0.5	-0.5
4	0.4	-0.6
5	0.33	-0.67
6	0.29	-0.71
7	0.25	-0.75
8	0.22	-0.77
9	0.2	-0.8
10	0.18	-0.82



EE 434 Homework 4

6-17 $\hat{\eta}_1$ | $\hat{\eta}_2$ | $\hat{\eta}_3$ $\gamma_2 \to j\beta_2$

$\leftarrow \lambda/4 \rightarrow$ $e^{\gamma_2 d} \to e^{j\frac{2\pi}{\lambda_2}\cdot\frac{\lambda_2}{4}} = j$

then $$\hat{Z}_1(0) = \eta_2 \frac{j(\hat{\eta}_3+\eta_2) - j(\hat{\eta}_3-\eta_2)}{j(\hat{\eta}_3+\eta_2) + j(\hat{\eta}_3-\eta_2)} = \frac{2j\eta_2\cdot\eta_2}{2j\hat{\eta}_3} = \frac{\eta_2^2}{\hat{\eta}_3} \leftarrow$$

6-18 $\epsilon_{r_3} = 2.56$ in above problem we need $\hat{Z}_1(0) = \eta_1$

$\therefore$ $\eta_2 = \sqrt{\eta_1\eta_3}$ or for all $\mu = \mu_0$

we want $\frac{\mu_0}{\epsilon_2} = \sqrt{\frac{\mu_0^2}{\epsilon_1\epsilon_3}}$; $\frac{1}{\epsilon_{r_2}} = \frac{1}{\sqrt{\epsilon_{r_1}\epsilon_{r_3}}}$

$$\boxed{\epsilon_{r_2} = \sqrt{1\times 2.56} = 1.6} \leftarrow$$

reciprocal because get same value for η_2 if η_1 + η_3 are interchanged ?

6-20 $\hat{\eta}_1$ | η_2 | $\hat{\eta}_3$ $e^{\gamma_2 d} \to e^{j\pi} = -1$

$\leftarrow \lambda/2 \rightarrow$

so $$\hat{Z}_1(0) = \eta_2 \frac{-(\hat{\eta}_3+\eta_2) - (\hat{\eta}_3-\eta_2)}{-(\hat{\eta}_3+\eta_2) + (\hat{\eta}_3-\eta_2)} = \frac{\eta_2(-2\hat{\eta}_3)}{-2\eta_2} = \hat{\eta}_3$$

same result whenever $e^{j\frac{2\pi}{\lambda}\cdot d} = -1$

or $d = n\frac{\lambda}{2}$; $n = 1, 2, 3, 4 \cdots$

EE 434 Homework 4 (Solutions - page 2)

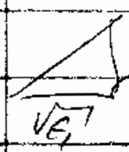
6-35 ① μ_0, ϵ_0 | ② $\mu_0, 3\epsilon_0$

$$\theta_{i_{\text{Brewster}}} = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$\therefore$ from ① → ② $\theta_{i_B} = 60°$

② → ① $\theta_{i_B} = 30°$

must be // polarized

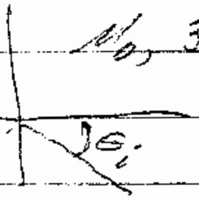


$$\Longleftrightarrow \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}} = 90° - \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

so $\theta_{i_{\text{Brewster}}}$ angles from opposite directions are always compliments!

EE 434 Homework 5

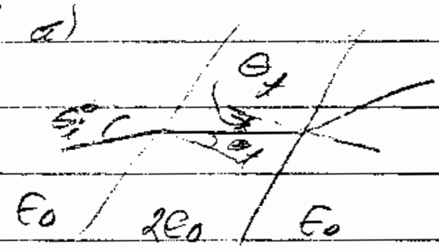
6.36 μ_0, ϵ_0 | $\mu_0, 3\epsilon_0$



$$\theta_{critical} = \sin^{-1}\sqrt{\frac{1}{3}} = \underline{\underline{35.26^\circ}}$$

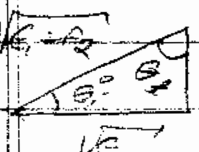
polarization not important

6.37 a)



$$\theta_i^\circ = \tan^{-1}\sqrt{\frac{\epsilon_2}{\epsilon_1}}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \sqrt{\frac{\epsilon_1}{\epsilon_2}}$$



$$\sin\theta_t = \frac{\sqrt{\epsilon_2}}{\sqrt{\epsilon_1+\epsilon_2}} \cdot \sqrt{\frac{\epsilon_1}{\epsilon_2}} = \frac{\sqrt{\epsilon_1}}{\sqrt{\epsilon_1+\epsilon_2}}$$

thus we see that $\boxed{\theta_t = 90^\circ - \theta_i^\circ}$

We showed in problem 6.35 that the Brewster angle for waves incident on the interface from either side are complementary angles. The angle of incidence at the second surface is θ_t so their will be no reflection at the surface because $\theta_t = 90^\circ - \theta_i^\circ$ as shown above

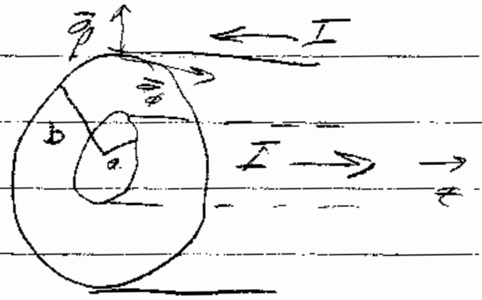
b) $\Gamma_{\parallel} = 0$; $\theta_i^\circ = \tan^{-1}\sqrt{2} = 54.73^\circ$

$$\Gamma_{\perp} = \frac{\cos\theta_i - \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_t}{\cos\theta_i + \sqrt{\frac{\epsilon_2}{\epsilon_1}}\cos\theta_t} \; ; \; \theta_t = \sin^{-1}\left(\sqrt{\frac{\epsilon_1}{\epsilon_2}}\sin\theta_i\right) = 35.26^\circ$$

$$\Gamma_{\perp} = \frac{0.5774 - 1.155}{0.5774 + 1.155} = \frac{-0.577}{1.732} = \boxed{-0.333}$$

EE 434 Homework 5 (solutions - page 2)

7.2



$$E_z = \frac{-J_z}{\sigma} \; ; \; J_z = \frac{I}{\pi c^2 - \pi b^2}$$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$\underset{@\rho=b}{\bar{E}\times\bar{H}} = \frac{J_z I}{\sigma 2\pi b}\bar{a}_\rho = \frac{I^2}{2\pi b \sigma (c^2-b^2)}\bar{a}_\rho$$

$$\underset{\text{outer conductor}}{P_{\text{into}}} = \int_{z=0}^{L}\int_{\phi=0}^{2\pi} \frac{I^2\,\bar{a}_\rho}{2\pi^2 b\sigma(c^2-b^2)} \cdot b\,d\phi\,dz$$

$$P = \frac{2\pi L I^2}{2\pi^2\sigma(c^2-b^2)} = I^2\frac{L}{\sigma\pi(c^2-b^2)} = I^2 R \;\nabla$$

EE 434 Homework 6

12.21

7-13 $\hat{E}_x = \hat{E}_m^+ e^{-j\beta z}[1+\Gamma]$ $\hat{H}_y = \frac{\hat{E}_m^+}{\eta} e^{-j\beta z}[1-\Gamma]$

$\bar{P}_{ave} = \frac{1}{2}Re\{\hat{\bar{E}} \times \hat{\bar{H}}^*\} = \frac{1}{2}Re\left\{\frac{|E_m^+|^2}{\eta}(1+\Gamma)(1-\Gamma^*)\right\}\bar{a}_z$

$\bar{P}_{ave} = \frac{|E_m^+|^2}{2\eta} Re\{1 + \cancel{\Gamma_r} + j\Gamma_i - \cancel{\Gamma_r} + j\Gamma_i - |\Gamma|^2\}\bar{a}_z$

$\bar{P}_{ave} = \frac{|E_m^+|^2}{2\eta}[1-|\Gamma|^2]\bar{a}_z = \bar{P}_{ave}^+ - \bar{P}_{ave}^-$

$\therefore \frac{|P_{ave}^-|}{|P_{ave}^+|} = |\Gamma|^2$

7-17 $P_{ave} = \frac{1}{2}Re\left\{\frac{|\hat{E}_m^+|^2 e^{-2\alpha z}}{\eta} e^{j\theta}[1+\Gamma][1-\Gamma^*]\right\}$

$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} e^{-2\alpha z} Re\{[\cos\theta + j\sin\theta][1 + \cancel{\Gamma_r} + j\Gamma_i - \cancel{\Gamma_r} + j\Gamma_i - |\Gamma|^2]\}$

$P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} e^{-2\alpha z}\{\cos\theta[1-|\Gamma|^2] - \sin\theta \, 2\Gamma_i\}$ (1)

for $\Gamma = 0$ this becomes $\boxed{P_{ave} = \frac{|\hat{E}_m^+|^2}{2\eta} e^{-2\alpha z}\cos\theta}$

b) lossless case $\hat{\eta} \to \eta$ i.e. $\theta \to 0$ and $\alpha \to 0$

giving: $\boxed{P_{ave} = \frac{|E_m^+|^2}{2\eta}[1-|\Gamma|^2] \text{ as in 7-13}}$

P_{ave} ~~positive wave~~ $= \frac{1}{2}Re\left\{\frac{\hat{E}_m^+ e^{-\alpha z} \cancel{e^{-j\beta z}} e^{+j\theta} \hat{E}_m^{+*} e^{-\alpha z} \cancel{e^{+j\beta z}}}{\eta}\right\} = \frac{|\hat{E}_m^+| e^{-2\alpha z}\cos\theta}{\eta}$

P_{ave} negative wave $= \frac{1}{2}Re\left\{\frac{\hat{E}_m^+ e^{-\alpha z} \cancel{e^{-j\beta z}} \Gamma \hat{E}_m^{+*} e^{-\alpha z} \cancel{e^{+j\beta z}} e^{j\theta}(-\Gamma^*)}{\eta}\right\} = \frac{|\hat{E}_m^+|^2 e^{-2\alpha z}|\Gamma|^2}{2\eta}\cos\theta$

summing these two terms would miss the $-\sin\theta[2\Gamma_i]$ term in the above expression ?

EE 434 Homework 6 (solution - page 2)

7-21 $\mathcal{P}_{av}$ of sun @ earth $= 1340\ W/m^2$

assuming single frequency wave we have:

$$\mathcal{P}_{av} = \frac{|\hat{E}_m^+|^2}{2\eta}\ ;\ \text{or}\ |\hat{E}_m| = \sqrt{2\times377\times1340} = 1.005\times10^3\ V/m$$

$$|\hat{H}_m| = \frac{|\hat{E}_m|}{\eta} = 2.66\ A/m$$

$$P_{total\ from\ sun} = 4\pi\,(1.48\times10^{11})^2\,1.34\times10^3 = \boxed{36.88\times10^{25}\ \text{Watts}}$$

EE 434 Homework 7

A. 1" radius ⊙ waveguide

$$f_{c_{lowest}} = \frac{c}{2\pi}\frac{P_{11}'}{a} = \frac{3\times10^8 \times 1.841\times10^2}{2\pi\times2.54} = \underline{3.46\times10^9\,Hz}$$

B. P_{max} for $0.4\times0.9''$ waveguide @ 10 GHz

$$\hat{E}_y = -jK\frac{\omega\mu a}{\pi}\sin\frac{\pi}{a}x = E_{y_{max}}\sin\frac{\pi}{a}x$$

$$\hat{H}_x = K_1'\frac{\beta_{10}a}{\pi}\sin\frac{\pi}{a}x = -E_{y_{max}}\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\sin\frac{\pi}{a}x$$

$$P_{avg}/m^2 = \frac{1}{2}Re\{\hat{E}\times\hat{H}^*\} = \frac{1}{2}Re\left\{\bar{a}_z E_{y_{max}}^2\sin^2\frac{\pi}{a}x\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\right\}$$

$$P_{avg} = \frac{1}{2}E_{y_{max}}^2\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\int_{y=0}^{b}\int_{x=0}^{a}\underbrace{\sin^2\frac{\pi}{a}x}_{\frac{1}{2}\left[1-\cos\frac{2\pi x}{a}\right]}\,dx\,dy$$

$$\text{so } P_{avg} = \frac{1}{2}E_{y_{max}}^2\sqrt{\frac{\epsilon}{\mu}}\sqrt{1-\left(\frac{f_c}{f}\right)^2}\,\frac{ba}{2} \quad \text{(Equation 8-90)}$$

$$P_{avg} = 1.16\times10^{-7}E_{y_{max}}^2$$

but $E_{y_{max}} \cong 3\times10^6\ V/m$ so $P_{avg_{max}} = 1.16\times10^{-7}\times9\times10^{12} \cong \underline{1.04\times10^6\,W}$

8.23 $f_c = \frac{1}{2\sqrt{\mu\epsilon}\,a}$ TE_{10} mode

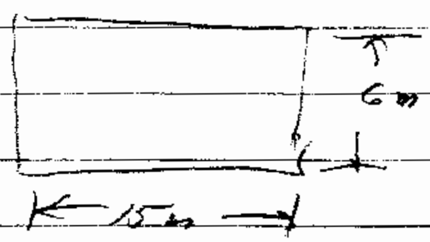
a) 5 GHz; $a_{min} = \frac{1}{2\sqrt{\mu\epsilon}\,f} = \frac{3\times10^8}{10\times10^9} = \underline{3\times10^{-2}\,m}$ ←

b) 5 MHz; $a_{min} = \frac{3\times10^8}{10\times10^6} = \underline{30\,m}$ ←

c) 5 KHz; $a_{min} = \frac{3\times10^8}{10\times10^3} = \underline{3\times10^4\,m}$ ←

EE 434 Homework 7 (solutions - page 2)

8.27



TE_{10} mode

$$f_c = \frac{1}{2\sqrt{\mu\epsilon}\,15} = \frac{3\times10^8}{30} = 10\,MHz$$

Vertical polarization

AM will not propagate } 535 → 1605 KHz

FM will propagate } 88 → 108 MHz

EE 434 Homework 8

11-11 $\Delta z = 0.01\lambda$; $\hat{I} = 5A$; $f = 300 MHz$

a) $\lambda = \frac{3\times10^8}{3\times10^8} = 1m$ $\therefore$ $\boxed{\Delta z = 0.01 m}$ ←

(far zone if $\beta_0 r >> 1$ from (11-30)
so $\beta_0 r = 20$ is reasonably in the far zone)

this is $\left[r = \frac{20}{\beta_0} = \frac{20}{2\pi}\lambda = 3.18 m \right]$ ←

b) $d\hat{E}_\theta = \eta\, d\hat{H}_\phi = j\frac{\omega\mu \hat{I}\Delta z\, e^{-j\beta r}}{4\pi r}\sin\theta$ ←

$\sin\theta = 1$ @ $\theta = 90°$

c) $\bar{P} = \frac{1}{2}Re\{d\hat{E}_\theta \bar{a}_\theta \times d\hat{H}^*_\phi \bar{a}_\phi\} = \frac{1}{2}\cdot\frac{\omega^2\mu^2 \hat{I}^2 \Delta z^2}{\eta\, 16\pi^2 r^2}\bar{a}_r$

$\bar{P} = \frac{4\pi^2 f^2 \mu^2 \hat{I}^2 \Delta z^2 \sqrt{\mu}\sqrt{\epsilon}\,\bar{a}_r}{\sqrt{\mu}\, 32\pi^2 r^2 \sqrt{\epsilon}} = \frac{\mu \hat{I}^2 \Delta z^2}{8\sqrt{\mu\epsilon}\,\lambda^2 r^2}\bar{a}_r$

$\boxed{P = \frac{\eta \hat{I}^2}{8 r^2}\left(\frac{\Delta z}{\lambda}\right)^2} = \frac{377\times25\times10^{-4}}{8\times3.18^2} = \boxed{0.0117 \text{ Watts}/m^2}$

11-14 $P_{rad} = \int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \sin^2\theta \frac{\eta \hat{I}^2}{8 r^2}\left(\frac{\Delta z}{\lambda}\right)^2 r^2 \sin\theta\, d\theta\, d\phi$

$\left[P_{rad} = \frac{2\pi\eta \hat{I}^2}{8_4}\left(\frac{\Delta z}{\lambda}\right)^2 \underbrace{\int_0^\pi \sin^3\theta\, d\theta}_{4/3} = \frac{\pi\eta \hat{I}^2}{3}\left(\frac{\Delta z}{\lambda}\right)^2 \right]$

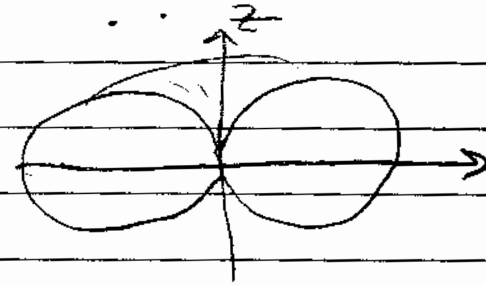
a) so $P_{rad} = \frac{\pi\times377\times25\times10^{-4}}{3} = \boxed{0.99 \text{ Watts}}$ ←

b) if $\Delta z \to 0.02 m$ $P_{rad} = 4\times0.99 = 3.95$ Watts ←

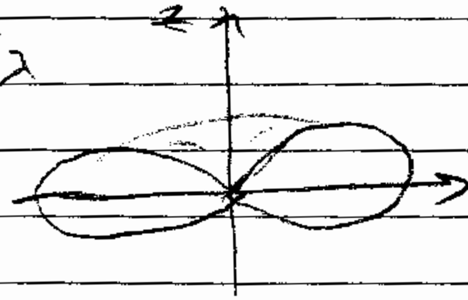
if $\Delta z = 0.005 m$ $P_{rad} = \frac{0.99}{4} = 0.25$ Watts ←

EE 434 Homework 8 (solutions-page 2)

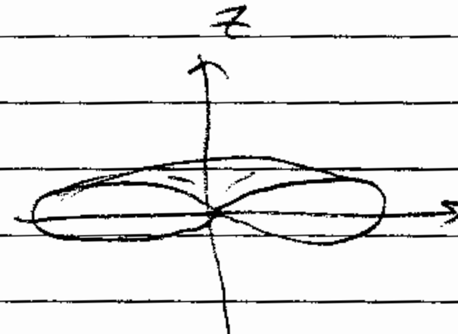
11-19 a) $L = \frac{\lambda}{2}$



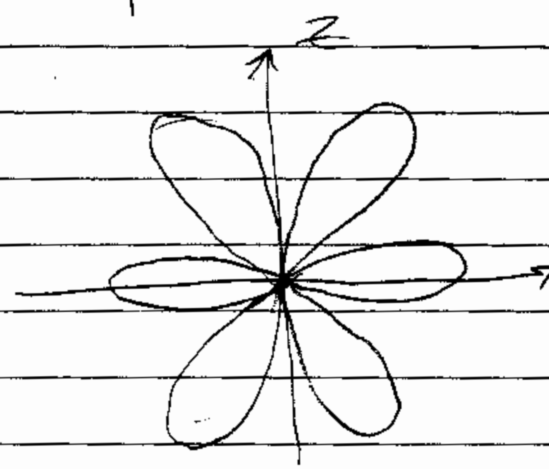
b) $L = \frac{3}{4}\lambda$



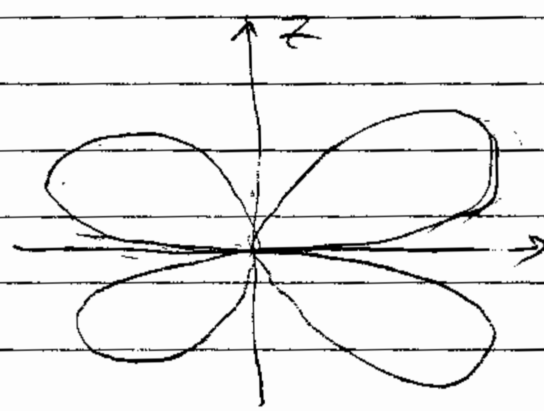
c) $L = \lambda$



11-20 a) $L = 1.5\lambda$



b) $L = 2\lambda$



EE 434 Homework 9

9.1, 9.2, 9.5
9.8, 9.10, 9.15

9.1 "dz" dipole ; $I = 10A$; $dz = \frac{\lambda}{50}$; $r = 10^3 m$

$$P_{max} = \frac{1}{2}\eta_0 \left\{ \frac{\beta I^2 dz^2 \omega\mu}{16\pi^2 r^2} \right\} = \frac{2\pi I^2 dz^2 \omega\mu}{2\lambda\, 16\pi^2 r^2}$$

but $\left[\omega\mu = \frac{2\pi f \sqrt{\mu}\sqrt{\mu\epsilon}}{\sqrt{\epsilon}} = \frac{2\pi\eta}{\lambda} \right]$

$$\therefore P_{max} = \frac{I^2 dz^2 \, 2\pi\eta}{\lambda^2 \, 16\pi r^2} = \frac{10^2 \times 120\pi}{(50)^2 \, 8 \times 10^6} = 1.88 \times 10^{-6} \, W/m^2$$

9.2 $L = 1m$; $f = 10^6$ $\therefore \lambda = \frac{3\times10^8}{10^6} = 300m$ (short dipole)

$I = 12A$, $r = 5\times10^3$

$$P(\theta = 30^\circ) = \frac{I^2\eta}{8r^2}\left(\frac{dz}{\lambda}\right)^2 \sin^2 30^\circ = \frac{12^2 \times 377}{8\times25\times10^6}\left(\frac{1}{300}\right)^2 \sin^2 30^\circ = 7.54\times10^{-10} \, W/m^2$$

9.5 $L = 2m$, $f = 10^6$ $\therefore \lambda = \frac{3\times10^8}{10^6} = 300m$ (short dipole)

Cu antenna $\therefore \sigma = 5.8\times10^7$ wire radius = 1mm

a) $$R_{rad} = \frac{2\pi\eta}{3}\left(\frac{dz}{\lambda}\right)^2 = \frac{2\pi\times377}{3}\left(\frac{2}{300}\right)^2 = \boxed{0.0351 \, \Omega}$$

$$R_{loss} = \frac{L}{2a}\sqrt{\frac{f\mu}{\pi\sigma}} = \frac{2}{2\times10^{-3}}\sqrt{\frac{10^6\times4\pi\times10^{-7}}{\pi\times5.8\times10^7}} = \boxed{0.083 \, \Omega}$$

so $$\xi = \frac{0.0351}{0.0351+0.083} = 0.297 \text{ or } \boxed{29.7\%}$$

b) $G = \xi D = 0.297\times1.5 = 0.446$ or $\boxed{-3.51 dB}$

c) $$20 = \frac{I^2\pi\eta}{3}\left(\frac{dz}{\lambda}\right)^2 \text{ or } I = \sqrt{\frac{60}{\pi\eta}}\left(\frac{\lambda}{dz}\right)$$

or $$I = \sqrt{\frac{60}{120\pi^2}}\left(\frac{\lambda}{dz}\right) = \frac{1}{\sqrt{2}\pi}(150) = \boxed{33.76A}$$

$$P_{transmitter} = \frac{P_{rad}}{\xi} = \frac{20}{0.297} = \boxed{67.34 \text{ Watts}}$$

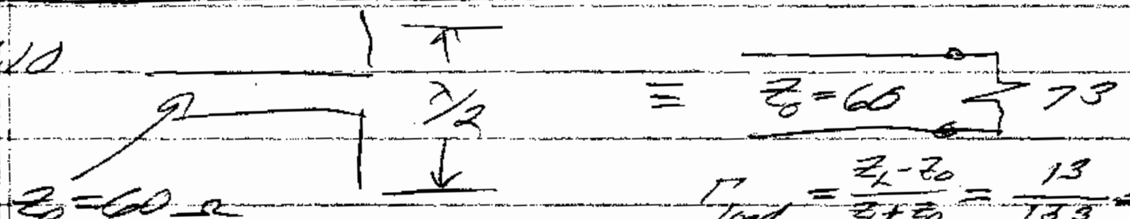
EE 434 Homework 9 (solution-pages)

9.8 $\xi = 90\%$; $D_{dB} = 6.7 dB$; $G = ?$

$D_{dB} = 10\log_{10} D$ $\therefore D = 10^{\frac{D_{dB}}{10}} = 4.677$; $G = \xi D = 4.2$

$\boxed{\text{or } G_{dB} = 6.24 dB}$

9.10



$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{13}{133} = 0.0977$

$SWR = \frac{|\hat{V}_{max}|}{|\hat{V}_{min}|} = \frac{1+|\Gamma|}{1-|\Gamma|} = \frac{1+0.0977}{1-0.0977} = \boxed{1.217}$

9.15 3GHz ; 1m dishes ; $P_r = 10^{-9} W$; $r = 4\times10^4$

$\lambda = \frac{3\times10^8}{3\times10^9} = 0.1 m$

$P_t = 10^{-9}(r\lambda)^2\left(\frac{1}{A_r A_t}\right) = \frac{10^{-9}\times16\times10^8\times10^{-2}\times16}{\pi^2} = \underline{25.94\times10^{-3} W}$

EE 434 Homework 10

9.16 $\lambda/2$ dipole ; $P_t = 10^3$; $f = 50\,MHz$; $G_r = 13\,dB$

$R = 3\times10^4$; $D_r = 19.95$; $\lambda = \frac{3\times10^8}{5\times10^7} = 6\,m$

$\therefore P_r = P_t D_r D_t \left(\frac{\lambda}{4\pi R}\right)^2 = 10^3 \times 19.95 \times 1.64 \left(\frac{6}{4\pi\times3\times10^4}\right)^2 = \underline{8.28\times10^{-6}\,W}$ ⟵

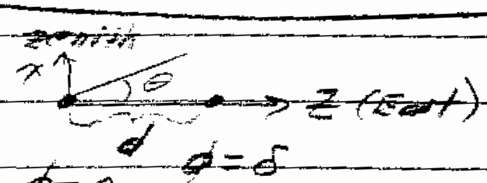
9.21 94 GHz ; antenna 1m × 0.1m ; $\lambda = \frac{3\times10^8}{94\times10^9} = 3.19\times10^{-3}\,m$

a) $BW_{elevation} \cong \frac{3.19\times10^{-3}}{0.1} = 3.19\times10^{-2}$ radians $= \boxed{1.8^\circ}$ ⟵

$BW_{azimuth} \cong 3.19\times10^{-3}$ radians $= \boxed{0.18^\circ}$ ⟵

b) 0.18° ; $d = 300\times3.19\times10^{-3} = \boxed{0.957\,m}$ ⟵

⟵ 300m ⟶

9-23 

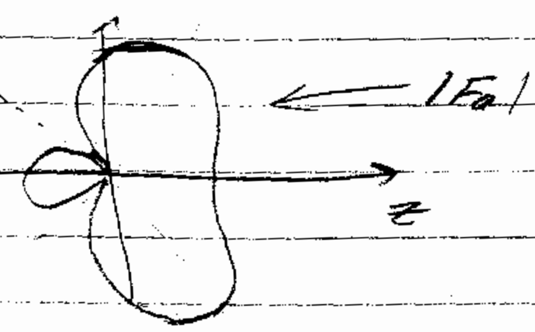
a) $a_0 = a_1 = 1$; $\delta = \frac{\pi}{4}$; $d = \frac{\lambda}{2}$

uniform array $\therefore$ $|F_a| = \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\frac{\psi}{2}}$ where $\psi = \frac{2\pi}{\lambda} d\cos\theta - \delta$

so $\boxed{\psi = \pi\cos\theta - \frac{\pi}{4}}$ and $|F_a| = \frac{\sin\psi)}{\sin\frac{\psi}{2}}$

maximum of array pattern @ $\psi = 0$ so $\left[\theta_{max} = \cos^{-1}\frac{1}{4} = 75.52^\circ\right]$

1^{st} zero when $\psi = -\pi = \pi\cos\theta - \frac{\pi}{4}$ or $\cos\theta = -\frac{3}{4}$; $\theta_{zero} = 138.6^\circ$



EE 434 Homework 10 (solutions - page 2)

9.23 b) $a_0=1$, $a_1=2$, $\delta=0$, $d=\lambda$

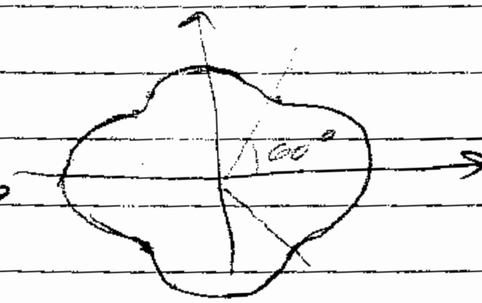
$F_a = 1+2e^{j\psi}$; $\psi = \frac{2\pi}{\lambda} d\cos\theta - \delta = 2\pi\cos\theta$

$\therefore F_a = 1+2e^{j2\pi\cos\theta} = 1+2\cos(2\pi\cos\theta)+j2\sin(2\pi\cos\theta)$

so $|F_a| = [1+2\cos(2\pi\cos\theta)]^2 + 4\sin^2(2\pi\cos\theta)$

$|F_a| = 1+4\cos(2\pi\cos\theta)+4 = 5+4\cos(2\pi\cos\theta)$

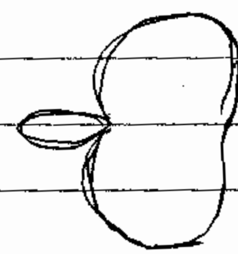
computer plot gives:

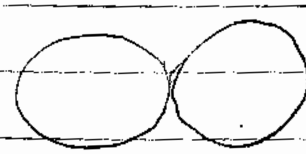


@ $\theta=0$ & $90°$ $|F_a|=9$

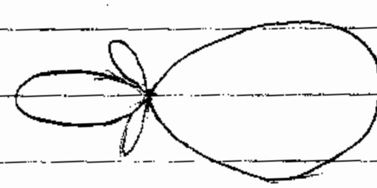
@ $\theta=60°$ $|F_a|=1$

9-24 from 9.23a)

$F_a =$ 

$f_{element} =$ 

total pattern $= f_{element} \times F_a$

$\therefore$ total pattern is 

EE 434 Homework 11

Broadcast Antenna Array $\left[F_a = (z - z_1)(z - z_2) \right]$

where $z = e^{j\psi} = e^{j\beta d \cos\theta}$

$\psi = 0$ (fed in phase) will provide maximum @ $\theta = 90°$

want zeroes when $\theta = 26°$ and $154°$

$\therefore \psi_1 = \frac{2\pi d}{\lambda} \cos 26° = \frac{\pi}{2} \cos 26° = 0.449\pi$

$\psi_2 = \frac{\pi}{2} \cos 154° = -0.449\pi$

so $F_a = (z - e^{j0.449\pi})(z - e^{-j0.449\pi}) = z^2 - z(e^{j0.449\pi} + e^{-j0.449\pi}) + 1$

$\boxed{\text{or } F_a = z^2 - 0.319z + 1}$ $\boxed{z = e^{j\frac{\pi}{2}\cos\theta}}$

(see attached plot)

9.18 a) $P_t = 10W$, $f = 6GHz$, $G_t = 20dB$, $G_r = 23dB$, $R = 2\times10^4 m$

$D(\theta,\phi) = \frac{P\,{}^{W}/_{m^2}\; 4\pi R^2}{P_t}$; $\lambda = \frac{3\times10^8}{6\times10^9} = 0.05m$

a) $P_{@\,receiver} = \frac{G_t P_t}{4\pi R^2} = \frac{10^2 \times 10}{4\pi \times 4\times10^8} = 0.0199\times10^{-5} = \boxed{1.99\times10^{-7} W/m^2}$

b) $P_{receiver} = P_t G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2 = 1.99\times10^{-7} \times 25\times10^{-4} \times 199.53 \left(\frac{1}{4\pi}\right) = \boxed{7.899\times10^{-9} W}$

c) $N = kTB = 1.38\times10^{-23} \times 10^3 \times 10^7 = 1.38\times10^{-13}$ $\therefore$ $\boxed{S/N = 5.72\times10^4 \text{ or } 47.6dB}$

9.20 Beamwidth $= 1.5°$ @ $20GHz$ $\quad 1.5° \times \frac{\pi}{180} = \frac{1.5\times10^{-2}}{dia}$; $dia = 0.573m$

$\lambda = \frac{3\times10^8}{20\times10^9} = 1.5\times10^{-2} m$; $D = \frac{4\pi}{\lambda^2} A_{eff} = \frac{4\pi^2 (0.573)^2}{2.25\times10^{-4}\times4} = 1.44\times10^4$

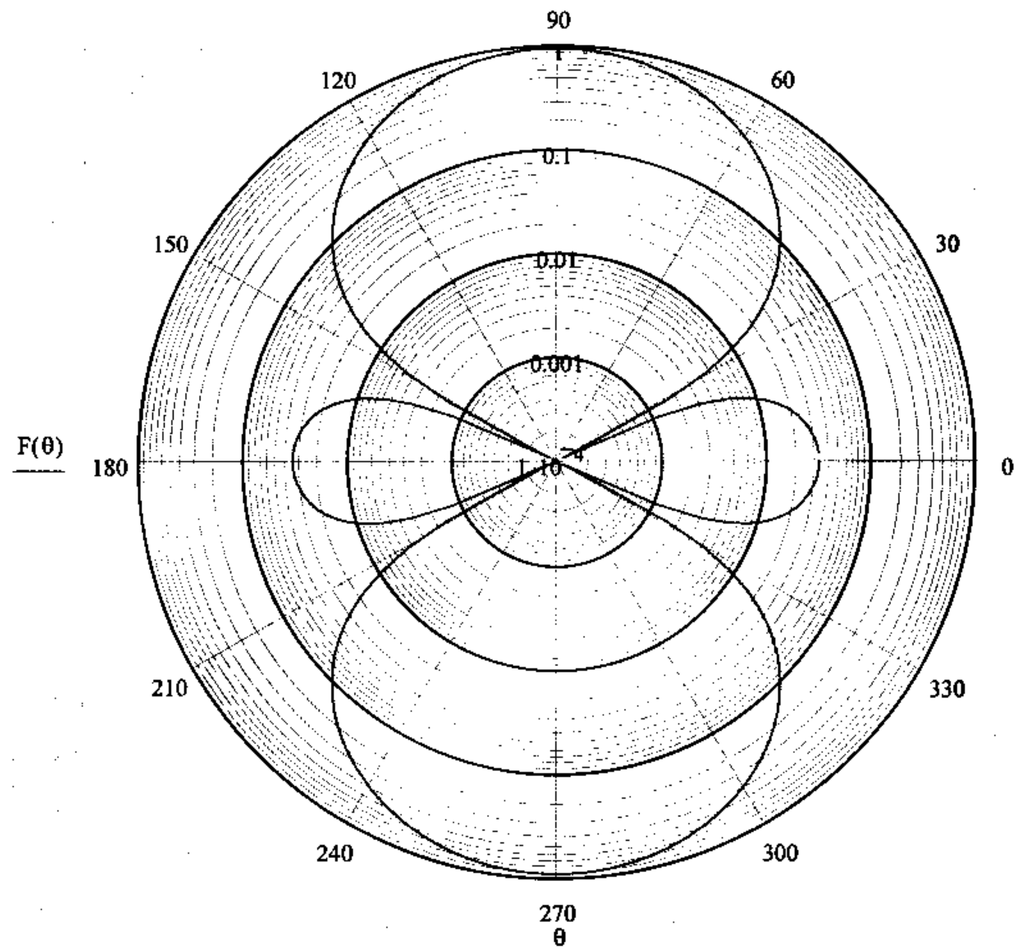
a) $\therefore$ $\boxed{D = 1.44\times10^4 \text{ or } 41.6dB}$

b) for twice the area D will be multiplied by 2 and beamwidth $\div \sqrt{2}$

c) for twice the frequency $\boxed{\text{beamwidth} \div 2}$ and $\boxed{D \text{ multiplied by } 4}$

Broadcast Array Special Problem

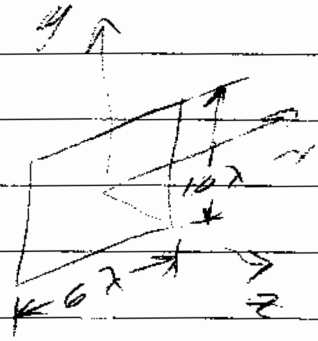
$j := \sqrt{-1};\quad \theta := 0, \frac{\pi}{100} .. 2\cdot\pi \quad z(\theta) := e^{j\cdot\frac{\pi}{2}\cdot\cos(\theta)} \quad z_1 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(26\cdot\frac{\pi}{180}\right)} \quad z_2 := e^{j\cdot\frac{\pi}{2}\cdot\cos\left(154\cdot\frac{\pi}{180}\right)} \quad F(\theta) := \left[\left|\frac{1}{\sqrt{3}}\cdot(z(\theta) - z_1)\cdot(z(\theta) - z_2)\right|\right]^2$



Problems

Homework 12

11-26



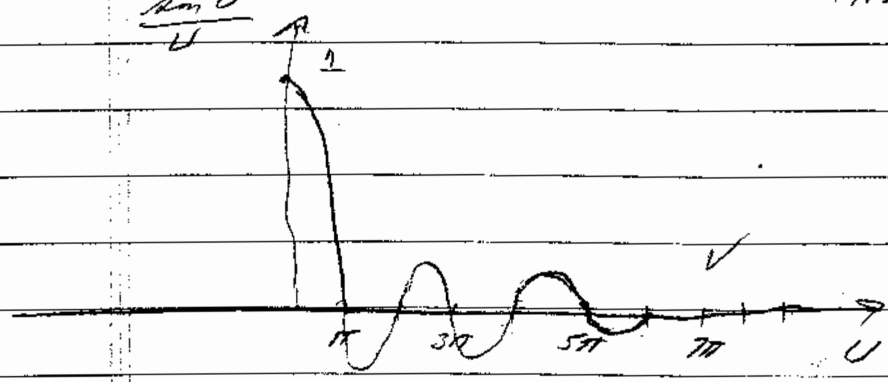
$\phi = 0$ pattern (xz plane)

$$\hat{E} = \frac{j\beta E_m^+}{4\pi r}\, ab\,(1+\cos\theta)\, e^{-j\beta r}\, \frac{\sin\left(\frac{\beta a}{2}\sin\theta\right)}{\frac{\beta a}{2}\sin\theta}\,\hat{a}_\theta$$

$$\frac{\sin U}{U} \rightarrow \frac{\sin\left(\frac{\beta a}{2}\sin\theta\right)}{\frac{\beta a}{2}\sin\theta}$$

(aperture is 6λ wide in this plane)

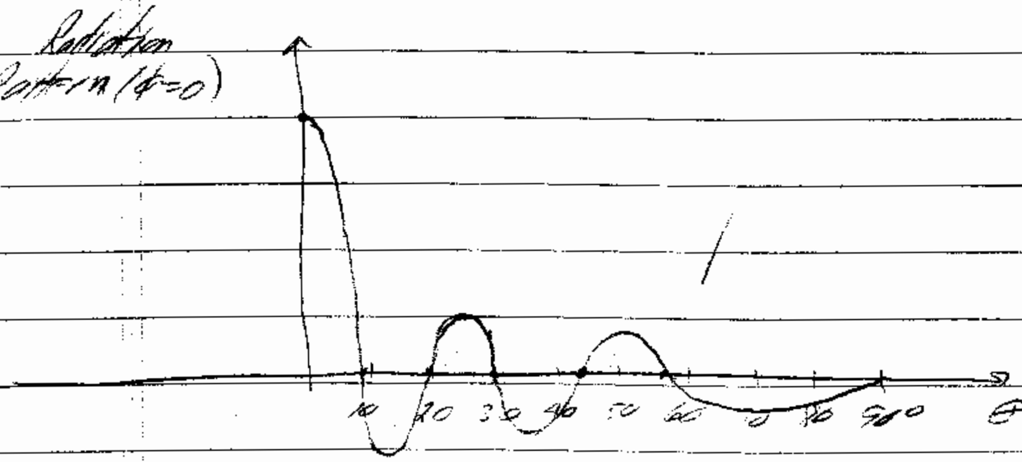
$\therefore \frac{\beta a}{2} = \frac{2\pi}{\lambda}\cdot\frac{6\lambda}{2} = 6\pi$ which is the visible range of U (i.e. $0 < U < 6\pi$ for $0 < \theta < \frac{\pi}{2}$)

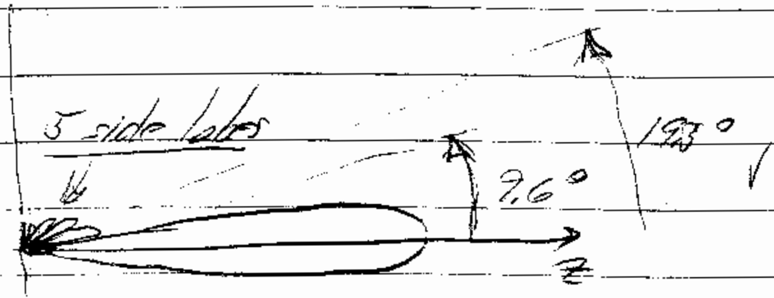


Zeros [$6\pi \sin\theta = n\pi$]

U	θ
π	9.6°
2π	19.5°
3π	30°
4π	42°
5π	56°
6π	90°

Radiation Pattern ($\phi = 0$)





Homework 12 (page 2)

11-26 (continued) beam width

From table 11-1 $\theta_{half\,power} \cong \frac{50^\circ}{6} = \underline{\underline{8.33^\circ}}$ ✓ 7

First sidelobe level down 13.2 dB ✓

11-29

5

11-38

4