

1. PR11.12d $\frac{(s+a)e^{-as}}{s+b} \cdot \frac{1}{s+b} \xrightarrow{a-b} \delta(t-a) + (a-b)e^{-b(t-a)}u(t-a)$

2. PR11.13b $\frac{4(s^2+1)}{s(s^2+4)} = \frac{K_1}{s} + \frac{K_2s+K_3}{(s+0)^2+2^2} \rightarrow K_1=1$
 $4s^2+1 = 1(s^2+4) + K_2s^2 + K_3s$
 $s^2: 4 = 1 + K_2 \Rightarrow K_2=3$
 $s: 0 = K_3$
 $\rightarrow \underline{u(t) + 3\cos(2t)u(t)}$

3. a) $\frac{d^2x}{dt^2} + 8\frac{dx}{dt} + 15x = (t-1)u(t-1), x(0)=1, \frac{dx(0)}{dt}=2$

LT $\Rightarrow [s^2X(s) - sx(0) - \frac{dx(0)}{dt}] + 8[sX(s) - x(0)] + 15X(s) = \frac{1}{s^2}(e^{-s})$
 $\Rightarrow X(s)(s^2+8s+15) - 5 - 2 - 8 = \frac{1}{s^2}e^{-s}$
 $\Rightarrow X(s)(s^2+8s+15) = 5+10 + \frac{1}{s^2}e^{-s}$
 $\Rightarrow X(s) = \frac{5+10}{s^2+8s+15} + \frac{1}{s^2} \frac{e^{-s}}{(s^2+8s+15)} = \frac{3.5}{s+3} - \frac{2.5}{s+5} + e^{-s} \left[\frac{-0.0356}{s} + \frac{0.0667}{s^2} + \frac{0.0556}{s+3} - \frac{0.02}{s+5} \right]$
 $\Rightarrow x(t) = (3.5e^{-3t} - 2.5e^{-5t})u(t) + (-0.0356 + 0.0667(t-1) + 0.0556e^{-3(t-1)} - 0.02e^{-5(t-1)})u(t-1)$

b) $\frac{d^2x}{dt^2} + 6\frac{dx}{dt} + 9x = 3u(t), x(0)=1, \frac{dx(0)}{dt}=2$

LT $\Rightarrow [s^2X(s) - s(1) - 2] + 6[sX(s) - 1] + 9X(s) = \frac{3}{s}$
 $\Rightarrow X(s)(s^2+6s+9) = 5+2+6 + \frac{3}{s} = \frac{s^2+8s+3}{s}$
 $\Rightarrow X(s) = \frac{s^2+8s+3}{s(s+3)^2} = \frac{K_1}{s} + \frac{K_2}{s+3} + \frac{K_3}{(s+3)^2} = \frac{1}{s} + \frac{2}{s+3} + \frac{4}{(s+3)^2}$
 $\Rightarrow \underline{x(t) = (\frac{1}{3} + \frac{2}{3}e^{-3t} + 4te^{-3t})u(t)}$

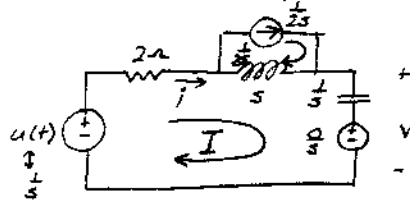
c) $\frac{d^2x}{dt^2} + 4\frac{dx}{dt} + 20x = 2u(t), x(0)=1, \frac{dx(0)}{dt}=2$

LT $\Rightarrow [s^2X(s) - s(1) - 2] + 4[sX(s) - 1] + 20X(s) = \frac{2}{s}$
 $\Rightarrow X(s)(s^2+4s+20) = 5+6 + \frac{2}{s} = \frac{s^2+6s+2}{s}$
 $\Rightarrow X(s) = \frac{s^2+6s+2}{s(s^2+4s+20)} = \frac{s^2+6s+2}{s(s+2+j4)(s+2-j4)} = \frac{K_1}{s} + \frac{K_2s+K_3}{(s+2)^2+4^2}$
 $\Rightarrow K_1 = \frac{1}{10}; s^2+6s+2 = \frac{1}{10}(s^2+4s+20) + K_2s^2+K_3s$
 $s^2: 1 = \frac{1}{10} + K_2 \Rightarrow K_2 = \frac{9}{10}$
 $s: 6 = \frac{4}{10} + K_3 \Rightarrow K_3 = \frac{60}{10} - \frac{4}{10} = \frac{56}{10}$

$\Rightarrow x(t) = \frac{1}{10}(1 + 9e^{-2t}\cos(4t) + 9.5e^{-2t}\sin(4t))u(t)$
 $= (0.1 + 0.9e^{-2t}\cos(4t) + 0.95e^{-2t}\sin(4t))u(t)$
 $= \underline{(0.1 + 1.31e^{-2t}\cos(4t - 46.5^\circ))u(t)}$



4. PR 11.36



$$\begin{aligned} \text{mesh current: } -\frac{1}{s} + 2I + s(I - \frac{1}{s}) + \frac{1}{s}I + 0 &= 0 \\ \Rightarrow I(2 + s + \frac{1}{s}) &= \frac{1}{s} + \frac{1}{s} \\ &= \frac{1 + 1/s}{s} \\ \Rightarrow I(s^2 + 2s + 1) &= \frac{1}{2}s + 1 \\ \Rightarrow I &= \frac{\frac{1}{2}s + 1}{s^2 + 2s + 1} = \frac{\frac{1}{2}s + 1}{(s + 1)^2} \end{aligned}$$

$$\begin{aligned} V(s) &= \frac{1}{s} I(s) + \frac{0}{s} \\ &= \frac{\frac{1}{2}s + 1}{(s + 1)^2 s} = \frac{K_1}{s} + \frac{K_2}{s + 1} + \frac{K_3}{(s + 1)^2} \end{aligned}$$

$$\Rightarrow K_1 = 1$$

$$\Rightarrow K_3 = \frac{\frac{1}{2} + 1}{-1} = -\frac{3}{2}$$

$$\rightarrow s = 1: \frac{\frac{1}{2} + 1}{4} = \frac{1}{1} + \frac{K_2}{2} + \frac{(-\frac{3}{2})}{4}$$

$$\Rightarrow \frac{3}{4} = 4 + 2K_2 - \frac{3}{4}$$

$$\Rightarrow \frac{3}{2} - 4 = 2K_2$$

$$\Rightarrow -2 = 2K_2$$

$$\Rightarrow K_2 = -1$$

$$\begin{aligned} \Rightarrow v(t) &= u(t) - e^{-t}u(t) - \frac{1}{2}te^{-t}u(t) \\ &= (1 - e^{-t} - \frac{1}{2}te^{-t})u(t) \quad \checkmark \end{aligned}$$

