EE 333 Homework 1 Solutions

1-2 a) $\bar{A}+\bar{B}+\bar{C} = 5\bar{a}_x + 5\bar{a}_y - 9\bar{a}_z$

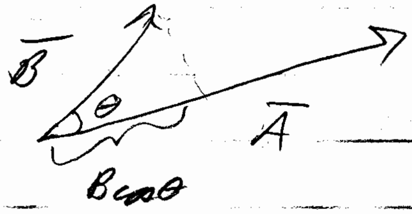
b) $|\bar{A}| = \sqrt{25+9+16} = 7{,}07$

$|\bar{D}| = \sqrt{25+25+1} = 7{,}14$

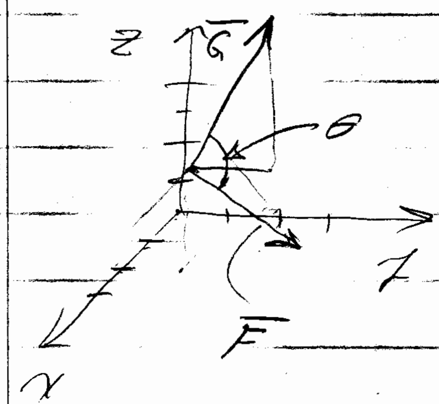
c) $2\bar{A} - \bar{C} = 10\bar{a}_x + 6\bar{a}_y + 14\bar{a}_z$

$|E| = \sqrt{10^2+6^2+14^2} = 18{,}22$

1-8



$\bar{A}\cdot\bar{B} = A\underbrace{B\cos\theta}_{\text{component of } B \text{ along } A}$



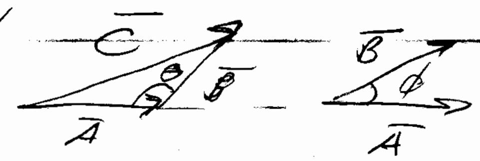
$\bar{F}\cdot\bar{G} = |F||G|\cos\theta = 800$

$\therefore\ F\cos\theta = \frac{800}{|G|} = \frac{800}{53{,}85} = 14{,}85$

$\cos\theta = \frac{800}{|F||G|} = \frac{800}{50\times 53{,}85} = 0{,}2971$

$\boxed{\therefore\ \theta = 72.71^\circ}$

1-11



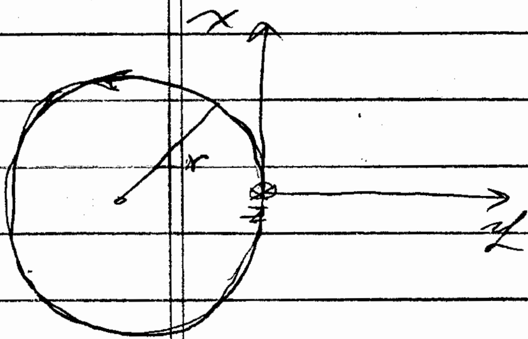
$\bar{A}\cdot\bar{B} = AB\cos\phi \qquad \theta = 180^\circ - \phi$

$C^2 = \bar{C}\cdot\bar{C} = (\bar{A}+\bar{B})\cdot(\bar{A}+\bar{B}) = A^2 + B^2 + 2AB\cos\phi$

or $C^2 = A^2+B^2+2AB\cos(180^\circ-\phi) = \boxed{A^2+B^2-2AB\cos\phi}$

EE 333 Homework 2 (Solutions)

1-25 $q = -e$, $\bar{v} = 10^5 \bar{a}_x$, $\bar{B} = 10^{-4} \bar{a}_z$



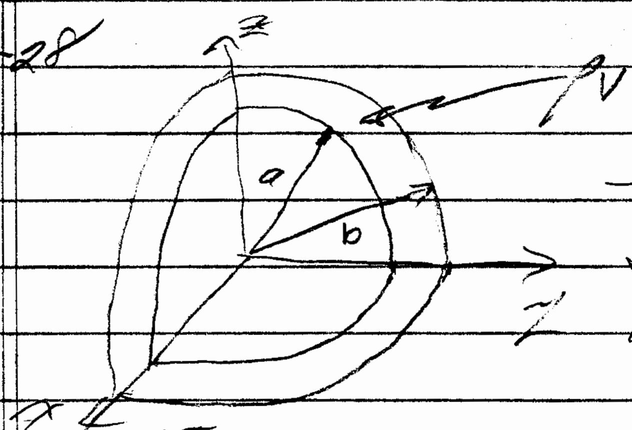
$\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})$

{ Constant force normal to velocity will produce a circular trajectory with $r = \frac{mv^2}{F}$ }

$\therefore r = \frac{9.1 \times 10^{-31} \times 10^{10}}{1.6 \times 10^{-19} \times 10^5 \times 10^{-4}} = \underline{\underline{5.69 \times 10^{-3}}}\ \text{m}$

for no deflection $\bar{E} = -\bar{v} \times \bar{B} = -10^5 \bar{a}_x \times 10^{-4} \bar{a}_z = \boxed{10\bar{a}_y}$

1-28



$\oint \epsilon_0 \bar{E} \cdot d\bar{s} = \int \rho_V dV$

$\rightarrow \bar{E} = E_r(r) \bar{a}_r$ from symmetry

$\boxed{\text{for } r < a \ \int \rho_V dv = 0 \ \therefore \bar{E} = 0}$

$\underline{\underline{a \le r \le b}}$ $\int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \epsilon_0 E_r \bar{a}_r \cdot r^2 \sin\theta \, d\theta \, d\phi = \int_{r=a}^{r} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} \rho_V r^2 \sin\theta \, d\theta \, d\phi \, dr$

$\therefore \epsilon_0 E_r 4\pi r^2 = \rho_V 4\pi \frac{r^3 - a^3}{3}$ so $E_r = \frac{\rho_V}{\epsilon_0 r^2} \left[\frac{r^3 - a^3}{3} \right]$

for $r > b$ right hand side becomes $\int_{r=a}^{b}$ so:

$$E_r = \frac{\rho_V}{\epsilon_0 r^2} \left[\frac{b^3 - a^3}{3} \right]$$

EE 333 Homework 3 (Solutions)

1.29 for $r<a$ $\rho_v = \rho_0(1+kr)$

$$Q = \int_{r=0}^{a}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0(1+kr)\, r^2 \sin\theta\, d\theta\, d\phi\, dr = 4\pi\rho_0 \left\{ \frac{a^3}{3} + k\frac{a^4}{4} \right\}$$

$$Q=0 \quad \text{if} \quad \boxed{k = -\frac{a^3}{3}\cdot\frac{4}{a^4} = -\frac{4}{3a}}$$

{for a spherically symmetric charge distribution $\bar{E} = E_r(r)\,\bar{a}_r$ $\oint \epsilon_0 \bar{E}\cdot d\bar{s}$ over a spherical surface for $r>a$ $\epsilon_0 E_r 4\pi r^2 = Q = 0$ $\therefore E = 0$}

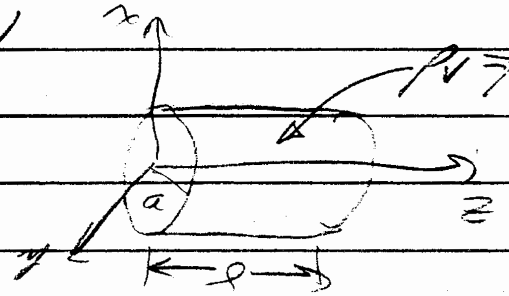
for $r<a$ $$\epsilon_0 E_r 4\pi r^2 = \int_{r=0}^{r}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} \rho_0\left(1-\frac{4r}{3a}\right) r^2 \sin\theta\, d\theta\, d\phi\, dr$$

$$= 4\pi\rho_0\left[\frac{r^3}{3} - \frac{\cancel{4}\, r^4}{3a\,\cancel{4}}\right]$$

$$\boxed{\text{so } E_r = \frac{\rho_0}{\epsilon_0}\left[\frac{r}{3} - \frac{r^2}{3a}\right]}$$

1-31

$\rho_v = \rho_0\left(\frac{\rho}{a}\right)^2$



$$Q = \int_{z=0}^{\ell}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{a} \rho_0\left(\frac{\rho}{a}\right)^2 \rho\, d\rho\, d\phi\, dz$$

a) or $$Q = 2\pi\ell\,\frac{\rho_0}{a^2}\cdot\frac{a^4}{4} = \boxed{\frac{\pi\ell\rho_0 a^2}{2}}$$

b) for $\rho>a$ $\epsilon_0 E_\rho 2\pi\rho\,\ell = \frac{\pi\ell\rho_0 a^2}{2}$

or $$\boxed{E_\rho = \frac{\rho_0 a^2}{4\epsilon_0\rho} = \frac{\rho_\ell}{2\pi\epsilon_0\rho}}$$

EE 333 Homework 3 (Solution page 2)

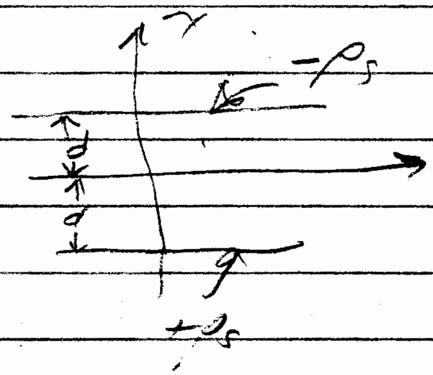
1-31 c) $\rho < a$

$$\int_{z=0}^{l}\int_{\phi=0}^{2\pi} \epsilon_0 E_\rho \, \rho \, d\phi \, dz = \int_{z=0}^{l}\int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \rho_0 \left(\frac{\rho}{a}\right)^2 d\rho \, \rho \, d\phi \, dz$$

$$\cancel{2\pi l}\,\epsilon_0 E_\rho \cancel{\rho} = \cancel{2\pi l}\,\frac{\rho_0}{a^2}\,\frac{\rho^{\cancel{4}\,3}}{4}$$

$$\boxed{E_\rho = \frac{\rho_0 \rho^3}{4\epsilon_0 a^2}}$$

1-32



a) between the plates both charges produce an E_x in the positive x direction $\therefore E_x = \frac{\rho_s}{\epsilon_0}$

for $x > d$ or $x < -d$ the fields cancel

i.e. $\boxed{E_x = 0}$

b) both planes have positive charge then

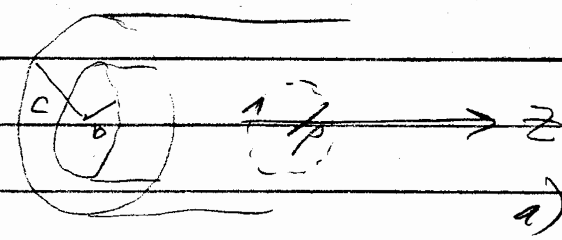
for $x > d$ $\quad E_x = \frac{\rho_s}{\epsilon_0}$

$x < -d$ $\quad E_x = -\frac{\rho_s}{\epsilon_0}$

$-d < x < d$ $\quad E_x = 0$

EE 333 Homework 4 (Solution)

1-33



$\bar{J} = \frac{I}{\text{cross sectional area}}$

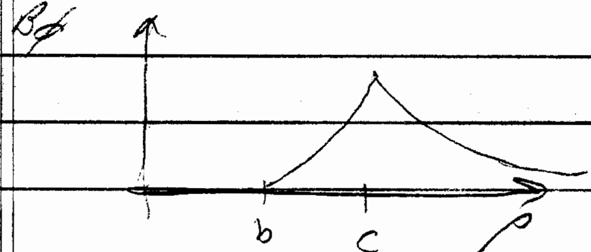
a) $\therefore \bar{J} = \frac{I}{\pi(c^2-b^2)}\, a_z$

for $\rho < b$ $\quad \oint \frac{\bar{B}}{\mu_0}\cdot d\bar{l} = 0 = B_\phi \frac{2\pi\rho}{\mu_0} \quad \therefore \boxed{B_\phi = 0}$

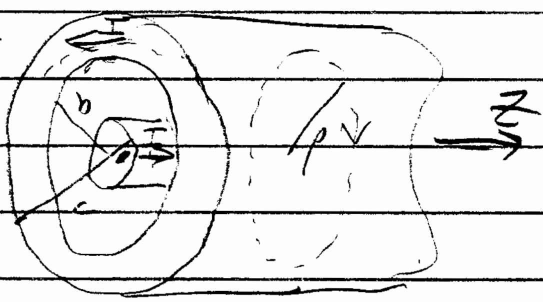
for $b<\rho<c$ $\quad \frac{2\pi\rho B_\phi}{\mu_0} = \int_{\phi=0}^{2\pi}\int_{\rho=b}^{\rho} \frac{I}{\pi(c^2-b^2)}\,\rho\, d\phi\, d\rho = \frac{I\,\cancel{2\pi}}{\cancel{2}\pi(c^2-b^2)}(\rho^2-b^2)$

$\therefore \boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}\cdot\frac{(\rho^2-b^2)}{(c^2-b^2)}}$

for $\rho > c$ $\quad \boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}}$ same as line current along z axis



1.34



a) for $\rho < a$; $\bar{J} = \frac{I}{\pi a^2}\,\bar{a}_z$

for $b<\rho<c$ $\quad \bar{J} = \frac{-I}{\pi(c^2-b^2)}\,\bar{a}_z$

from $J = \frac{I}{\text{area}}$

b) for $\rho<a$ $\quad \frac{2\pi\rho B_\phi}{\mu_0} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \frac{I}{\pi a^2}\,\rho\, d\phi\, d\rho = \frac{\cancel{2\pi} I}{\cancel{2}\pi a^2}\rho^2$

or $\boxed{B_\phi = \frac{\mu_0 I \rho}{2\pi a^2}}$

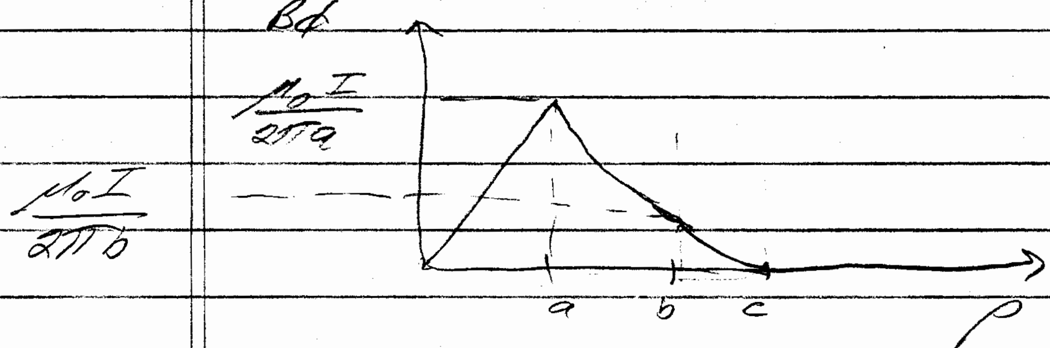
for $a<\rho<b$ $\quad \frac{2\pi\rho B_\phi}{\mu_0} = I$ or $\boxed{B_\phi = \frac{\mu_0 I}{2\pi\rho}}$

for $b<\rho<c$; $\frac{2\pi\rho B_\phi}{\mu_0} = -\int_{\phi=0}^{2\pi}\int_{\rho=b}^{\rho} \frac{I}{\pi(c^2-b^2)}\rho\, d\phi\, d\rho + I$

$$B_\phi = \frac{\mu_0}{2\pi\rho}\left\{ I - \frac{\cancel{2\pi} I}{\cancel{2\pi}(c^2-b^2)}(\rho^2-b^2) \right\} = \boxed{\frac{\mu_0 I}{2\pi\rho}\left\{\frac{c^2-\rho^2}{c^2-b^2}\right\}}$$

for $\rho > c$ $\quad B_\phi = 0$ (no current through circular contour)

c) $a = 3mm$, $b = 6mm$, $C = 8mm$, $I = 100$ Ampere



$$B_\phi(\rho=a) = \frac{\cancel{4\pi}^{2}\times10^{-7}\times10^2}{\cancel{2\pi}\,3\times10^{-3}} = 0.667\times10^{-2}$$

$$B_\phi(\rho=b) = \frac{1}{2} \text{ of above } = 0.333\times10^{-2}$$

$$|J|_{\rho<a} = \frac{100}{\pi\, 9\times10^{-6}} = 3.53\times10^{6}\ A/m^2$$

$$|J|_{b<\rho<c} = \frac{-100}{\pi(64\times10^{-6}-36\times10^{-6})} = \frac{10^2}{\pi\, 28\times10^{-6}} = 1.13\times10^{6}\ \frac{A}{m^2}$$

EE 333 Homework 4 (solutions page 3)

1-35 for $\rho < b$ B_ϕ only due to current in center conductor which is example 1-15

for $b < \rho < c$ superposition gives

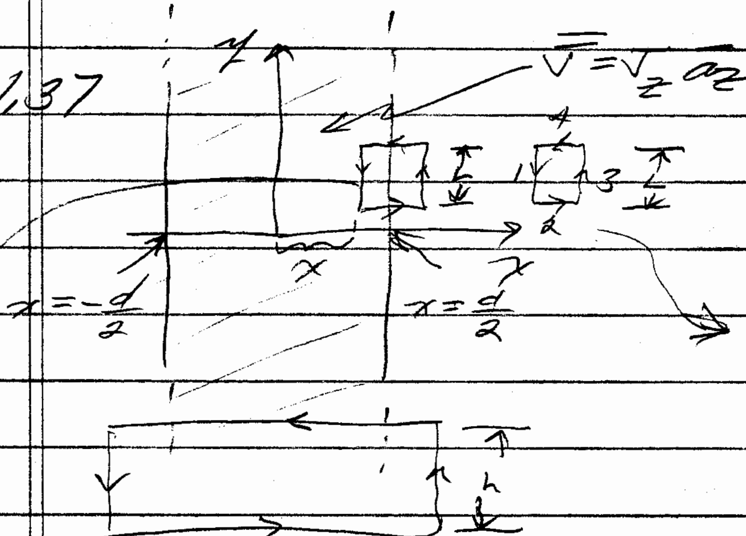
$$B_\phi = \frac{\mu_0 I}{2\pi\rho} - \frac{\mu_0 I}{2\pi\rho}\cdot\frac{\rho^2-b^2}{c^2-b^2} = \frac{\mu_0 I}{2\pi\rho}\left\{\frac{c^2-\rho^2}{c^2-b^2}\right\} \quad \text{QED}$$

for $\rho > c$

$$B_\phi = \frac{\mu_0 I}{2\pi\rho} - \frac{\mu_0 I}{2\pi\rho} = 0\ !$$

EE 333 Homework 5 (Solutions)

1.37



$\bar{J} = J_z a_z$

Relating back to line currents we see that there is only a B_y component

$$\oint \frac{\bar{B}}{\mu_0}\cdot d\bar{l} = 0 = \left(-B_{y_1} L + B_{y_2} L\right)\frac{1}{\mu_0}$$

$\therefore$ outside of current $B_y = \text{constant}$

for this contour $\frac{2 B_y \ell}{\mu_0} = d \ell J_z$; $\boxed{B_y = \frac{\mu_0 d J_z}{2} \quad x > \frac{d}{2}}$

$$\frac{\cancel{\mu_0} d J_z \ell}{\cancel{\mu_0} 2} - \frac{B_y(x)\ell}{\mu_0} = J_z\left(\frac{d}{2} - x\right)\ell$$

$$B_y = -\mu_0 J_z\left(\frac{d}{2} - x\right) + \frac{d J_z \mu_0}{2} = \boxed{\mu_0 J_z x \quad 0 < x < \frac{d}{2}}$$

negative of these answers for $x < 0$

2.17 $E_r = \frac{\rho_0}{\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{3a}\right)$

$$\nabla\cdot E_r = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{\rho_0}{\epsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{3a}\right)\right] = \frac{\rho_0}{r^2\epsilon_0}\left(r^2 - \frac{4}{3}\frac{r^3}{a}\right)$$

$$\nabla\cdot E_r = \frac{\rho_0}{\epsilon_0}\left(1 - \frac{4r}{3a}\right) = \frac{\rho_v}{\epsilon_0}$$

EE 333 Homework 6 (Solutions)

2.41 $\hat{E}_x^+ = 1885\, e^{-j\beta_0 z}$; $f = 100\,MHz$

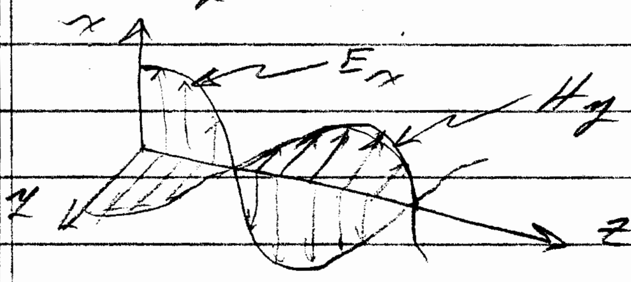
a) Magnitude = 1885 V/m
Traveling in $+\bar{a}_z$ direction
Linear $\bar{a}_x$ polarization

b) $E_x^+(z,t) = Re\{\hat{E}_x^+ e^{j2\pi 10^8 t}\} = 1885\cos(2\pi\times10^8 t - \beta_0 z)$
where $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$; $\lambda = \frac{2\pi}{\beta_0} = \frac{3\times10^8}{10^8} = \boxed{3\,m}$

c) $\hat{H}_y^+ = \frac{1885}{377} e^{-j\beta_0 z} = \boxed{5\, e^{-j\beta_0 z}}$

$\hat{B}_y^+ = \mu_0 \hat{H}^+ = 20\pi\times10^{-7} e^{-j\beta_0 z}$

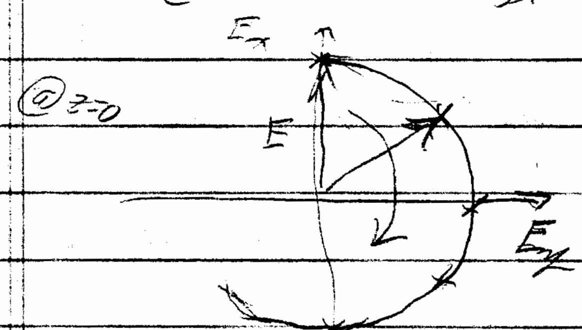
$\therefore\ H_y^+(z,t) = 5\cos(2\pi\times10^8 t - \beta_0 z)$

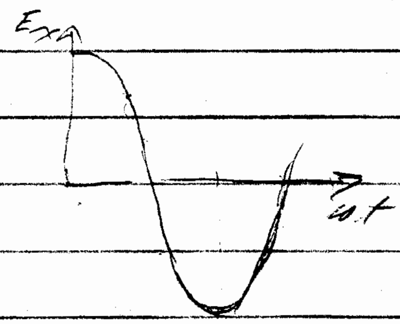


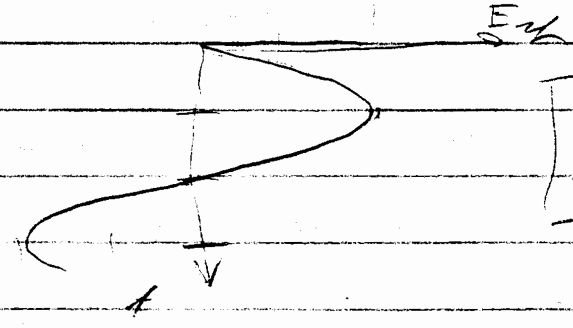
2.46 $\hat{E}(z) = 500\, e^{-j\beta_0 z}(\bar{a}_x - j\bar{a}_y)$

$E(z,t) = 500\cos(\omega t - \beta_0 z)\bar{a}_x + 500\sin(\omega t - \beta_0 z)\bar{a}_y$

or @ $z=0$ $E(z,t) = 500\cos(\omega t)\bar{a}_x + 500\sin(\omega t)\bar{a}_y$







CW circular

$$\boxed{\hat{H}(z) = \frac{500}{\mu_0} e^{-j\beta_0 z}(\bar{a}_y + j\bar{a}_x)}$$

EE 333 Homework 6 (Solution - page 2)

3.1 $n = 10^{29}$ electrons/m^3 ; $\sigma = 5.8\times10^{7}$ mho/m

a) $\mu_e = \frac{e\tau_c}{m}$; $\sigma = \frac{ne^2}{m}\tau_c$

$\therefore \mu_e = \sigma\left(\frac{1}{ne}\right) = \frac{5.8\times10^{7}}{10^{29}\times1.6\times10^{-19}} = \boxed{3.625\times10^{-3}}$

b) charge density $= -10^{29}\frac{\text{elec}}{m^3}\times\frac{1.6\times10^{-19}\,C}{\text{elec}}\times\frac{1\,m^3}{10^{9}\,mm^3} = \boxed{-16\ C/mm^3}$

c) $\overline{E} = 1\,\overline{a}_x$; $\overline{v}_d = -\mu_e\overline{E} = -3.625\times10^{-3}\times1\,\overline{a}_x = \boxed{-3.625\times10^{-3}\ m/sec}$

$\boxed{\overline{J} = \sigma\overline{E} = 5.8\times10^{7}\,\overline{a}_x\ A/m^2}$

$\overline{E}\ \longrightarrow$

$\overline{J}\ \longrightarrow$

$\overline{v}_d\ \longleftarrow$

negative charge on electrons so $\overline{v}_d$ & $\overline{J}$ in opposite directions

EE 333 Homework 7 (Solutions)

3-3 He 10^{25} atoms/m^3, $\chi_e = 1.5\times10^{-4}$, $E = 10^3$

$\bar{D} = \epsilon_0 \bar{E} + \bar{P}$ $\quad \bar{P} = \epsilon_0 \chi_e \bar{E}$ $\quad \therefore \bar{D} = \epsilon_0 (1+\chi_e)\bar{E}$

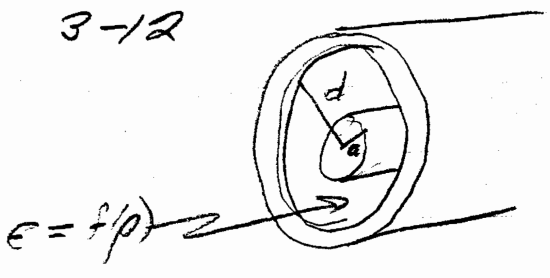
a) $P = \epsilon_0 \chi_e E = \frac{10^{-9}}{36\pi}\times 1.5\times10^{-4}\times10^{3} = 0.0133\times10^{-10} = \underline{\underline{1.33\times10^{-12}}}$

b) $\rho_+ = 2\times10^{25}\times 1.6\times10^{-19} = 3.2\times10^{6}\ C/m^3$; $P = \rho_+ d_{ave}$

$\therefore \quad d_{ave} = \frac{1.33\times10^{-12}}{3.2\times10^{6}} = \underline{\underline{0.41\times10^{-18}\ m}}$ ←

c) $\epsilon_r = 1 + \chi_e = \underline{\underline{1.00015}}$ ←

3-12



a) $\oint \bar{D}\cdot d\bar{s} = \int \rho_v\, dv$ use circular cylindrical can $a < \rho < d$

$$\int_{\phi=0}^{2\pi}\int_{z=0}^{\ell} D_\rho\, \rho\, d\phi\, dz = Q$$

$$2\pi\rho D_\rho \ell = Q$$

so $E_\rho = \frac{Q}{2\pi\epsilon\rho\ell}$ constant if $\epsilon = \frac{K}{\rho}$

problem ask for $\epsilon = \epsilon_1$ @ $\rho = d$ $\therefore$ $\boxed{\epsilon = \frac{\epsilon_1 d}{\rho}}$

so $E_\rho = \frac{Q\cancel{\rho}}{2\pi\epsilon_1 d\cancel{\rho}\ell} = \boxed{\frac{Q}{2\pi\epsilon_1 d\ell}}$ ←

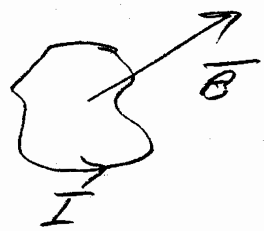
b) $D_\rho = \epsilon_0 E_\rho + P_\rho$ so $P_\rho = D_\rho - \epsilon_0 E_\rho = \frac{Q}{2\pi\rho\ell} - \frac{Q\epsilon_0}{2\pi\epsilon_1 d\ell}$

$\boxed{P_\rho = \frac{Q}{2\pi\ell}\left(\frac{1}{\rho} - \frac{\epsilon_0}{\epsilon_1 d}\right)}$ ←

$\rho_p = -\nabla\cdot\bar{P} = +\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\frac{\rho Q\epsilon_0}{2\pi\ell\epsilon_1 d}\right) = \boxed{\frac{Q\epsilon_0}{2\pi\epsilon_1 \ell d\rho}}$ ←

c) use layers that approximate the $\frac{\epsilon_1 d}{\rho}$ variation in ϵ.

3.16

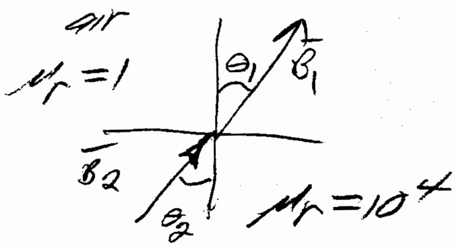


$$\bar{F} = \oint dq\,\bar{u} \times \bar{B}$$

$$\bar{F} = \oint I\,\overline{dl} \times \bar{B} = -I\oint \bar{B} \times \overline{dl}$$

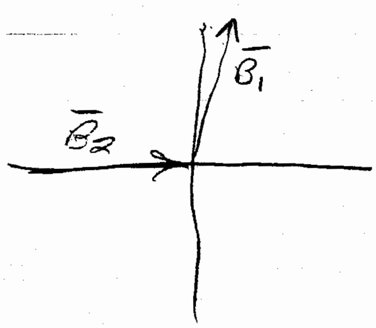
$$\bar{F} = -I\,\bar{B} \times \oint \overline{dl} \equiv 0$$

3.25 $H_{t_1} = H_{t_2}$ and $B_{n_1} = B_{n_2}$



$$\tan\theta_2 = \frac{\mu_2}{\mu_1}\tan\theta_1 \qquad (3.76)$$

or $\theta_1 = \tan^{-1}\left[10^{-4}\tan\theta_2\right]$

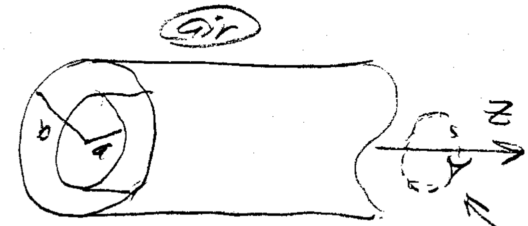


θ_2	θ_1
0	0
45°	5.7×10^{-3} °
89°	0.328°
89.9°	3.27°
89.9675	10°

EE 333 Homework 8 (Solution)

3-28

air



I carried in conductor of radius a ($\mu=\mu_0$)
for $a<\rho<b$ $\mu=\mu$

$\oint \bar{H}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{s}$ integrate around circular contour of radius ρ.

a) for $\rho<a$ $H_\phi 2\pi\rho = \frac{I}{\pi a^2}\pi\rho^2$ or $H_\phi = \frac{I\rho}{2\pi a^2}$

$$B_\phi = \frac{\mu_0 I\rho}{2\pi a^2}$$

for $a<\rho<b$ $H_\phi 2\pi\rho = I$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$B_\phi = \frac{\mu I}{2\pi\rho} = \frac{\mu_r\mu_0 I}{2\pi\rho}$$

for $\rho>b$

$$H_\phi = \frac{I}{2\pi\rho}$$

$$B_\phi = \frac{\mu_0 I}{2\pi\rho}$$

b) $\bar{M} = \frac{\bar{B}}{\mu_0} - \bar{H} = \left(\frac{\mu_r I}{2\pi\rho} - \frac{I}{2\pi\rho}\right)\bar{a}_\phi = \frac{I}{2\pi\rho}(\mu_r-1)\bar{a}_\phi$

H_ϕ is continuous across each boundary (H_{tang} is continuous)

B_ϕ is discontinuous

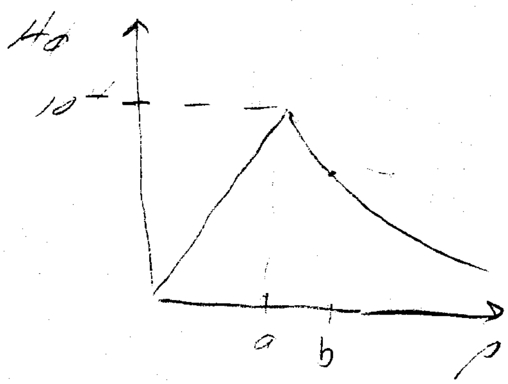
@ $\rho=a$ $H_\phi = \frac{628}{2\pi\times10^{-2}} = 10^4$; $B_\phi = 24\pi\times10^{-3}$ ($\rho=a^+$)

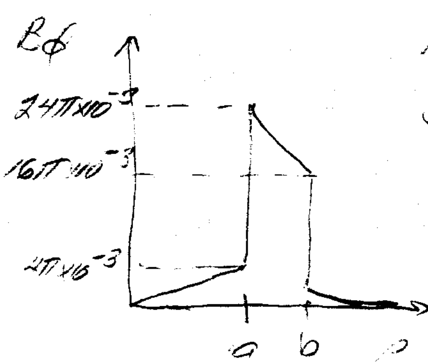
$B_\phi = 4\pi\times10^{-3}$ ($\rho=a^-$)

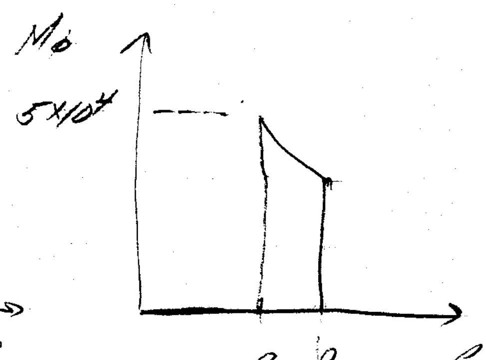
$\rho=b$ $H_\phi = \frac{628}{2\pi\,1.5\times10^{-2}} = \frac{2}{3}\times10^4$; $B_\phi(\rho=b^-) = 4\pi\times10^{-7}\times6\times\frac{2}{3}\times10^4 = 16\pi\times10^{-3}$

$B_\phi(\rho=b^+) = 4\pi\times10^{-7}\times\frac{2}{3}\times10^4 = \frac{8}{3}\pi\times10^{-3}$

$M=0$ for $\rho<a$ and $\rho>b$ $M=5H$ for $a<\rho<b$







EE 333 Homework 8 (Solutions page 2)

$\bar{n} \times (\bar{M}_1 - \bar{M}_2) = \bar{J}_{sm}$

3-28 continued $\bar{J}_m = \nabla \times \bar{M}$

$$\therefore \bar{J}_m = \cancel{\bar{a}_\rho \left(-\frac{\partial M_\phi}{\partial z}\right)} + \bar{a}_z \left(\frac{1}{\rho}\frac{\partial}{\partial \rho}(\rho M_\phi)\right) = 0$$

$$\bar{J}_{sm}(\rho = a) = \bar{a}_\rho \times \bar{M} = 5 \times 10^4 \, \bar{a}_z$$

$$\bar{J}_{sm}(\rho = b) = -\bar{a}_\rho \times \bar{M} = -\frac{10}{3} \times 10^4 \, \bar{a}_z$$

3.29

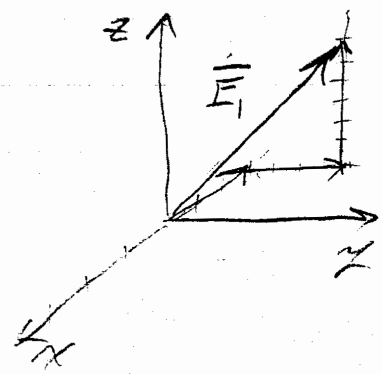
Region 1 $\epsilon = \epsilon_0$ $\quad \bar{n} = \bar{a}_z$ $\quad \bar{E}_1 = -15\bar{a}_x + 20\bar{a}_y + 30\bar{a}_z$ $\quad \left\{ \begin{array}{l} E_{tang_1} = E_{tang_2} \\ D_{norm_1} = D_{norm_2} \end{array} \right\}$

Region 2 $\epsilon = 4\epsilon_0$

a) $\bar{E}_2 = -15\bar{a}_x + 20\bar{a}_y + \frac{15}{2}\bar{a}_z$

$\bar{D}_1 = \epsilon_0 \bar{E}_1$

$\bar{D}_2 = 4\epsilon_0 \bar{E}_2$



b) $\bar{n} \cdot \bar{E}_1 = |E_1| \cos\theta_1 = 30$

$$\therefore \theta_1 = \cos^{-1} \frac{30}{\sqrt{15^2 + 20^2 + 30^2}} = \boxed{39.8^\circ}$$

$$\theta_2 = \cos^{-1} \frac{15/2}{\sqrt{15^2 + 20^2 + 7.5^2}} = \boxed{73.3^\circ}$$

ck: $\tan\theta_2 = \frac{\epsilon_2}{\epsilon_1} \tan\theta_1$

$3.333 = 4 \times 0.833 = 3.33$!

EE 333 Homework 8 (solutions page 3)

3.33 $\alpha = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2} - 1\right]^{1/2}$

$\beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\quad '' \quad + 1\right]^{1/2}$

$\eta = \sqrt{\frac{\mu}{\epsilon - j\frac{\sigma}{\omega}}}$

for $\sigma = 0$, $\mu = \mu_0$, + $\epsilon = \epsilon_0$ this gives:

$\alpha = 0$; $\beta = \omega\sqrt{\mu_0\epsilon_0}$; $\eta = \sqrt{\frac{\mu_0}{\epsilon_0}}$

3.38 $\hat{E}_m^+ = 1$ @ $z = 0$ sea water $\epsilon_r = 81$, $\sigma = 4$

$f = 10^4$

$\frac{\sigma}{\omega\epsilon} = \frac{\cancel{4}^{2} \times 36\cancel{\pi}}{\cancel{2}\cancel{\pi} \times 10^4 \times 10^{-9}} = 72 \times 10^5$ ∴ good conductor

$\alpha \doteq \beta = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\sqrt{\frac{\sigma}{\omega\epsilon}} = \sqrt{\frac{\omega\sigma\mu}{2}} = \sqrt{\frac{\cancel{2}\pi \times 10^4 \times 4 \times 4\pi \times 10^{-7}}{\cancel{2}}} = 4\pi \times 0.0316 = \underline{\underline{0.397}}$

$e^{-\alpha z} = 0.05$; $-\alpha z = \ln 0.05$; $\boxed{z = -\frac{\ln 0.05}{\alpha} \doteq 7.5 m}$

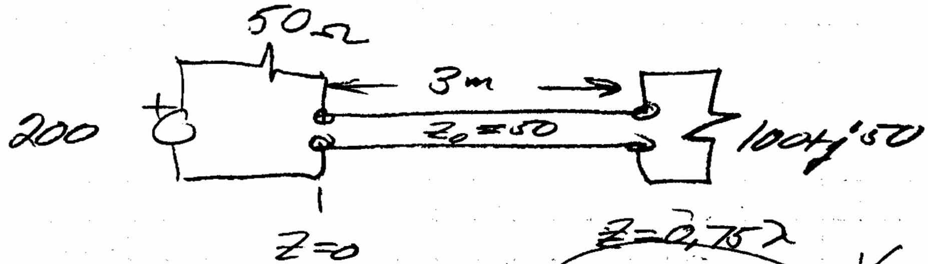
$f = 10^3$ $\frac{\sigma}{\omega\epsilon} = 72 \times 10^6$ ∴ good conductor

$\alpha = \beta = \sqrt{\frac{\cancel{2}\pi \times 10^3 \times 4 \times 4\pi \times 10^{-7}}{\cancel{2}}} = 4\pi \times 10^{-2} = 0.1257$

$\boxed{z = -\frac{\ln 0.05}{0.1257} = 23.8 m}$

RF communication in sea water is only good @ very low frequencies ... and even then only over very short distances.

Problem 10-1 EE 333 Problems



$f = 50\,MHz$

$\epsilon_r = 2.25$

$v_p = 3\times10^8 \frac{1}{\sqrt{2.25}} = 2\times10^8\ m/sec$ ∴ length of line $= \frac{3\times5\times10^7}{2\times10^8} = 7.5\times10^{-1}\lambda$

$\lambda = \frac{2\times10^8}{0.5\times10^8} = 4m$

$\Gamma_{load} = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50+j50}{150+j50} = 0.4+j0.2 = 0.4472\,e^{j26.56°}$

$\Gamma_{in} = \Gamma_{load}\, e^{-2\gamma\ell} = (0.4+j0.2)\, e^{-j2\frac{2\pi}{\lambda}\cdot\frac{3\lambda}{4}} = -(0.4+j0.2)$

∴ approximately 45% voltage reflection (20% power reflection)

$Z_{in} = 50\frac{1+\Gamma_{in}}{1-\Gamma_{in}} = 50\frac{0.6-j0.2}{1.4+j0.2} = 20-j10$

$\hat{I}_{in} = \frac{200}{50+Z_{in}} = \frac{200}{70-j10} = 2.828\,e^{j8.13°}$

$P_{in} = \frac{1}{2}|\hat{I}_{in}|^2 R_{in} = \frac{1}{2}(2.828)^2\, 20 = 79.98$ watts

$\hat{I} = \hat{I}_{in}^{+} e^{-j\beta z}[1-\Gamma(z)]$ ∴ $\hat{I}_{in}^{+} = \hat{I}_{in}[1-\Gamma_{in}]^{-1}$

and $I_{load} = \hat{I}_{in}^{+} e^{-j\frac{2\pi}{\lambda}\cdot\frac{3\lambda}{4}}[1-\Gamma_{load}] = \frac{I_{in}\, e^{-j\frac{3\pi}{2}}}{1-\Gamma_{in}}[1-\Gamma_{load}]$

$= 2.828\,e^{j8.13°}\, j\, \frac{0.6-j0.2}{1.4+j0.2}$

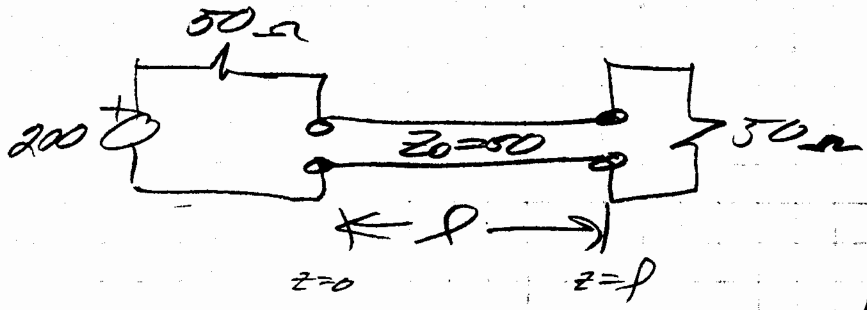
or $I_{load} = 1.2647\,e^{j71.56°}$

so $P_{load} = \frac{1}{2}(1.2647)^2\, 100 = 79.97$ watts

11

EE 333 Problems

10-3



$f = 30\,MHz$

$\ell = 3\,m$

$\epsilon_r = 2.25$

$\Gamma_{load} = 0 \quad \therefore \; \Gamma(z) = 0$ everywhere!

$Z(z) = 50 = Z_0 \frac{1+\Gamma}{1-\Gamma}$

$$\hat{V} = \hat{V}_m^+ e^{-j\beta z} \quad \left(\frac{2\pi}{\lambda}\cdot z\right) \qquad = V_m^+$$

$$\hat{I} = \frac{\hat{V}_m^+}{Z_0} e^{-j\beta z} \qquad = \frac{V_+}{Z_0}$$

when $z = 0, \lambda, 2\lambda, \ldots$

$\hat{I}_m^+ = \hat{I}_{in} = \frac{200}{100} = 2 \; ; \; \hat{V}_{in} = 100 = \hat{V}_m^+$

$P_{in} = \frac{1}{2} Re\{\hat{V}_{in} \hat{I}_{in}^*\} = 100$ watts

$$\hat{V}(z) = 100\, e^{-j\frac{2\pi}{\lambda} z}$$

$$\hat{I}(z) = 2\, e^{-j\frac{2\pi}{\lambda} z}$$

$$\hat{V}_{load} = 100\, e^{-j\frac{2\pi}{\cancel{\lambda}}\cdot\frac{3}{\cancel{4}\,2}\cancel{\lambda}} = 100j$$

9

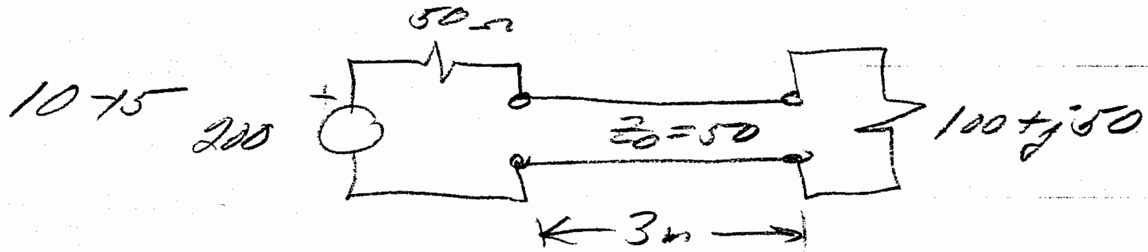
$\hat{I}_{load} = 2j$

$$P_{load} = \frac{1}{2} Re\{100j(-2j)\} = 100$$

$$= \frac{1}{2} |\hat{I}|^2 50 = 100$$

$$= \frac{1}{2} \frac{|\hat{V}|^2}{50} = 100$$

EE 333 Homework 10 (Solution)



$\epsilon_r = 2.25$; $f = 50MHz$

$v_{phase} = \frac{3\times10^8}{\sqrt{2.25}} = 2\times10^8$ ∴ $\lambda = \frac{2\times10^8}{5\times10^7} = 4m$

line length $= \frac{3}{4}\lambda$

$\boxed{Z(l)_{norm} = 2+j1}$ from chart $\boxed{\Gamma(l) = 0.45e^{j26°}}$

λ on chart @ input $= 0.213 + 0.25 = 0.463$

from chart $Z_{in_{norm}} = 0.41 - j0.2$ or $\boxed{Z_{in} = 20.5 - j10}$ ←

$\boxed{\Gamma(0) = 0.45e^{-j153°}}$

Problem 10-15

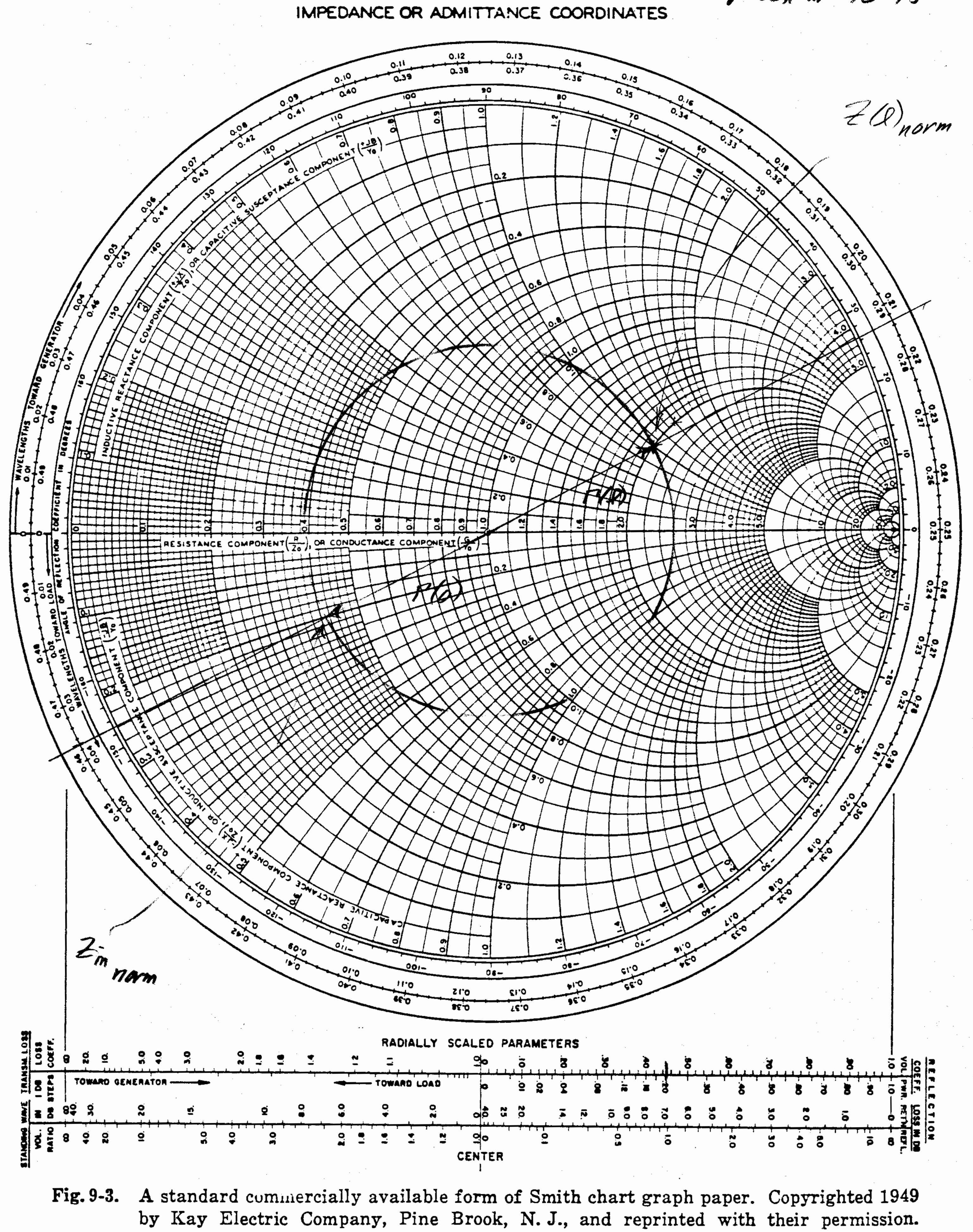
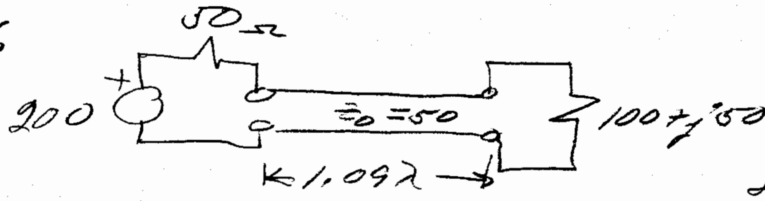


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 333 Homework 11 (Solutions)

10.16



$Z_{L_{norm}} = 2 + j1$

See chart $\lambda = 0.214 + 0.09 = 0.304\lambda$ rotation

$\Gamma(l) = \underline{0.455 e^{j26°}}$ ←

$\Gamma(0) = \underline{0.455 e^{-j39°}}$ ← $Z_{in_{norm}} = 1.59 - j1.12$; $Z_{in} = \boxed{79.5 - j56}$ ←

10.17 same as 10.16 but $\alpha = 0.00197$ Np/m ; $l = 11.5$ m

$e^{-2\alpha l} = e^{-2 \times 1.97 \times 10^{-3} \times 11.5} = \boxed{0.956}$ ←

∴ @ input $|\Gamma| = 0.956 \times 0.455 = \underline{0.435}$ ←

rotation not affected

from chart $Z_{in_{norm}} = 1.58 - j1.03$; $\boxed{Z_{in} = 79 - j51.5}$ ←

10.21 $Z_L = 36 + j20$; $Z_{L_{norm}} = 0.72 + j0.4$

$Z_0 = 50\,\Omega$

from chart $\boxed{Y_{L_{norm}} = 1.06 - j0.58}$ ←

$$\frac{1}{Z_{L_{norm}}} = \frac{1}{0.72 + j0.4} = \frac{1}{0.82365 e^{j29.05°}} = \boxed{1.06 - j0.59 \quad QED}$$ ←

10.23 $Y_{in_2} = G_2 + jB_2$; $Y_{in_3} = G_3 + jB_3$ $\hat{I} = \hat{V} Y$

$$P_{ave} = \frac{1}{2} Re\{\hat{V}\hat{I}^*\} = \frac{1}{2} Re\{\hat{V}\hat{V}^* Y^*\} = \frac{1}{2} Re\{|\hat{V}|^2 (G - jB)\}$$

so $\boxed{\frac{P_{ave_2}}{P_{ave_3}} = \frac{G_2}{G_3}}$ $G_2 = 0.0366$

$G_3 = 0.012$

$$\frac{1}{2}|\hat{V}|^2(G_2 + G_3) = 8.5 = \frac{1}{2}|\hat{V}|^2 G_2 \left(1 + \frac{G_3}{G_2}\right) = P_{ave_2}\left(1 + \frac{G_3}{G_2}\right)$$

∴ $P_{ave_2} = \frac{8.5}{1 + \frac{G_3}{G_2}} = \boxed{6.4 \text{ Watts}}$ ←

$P_{ave_3} = 8.5 - 6.4 = \boxed{2.1 \text{ Watts}}$ ←

Problem 10.16

IMPEDANCE OR ADMITTANCE COORDINATES

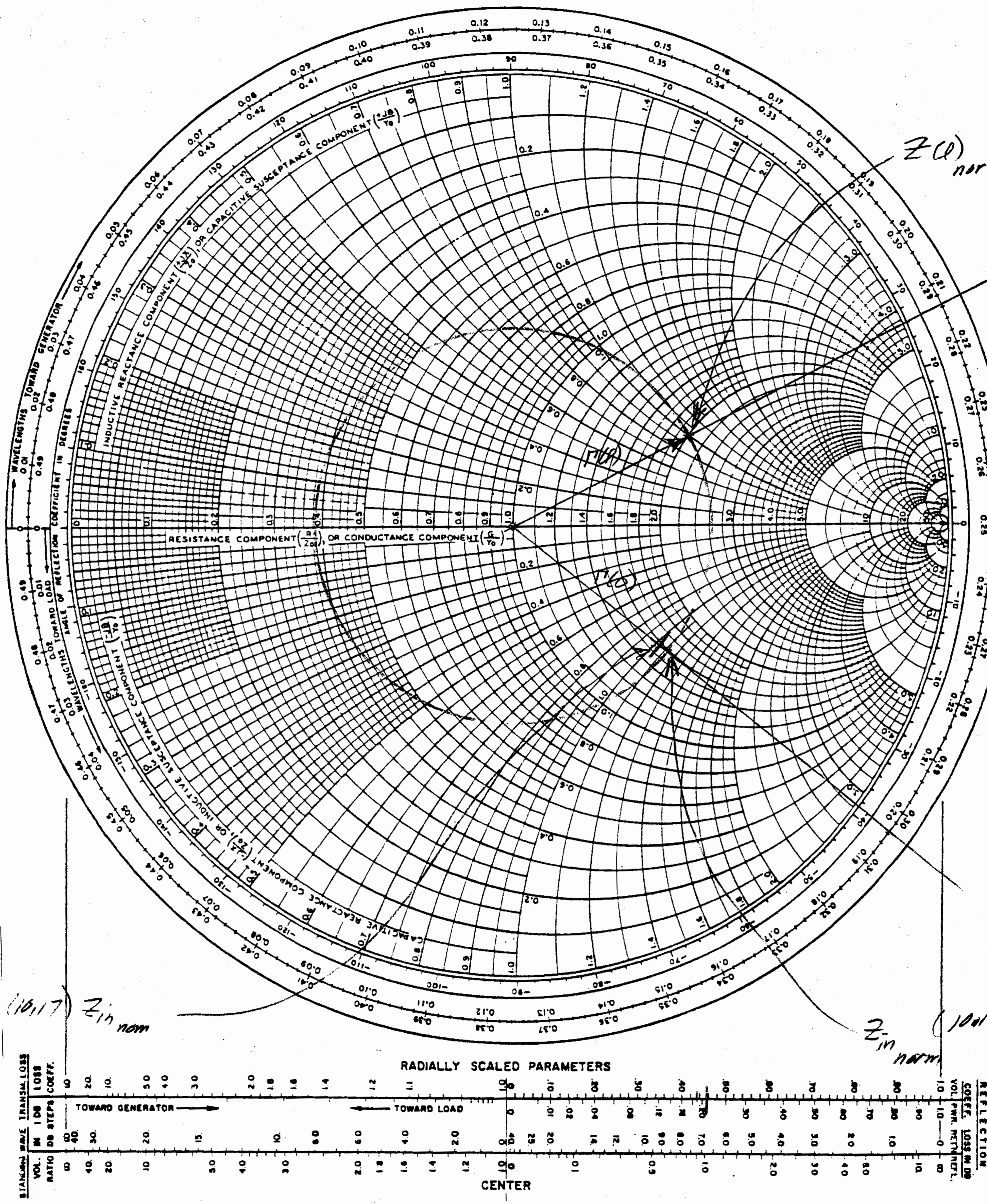
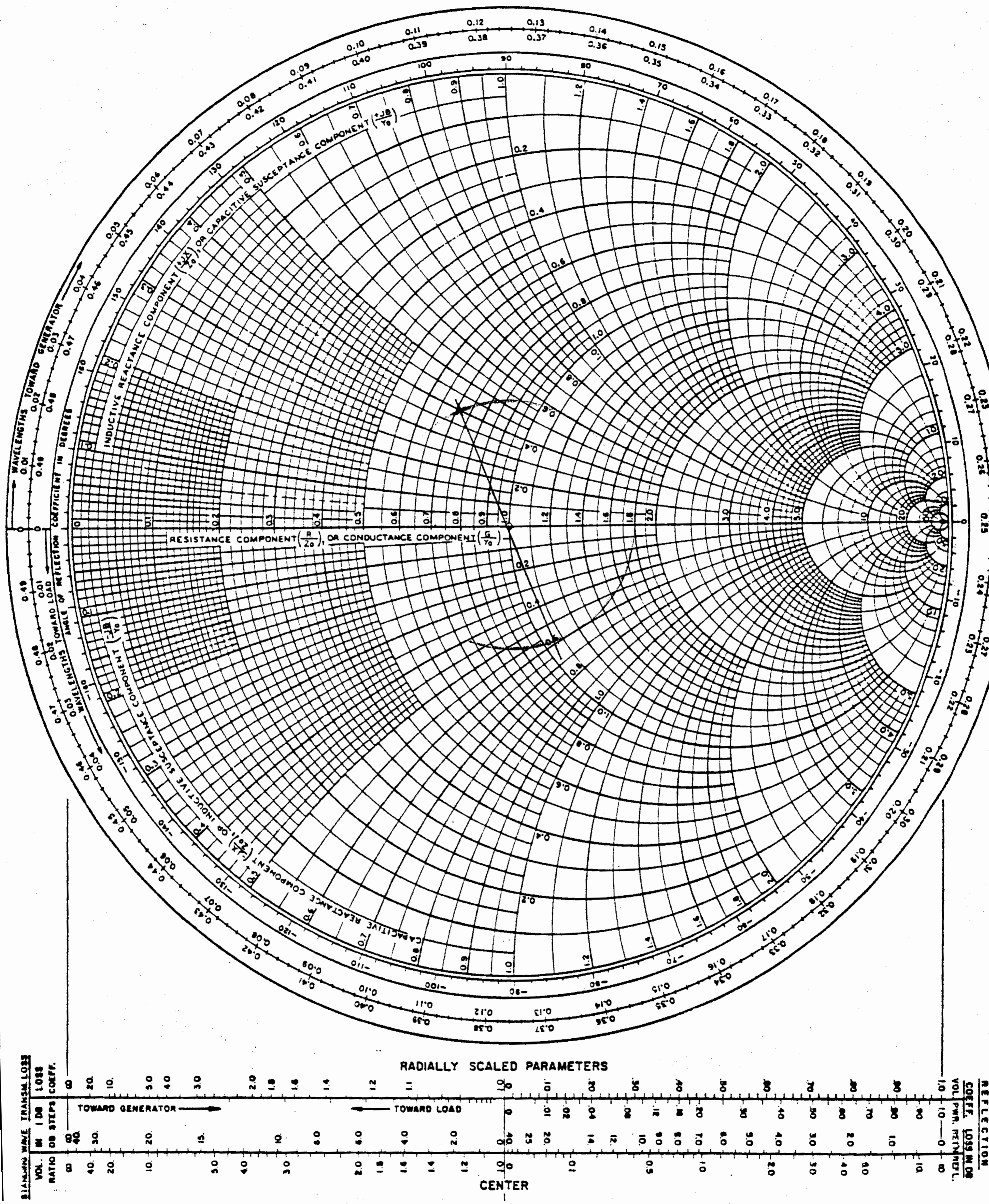


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

IMPEDANCE OR ADMITTANCE COORDINATES

Problem 10.21



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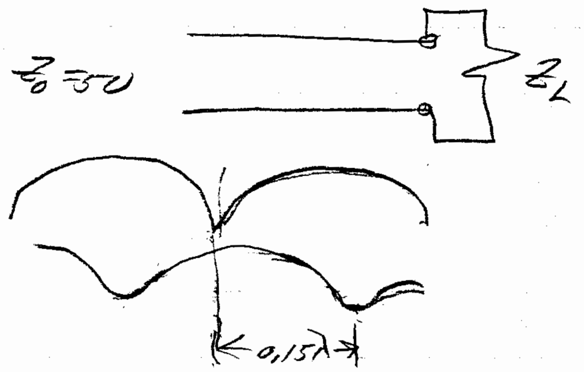
EE333   Homework 12   (Solutions)

10-27

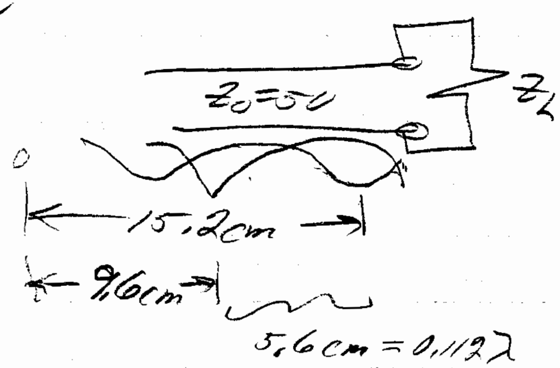


SWR = 4.0

from chart $Z_{L_{norm}} = 0.67 + j1.14$

$\therefore \boxed{Z_L = 33.5 + j57}$

10.28



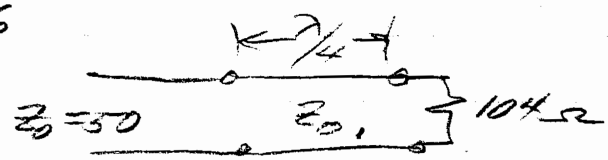
f = 600MHz ; SWR = 3.5

$\lambda = \frac{3\times10^8}{6\times10^8} = 0.5m$

from chart $Z_{L_{norm}} = 0.475 + j0.73$

$\boxed{\text{or } Z_L = 23.75 + j36.5}$

10.36

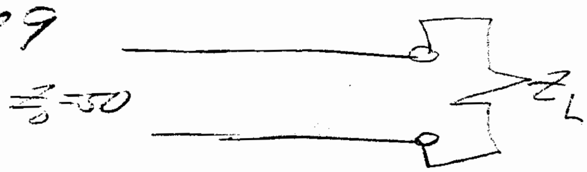


f = 300MHz , $\epsilon_r = 2.26$

$\boxed{Z_{01} = \sqrt{50\times104} = 72.1\Omega}$

$$\lambda = \frac{3\times10^8}{\sqrt{2.26}\ 5\times10^8} = \frac{0.6}{1.5} = 0.4m \quad \therefore \boxed{\lambda/4 = 0.1m}$$

10.39



f = 500MHz ; λ = 60cm

SWR = 3.5

(see chart)

place stub $(0.25-0.171)\lambda = 0.08\lambda = 4.8cm$ either direction from V_{min}

a) for stub toward the load $Y_{norm_{stub}} = -j1.3$; stub length $= 0.355 - 0.25 = 0.105\lambda$

or stub $\boxed{\text{length} = 6.3\text{cm}}$

b) SWR from generator to stub = 1

SWR from stub to load = 3.5

c) for stub toward the generator $Y_{norm_{stub}} = +j1.3$; stub length $= (0.145+0.25)\lambda \doteq \boxed{23.7\text{cm}}$

IMPEDANCE OR ADMITTANCE COORDINATES

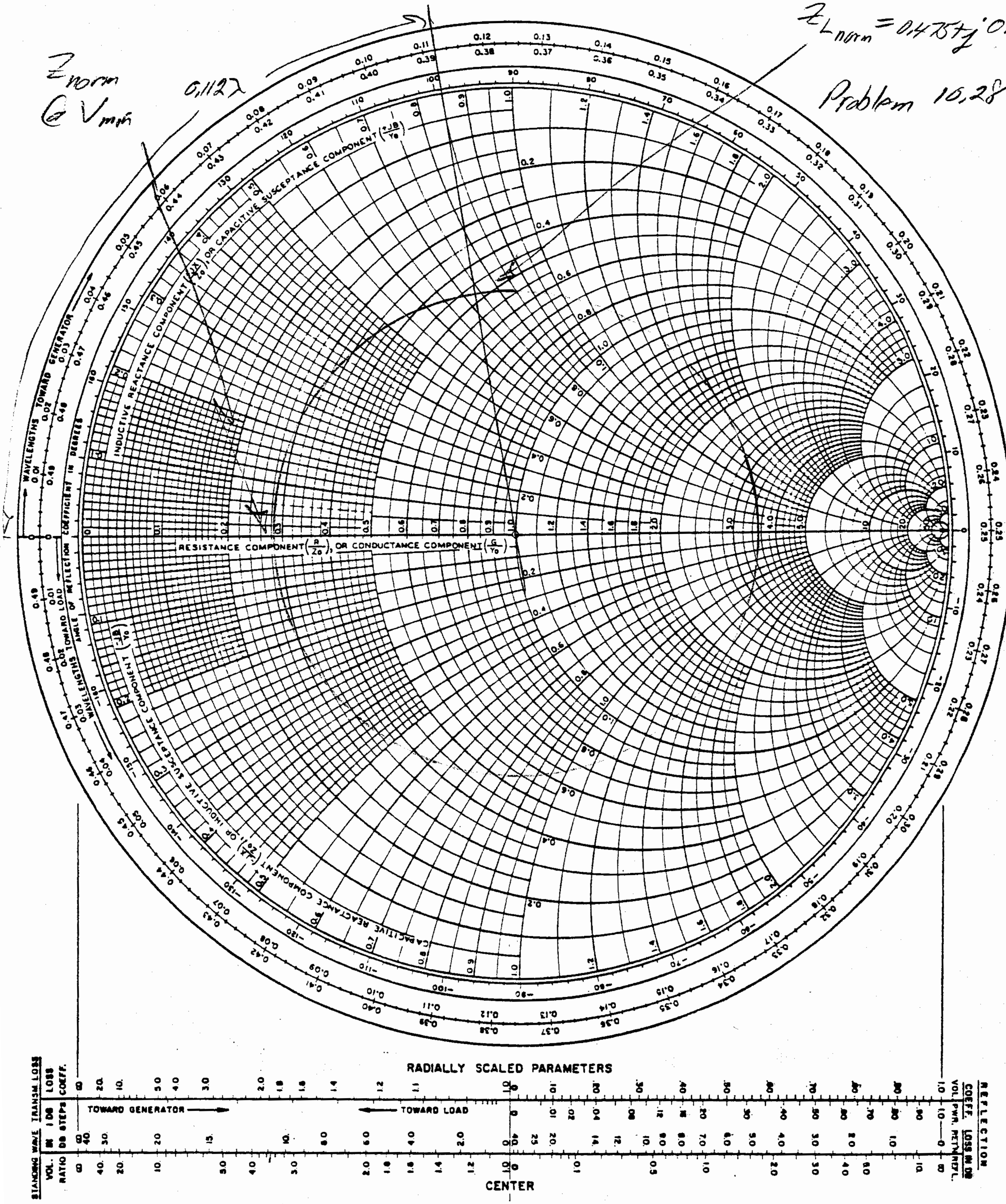


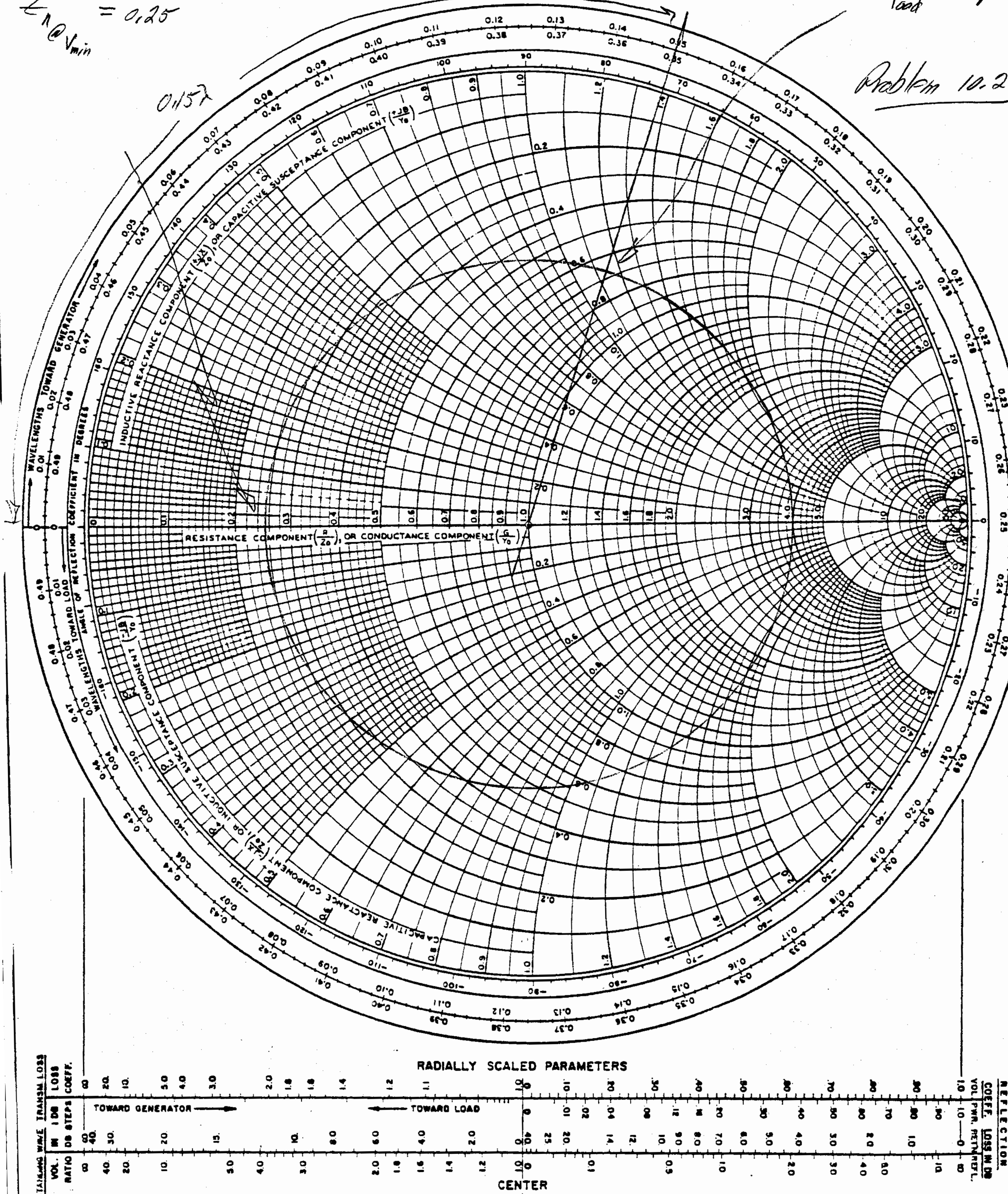
Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

IMPEDANCE OR ADMITTANCE COORDINATES

$Z_{n\,@V_{min}} = 0.25$

$Z_{n_{load}} = 0.67 + j1.14$

Problem 10.27



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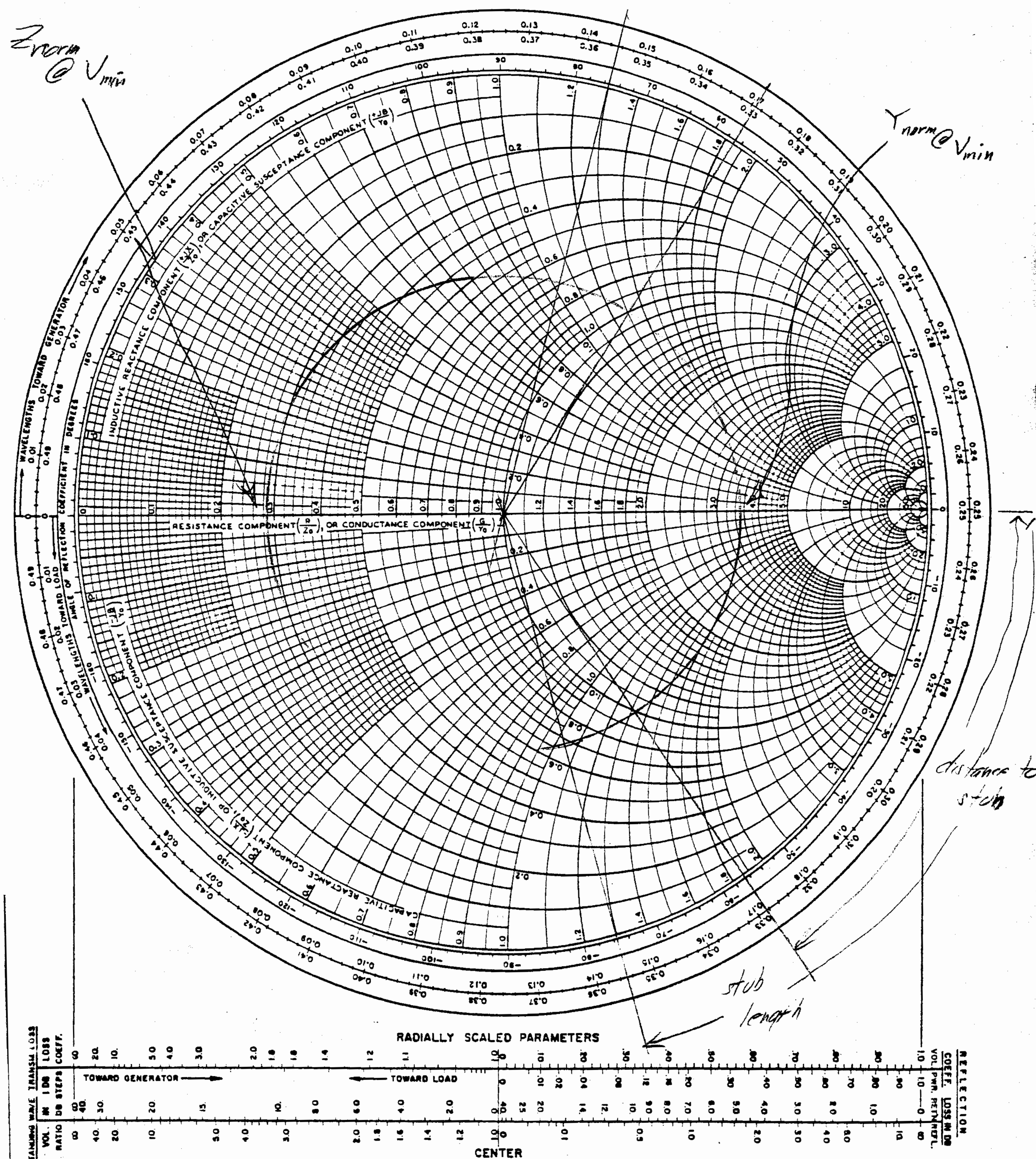
Problem 10.39

IMPEDANCE OR ADMITTANCE COORDINATES



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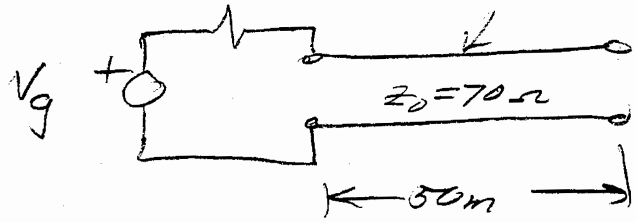
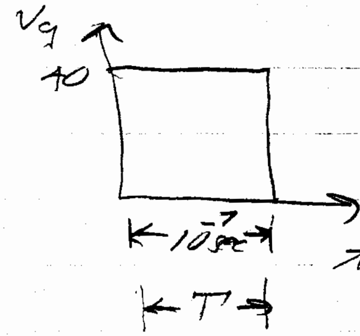
9-36





a) $\boxed{u = \dfrac{3\times10^8}{\sqrt{2.26}} = 2\times10^8 \text{ m/sec}}$

$$V(z=0,t) = 40\,\frac{70}{170}\left[u(t)-u(t-T)\right] = 16.47\left[u(t)-u(t-T)\right]$$

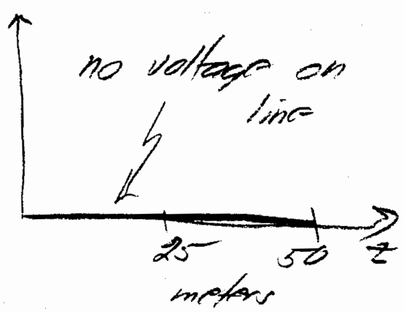
$$I(z=0,t) = \frac{V(z=0,t)}{70} = \frac{40}{170}\left[u(t)-u(t-T)\right] = 0.235\left[u(t)-u(t-T)\right]$$

b) $V^+\left(t-\frac{z}{u}\right) = V_g\left(t-\frac{z}{u}\right)\times\frac{70}{170}$

$I^+\left(t-\frac{z}{u}\right) = \frac{1}{170}V_g\left(t-\frac{z}{u}\right)$

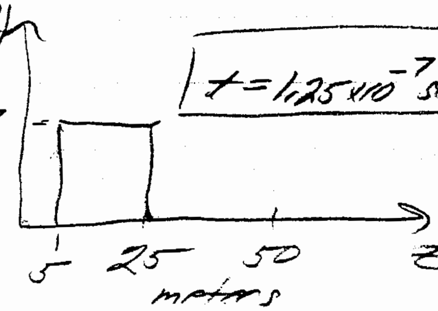
$V(z,t=0)$

t = 0



$V(z,t=\frac{1}{2}\frac{l}{u})$

$\boxed{t = 1.25\times10^{-7}\text{ sec}}$

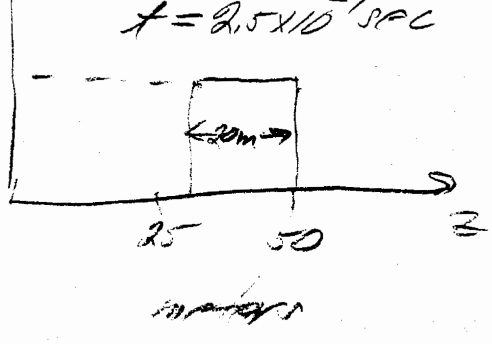


$$\frac{l}{u} = \frac{50}{2\times10^8} = 25\times10^{-8} = 2.5\times10^{-7}\text{ sec}$$

in 10^{-7} seconds wave travels $2\times10^8\times10^{-7} = \boxed{20\text{m}}$

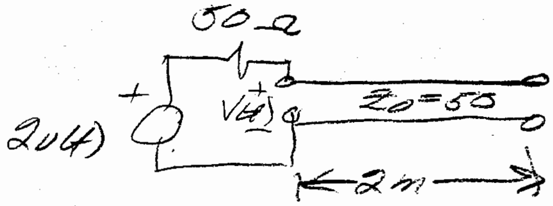
$V(z,t=\frac{l}{u})$

$t = 2.5\times10^{-7}$ sec

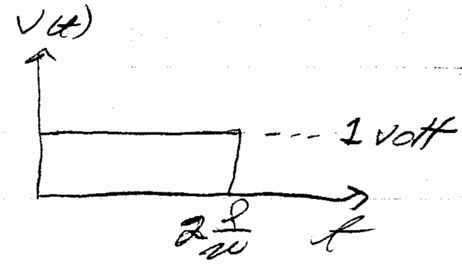


EE 333 Homework 13 (Solutions)

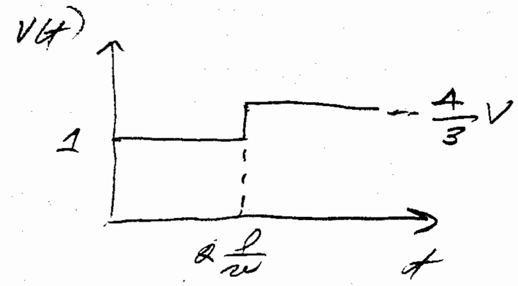
9-38



a) (short)



b)

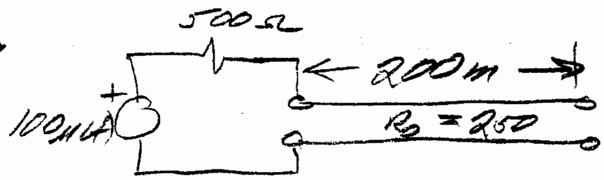


$R_L = 100$

$\Gamma(l) = \frac{100-50}{100+50} = \frac{1}{3}$

$$\boxed{\frac{l}{u} = \frac{2}{3\times10^8} = \frac{2}{3}\times10^{-8}\,\text{sec}}$$

9-39

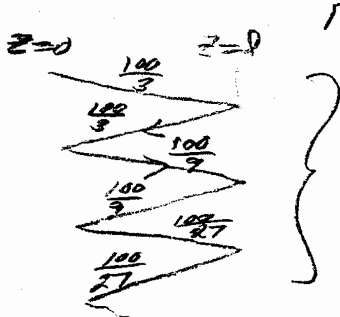


$V^+(t, z=0) = 100\,\frac{250}{750}$

$V^+(t, z=0) = \frac{100}{3}$

$\Gamma(0) = \frac{1}{3}$ $\Gamma(l) = 1$

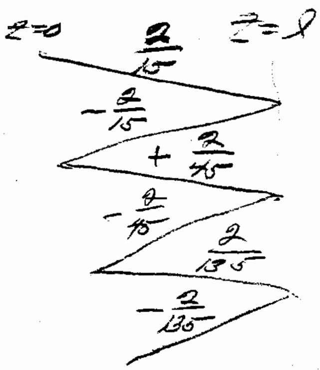
a)



b) $V(z,t) = 100$ as $t \to \infty$

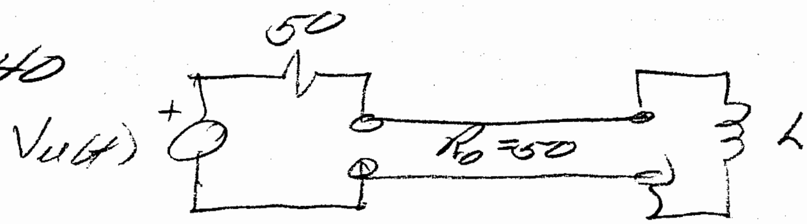
$V(z,t) = V^+\left(t-\frac{z}{u}\right) + V^-\left(t+\frac{z}{u}\right)$

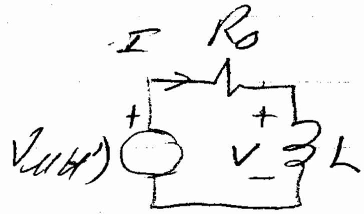
c) $I(z,t) = \frac{1}{R_0}\left\{V^+\left(t-\frac{z}{u}\right) - V^-\left(t+\frac{z}{u}\right)\right\}$



EE 333 Homework 13 (Solutions - page 2)

9-40





@ load $V^+(t') = \frac{V}{2} u(t')$

for $t' > 0$ $V = IR_0 + L\frac{dI}{dt}$

$I_{homo} = Ae^{st}$; $0 = R_0 + Ls \Rightarrow s = -\frac{R_0}{L}$; $I_h = Ae^{-\frac{R_0 t'}{L}}$

$I_{particular} = K$ so $V = R_0 K$ or $K = \frac{V}{R_0}$

this gives $I = I_h + I_p = \frac{V}{R_0} + Ae^{-\frac{R_0}{L}t'}$

assuming no initial energy storage (i.e. $I(t'=0) = 0$)

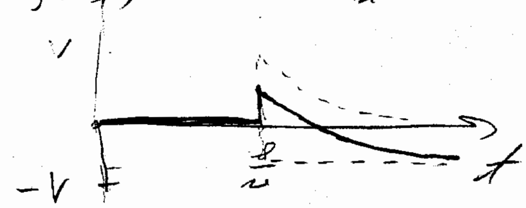
we have $I = \frac{V}{R_0}\left(1 - e^{-\frac{R_0}{L}t'}\right) = \frac{V^+(t-\frac{l}{u})}{R_0} - \frac{V^-(t+\frac{l}{u})}{R_0}$

giving: $V^-(t+\frac{l}{u}) = V^+(t-\frac{l}{u}) - R_0 I(z=l, t)$

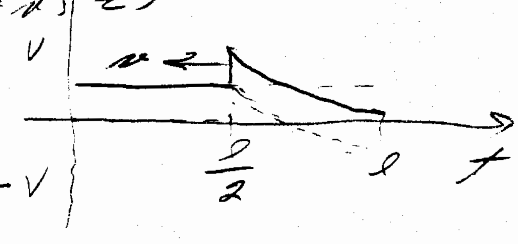
or $V^-(t+\frac{l}{u}) = \frac{V}{2} u(t-\frac{l}{u}) - V\left(1 - e^{-\frac{R_0}{L}(t-\frac{l}{u})}\right) u(t-\frac{l}{u})$

$V^-(t+\frac{l}{u}) = -\frac{V}{2} u(t-\frac{l}{u}) + Ve^{-\frac{R_0}{L}(t-\frac{l}{u})} u(t-\frac{l}{u})$

$V^-(t, z=l)$

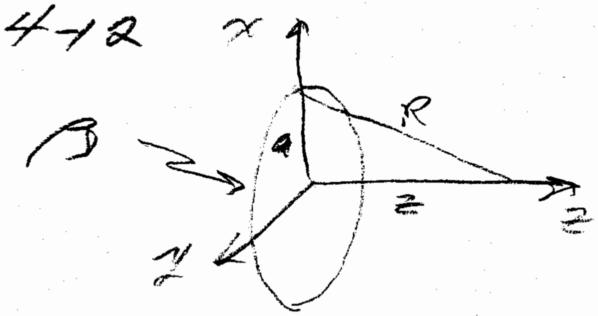


$V(t=\frac{3}{2}\frac{l}{u}, z)$



EE 333 Homework 14 (Solution)

4-12



a) $\Phi = \int_{\phi=0}^{2\pi} \frac{\rho_\ell\, a\, d\phi}{4\pi\epsilon R} = \frac{\rho_\ell\, a}{2\epsilon\sqrt{a^2+z^2}}$

$\bar{E} = -\nabla\Phi = -\bar{a}_z \frac{\partial\Phi}{\partial z} = \bar{a}_z \frac{\rho_\ell\, a\, 2z}{4\epsilon(a^2+z^2)^{3/2}}$

b) $\Phi = -\int_{\rho=\infty}^{z} \frac{\rho_\ell\, a\, z}{2\epsilon(a^2+z^2)^{3/2}}\, dz = -\int_{\infty}^{a^2+z^2} \frac{\rho_\ell\, a\, du}{4\epsilon u^{3/2}} = \frac{\rho_\ell\, a}{2\epsilon u^{1/2}}\Big|_{\infty}^{a^2+z^2}$

$u = a^2+z^2$
$du = 2z\,dz$

$\boxed{\text{or } \Phi = \frac{\rho_\ell\, a}{2\epsilon\sqrt{a^2+z^2}}}$

4-15 a) $\bar{E} = \frac{q}{2\pi\epsilon\rho\ell}\bar{a}_\rho$; $\Phi(\rho) = -\int_b^{\rho} \frac{q\, d\rho}{2\pi\epsilon\rho\ell} = \frac{q}{2\pi\epsilon\ell}\ln\frac{b}{\rho}$

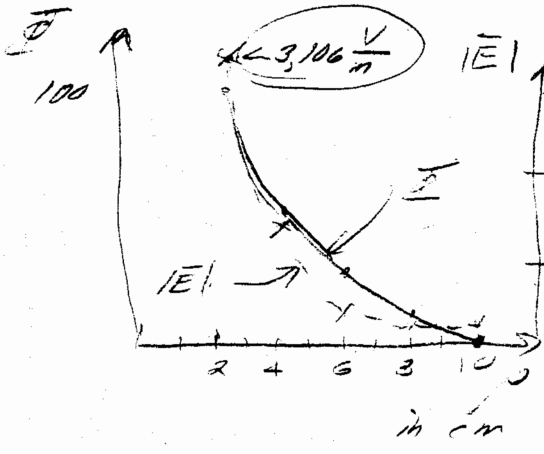
from book $C = \frac{Q}{V} = \frac{2\pi\epsilon\ell}{\ln\frac{b}{a}} = \frac{q}{V}$ $\therefore \Phi(\rho) = \boxed{V\frac{\ln\frac{b}{\rho}}{\ln\frac{b}{a}}}$

$\bar{E} = -\nabla\Phi = -\bar{a}_\rho\frac{\partial\Phi}{\partial\rho} = +\frac{V}{\ln\frac{b}{a}}\cdot\frac{b}{\rho}\cdot\frac{1}{b}\frac{\bar{a}}{\rho} = \boxed{\frac{V}{\ln\frac{b}{a}}\cdot\frac{\bar{a}_\rho}{\rho}}$

c) $V=100$, $a=2\text{cm}$, $b=10\text{cm}$, $\epsilon_r=2$

$C/m = \frac{2\pi\epsilon}{\ln\frac{b}{a}} = \frac{\cancel{2\pi}\times10^{-9}\times 2}{\cancel{36\pi}\ \ln 5}$ (9, 1.609) $= 0.06\times10^{-9} = \boxed{60\,pF/m}$

$\Phi = 100\frac{\ln\frac{b}{\rho}}{\ln(5)} = 62.13\ln\frac{b}{\rho}$



$|E| = \frac{100}{\ln\frac{b}{a}}\cdot\frac{1}{\rho} = 62.13\left(\frac{1}{\rho}\right)$

$\frac{\Phi}{100}\ln 5 = \ln\frac{b}{\rho}$ or $\frac{b}{\rho} = e^{0.0161\Phi}$

or $\boxed{\rho = \frac{0.1}{e^{0.0161\Phi}}}$

EE 333 Homework 14 (Solutions - page 2)

Φ	ρ
0	0.1
25	0.0669
50	0.0447
75	0.0299
100	0.02

@ $\rho = 0.06m$ $\boxed{\Phi = 31.74 \text{ Volts}}$

4-16

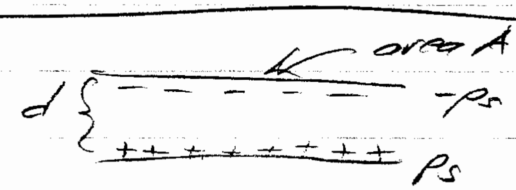
for coax. $C/m = \frac{2\pi\epsilon}{\ln\frac{b}{a}} = 96.8\times10^{-12}$

$$\ln\frac{b}{a} = \frac{2\cancel{\pi}\times10^{-9}\times 2.26}{\underset{18}{\cancel{36\pi}}\times 96.8\times10^{-12}} = 1.297 \quad ; \quad \frac{b}{a} = 3.6585$$

so $\boxed{b = 0.713\text{in} \text{ or } 1.81\text{cm}}$

4-21

$W_e = \frac{1}{2}\int_v \bar{D}\cdot\bar{E}\,dv$



$D = \rho_s$

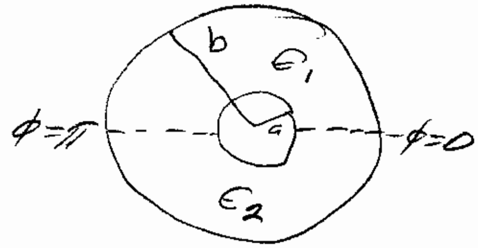
$\therefore\ W_e = \frac{1}{2} dA\,\rho_s^2\,\frac{1}{\epsilon} = \boxed{\frac{dA\rho_s^2}{2\epsilon}}$

b) $W_e = \frac{1}{2}CV^2$; $V = Ed = \frac{\rho_s d}{\epsilon}$

$$\therefore\ C = \frac{2W_e}{V^2} = \frac{\frac{\cancel{d}A\cancel{\rho_s^2}}{\cancel{\epsilon}}}{\frac{\cancel{\rho_s^2}d^{\cancel{2}}}{\epsilon^{\cancel{2}}}} = \boxed{\frac{A\epsilon}{d}}$$

EE 333 Homework 15 (Solution)

4.28



$\Phi(a)=V$; $\Phi(b)=0$

a) from example 4-12

$$\Phi(\rho)=V\,\frac{\ln\frac{b}{\rho}}{\ln\frac{b}{a}}$$

$$\bar{E}=-\nabla\Phi=\bar{a}_\rho\,\frac{1}{\rho}\cdot\frac{V}{\ln\frac{b}{a}}$$

These solution are valid. They give the correct potentials at $\rho=a$ and $\rho=b$ and ~~tangential~~ $\bar{E}$ is continuous at $\phi=0$ and $\phi=\pi$.

a) in Region 1 $\quad \bar{D}_1=\epsilon_1\,\frac{1}{\rho}\cdot\frac{V}{\ln\frac{b}{a}}\,\bar{a}_\rho$

in Region 2 $\quad \bar{D}_2=\epsilon_2\,\frac{1}{\rho}\,\frac{V}{\ln\frac{b}{a}}\,\bar{a}_\rho$

b) q in length ℓ = $\frac{\pi \cancel{a}\ell V}{\cancel{a}\ln\frac{b}{a}}\{\epsilon_1+\epsilon_2\}$; $\boxed{C=\frac{\pi\ell}{\ln\frac{b}{a}}(\epsilon_1+\epsilon_2)}$

$C_{top}=\frac{\pi\ell\epsilon_1}{\ln\frac{b}{a}}$; $C_{bottom}=\frac{\pi\ell\epsilon_2}{\ln\frac{b}{a}}$

if $\boxed{\epsilon=\frac{\epsilon_1+\epsilon_2}{2} \qquad C=\frac{2\pi\ell}{\ln\frac{b}{a}}\left\{\frac{\epsilon_1+\epsilon_2}{2}\right\}}$ Q.E.D.

for $\epsilon_{r_1}=1$; $\epsilon_{r_2}=2.26$; $a=2\,mm$, $b=7\,mm$

$$C/\ell=\frac{\cancel{\pi}\,10^{-9}}{\ln\left(\frac{7}{2}\right)36\cancel{\pi}}(1+2.26)=0.0723\times10^{-9}=\boxed{72.3\ pF/m}$$

EE 333 Homework 15 (Solutions-page 2)

4.29 $$U_e = \frac{1}{2}\iiint \bar{D}\cdot\bar{E}\,dv = \frac{1}{2}\int_{\phi=0}^{\pi}\int_{\rho=a}^{b}\int_{z=0}^{l} \epsilon_1 \frac{V^2 \rho\, d\phi\, d\rho\, dz}{\rho^2 (\ln\frac{b}{a})^2} + \frac{1}{2}\int_{\phi=\pi}^{2\pi}\int_{\rho=a}^{b}\int_{z=0}^{l} \epsilon_2 \frac{V^2 \rho\, d\phi\, d\rho\, dz}{\rho^2 (\ln\frac{b}{a})^2}$$

or $$U_e = \frac{\pi}{2} l \frac{V^2 \ln\frac{b}{a}}{(\ln\frac{b}{a})^2}(\epsilon_1+\epsilon_2) = \frac{1}{2} C V^2 \leftarrow$$

$$\therefore \quad C = \frac{\pi l}{\ln\frac{b}{a}}(\epsilon_1+\epsilon_2) \leftarrow$$

4.61 $30 \rightarrow 500\,pF$ in 0 to 260° $\quad V = 5000V$

$$T = \frac{\partial U_e}{\partial \theta} = \frac{\Delta U_e}{\Delta \theta}$$

$$U_e = \frac{1}{2} C V^2 \quad \therefore \quad \Delta U_e = \frac{V^2}{2}(30-500) = -V^2\frac{470}{2}$$

$$T = -\frac{25\times10^6 \times 470\times10^{-12}}{2\times 260 \times \pi/180} = 1294\times10^{-6} = \underline{1.294\times10^{-3}\ N\text{-}m}$$