EE 333 — Test 1 — Solutions

1. a) $v_{phase} = \frac{-a}{b}$

   b) $\lambda = \frac{2\pi}{b}$

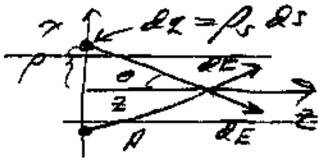
   c) $-x$ direction

---

2. $\overline{A} \cdot \overline{B} = \rho^2 \cos^2\phi + \rho^2 \sin^2\phi - \rho^2 = \rho^2 - \rho^2 = 0$

---

3. a) looking @ diametrically opposed differential surface elements



we see there is only E_z on the z axis

$$dE_z = \frac{\rho_s\, ds \cos\theta}{4\pi\epsilon_0 R^2} = \frac{\rho_s\, z\, \rho\, d\phi\, d\rho}{4\pi\epsilon_0 (\rho^2+z^2)^{3/2}}$$

$$\boxed{E_z} = \int_{\rho=0}^{a} \int_{\phi=0}^{2\pi} \frac{\rho_s\, z\, \rho\, d\phi\, d\rho}{4\pi\epsilon_0 (\rho^2+z^2)^{3/2}} = \boxed{\frac{2\pi\rho_s z}{4\pi\epsilon_0} \int_0^a \frac{\rho\, d\rho}{(\rho^2+z^2)^{3/2}}}$$

let $u = \rho^2 + z^2 \quad du = 2\rho\, d\rho$

$$E_z = \frac{z\rho_s}{2\epsilon_0} \int_{z^2}^{a^2+z^2} \frac{du}{2u^{3/2}} = \frac{-z\rho_s 2}{4\epsilon_0} u^{-\frac{1}{2}} \Big|_{z^2}^{a^2+z^2}$$

$$E_z = \frac{\rho_s z}{2\epsilon_0}\left\{\frac{1}{z} - \frac{1}{(a^2+z^2)^{1/2}}\right\}$$

---

4. a) $J_z = \frac{I}{\pi a^2}$ (use circular contour centered on z axis)

$$\oint \frac{B_\phi \overline{a}_\phi}{\mu_0} \cdot d\overline{l} = \int J_z \overline{a}_z \cdot ds\, \overline{a}_z \Rightarrow \int_{\phi=0}^{2\pi} \frac{B_\phi\, \rho\, d\phi}{\mu_0} = \int_{\phi=0}^{2\pi}\int_{\rho=0}^{\rho} \frac{I}{\pi a^2} \cdot \rho\, d\phi\, d\rho$$

$$\frac{2\pi B_\phi \rho}{\mu_0} = \frac{I\, 2\pi}{\pi a^2} \frac{\rho^2}{2} \quad ; \quad B_\phi = \frac{\mu_0 I \rho}{2\pi a^2}$$

$\overline{F} = q\overline{v} \times \overline{B}$ — charge per unit volume $= \rho_v$

$$\boxed{\therefore\ \overline{F}/_{\text{unit volume}} = \rho_v\, \overline{v} \times \overline{B}}$$

also know $J = \rho_v \overline{v}$ so $\overline{F}/_{\text{unit volume}} = \overline{J} \times \overline{B} = \frac{I}{\pi a^2} \cdot \frac{\mu_0 I \rho}{2\pi a^2} (-\overline{a}_\rho)$

$$\left[\ \overline{F}/_{\text{unit volume}} = \frac{-\mu_0 I^2 \rho}{2\pi^2 a^4} \overline{a}_\rho\ \right]$$

b. If charge was displaced toward z axis electrostatic force would force it back.