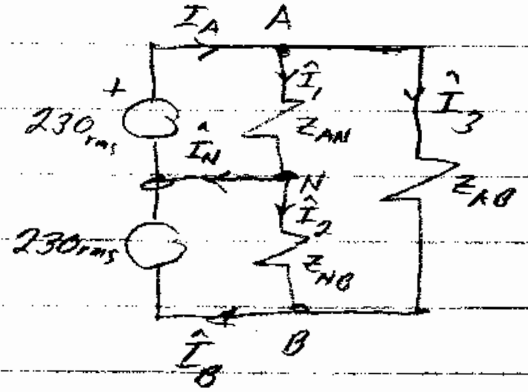
EE 212 Homework 1

12.10



$\hat{S} = \hat{V}_{rms} \hat{I}_{rms}^{*}$

$\therefore \hat{I}_1 = \left(\frac{\hat{S}_{AN}}{230}\right)^{*} = \frac{10{,}000 e^{-j40^\circ}}{230} = 43.478 e^{-j40^\circ}$

$\hat{I}_2 = \left(\frac{\hat{S}_{NB}}{230}\right)^{*} = \frac{9\times10^{3} e^{-j10^\circ}}{230} = 34.78 e^{-j10^\circ}$

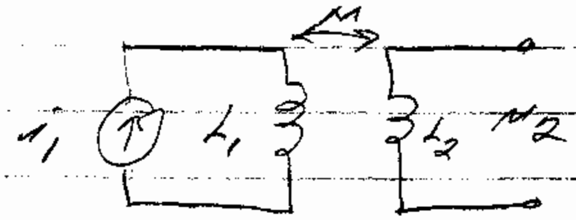
$\hat{I}_3 = \left(\frac{\hat{S}_{AB}}{230}\right)^{*} = \frac{4\times10^{3} e^{j80^\circ}}{460} = 8.696 e^{j80^\circ}$

$\hat{I}_A = \hat{I}_1 + \hat{I}_3 = 34.816 - j19.383 = \boxed{39.85 e^{-j29^\circ}}$ ←

$\hat{I}_B = \hat{I}_2 + \hat{I}_3 = 35.76 + j2.52 = \boxed{35.85 e^{j4^\circ}}$ ←

$\hat{I}_N = \hat{I}_1 - \hat{I}_2 = -0.946 - j21.91 = \boxed{-21.98 e^{j87.53^\circ}}$ ←

13.1



$i_1(t) = 400 \cos 120\pi t$

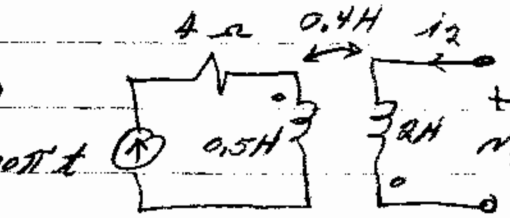
$v_2(t) = 100 \cos(120\pi t + \theta$

but $v_2 = \pm M \frac{di}{dt} = \pm M\, 400\times120\pi(-\sin 120\pi t)$

so $M = \frac{100}{400\times120\pi} = 0.663\,mH$

EE 211 Homework 2

12.8 a)



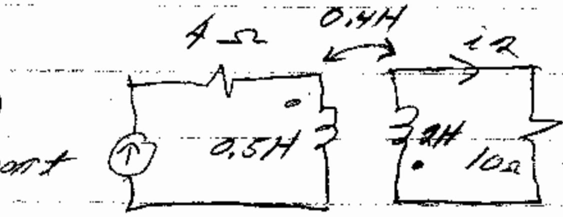
ck sign

$$v_2 = 2\overset{\overset{0}{\|}}{\frac{di_2}{dt}} - 0.4\frac{d}{dt}(\cos 100\pi t)$$

$$\boxed{v_2 = 40\pi \sin 100\pi t} \leftarrow$$

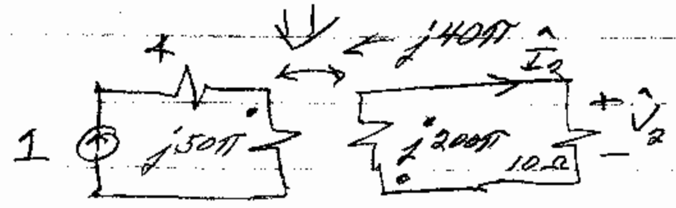
$$\underline{\text{or } v_2 = 40\pi\cos(100\pi t - 90°)} \leftarrow$$

b)



$$v_2 = -2\frac{di_2}{dt} + 0.4\frac{d}{dt}(\cos 100\pi t)$$

$$v_2 = +10\, i_2$$

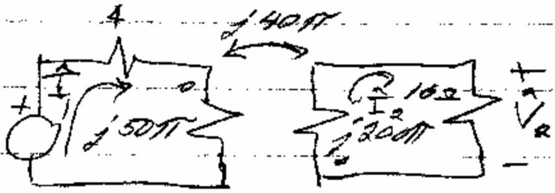


$$\hat{I}_2 j200\pi + j40\pi\times 1 + 10\hat{I}_2 = 0 \;;\; \hat{I}_2 = \frac{-j40\pi}{10+j200\pi}$$

$$\hat{V}_2 = 10\hat{I}_2 = \frac{-j400\pi}{10+j200\pi} = \frac{-j2}{\frac{1}{20\pi}+j} = \frac{2e^{-j90°}}{1e^{j89.09°}} = \underline{\underline{2e^{-j179.09°}}}$$

$$\text{or } \boxed{v_2(t) = 2\cos(100\pi t - 179.09°)}$$

c)



$$4\hat{I}_1 + j50\pi\hat{I}_1 + j40\pi\hat{I}_2 = 1$$

$$j200\pi\hat{I}_2 + j40\pi\hat{I}_1 + 10\hat{I}_2 = 0$$

or

$$\hat{I}_1(4+j50\pi) + j40\pi\hat{I}_2 = 1 \qquad (1)$$

$$j40\pi\hat{I}_1 + (10+j200\pi)\hat{I}_2 = 0 \qquad (2)$$

$$\hat{I}_2 = \frac{\begin{vmatrix} 4+j50\pi & 1 \\ j40\pi & 0 \end{vmatrix}}{\begin{vmatrix} 4+j50\pi & j40\pi \\ j40\pi & 10+j200\pi \end{vmatrix}} = \frac{-j40\pi}{40 - 10,000\pi^2 + j(500\pi+800\pi) + 1600\pi^2}$$

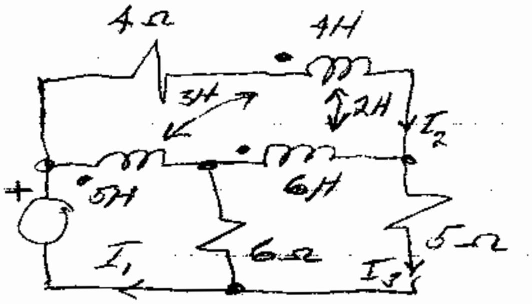
$$\text{or } \hat{V}_2 = 10\hat{I}_2 = \frac{10\times 40\pi\, e^{-j90°}}{-82,864.7 + j4084} = \frac{1,256.6\, e^{-j90°}}{82,965\, e^{j177.18°}} = 0.015\, e^{-j267.18}$$

$$\boxed{v_2(t) = 0.015\cos(100\pi t + 92.82°)} \leftarrow$$

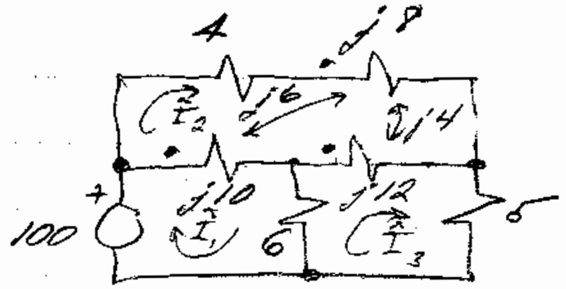
EE 212 Homework 3

13.15

100cos2t



⇒



$$100 = (\hat I_1 - \hat I_2) j10 + j6\hat I_2 + (\hat I_1 - \hat I_3)6$$

$$0 = 4\hat I_2 + j8\hat I_2 + (\hat I_1 - \hat I_2) j6 + (\hat I_3 - \hat I_2) j4 + (\hat I_2 - \hat I_3) j12$$

$$- \hat I_2 j4 + (\hat I_2 - \hat I_1) j10 - j6\hat I_2 = 0$$

$$0 = (\hat I_3 - \hat I_1)6 + (\hat I_3 - \hat I_2) j12 + \hat I_2 j4 + 5\hat I_3 = 0$$

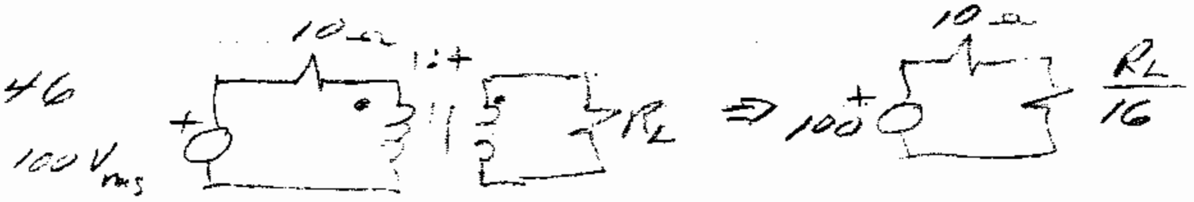
$$\text{or} \left[\begin{array}{l} \hat I_1(6+j10) + \hat I_2(-j4) - \hat I_3 6 = 100 \\ \hat I_1(-j4) + \hat I_2(4+j10) + \hat I_3(-j8) = 0 \\ -6\hat I_1 + \hat I_2(-j8) + \hat I_3(11+j12) = 0 \end{array} \right]$$

from computer

$$\boxed{\hat I_3 = 4.32 e^{-j54.3^\circ}} \leftarrow$$

EE 212 Homework 4

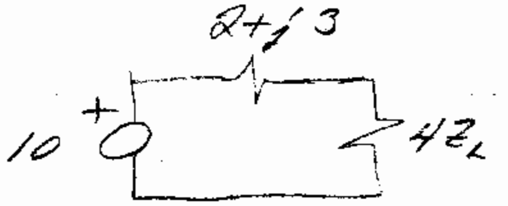
13.46



for maximum power $\frac{R_L}{16} = 10$ or $\boxed{R_L = 160\,\Omega}$

$P_{max} = \frac{50^2}{10} = \frac{2500}{10} = \boxed{250\text{ Watts}}$ ←

Special

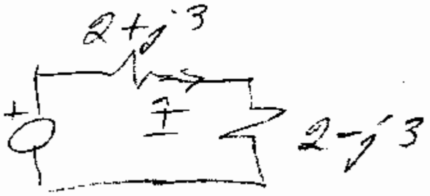


for maximum power transfer

$4Z_L = 2-j3$

a) or $\boxed{Z_L = \frac{2-j3}{4}}$ ←

b)



$\hat{I} = \frac{10}{4}$

$P_{max} = |\hat{I}|^2 \times 2 = \frac{100}{16} \times 2$

$\boxed{P_{max} = \frac{25}{2}\text{ Watts}}$

EE 212 Homework 5

14.18 a) $\mathcal{L}\{3u(t)\} = \int_{0^-}^{\infty} 3e^{-st}dt = -\frac{3}{s}e^{-st}\Big|_0^{\infty} = \boxed{\frac{3}{s}} \leftarrow$

b) $\mathcal{L}\{3u(t-3)\} = \int_{0^-}^{\infty} 3u(t-3)e^{-st}dt = \int_{3}^{\infty} 3e^{-st}dt$

$= \frac{-3}{s}e^{-st}\Big|_3^{\infty} = \boxed{\frac{3}{s}e^{-3s}} \leftarrow$

c) $\mathcal{L}\{3u(t-3)-3\} = \frac{3}{s}e^{-3s} - \frac{3}{s} = \frac{3}{s}\left[e^{-3s}-1\right] \leftarrow$

[using parts a) and b)]

d) $\mathcal{L}\{3u(3-t)\} = \int_{0^-}^{\infty} 3u(3-t)e^{-st}dt = \int_{0}^{3} 3e^{-st}dt$

$= -\frac{3}{s}e^{-st}\Big|_0^3 = +\frac{3}{s}\left[1-e^{-3s}\right] \leftarrow$

14.19

a) $\mathcal{L}\{2+3u(t)\} = \boxed{\frac{5}{s}}$ from a) above $\leftarrow$

b) $\mathcal{L}\{3e^{-8t}\} = \int_{0^-}^{\infty} 3e^{-8t}e^{-st}dt = 3\left[\frac{-1}{s+8}\right]e^{-st}\Big|_0^{\infty}$

$\boxed{= \frac{3}{s+8}} \leftarrow$

c) $\mathcal{L}\{u(-t)\} = \int_{0}^{\infty} 0 \times e^{-st}dt = \boxed{0} \leftarrow$

d) $\mathcal{L}\{K\} = \int_{0^-}^{\infty} Ke^{-st}dt = K\left(\frac{-1}{s}\right)e^{-st}\Big|_0^{\infty} = \boxed{\frac{K}{s}} \leftarrow$

EE 212 Homework 6

14.25 a) $F(s) = \frac{1}{s+3} \therefore f(t) = \boxed{e^{-3t}u(t)}$

b) $F(s) = 1 \therefore f(t) = \boxed{\delta(t)}$

c) $F(s) = \frac{1}{s^2} \therefore f(t) = \boxed{t\,u(t)}$

d) $F(s) = 275 \therefore f(t) = 275\,\delta(t)$

e) $F(s) = \frac{s^2}{s^3} = \frac{1}{s} \therefore f(t) = u(t)$

EE 212 Homework 7

14.32 a) $F(s) = 3 + \frac{1}{s} \Rightarrow f(t) = \boxed{3\delta(t) + u(t)}$

b) $F(s) = 3 + \frac{1}{s^2} \Rightarrow f(t) = 3\delta(t) + t\,u(t)$

c) $F(s) = \frac{1}{(s+3)(s+4)} = \frac{1}{s+3} + \frac{-1}{s+4} \Rightarrow f(t) = \{e^{-3t} - e^{-4t}\}u(t)$

d) $F(s) = \frac{1}{(s+3)(s+4)(s+5)} = \frac{1/2}{s+3} + \frac{-1}{s+4} + \frac{1/2}{s+5}$

$\therefore \; f(t) = \{\frac{1}{2}e^{-3t} - e^{-4t} + \frac{1}{2}e^{-5t}\}u(t)$

14.35 $V(s) = \frac{5}{s}$ so $v(t) = 5u(t)$

$i_{2k\Omega} = \frac{v}{2\times10^3} = \frac{5}{2\times10^3}u(t)$

so $\boxed{i_{2k\Omega} = \frac{5}{2}\times10^{-3} \text{ for all } t > 0}$ ⇐

EE 212 Homework 8

14.39 a) $F(s) = \frac{5}{s+1}$; $f(t) = \boxed{5e^{-t}u(t)}$

b) $F(s = \frac{5}{s+1} - \frac{2}{s+4}$; $f(t) = \boxed{(5e^{-t} - 2e^{-4t})u(t)}$

c) $F(s) = \frac{18}{(s+1)(s+4)} = \frac{6}{s+1} + \frac{-6}{s+4}$; $\boxed{f(t) = (6e^{-t} - 6e^{-4t})u(t)}$

d) $F(s) = \frac{18s}{(s+1)(s+4)} = \frac{-6}{s+1} + \frac{24}{s+4}$; $\boxed{f(t) = (-6e^{-t} + 24e^{-4t})u(t)}$

e) $F(s) = \frac{18s^2}{(s+1)(s+4)} = \frac{18s^2}{s^2+5s+4} = 18 - \frac{90s+72}{(s+1)(s+4)}$

$$\left\{ \begin{array}{r|l} & 18 \\ \hline s^2+5s+4 & 18s^2 \\ & 18s^2+90s+72 \end{array} \right\}$$

$F(s) = 18 - \frac{-6}{s+1} - \frac{96}{s+4}$; $f(t) = \boxed{18\delta(t) + (6e^{-t} - 96e^{-4t})u(t)}$

14.43 a) $F(s) = \frac{(s+1)(s+2)}{s(s+3)} = \frac{s^2+3s+2}{s^2+3s} = 1 + \frac{2}{s(s+3)}$

$$\left\{ \begin{array}{r|l} & 1 \\ \hline s^2+3s & s^2+3s+2 \\ & s^2+3s \end{array} \right\}$$

$F(s) = 1 + \frac{\frac{2}{3}}{s} + \frac{-\frac{2}{3}}{s+3}$

$\boxed{f(t) = \delta(t) + \frac{2}{3}(1 - e^{-3t})u(t)}$ ←

b) $F(s) = \frac{s+2}{s^2(s^2+4)} = \frac{As+B}{s^2} + \frac{Cs+D}{s^2+4}$

$B = \frac{1}{2}$; $s+2 = (As + \frac{1}{2})(s^2+4) + (Cs+D)s^2$

s^3 terms: $0 = A + C \Rightarrow C = -\frac{1}{4}$

s^2 terms: $0 = \frac{1}{2} + D \Rightarrow D = -\frac{1}{2}$

s terms: $1 = 4A \Rightarrow A = \frac{1}{4}$

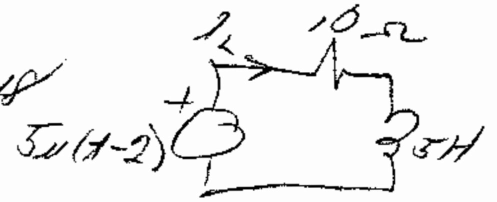
constant terms: $2 = 2$ ✓

$F(s) = \frac{\frac{1}{4}s + \frac{1}{2}}{s^2} + \frac{-\frac{1}{4}s - \frac{1}{2}\cdot\frac{2}{2}}{s^2+4}$

$\boxed{f(t) = \frac{1}{4}u(t) + \frac{1}{2}tu(t) - \left(\frac{1}{4}\cos 2t + \frac{1}{4}\sin 2t\right)u(t)}$

EE 212 Homework 9

14.48



$i_L(0) = 5\times10^{-3}$

a) $\boxed{5u(t-2) = 10 i_L + 5\frac{di_L}{dt}}$ $\longleftarrow$

b) $$\frac{5e^{-2s}}{s} = 10\,I(s) + 5\left\{s\,I(s) - 5\times10^{-3}\right\}$$

$$\therefore\ I(s) = \frac{\frac{5e^{-2s}}{s} + 25\times10^{-3}}{5s+10} = \frac{5e^{-2s} + 25\times10^{-3}s}{5s(s+2)}$$

$$\text{or}\ \ I(s) = \frac{e^{-2s}+5\times10^{-3}s}{s(s+2)} = \frac{e^{-2s}}{s(s+2)} + \frac{5\times10^{-3}}{s+2}$$

$$\boxed{I(s) = e^{-2s}\left[\frac{\frac{1}{2}}{s} + \frac{-\frac{1}{2}}{s+2}\right] + \frac{5\times10^{-3}}{s+2}} \longleftarrow$$

c) $\boxed{i_L(t) = \frac{1}{2}u(t-2) - \frac{1}{2}e^{-2(t-2)}u(t-2) + 5\times10^{-3}e^{-2t}u(t)}$

Special

$$F(s) = \frac{-5s^2-30s+15}{s^3+2s^2+5s} = \frac{-5s^2-30s+15}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$A = 3$ $\left\{-5s^2-30s+15 = 3(s^2+2s+5)+Bs^2+Cs\right\}$

$\therefore$ $-5 = 3 + B \Rightarrow B = -8$

$-30 = 6 + C \Rightarrow C = -36$

$15 = 15$

$$F(s) = \frac{3}{s} - \frac{8s+36}{(s+1)^2+4} = \frac{3}{s} - \frac{8(s+1)+28\left(\frac{2}{2}\right)}{(s+1)^2+4}$$

$$\boxed{f(t) = 3u(t) - \left(8e^{-t}\cos 2t - 14e^{-t}\sin 2t\right)u(t)}$$

EE212 Homework 10

15.110

with no source before $t=0$
$i(0)=0$

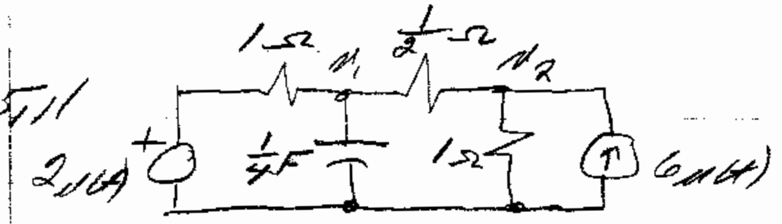
$$V(s) = \frac{\overset{5}{\cancel{25}}\times10^{-6}}{\cancel{s}} \times \frac{\frac{\cancel{s}}{\cancel{5}}}{2+\frac{s}{10}} = \frac{5\times10^{-6}}{2+\frac{s}{10}} = \frac{5\times10^{-5}}{s+20}$$

$$\therefore\ v(t) = 5\times10^{-5} e^{-20t} u(t) \leftarrow$$

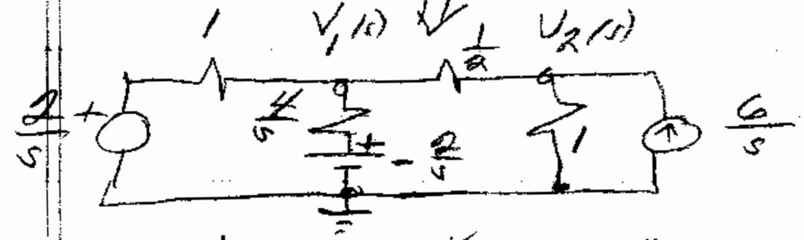
$$P_{resistor}(t) = \frac{v^2}{2} = 12.5\times10^{-10} e^{-40t} = \boxed{1.25\times10^{-9} e^{-40t}\, W} \leftarrow$$

EE 212 Homework 11

15.11



$v_1(0^-) = -2$



$$\frac{V_1 - \frac{2}{s}}{1} + \frac{V_1 + \frac{2}{s}}{4/s} + \frac{V_1 - V_2}{1/2} = 0 \quad \Big\} \quad \begin{aligned} &V_1\left(3 + \frac{s}{4}\right) - 2V_2 = -\frac{1}{2} + \frac{2}{s} \\ &-2V_1 + 3V_2 = \frac{6}{s} \end{aligned}$$

$$\frac{V_2 - V_1}{1/2} + \frac{V_2}{1} = \frac{6}{s}$$

$$V_1(12 + s) - 8V_2 = -2 + \frac{8}{s}$$

or $V_1(-2s) + V_2(3s) = 6$

$$V_1 = \frac{\begin{vmatrix} -2+\frac{8}{s} & -8 \\ 6 & 3s \end{vmatrix}}{\begin{vmatrix} 12+s & -8 \\ -2s & 3s \end{vmatrix}} = \frac{\overbrace{\cancel{-6s+24+48}}^{72}}{\underset{3s^2+20s}{36s+3s^2-16s}} = \frac{72-6s}{s(20+3s)}$$

$$V_1 = \frac{24-2s}{s\left(s+\frac{20}{3}\right)} = \frac{\frac{18}{5}}{s} - \frac{\frac{28}{5}}{s+\frac{20}{3}} \quad ◁—$$

$$V_2 = \frac{\begin{vmatrix} 12+s & -2+\frac{8}{5} \\ -2s & 6 \end{vmatrix}}{s(20+3s)} = \frac{72+6s-4s+16}{s(20+3s)} = \frac{88+2s}{3s\left(s+\frac{20}{3}\right)} = \frac{\frac{88}{3}+\frac{2}{3}s}{s\left(s+\frac{20}{3}\right)}$$

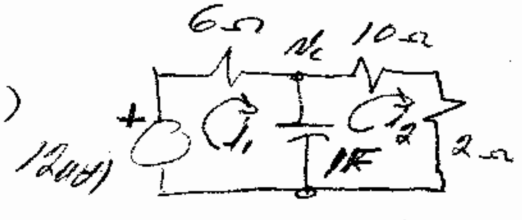
$$V_2 = \frac{\frac{22}{5}}{s} - \frac{\frac{56}{15}}{s+\frac{20}{3}} \quad ◁—$$

$$\therefore\ v_1(t) = \left\{\frac{18}{5} - \frac{28}{5}e^{-\frac{20t}{3}}\right\}u(t) = \left\{3.6 - 5.6e^{-6.67t}\right\}u(t) \longleftarrow$$

$$v_2(t) = \left\{\frac{22}{5} - \frac{56}{15}e^{-\frac{20t}{3}}\right\}u(t) = \left\{4.4 - 3.73e^{-6.67t}\right\}u(t) \longleftarrow$$

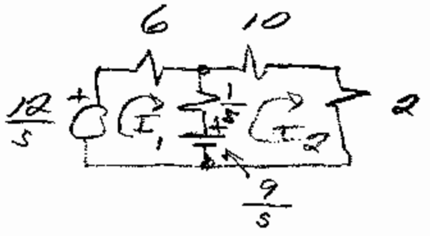
EE 212 Homework 12

15,13 a)



$v_c(0) = 9V$

$\Rightarrow$



$$\left.\begin{array}{l}\frac{12}{s} = 6I_1 + \frac{1}{s}(I_1 - I_2) + \frac{9}{s} \\ \frac{9}{s} = (I_2 - I_1)\frac{1}{s} + 12 I_2\end{array}\right\} \begin{array}{l} I_1(6+\frac{1}{s}) - I_2 \frac{1}{s} = \frac{3}{s} \\ -I_1 \frac{1}{s} + I_2(12 + \frac{1}{s}) = \frac{9}{s}\end{array}$$

multiplying by s gives: $\left[\begin{array}{l} I_1(6s+1) - I_2 = 3 \\ -I_1 + I_2(12s+1) = 9\end{array}\right]$

$$I_1 = \frac{\begin{vmatrix} 3 & -1 \\ 9 & 12s+1\end{vmatrix}}{\begin{vmatrix} 6s+1 & -1 \\ -1 & 12s+1\end{vmatrix}} = \frac{36s+3+9}{72s^2+6s+12s+\cancel{1}-\cancel{1}} = \frac{36s+12}{s(72s+18)} = \frac{6s+2}{s(12s+3)}$$

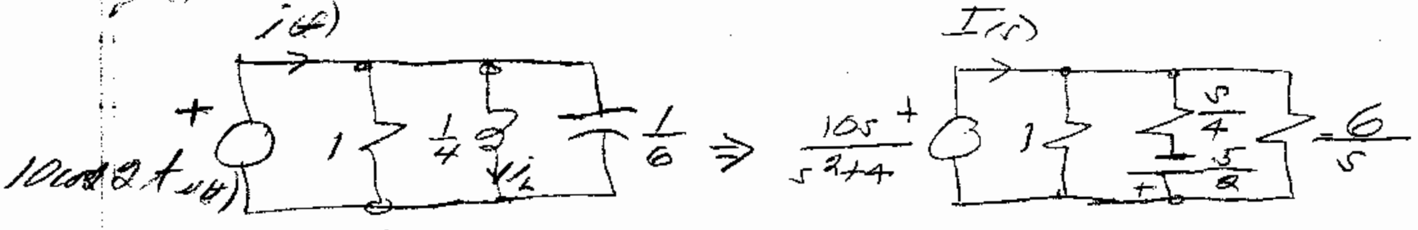
$$I_1 = \frac{6s+2}{12s(s+\frac{1}{4})} = \frac{\frac{2}{3}}{s} + \frac{-\frac{1}{6}}{s+\frac{1}{4}} \quad \therefore \boxed{i_1(t) = \{\tfrac{2}{3} - \tfrac{1}{6}e^{-\frac{t}{4}}\}u(t)}$$

$$I_2 = \frac{\begin{vmatrix} 6s+1 & 3 \\ -1 & 9\end{vmatrix}}{s(72s+18)} = \frac{54s+\overset{12}{9+3}}{s(72s+18)} = \frac{9s+2}{12s(s+\frac{1}{4})} = \frac{\frac{2}{3}}{s} + \frac{\frac{1}{12}}{s+\frac{1}{4}}$$

so $i_2(t) = \{\frac{2}{3} + \frac{1}{12}e^{-\frac{t}{4}}\}u(t) \leftarrow$

Homework 13

Special



$i_L(0) = 10$

$$I(s) = \frac{10s}{s^2+4} + \frac{\frac{10s}{s^2+4} + \frac{5}{2}}{\frac{5}{4}} + \frac{\frac{10s}{s^2+4}}{\frac{6}{s}}$$

$$I(s) = \frac{10s}{s^2+4} + \frac{40}{s^2+4} + \frac{10}{s} + \frac{10s^2}{6(s^2+4)}$$

$-\frac{20}{3} + \frac{120}{3}$

$$I(s) = \frac{10s}{s^2+4} + \frac{40}{s^2+4} + \frac{10}{s} + \frac{5}{3}\left[1 - \frac{4}{s^2+4}\right]$$

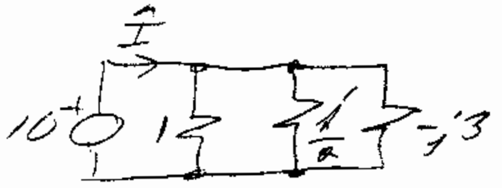
$$I(s) = \frac{10s}{s^2+4} + \frac{\overset{50}{\cancel{100}}}{\cancel{2}\times 3} \cdot \frac{2}{s^2+4} + \frac{10}{s} + \frac{5}{3}$$

$$\boxed{i(t) = \left\{10\cos 2t + \frac{50}{3}\sin 2t + 10\right\}u(t) + \frac{5}{3}\delta(t)}$$

or $i(t) = \left\{\sqrt{10^2 + \left(\frac{50}{3}\right)^2}\cos\left(2t - \tan^{-1}\frac{5}{3}\right) + 10\right\}u(t) + \frac{5}{3}\delta(t)$

---

check of sinusoidal steady state

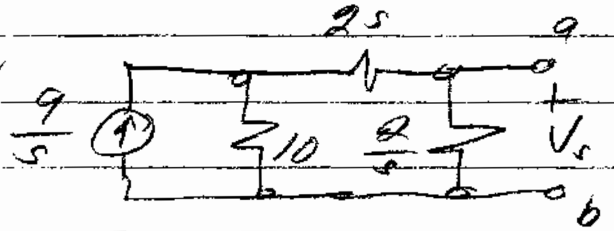


$$\hat{I} = 10 - j20 + j\frac{10}{3} = 10 - j\left(\frac{50}{3}\right) = \sqrt{10^2 + \left(\frac{50}{3}\right)^2}\, e^{-j\tan^{-1}\left(\frac{5}{3}\right)}$$

$$i(t) = \mathrm{Re}\left\{\hat{I} e^{j2t}\right\} = \sqrt{10^2 + \left(\frac{50}{3}\right)^2}\cos\left(2t - \tan^{-1}\frac{5}{3}\right)$$

EE 212 Homework 14

15-28 ✓



$$V_{Th}(s) = \frac{9}{s} \cdot \frac{10}{10+2s+\frac{2}{s}} \cdot \frac{2}{s} = \frac{9}{s} \cdot \frac{20}{2s^2+10s+2}$$

$$\boxed{V_{Th}(s) = \frac{90}{s(s^2+5s+1)}}$$

$$Z_{Th} = \frac{\frac{2}{s}(2s+10)}{\frac{2}{s}+2s+10} = \frac{4s+20}{2s^2+10+2} \boxed{= \frac{2s+10}{s^2+5s+1}}$$

EE 212 Homework 15

15.42 $f_1(t) = e^{-5t}u(t)$; $f_2(t) = (1-e^{-2t})u(t)$

$$y(t) = \int_0^t e^{-5(t-\tau)}u(t-\tau)(1-e^{-2\tau})u(\tau)\,d\tau$$

$$= e^{-5t}\int_0^t e^{5\tau}(1-e^{-2\tau})\,d\tau = e^{-5t}\left\{\frac{1}{5}e^{5\tau} - \frac{1}{3}e^{3\tau}\right\}_0^t u(t)$$

$$y(t) = e^{-5t}\left\{\frac{1}{5}\left[e^{5t} - \frac{1}{5}\right] - \frac{1}{3}\left[e^{3t} - 1\right]\right\}u(t)$$

$$y(t) = \left\{\frac{1}{5}\left[1 - e^{-5t}\right] - \frac{1}{3}\left[e^{-2t} - e^{-5t}\right]\right\}u(t)$$

$$\boxed{y(t) = \left\{\frac{1}{5} + \frac{2}{15}e^{-5t} - \frac{1}{3}e^{-2t}\right\}u(t)} \leftarrow$$

$F_1(s) = \frac{1}{s+5}$; $F_2(s) = \frac{1}{s} - \frac{1}{s+2}$

$$\therefore Y(s) = F_1(s)F_2(s) = \frac{1}{s}\cdot\frac{1}{s+5} - \frac{1}{(s+2)(s+5)}$$

$$Y(s) = \frac{\frac{1}{5}}{s} + \frac{-\frac{1}{5}}{s+5} - \frac{\frac{1}{3}}{s+2} - \frac{-\frac{1}{3}}{s+5} = \frac{\frac{1}{5}}{s} - \frac{\frac{1}{3}}{s+2} + \frac{\frac{2}{15}}{s+5}$$

so $\boxed{y(t) = \left\{\frac{1}{5} - \frac{1}{3}e^{-2t} + \frac{2}{15}e^{-5t}\right\}u(t)}$

EE 212 Homework 16

15.44 $h(t) = 2e^{-3t}u(t)$; $x(t) = u(t) - \delta(t)$

a) $$y(t) = \int_0^t 2e^{-3(t-z)}u(t-z)\left[u(z) - \delta(z)\right]dz$$

$$= 2e^{-3t}\left(\frac{1}{3}\right)e^{3z}\Big|_0^t - 2e^{-3t}\int_0^t e^{3z}\delta(z)\,dz$$

$$y(t) = \frac{2}{3}e^{-3t}\left[e^{3t} - 1\right]u(t) - 2e^{-3t}u(t)$$

$$y(t) = \boxed{\left\{\frac{2}{3} - \frac{8}{3}e^{-3t}\right\}u(t)} \leftarrow$$

b) $H(s) = \frac{2}{s+3}$; $X(s) = \frac{1}{s} - 1$

$$Y(s) = H(s)X(s) = \frac{2}{s(s+3)} - \frac{2}{s+3} = \frac{\quad}{s} + \frac{\quad}{s+3} - \frac{2}{s+3}$$

$$Y(s) = \frac{\frac{2}{3}}{s} + \frac{-\frac{2}{3} - 2}{s+3} = \frac{\frac{2}{3}}{s} - \frac{\frac{8}{3}}{s+3}$$

so $$y(t) = \boxed{\left\{\frac{2}{3} - \frac{8}{3}e^{-3t}\right\}u(t)} \leftarrow$$

Homework 17

16,8

$$Z_{in} = 2 + \frac{\frac{5}{j\omega 10^{-2}}}{5 + \frac{1}{j\omega 10^{-2}}} + \frac{5j\omega 10^{-2}}{5 + j\omega 10^{-2}} = 2 + \frac{5}{1 + j\omega 5\times 10^{-2}} + \frac{j\omega 5\times 10^{-}}{5 + j\omega 10^{-}}$$

$$Z_{in} = 2 + \frac{5(1 - j\omega 5\times 10^{-2})}{1 + 25\omega^2\times 10^{-4}} + \frac{j\omega 5\times 10^{-2}(5 - j\omega 10^{-2})}{25 + \omega^2\times 10^{-4}}$$

@ resonance $$\frac{-\cancel{25\omega\times 10^{-2}}}{1 + 25\omega^2\times 10^{-4}} + \frac{\omega\cancel{25\times 10^{-2}}}{25 + \omega^2\times 10^{-4}} = 0$$

or $25 + \omega^2\times 10^{-4} = 1 + 25\omega^2\times 10^{-4} \Rightarrow \cancel{24} = \cancel{24}\omega^2\times 10^{-4}$

$$\boxed{\omega_0 = \sqrt{\frac{1}{10^{-4}}} = 10^2 \quad \text{or} \quad f_0 = 15,9\ \text{Hz}}$$

$$Z_{in}(\omega_0) = 2 + \frac{5}{1+25} + \frac{5\times 10^4\times 10^{-4}}{25+1} = \frac{52+10}{26} = \frac{62}{26} = \boxed{2,38\ \Omega}$$

16,25 $BW = 10^6$ Hz, $f_1 = 5,5$ kHz

a) $\omega_1 = 2\pi\times 5,5\times 10^3$

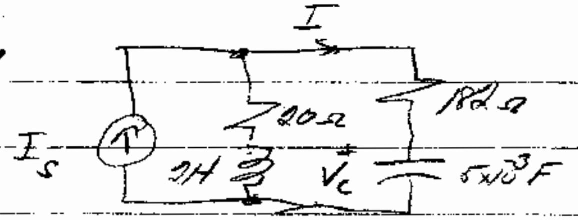
$$f_2 = f_1 + BW = 5,5\times 10^3 + 10^6 = \boxed{1,0055\times 10^6\ \text{Hz}}$$

b) $$f_0 = \sqrt{f_2 f_1} = \sqrt{5,5\times 10^3\times 1,0055\times 10^6} = \boxed{74,37\times 10^3\ \text{Hz}}$$

c) $$Q_0 = \frac{f_0}{BW} = \frac{74,37\times 10^3}{10^6} = \boxed{0,074} \leftarrow$$

EE 212 Homework 18

16.5 7



$H(j\omega) = \frac{\hat{V}_c}{\hat{I}_s}$

$H(s) = \frac{V_c(s)}{I_s(s)}$

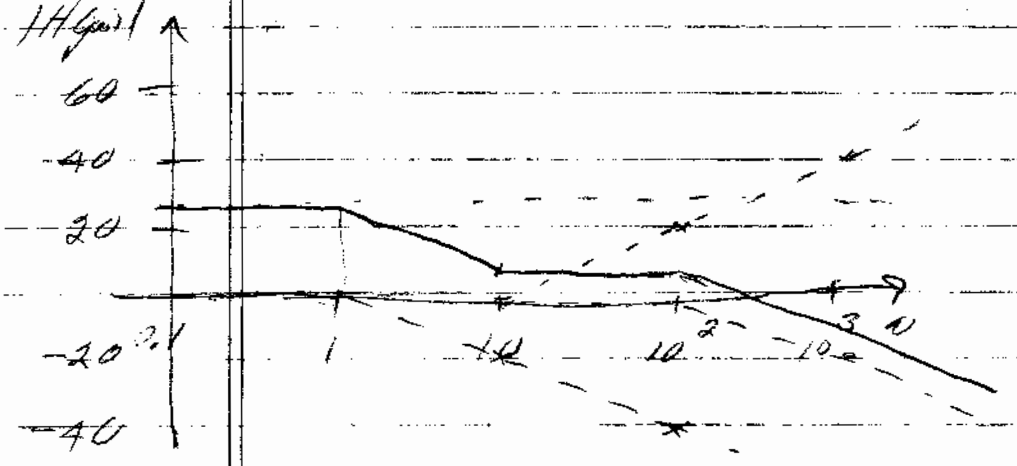
$$I(s) = I_s \frac{20+2s}{202+2s+\frac{1}{s\times5\times10^{-3}}}$$

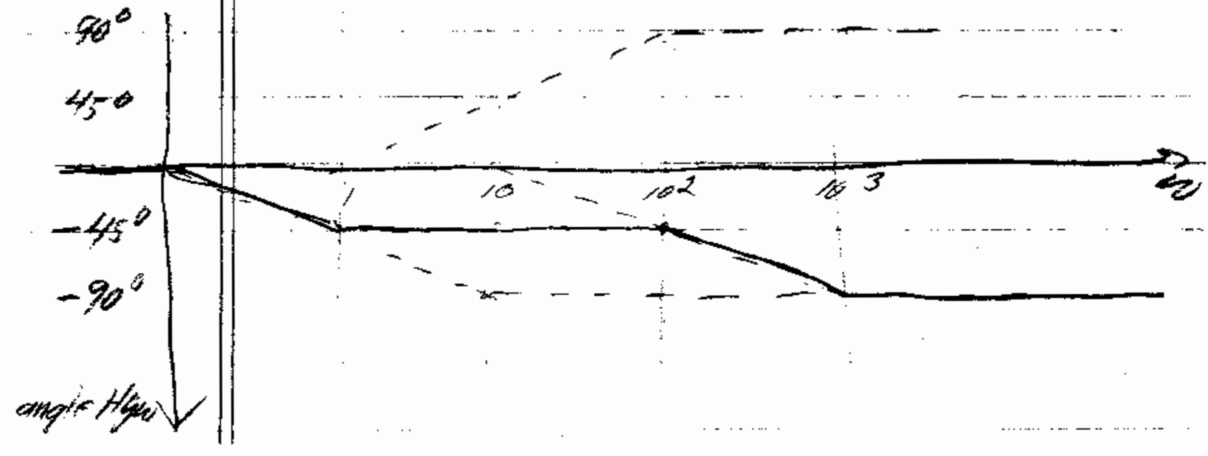
$$V_c(s) = I(s)\frac{1}{s\times5\times10^{-3}} = \frac{(20+2s)\frac{I_s}{\cancel{s\times5\times10^{-3}}}}{\frac{1.01s+10^{-2}s^2+1}{\cancel{s\times5\times10^{-3}}}} = \frac{(20+2s)I_s}{10^{-2}s^2+1.01s+1}$$

$$\therefore H(s) = \frac{10^2(20+2s)}{s^2+101s+1} = \frac{2\times10^2(s+10)}{(s+1)(s+100)}$$

$$H(j\omega) = 20\frac{\left(1+j\frac{\omega}{10}\right)}{\left(1+j\frac{\omega}{1}\right)\left(1+j\frac{\omega}{100}\right)}$$

$\boxed{20\log_{10}20 = 26dB}$

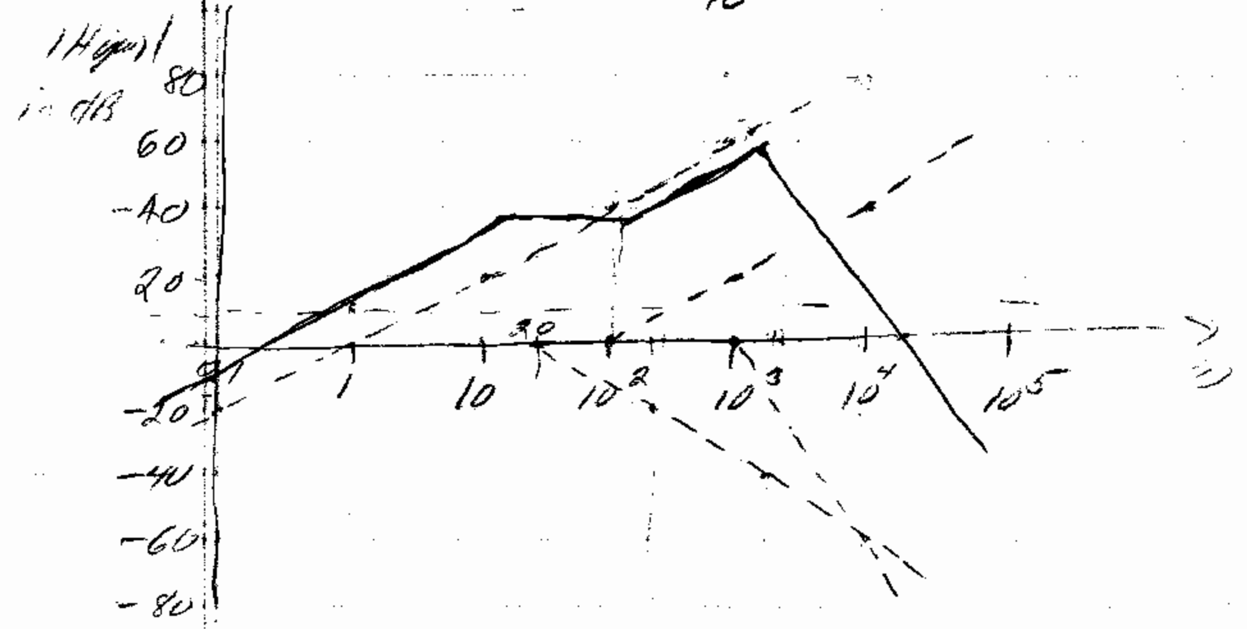




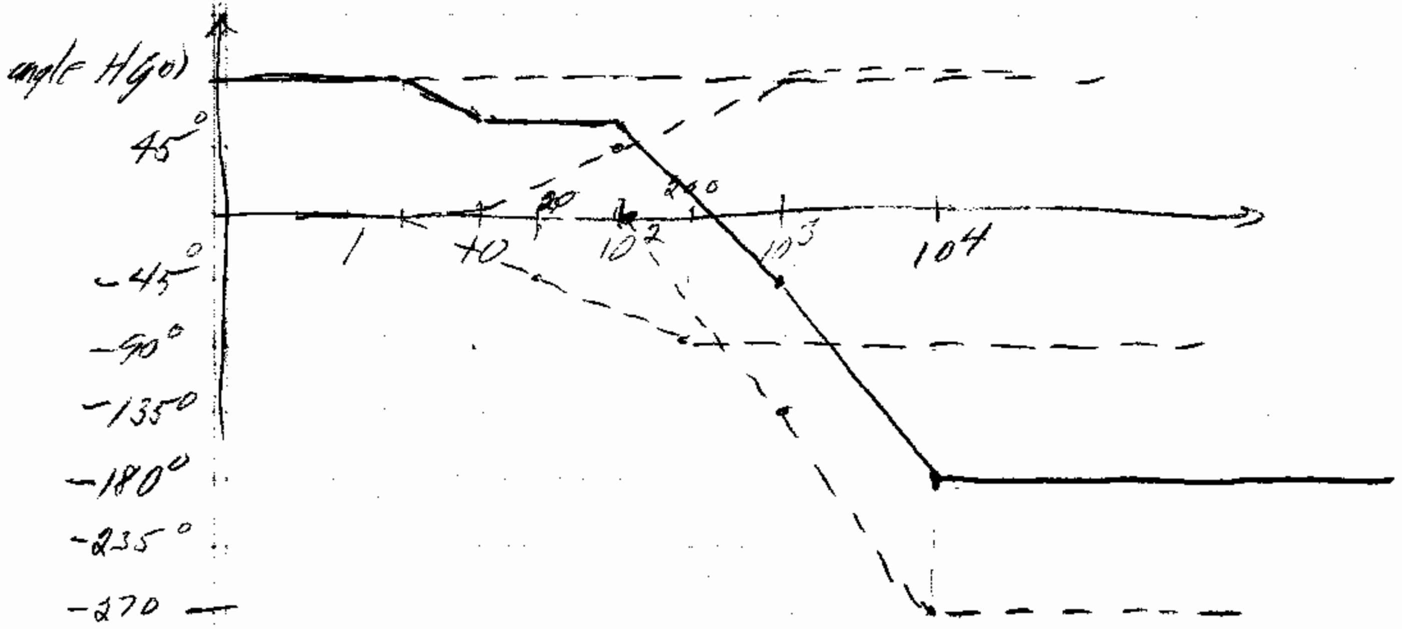
EE 212 Homework 19

16.58a) $H(s) = \frac{5\times10^8 s(s+100)}{(s+80)(s+10^3)^3} = \frac{5\times10^8\times10^2 s(1+\frac{s}{100})}{80\times10^9(1+\frac{s}{80})(1+\frac{s}{10^3})^3}$

$H(j\omega) = \frac{5}{8}\frac{j\omega(1+\frac{j\omega}{100})}{(1+\frac{j\omega}{80})(1+\frac{j\omega}{10^3})^3}$; $20\log_{10}(\frac{5}{8}) = 7.96$

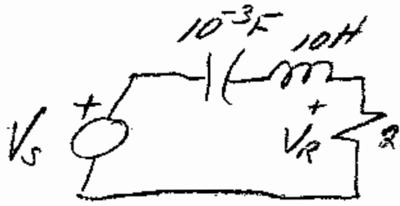


16,59a)



EE 212 Homework 20

16.61



$$\frac{V_R(s)}{V_s(s)} = \frac{25}{25+10s+\frac{1}{10^{-3}s}} = H(s)$$

a) $$H(s) = \frac{25s}{10s^2+25s+10^3} = \frac{25}{10^3}\,\frac{s}{1+25\times10^{-3}s+\frac{s^2}{100}}$$

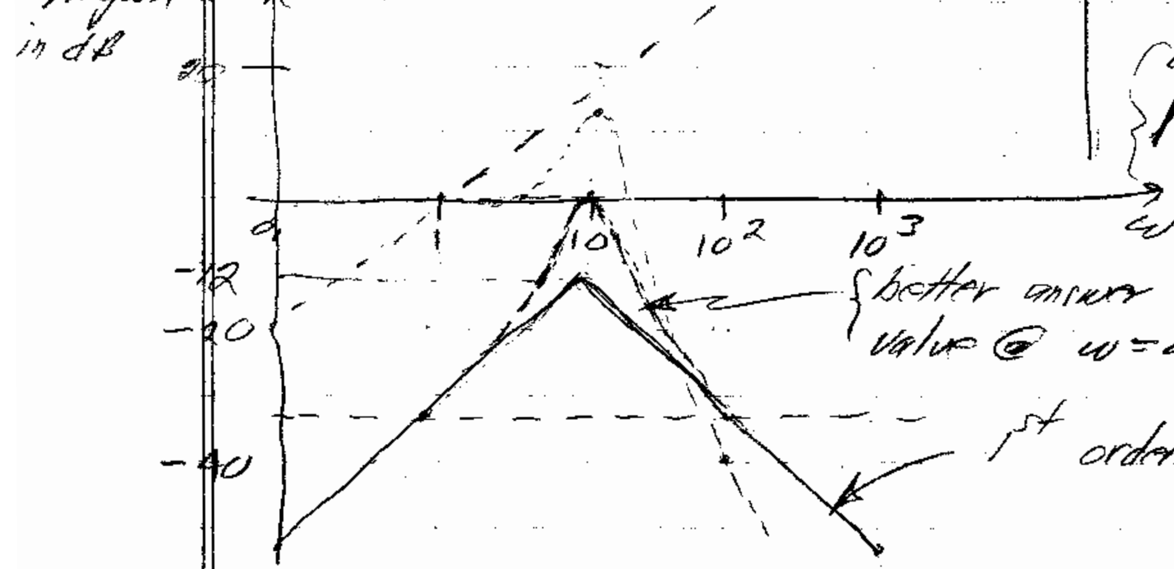
$\boxed{\omega_0 = 10}$; $\frac{2\zeta}{\omega_0} = 25\times10^{-3}$; so $\zeta = \frac{25\times10^{-3}\times10}{2}$

$\boxed{\zeta = 0,125}$

b) $$H(j\omega) = \frac{0,025\,j\omega}{1+2\left(\frac{1}{8}\right)\frac{1}{10}j\omega-\frac{\omega^2}{10^2}}$$

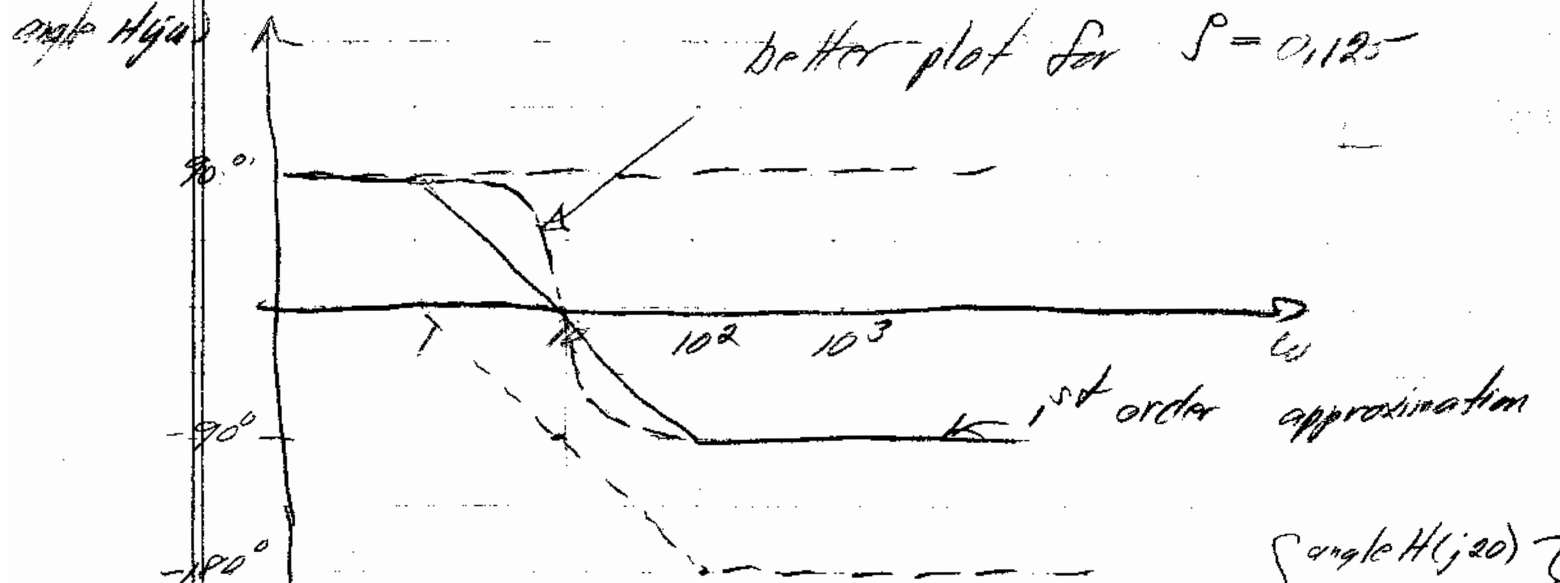
$\left[20\log_{10}0.025\right] = -32$

at $\omega=\omega_0$
|denominator of $H(j\omega)$|
$= 20\log_{10}\frac{1}{4} = -12$



better answer using actual value @ $\omega=\omega_0$

1^{st} order appoximation



better plot for $\zeta = 0,125$

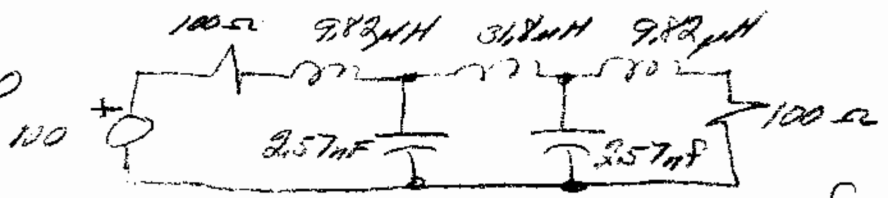
1^{st} order approximation

c) @ $\omega=20$ $|H(j20)| = \left|\frac{j0.5}{1-4+j0.5}\right| \Rightarrow |H(j20)|_{\text{in dB}} = -15.68$; $\left\{ \text{angle } H(j20) = \underline{\underline{-80,54^{\circ}}} \right\}$

16.62

EE 212 Homework 21

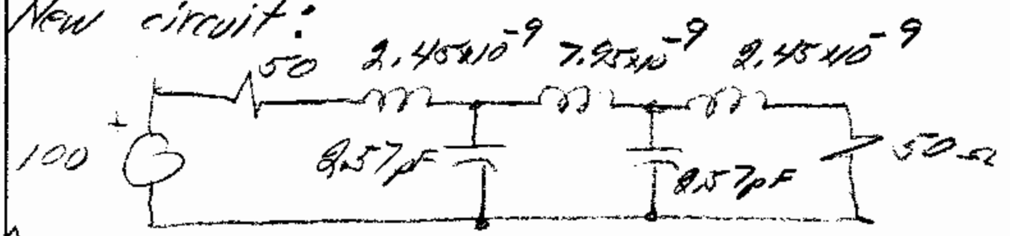
16.50

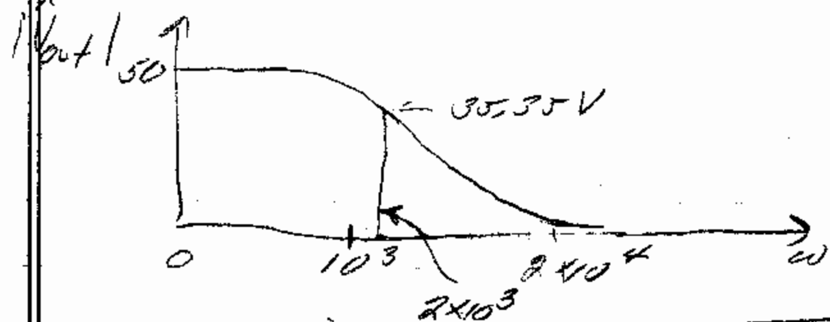


Magnitude scaling factor $= \frac{1}{2}$ $\left\{ R' = \frac{R}{2},\ L' = \frac{L}{2},\ C' = 2C \right\}$

frequency scaling factor $= 2\times10^3$ $\left\{ R' = R,\ L' = L/2\times10^3,\ C' = C/2\times10^3 \right.$

New circuit:





OK for original cutoff frequency = 1

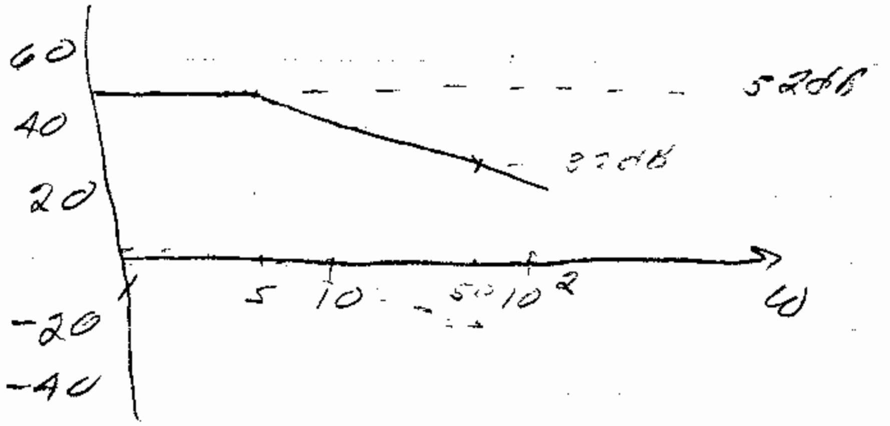
(see next page for $f_c = 10^9$)

16.62 1st stage $H_1(s) = -10$; 2nd stage $H_2(s) = -10$

3rd stage $H_3(s) = -\frac{\frac{2\times10^5\left(\frac{1}{5\times10^{-6}}\right)}{2\times10^5+\frac{1}{5\times10^{-6}}}}{5\times10^4} = -\frac{2\times10^5}{5\times10^4}\cdot\frac{1}{1+s\times2\times10^{-1}}$

∴ total $H(s) = H_1(s)H_2(s)H_3(s) = -\frac{2}{5}\times10^{+3}\frac{1}{1+s\times0.2} = \boxed{-400\frac{1}{1+\frac{s}{5}}}$

$20\log_{10}400 = 52dB$

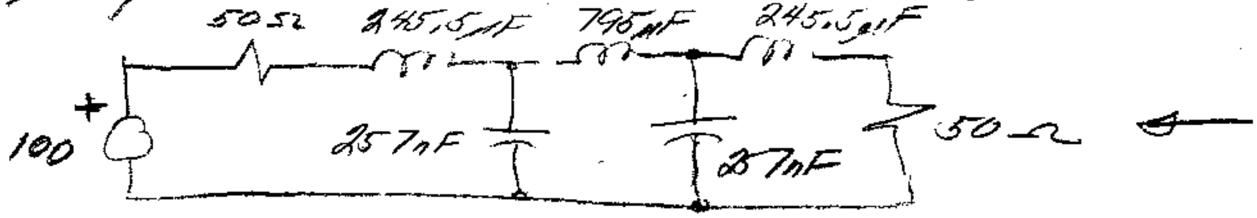


EE 212 Homework 21 (continued)

16.50 for original $f_{cutoff} = 10^6 Hz$

Magnitude scaling factor $= \frac{1}{2}$

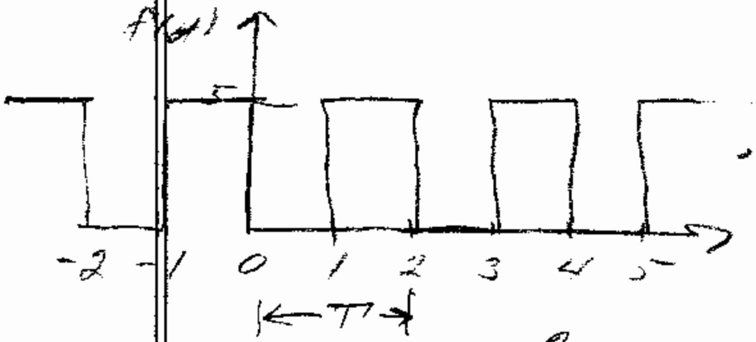
frequency scaling factor $= \frac{20 \times 10^3}{10^6} = 2 \times 10^{-2}$



plot the same as previous page!

EE 212 Homework 22

18.8 $f(t) = 5u(t-1) - 5u(t-2) + 5u(t-3) - 5u(t-4) \cdots$



$T = 2$; $\omega_0 = \pi$

$$\boxed{a_0 = \frac{1}{2}\int_1^2 5\,dt = \frac{5}{2}} \longleftarrow$$

$$a_n = \frac{2}{2}\int_1^2 5\cos n\pi t\,dt = 5\left(\frac{1}{n\pi}\right)\sin n\pi t\Big|_1^2$$

$$\boxed{a_n = \frac{5}{n\pi}\left[\sin 2\pi n - \sin n\pi\right] = 0} \longleftarrow$$

$$b_n = \frac{2}{2}\int_1^2 5\sin n\pi t\,dt = 5\left(-\frac{1}{n\pi}\right)\cos n\pi t\Big|_1^2$$

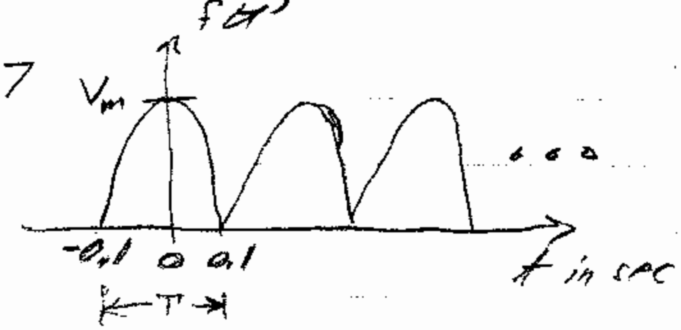
$$b_n = \frac{5}{n\pi}\Big[\underset{(-1)^n}{\cos n\pi} - \underbrace{\cos 2n\pi}_{1}\Big]$$

$$\boxed{b_n = \frac{5}{n\pi}\left[(-1)^n - 1\right]} \longleftarrow$$

$$\boxed{\begin{array}{c} \text{so } b_1 = -\frac{10}{\pi} \\ b_2 = 0 \\ \vdots \end{array}}$$

EE 212 Homework 23

18-17



$T = 0.2$ sec

$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\times10^{-1}} = 10\pi$

$\boxed{\text{Even function so } b_n = 0}$

$$a_0 = \frac{V_m}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \cos\frac{\pi}{T}t\,dt = \frac{V_m}{T}\cdot\frac{T}{\pi}\sin\frac{\pi t}{T}\Big|_{-\frac{T}{2}}^{\frac{T}{2}} = \boxed{\frac{V_m 2}{\pi}} \leftarrow$$

$$a_n = \frac{4V_m}{T}\int_0^{\frac{T}{2}} \cos n\frac{2\pi}{T}t\cos\frac{\pi}{T}t\,dt = \frac{2V_m}{T}\int_0^{\frac{T}{2}}\left[\cos\left(\frac{2n\pi}{T}+\frac{\pi}{T}\right)t + \cos\left(\frac{2n\pi}{T}-\frac{\pi}{T}\right)t\right]dt$$

$$a_n = \frac{2V_m}{T}\left\{\frac{\sin\left(\frac{2n\pi}{T}+\frac{\pi}{T}\right)t}{\frac{2n\pi}{T}+\frac{\pi}{T}} + \frac{\sin\left(\frac{2n\pi}{T}-\frac{\pi}{T}\right)t}{\frac{2n\pi}{T}-\frac{\pi}{T}}\right\}_0^{\frac{T}{2}}$$

$$a_n = \frac{2V_m}{T}\left\{\frac{\sin\frac{2n\pi t}{T}\cos\frac{\pi t}{T}+\cos\frac{2n\pi t}{T}\sin\frac{\pi t}{T}}{\frac{\pi}{T}(1+2n)} - \frac{\sin\frac{2n\pi t}{T}\cos\frac{\pi t}{T}-\cos\frac{2n\pi t}{T}\sin\frac{\pi t}{T}}{\frac{\pi}{T}(1-2n)}\right\}_0$$

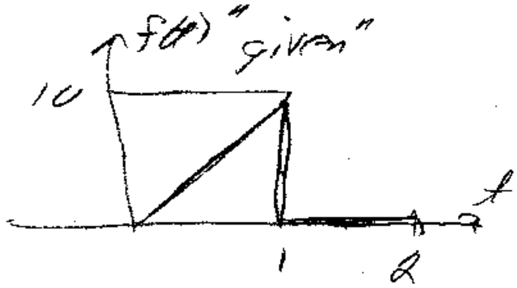
$$a_n = \frac{2V_m}{\pi}\left\{\frac{+\cos n\pi}{1+2n} + \frac{\cos n\pi}{1-2n}\right\} = \frac{2V_m\cos n\pi}{\pi}\left\{\frac{2}{1-(2n)^2}\right\}$$

$$a_n = \frac{4V_m}{\pi}\left\{\frac{1}{1-(2n)^2}\right\}\cos(n\pi)$$

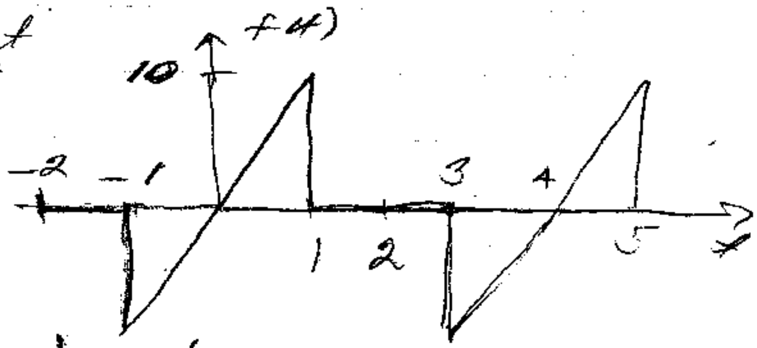
So $f(t) = \frac{2V_m}{\pi} + \frac{4V_m}{3\pi}\cos 10\pi t - \frac{4V_m}{15\pi}\cos 20\pi t + \frac{4V_m}{35\pi}\cos 30\pi t + \cdots$

EE 212   Homework 24

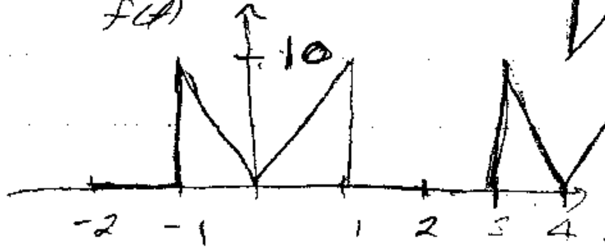
11.24



a) Odd function T = 4



b)

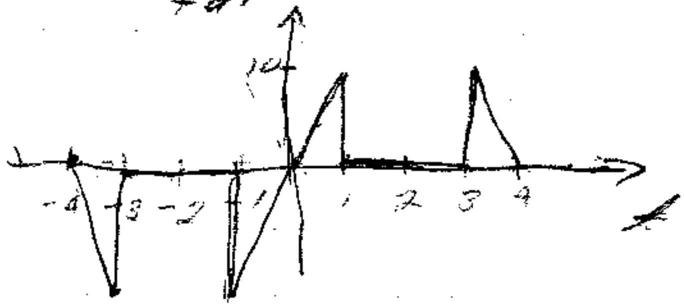


even function T = 4

c) odd function T=8
half wave symmetry

$\therefore f(t-\frac{T}{2}) = -f(t)$

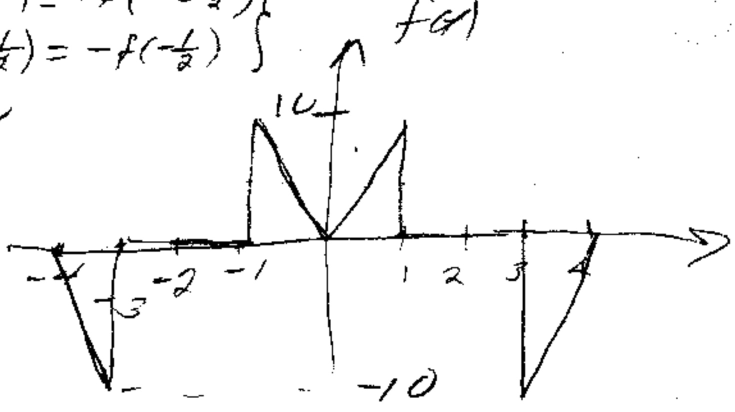
or $f(t-4) = -f(t)$



$$\left\{ \begin{array}{l} f(\frac{1}{2}) = -f(-3\frac{1}{2}) \\ f(3\frac{1}{2}) = -f(-\frac{1}{2}) \end{array} \right\}$$

d) Even function T=8
half wave symmetry

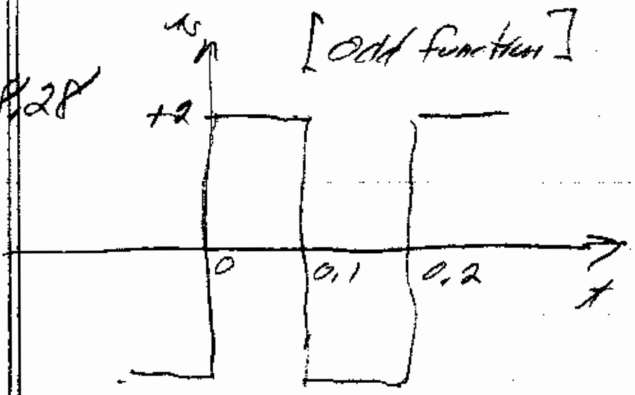
$f(t-4) = -f(t)$



EE 212 Homework 25

18.28

[Odd function]



$T = 0.2$

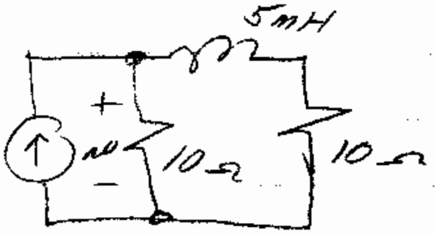
$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2\times10^{-1}} = 10\pi$

$a_0 = 0$

$a_n = 0$

$$b_n = \frac{2\times2}{2\times10^{-1}}\int_0^{0.1} 2\sin n10\pi t\,dt = 40\left(\frac{-1}{10n\pi}\right)\cos 10n\pi t\Big|_0^{\frac{1}{10}}$$

$$b_n = \frac{-4}{n\pi}\{\cos(n\pi) - 1\} = \frac{4}{n\pi}\{1-(-1)^n\} = \frac{8}{n\pi} \text{ for } n \text{ odd}$$



$$\hat{V} = \hat{I}_s\,\frac{10(10+jn\omega_0 5\times10^{-3})}{20+jn\omega_0 5\times10^{-3}}$$

$$\boxed{\hat{I}_s = -j\frac{8}{n\pi}} \leftarrow -90^\circ \text{ phase shift for sin functions}$$

$$\hat{V} = -j\frac{80}{n\pi}\times\frac{10+jn\pi\times0.05}{20+jn\pi\times0.05} = -j\frac{40}{n\pi}\times\frac{1+jn\pi 0.005}{1+jn\pi 0.0025}$$

50×10^{-3}

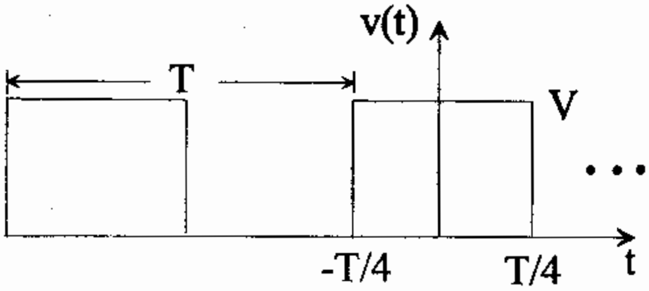
$$\text{so } v(t) = \sum_{n_{odd}=1}^{\infty}\left(\frac{40}{n\pi}\right)\frac{\sqrt{1+(n\pi\times0.005)^2}}{\sqrt{1+(n\pi\times0.0025)^2}}\cos\left[10n\pi t - 90^\circ + \tan^{-1}(n\pi\times0.005) - \tan^{-1}(n\pi\times0.0025)\right]$$

K. H. Bond
(solutions)

EE 212 Spring 2009
Assignment 26
Due Monday April 20

1. a) Calculate the "rms" value of the square shown below.

   b) Use the first 5 non zero terms of the Fourier series plus the dc term to calculate the "rms" value of the square wave and compare the results with part a).



a) $$V_{rms} = \sqrt{\frac{1}{T}\int_{-\frac{T}{4}}^{\frac{T}{4}} V^2 dt} = \sqrt{\frac{V^2}{T}\cdot\frac{T}{2}} = \frac{V}{\sqrt{2}} = 0{,}707\,V \leftarrow$$

b) From notes $v(t) = \frac{V}{2} + \sum_{n=1}^{\infty} \frac{2V}{\pi n} \sin\frac{n\pi}{2} \cos n\omega_0 t$

$$\therefore V_{rms} = \sqrt{V_{dc}^2 + \sum_{n=1}^{\infty} \frac{V_n^2}{2}} = \left\{\frac{V^2}{4} + \frac{1}{2}\left[\left(\frac{2V}{\pi}\right)^2 + \left(\frac{2V}{3\pi}\right)^2 + \left(\frac{2V}{5\pi}\right)^2 + \left(\frac{2V}{7\pi}\right)^2 + \left(\frac{2V}{9\pi}\right)^2\right]\right\}$$

$$V_{rms} = V\left\{\frac{1}{4} + \frac{1}{2}\left[0{,}405 + 0{,}045 + 0{,}0162 + 0{,}00827 + 0{,}005\right]\right\}^{1/2}$$

$$V_{rms} = V\left\{\frac{1}{4} + 0.2397\right\}^{1/2} = V\,0.6998$$

$$\%\ error = \frac{0.707 - 0.6998}{0.707} \times 100 = 1.02\%$$

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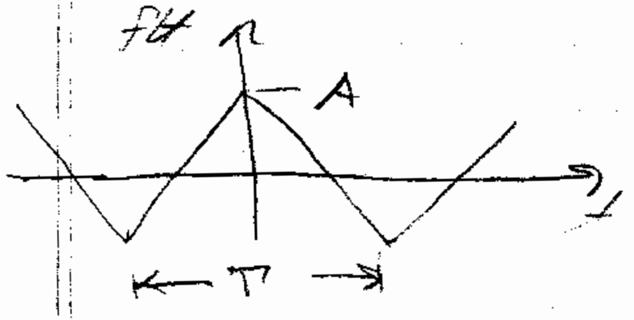

find $\hat{C}_n$

$$\hat{C}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\, e^{-jn\omega_0 t}\, dt$$

$$\hat{C}_n = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} f(t)\left[\cos n\omega_0 t - j \overset{0}{\sin n\omega_0 t}\right] dt = \frac{2}{T}\int_0^{\frac{T}{2}} \left(-\frac{4A}{T}t + A\right)\cos n\omega_0 t\, dt$$

$$\hat{C}_n = -\frac{8A}{T^2}\int_0^{\frac{T}{2}} t\cos n\omega_0 t\, dt + \frac{2A}{T}\int_0^{\frac{T}{2}} \cos n\omega_0 t\, dt$$

$u = t \quad dv = \cos n\omega_0 t\, dt$

$du = dt \quad v = \frac{1}{n\omega_0}\sin n\omega_0 t$

$$\hat{C}_n = \frac{-8A}{T^2}\left\{\frac{t}{n\omega_0}\sin n\omega_0 t\Big|_0^{\frac{T}{2}} - \int_0^{\frac{T}{2}} \frac{1}{n\omega_0}\sin n\omega_0 t\, dt\right\} + \frac{2A}{T}\cdot\frac{1}{n\omega_0}\sin n\omega_0 t\Big|_0^{\frac{T}{2}}$$

$$\hat{C}_n = -\frac{8A}{T^2}\left\{\frac{\cancel{T^2}}{2n2\pi}\overbrace{\sin\left[n\frac{2\pi}{\cancel{T}}\cdot\frac{\cancel{T}}{2}\right]}^{0} + \frac{\cancel{T^2}}{n^2 4\pi^2}\cos(n\omega_0 t)\Big|_0^{\frac{T}{2}}\right\} + \frac{2AT}{nT2\pi}\left\{\overbrace{\sin n\frac{2\pi}{T}\cdot\frac{T}{2}}^{0}\right\}$$

$$\hat{C}_n = -\frac{2A}{n^2\pi^2}\left[\cos\left(\frac{2\pi n}{\cancel{T}}\cdot\frac{\cancel{T}}{2}\right) - 1\right] = \frac{2A}{n^2\pi^2}\left[1 - \cos(n\pi)\right] \leftarrow$$

$$\left\{\begin{array}{ll} \text{or} \quad \hat{C}_n = \frac{4A}{n^2\pi^2} & \text{for } n \text{ odd} \\ \bar{C}_n = 0 & \text{for } n \text{ even} \end{array}\right\} \leftarrow$$

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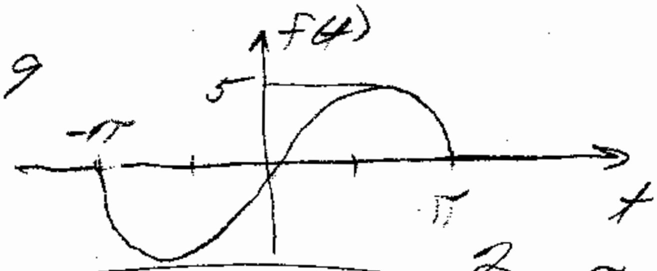
18.39

⑥



$f(t)=5\sin t \quad -\pi<t<\pi$

2 ✓

$$\boxed{F(j\omega)=\int_{-\pi}^{\pi}5\sin t\, e^{-j\omega t}dt}=\int_{-\pi}^{\pi}5\left\{\frac{e^{jt}-e^{-jt}}{2j}\right\}e^{-j\omega t}dt$$

$$F(j\omega)=\frac{5}{2j}\left\{\int_{-\pi}^{\pi}e^{-j(1+\omega)t}dt-\int_{-\pi}^{\pi}e^{-j(\omega-1)t}dt\right\}$$

$$F(j\omega)=\frac{5\cancel{j}}{2}\left\{-\frac{1}{\cancel{j}(1+\omega)}e^{-j(1+\omega)t}+\frac{1}{(\omega-1)\cancel{j}}e^{-j(\omega-1)t}\right\}_{t=-\pi}^{\pi}$$

$$F(j\omega)=\frac{5}{2}\left\{\frac{-1}{1+\omega}e^{-j(1+\omega)\pi}+\frac{1}{1+\omega}e^{j(1+\omega)\pi}+\frac{1}{\omega-1}e^{-j(\omega-1)\pi}-\frac{1}{\omega-1}e^{j(\omega-1)\pi}\right\}$$

$$F(j\omega)=\frac{5}{2}\left\{\underset{-1}{e^{j\pi}}\left[\frac{e^{j\omega\pi}}{1+\omega}+\frac{e^{-j\omega\pi}}{\omega-1}\right]+e^{-j\pi}\left[\frac{-e^{-j\omega\pi}}{1+\omega}-\frac{e^{j\omega\pi}}{\omega-1}\right]\right\}$$

$$=\frac{5}{2}\left\{e^{j\pi}\left[\frac{(\omega-1)e^{j\omega\pi}+(\omega+1)e^{-j\omega\pi}}{\omega^2-1}\right]-\overset{-1}{e^{-j\pi}}\left[\frac{(\omega-1)e^{-j\omega\pi}+(\omega+1)e^{j\omega\pi}}{\omega^2-1}\right]\right\}$$

$$=\frac{5}{2(\omega^2-1)}\left\{e^{j\pi}\left[2\omega\cos\omega\pi-2j\sin\omega\pi\right]-e^{-j\pi}\left[2\omega\cos\omega\pi+2j\sin\omega\pi\right]\right\}$$

2

$$=\frac{5}{\cancel{2}(\omega^2-1)}\left\{\cancel{2}\omega\cos\omega\pi(2j)\sin\pi-\cancel{2}j\sin\omega\pi(2)\cos\pi\right\}$$

$$=\frac{10j}{\omega^2-1}\left\{\omega\cos\omega\pi\overset{0}{\sin\pi}-\sin\omega\pi\overset{-1}{\cos\pi}\right\}$$

$$\boxed{F(j\omega)=\frac{10j}{\omega^2-1}\sin\omega\pi}$$

✓

can be done with a lot less algebra!