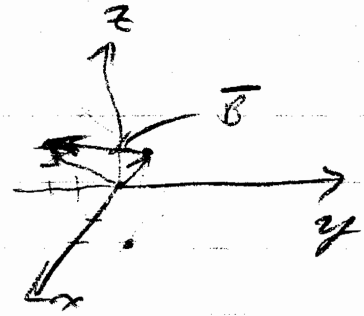
EE 333 Assignment 1 (page 1)

1-2 $\bar{A} = \bar{a}_x + 2\bar{a}_y - 3\bar{a}_z$



$\boxed{a)\ \bar{B} = -2\bar{a}_x - 3\bar{a}_y - \bar{a}_z}$ ←

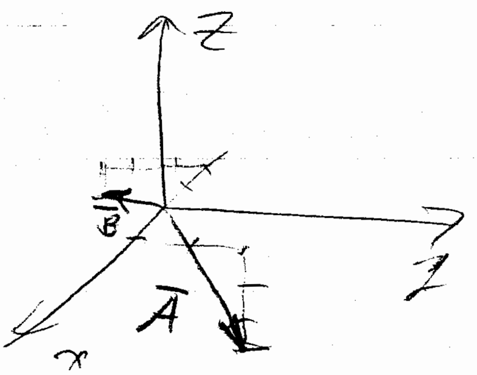
b) $|\bar{B}\cdot\bar{A}| = |-2-6+3| = 5$

projection of $\bar{B}$ on $\bar{A} = \dfrac{|\bar{B}\cdot\bar{A}|}{|\bar{A}|} = \dfrac{5}{\sqrt{1+4+9}} = \boxed{\dfrac{5}{\sqrt{14}}}$ ←

c) $\bar{A}\cdot\bar{B} = |\bar{A}||\bar{B}|\cos\theta = -5$

$\therefore \cos\theta = \dfrac{-5}{\sqrt{14}\sqrt{14}} = -0.357$ (2^{nd} or 3^{rd} quadrant)

$\boxed{\theta = \pm 110.92^\circ}$



d) unit vector $= \dfrac{\bar{A}\times\bar{B}}{|\bar{A}\times\bar{B}|} = \dfrac{\begin{vmatrix} \bar{a}_x & \bar{a}_y & \bar{a}_z \\ 1 & 2 & -3 \\ -2 & -3 & -1 \end{vmatrix}}{|\bar{A}\times\bar{B}|} = \dfrac{\bar{a}_x(-11) - \bar{a}_y(-7) + \bar{a}_z(1)}{\sqrt{121+49+1}}$

$\boxed{\text{unit vector} = -0.841\bar{a}_x + 0.535\bar{a}_y + 0.076\bar{a}_z}$

1-15 $F = \dfrac{q_1 q_2}{4\pi\epsilon_0 R^2} = \dfrac{1.6^2 \times 10^{-38}\ 9\ \cancel{36\pi}}{\cancel{4\pi}\times 10^{-9}\ 10^{-20}} = \boxed{23.04\times 10^{-9}\,N}$ ←

EE 333 Assignment 1 (page 2)

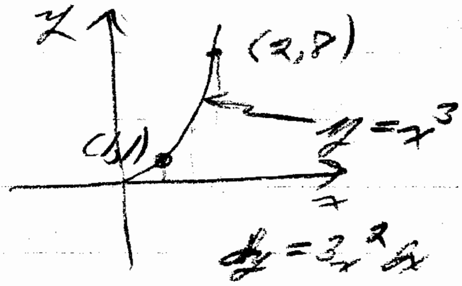
1-24 $\bar B = B_0(\bar a_x + 2\bar a_y - 4\bar a_z)$; $\bar v = v_0(3\bar a_x - \bar a_y + 2\bar a_z)$

$\bar F = q(\bar E + \bar v \times \bar B)$ for $\bar F = 0$

$$\left\{ \frac{-\bar E}{v_0 B_0} = + \begin{vmatrix} \bar a_x & \bar a_y & \bar a_z \\ 3 & -1 & 2 \\ 1 & 2 & -4 \end{vmatrix} = -\bar a_y(-12-2) + \bar a_z(6+1) = \boxed{14\bar a_y + 7\bar a_z} \right.$$

$$\therefore \boxed{\bar E = v_0 B_0 \{-14\bar a_y - 7\bar a_z\}}$$

1-27 $\bar E = (5xy - 6x^2)\bar a_x + (2y - 4x)\bar a_y$



$$\text{Work} = Q\int_{x=1}^{2} \bar E \cdot d\bar l = Q\int_{x=1}^{2} \bar E \cdot (dx\,\bar a_x + 3x^2 dx\,\bar a_y)$$

$$\text{Work} = Q\int_{x=1}^{2} \{(5xy - 6x^2)dx + (2y - 4x)3x^2 dx\}$$

$$= Q\int_{x=1}^{2} (5x^4 - 6x^2 + 6x^5 - 12x^3)dx$$

$$= Q\left\{\cancel{5}\frac{x^5}{\cancel{5}} - \cancel{6}^{\,2}\frac{x^3}{\cancel{3}} + \cancel{6}\frac{x^6}{\cancel{6}} - \cancel{12}^{\,3}\frac{x^4}{\cancel{4}}\right\}_1^2$$

$$= Q\{32 - 1 - 2(8-1) + 64 - 1 - 3(16-1)\} = Q\{31 - 14 + 63 - 45\}$$

$$\boxed{\text{Work} = Q\{35\} = 35\times10^{-6}\ \text{Joules}}$$

EE 333 Assignment 1 (page 3)

1-37 $\rho_v = \rho_0\left(1-\frac{r^2}{a^2}\right) \quad r<a$

$\rho_v = 0 \quad r>a$

for $0<\rho<a$ $\quad \oint \epsilon_0 \bar{E}\cdot\bar{dS} = \int_{vol} \rho_0\left(1-\frac{r^2}{a^2}\right) dv$

$\therefore \epsilon_0 E_r \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi} r^2 \sin\theta\, d\phi\, d\theta = \int_{\theta=0}^{\pi}\int_{\phi=0}^{2\pi}\int_{r=0}^{r} \rho_0\left(1-\frac{r^2}{a^2}\right) r^2 \sin\theta\, dr\, d\phi\, d\theta$

$\epsilon_0 E_r r^2 \cancel{4\pi} = \cancel{4\pi}\rho_0 \int_{r=0}^{r}\left(1-\frac{r^2}{a^2}\right) r^2\, dr$

$= 4\pi\rho_0 \left\{\frac{r^3}{3} - \frac{r^5}{5a^2}\right\}_0^r$

or $\boxed{E_r = \frac{\rho_0}{\epsilon_0}\left\{\frac{r}{3} - \frac{r^3}{5a^2}\right\}} \quad 0<r<a$

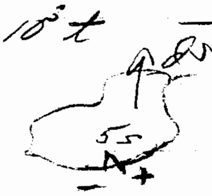
for $r>a$ $\quad Q_{enclosed}$ = above rhs with integral on r from 0 to a

$\therefore \epsilon_0 E_r r^2 \cancel{4\pi} = \cancel{4\pi}\rho_0\left\{\frac{a^3}{3} - \frac{a^3}{5}\right\} \qquad \frac{1}{3}-\frac{1}{5} = \frac{2}{15}$

$\boxed{E_r = \frac{2\rho_0 a^3}{15\epsilon_0 r^2} \quad \text{for} \quad r>a}$

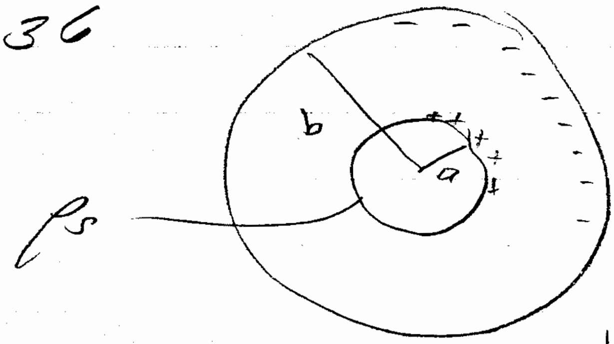
1-40 $emf = -\frac{d}{dt}\int \bar{B}\cdot\bar{dS} = -\frac{d}{dt}(0.02\sin 10^3 t) = -20\cos 10^3 t$

I = clockwise current $= \frac{emf}{5} = 4\cos 10^3 t$



EE 333   1-37   Assignment 2

1-36



a) for $a < r < b$

$$\cancel{4\pi} r^2 \epsilon_0 E_r = \rho_s \cancel{4\pi} a^2$$

$$\boxed{\text{or } E_r = \frac{\rho_s a^2}{\epsilon_0 r^2}}$$

$$\boxed{\text{for } r > b \quad E_r = 0}$$

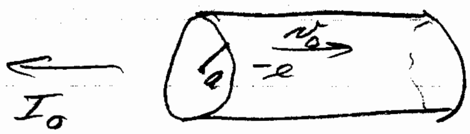
b) $\bar{J}_D = \dfrac{\partial(\epsilon_0 \bar{E})}{\partial t}$ $\rho_s(t) = 2\times10^{-9}\cos 10^5 t$ C/m^2

$$\therefore \bar{J}_D = \frac{\partial}{\partial t}\left(\frac{a^2}{r^2}\, 2\times10^{-9}\cos 10^5 t\right)\bar{a}_r = \boxed{-\frac{2\times10^{-4}\, a^2}{r^2}\sin 10^5 t\, \bar{a}_r}$$

1-37 see assignment one.

EE 333   Assignment 3

1-38



$v_0 = 10^7$ m/sec $a = 1\times10^{-3}$

$I_0 = 10^{-2}$ A

$I_0 = J\pi a^2 = \rho_V 10^7 \pi a^2$ or $\boxed{\rho_V = \frac{10^{-2}}{10^7 \pi \times 10^{-6}} = \frac{10^{-3}}{\pi}}$

using a circular cylindrical Gaussian surface of length L we obtain

$\epsilon_0 E_\rho \cancel{\pi} 2\rho \cancel{L} = \rho_V \cancel{\pi} \rho^2 \cancel{L}$ for $r < a$

$$\boxed{E_\rho = \frac{\rho_V \rho}{\epsilon_0 2} = \frac{10^{-3}\rho}{2\pi\epsilon_0} \quad \text{for } r<a}$$

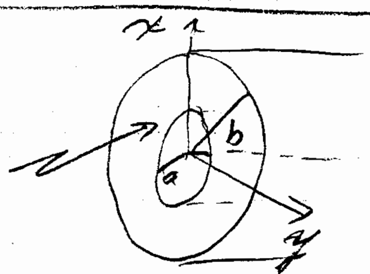
$$E_\rho = \frac{10^{-3}\rho\, \cancel{36\pi}^{18}}{\cancel{2\pi}\cdot 10^{-9}} = \underline{18\times10^6 \rho}$$

$\epsilon_0 E_\rho \cancel{\pi} 2\rho \cancel{L} = \rho_V \cancel{\pi} a^2 \cancel{L}$ for $r > a$

$$\boxed{E_\rho = \frac{\rho_V a^2}{\epsilon_0 2\rho} = \frac{10^{-3}\times10^{-6}\,\cancel{36\pi}^{18}}{\cancel{\pi}\, 2\times10^{-9}\rho} = 18/\rho \text{ V/m}}$$

1-44

$\bar J = 2\rho \bar a_z$



$\oint \frac{\bar B}{\mu_0}\cdot d\bar l = \int \bar J \cdot d\bar s$

a) for $\rho < a$ $\int \bar J\cdot d\bar s = 0$ $\therefore B_\phi = 0$

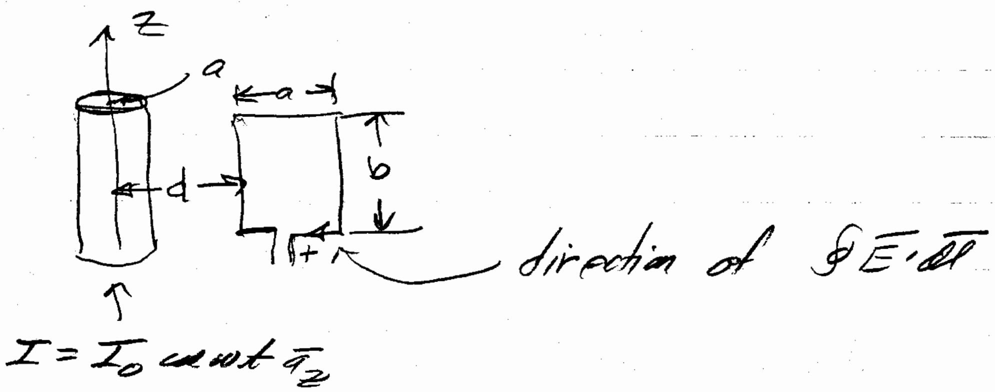
b) $\int_{\phi=0}^{2\pi} \frac{B_\phi}{\mu_0}\rho\, d\phi = \int_{\rho=a}^{\rho}\int_{\phi=0}^{2\pi} 2\rho \bar a_z \cdot \rho\, d\phi\, d\rho\, \bar a_z = 4\pi \frac{\rho^3}{3}\Big|_a^{\rho} = \frac{4\pi}{3}[\rho^3 - a^3]$

so $B_\phi = \frac{2\mu_0}{3}\left[\rho^2 - \frac{a^3}{\rho}\right]$ ← $a<\rho<b$

c) $\cancel{2\pi}\rho B_\phi/\mu_0 = \frac{\cancel{4\pi}^{2}}{3}[b^3 - a^3]$

$B_\phi = \frac{2\mu_0}{3\rho}[b^3 - a^3]$ ← $\rho > b$

1-45



a) for $\rho > a$ $\boxed{B_\phi = \frac{\mu_0 I_0 \cos\omega t}{2\pi\rho}}$

b) [i] Ψ = magnetic flux through loop $= \int_{\rho=d}^{d+a} \int_{z=0}^{b} \frac{\mu_0 I_0 \cos\omega t}{2\pi\rho}\, dz\, d\rho$

$$\boxed{\Psi = \frac{b\,\mu_0 I_0 \cos\omega t}{2\pi} \ln\left(\frac{d+a}{d}\right)}$$

$$\boxed{emf = \oint \bar{E}\cdot d\bar{l} = -\frac{d\Psi}{dt} = \frac{\omega b \mu_0 I_0 \sin\omega t}{2\pi} \ln\left(\frac{d+a}{d}\right)}$$

EE 333 Homework 4

2.9 a) $\overline{A} = yz\,\overline{a}_x + xz\,\overline{a}_y + xy\,\overline{a}_z$; $\nabla\cdot\overline{A} = 0$

b) $\overline{B} = \rho\,\overline{a}_z$; $\frac{\partial B_z}{\partial z} = 0 = \nabla\cdot\overline{B}$

c) $\overline{C} = r\,\overline{a}_r$; $\frac{1}{r^2}\frac{\partial (r^3)}{\partial r} = 3 = \nabla\cdot\overline{C}$

d) $\overline{D} = 2r^2\,\overline{a}_r$; $\nabla\cdot\overline{D} = \frac{1}{r^2}\,2\frac{dr^4}{dr} = 8r$; $\nabla\cdot\overline{D}(r=3) = 24$

e) $\overline{E} = 3x\,\overline{a}_x + (y-3)\overline{a}_y + (2-z)\overline{a}_z$; $\nabla\cdot\overline{E} = 3+1-1 = 3$

2.17 $\overline{B} = \frac{1}{r^2}\sin\phi\cos^2\theta\,\overline{a}_r$ $\qquad \nabla\times\overline{B} = \mu_0\overline{J}$

$\nabla\cdot\overline{B} = \frac{1}{r^2}\frac{\partial}{\partial r}(\sin\phi\cos^2\theta) = 0$ $\therefore$ could be $\overline{B}$ field

$$\overline{J} = \frac{1}{\mu_0}\left\{\frac{\cos\phi\cos^2\theta}{r^3\sin\theta}\overline{a}_\theta + \frac{2\sin\phi\cos\theta\sin\theta}{r^3}\overline{a}_\phi\right\} \leftarrow$$

2.28 a) λ = length of one cycle in space $\lambda = \frac{2\pi}{\beta_0}$

phase factor $e^{\pm j\beta_0 z}$ phase as a function of z

phase velocity = velocity of constant phase point $= \frac{\omega}{\beta_0}$

Intrinsic impedance $= \frac{|E|}{|H|} = \sqrt{\frac{\mu_0}{\epsilon_0}}$

b) $\overline{E}(z,t) = 37.7\cos(6\pi\times10^8 t + 2\pi z)\,\overline{a}_x$

i) $\omega = 6\pi\times10^8 = 2\pi f$ $\therefore$ $\boxed{f = 3\times10^8}$

ii) $\lambda = \frac{2\pi}{\beta} = 1$ meter

iii) $v_{phase} = \frac{\omega}{\beta} = \frac{6\pi\times10^8}{2\pi} = 3\times10^8$ m/sec

iv) negative z direction

v) $\overline{H}(z,t) = 0.1\cos(6\pi\times10^8 t + 2\pi z)(-\overline{a}_y)$

2.29 $\overline{H} = \frac{1}{3\pi} \cos(\omega t - 30z)\,\overline{a_y}$ ⟵

free space so $\eta = \eta_0 = 120\pi$ and $\boxed{v_p = \frac{\omega}{\beta_0} = 3\times10^8}$

$\therefore\ \overline{E} = 40\cos(\omega t - 30z)\,\overline{a_x}$ ⟵

$\omega = 3\times10^8 \beta = 9\times10^9 = 2\pi f$

$$\boxed{f = \frac{9\times10^9}{2\pi}}$$

$$\boxed{\lambda = \frac{2\pi}{30} = \frac{\pi}{15}\ m}$$

$$\boxed{\begin{aligned}\overline{H} &= \frac{1}{3\pi}\cos(9\times10^9 t - 30z)\,\overline{a_y}\\ \overline{E} &= 40\cos(9\times10^9 t - 30z)\,\overline{a_x}\end{aligned}}$$

EE 333 Homework 5

2.32 200MHz, positive z direction, E_x only
maximum when $z=1, t=0$ of 150 V/m, free space

$$\bar{E} = 150 \cos\left(2\pi \times 2\times10^8 t - \frac{4\pi\times10^8}{3\times10^8} z + \frac{4\pi}{3}\right) \bar{a}_x$$

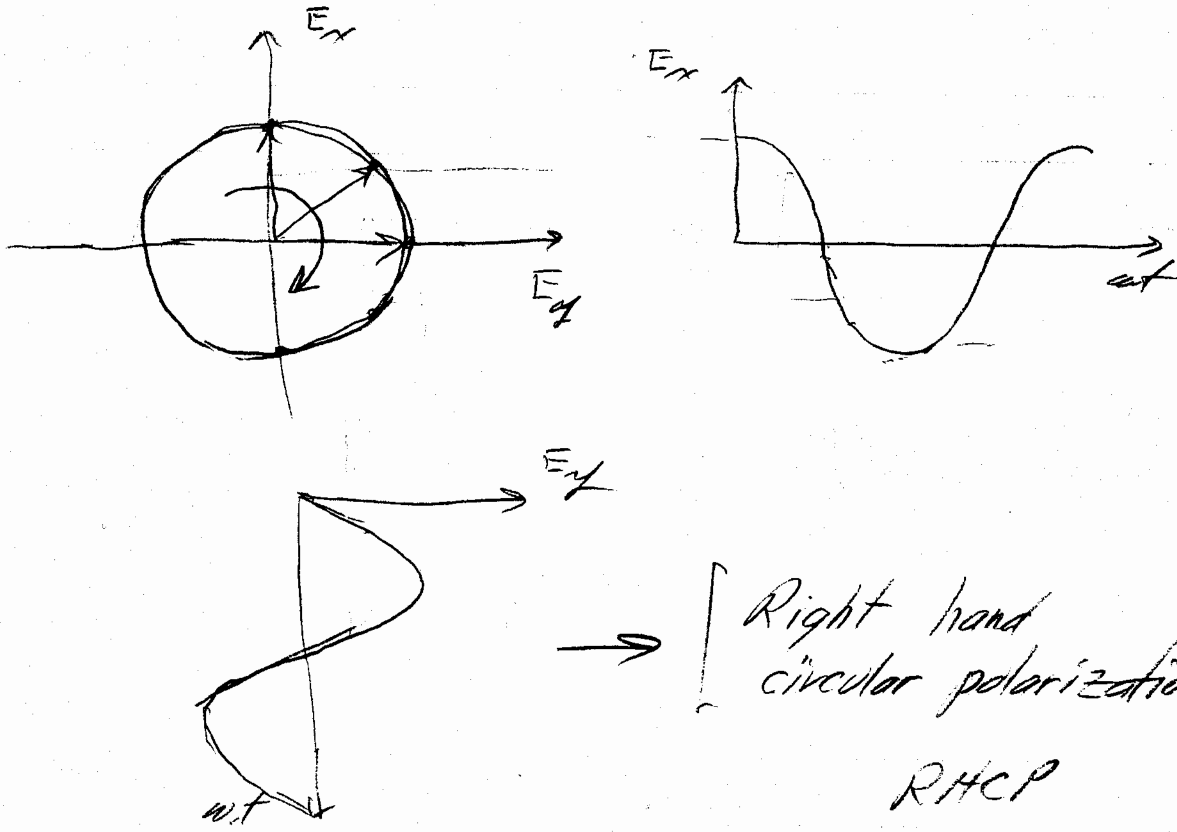
$$\bar{E} = 150 \cos\left(4\pi\times10^8 t - \frac{4\pi}{3} z + \frac{4\pi}{3}\right) \bar{a}_x \longleftarrow$$

Special: $\hat{\bar{E}} = 500 e^{-j\beta_0 z} (\bar{a}_x - j\bar{a}_y)$

a) $\bar{E} = 500 \cos(\omega t - \beta_0 z)\bar{a}_x + 500 \sin(\omega t - \beta_0 z)\bar{a}_y \longleftarrow$

b) $\hat{\bar{H}} = \frac{500}{\eta} e^{-j\beta_0 z} (\bar{a}_y + j\bar{a}_x) \longleftarrow$

c)



$\rightarrow$ [Right hand circular polarization]

RHCP

2.23 $\quad \bar{E} = \frac{\rho_0 \rho^3}{4\epsilon_0 a^2} \bar{a}_\rho \quad ; \bar{B} = \frac{\mu_0 J_0 \rho^2}{3a} \bar{a}_\phi$

4

$$\nabla \cdot \epsilon_0 \bar{E} = \rho_v = \frac{1}{\rho} \frac{\partial}{\partial \rho} \left(\frac{\rho_0 \rho^4 \epsilon_0}{4\epsilon_0 a^2} \right) = \frac{\cancel{\epsilon_0}}{\cancel{\rho}} \cdot \frac{\rho_0 \cancel{4} \rho^{\cancel{3}\,2}}{\cancel{4\epsilon_0} a^2} = \boxed{\frac{\rho_0 \rho^2}{a^2}} \quad 2$$

$$\nabla \times \frac{\bar{B}}{\mu_0} = \bar{J} = \frac{\cancel{\mu_0} J_0}{\cancel{\mu_0} 3a} \cdot \frac{1}{\rho} \frac{\partial \rho^3}{\partial \rho} = \boxed{\frac{J_0 \rho}{a} \bar{a}_z} \quad 2$$

EE 333 Assignment 6

3.1 $\bar{E} = 3z^2 y \cos(10^8 t)\,\bar{a}_x$ Lucite $\epsilon_r = 2.56$

a) $\bar{P} = ?$ $\epsilon_0 \bar{E} + \bar{P} = \bar{D} = \epsilon_0 \epsilon_r \bar{E}$

$\therefore \bar{P} = \epsilon_0 \epsilon_r \bar{E} - \epsilon_0 \bar{E} = \epsilon_0 \bar{E}(\epsilon_r - 1)$

$$\bar{P} = \frac{1.56 \times 10^{-9} \times 3}{36\pi} z^2 y \cos(10^8 t)\,\bar{a}_x \leftarrow$$

$$\bar{P} = 4.14 \times 10^{-11} z^2 y \cos(10^8 t)\,\bar{a}_x \leftarrow$$

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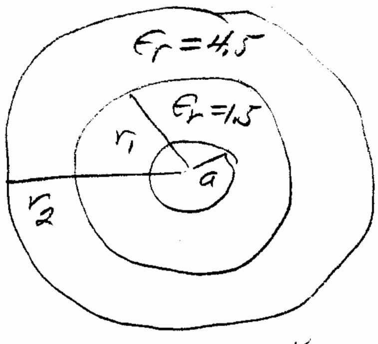
b) $\rho_p = -\nabla \cdot \bar{P} = 0$ $\leftarrow$

$\bar{P}$ not a function of x and there is only a P_x (uniform polarization leads to no ρ_p)

c) $\bar{J}_p = \frac{\partial \bar{P}}{\partial t} = -4.14 \times 10^{-3} z^2 y \sin 10^8 t\,\bar{a}_x$ $\leftarrow$

3.2 a)



"power coax"

ρ_ℓ C/m ; for circular cylindrical surface.

a) $\oint \bar{D} \cdot d\bar{S} = \rho_\ell L$

or $D_\rho 2\pi\rho L = \rho_\ell L$

$$\boxed{D_\rho = \frac{\rho_\ell}{2\pi\rho} \text{ everywhere}}$$

$$E_\rho = \frac{\rho_\ell}{1.5\epsilon_0 2\pi\rho} \qquad a \le \rho \le r_1$$

$$E_\rho = \frac{\rho_\ell}{4.5\epsilon_0 2\pi\rho} \qquad r_1 \le \rho \le r_2$$

$\frac{3.5}{4.5} = \frac{7/2}{9/2}$

$$P_\rho = \epsilon_0(\epsilon_r - 1)\bar{E} \qquad \begin{cases} P_\rho = \dfrac{\rho_\ell}{6\pi\rho} & a \le \rho \le r_1 \\ P_\rho = \dfrac{7\rho_\ell}{18\pi\rho} & r_1 < \rho \le r_2 \end{cases}$$

3.2 a) (continued) Outside of cable $\bar{E} = \bar{D} = \bar{P} = 0$ ◄

c) ρ_p for $r_1 < \rho \leq r_2$

$$\rho_p = -\nabla \cdot \bar{P} = -\frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\frac{7\rho_\ell}{18\pi}\right) = 0 \; !$$

Special Cu 10^{29} electrons$/m^3$, $\sigma = 5.8 \times 10^7$ mho$/m$

a) $\mu_e = -\frac{e\tau_c}{m}$; $\sigma = \frac{ne^2\tau_c}{m} = -ne\mu_e$

$$\therefore \mu_e = \frac{\sigma}{ne} = \frac{+5.8 \times 10^7}{10^{29} \times 1.6 \times 10^{-19}} = +\underline{\underline{3.625 \times 10^{-3}}} \;◄$$

b) $\rho_v = 10^{29}(-1.6 \times 10^{-19}) = -1.6 \times 10^{10}\ C/m^3 = \underline{\underline{-16\ C/mm^3}}$ ◄

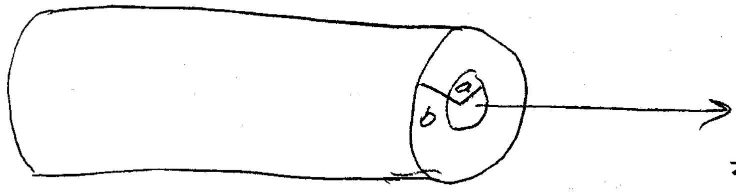
c) $\bar{v}_d = -\mu_e \bar{E} = -3.625 \times 10^{-3}\ \bar{a}_x$

d) $\bar{J} = \rho_v \bar{v}_d = 1.6 \times 10^{10} \times 3.625 \times 10^{-3}\ \bar{a}_x = \underline{\underline{5.8 \times 10^7\ A/m^2\ \bar{a}_x}}$

$$\bar{J} = 58\ A/mm^2\ \bar{a}_x$$

EE 333 Assignment 7

3.5



for $\rho \le a$ $\bar{J} = \frac{1}{2}\bar{a}_z$

for $a < \rho \le b$ $\bar{J} = -\frac{\rho}{2a}\bar{a}_z$

for $\rho \le a$ $\mu = \mu_0\mu_{r_1}$; $a < \rho \le b$ $\mu = \mu_0\mu_{r_2}$

a) Find $\bar{H}$ $\oint \bar{H}\cdot d\bar{l} = \int \bar{J}\cdot d\bar{s}$

$\boxed{\text{for } \rho \le a}$ $2\pi\rho H_{\phi_1} = \int_{\rho=0}^{\rho}\int_{\phi=0}^{2\pi} \frac{1}{2}\rho\, d\phi\, d\rho = \frac{2\pi\rho^2}{4}$

so $\boxed{H_{\phi_1} = \frac{\rho}{4}}$

$\boxed{\text{for } a < \rho \le b}$ $2\pi\rho H_{\phi_2} = \int_{\rho=0}^{a}\int_{\phi=0}^{2\pi} \frac{1}{2}\rho\, d\phi\, d\rho - \int_{\phi=0}^{2\pi}\int_{\rho=a}^{\rho} \frac{\rho}{2a}\rho\, d\phi\, d\rho$

$= 2\pi\frac{a^2}{4} - 2\pi\frac{\rho^3}{6a}\Big|_a^{\rho} = 2\pi\frac{a^2}{4} - 2\pi\frac{\rho^2}{6a} + 2\pi\frac{a^2}{6}$

$\frac{1}{4} + \frac{1}{6} = \frac{3}{12} + \frac{2}{12}$

$\therefore H_{\phi_2} = \frac{a^2}{4\rho} + \frac{a^2}{6\rho} - \frac{\rho^2}{6a} = \boxed{\frac{a^2}{\rho}\left(\frac{5}{12}\right) - \frac{\rho^2}{6a}}$

b) $\rho \le a$ $B_{\phi_1} = \mu_0\mu_{r_1}H_{\phi_1}$

$a < \rho \le b$ $B_{\phi_2} = \mu_0\mu_{r_2}H_{\phi_2}$

c) $\bar{M} = \chi_m\bar{H}$ $1 + \chi_m = \mu_r$ or $\chi_m = \mu_r - 1$

$\therefore 0 < \rho \le a$ $\bar{M} = (\mu_{r_1} - 1)\frac{\rho}{4}$

$\bar{J}_m = \nabla\times\bar{M} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[\frac{\rho^2}{4}\right]\bar{a}_z = \frac{1}{2}\bar{a}_z$ } not required

c) (continued)

$$\overline{M}_2 = (\mu_{r_2} - 1) H_{\phi_2} \overline{a_\phi} \qquad a < \rho \le b$$

$$\frac{\overline{J_{m_2}}}{(\mu_{r_2}-1)} = \frac{1}{\rho}\frac{\partial}{\partial\rho}\left[-\frac{\rho^3}{6a}\right]\overline{a_z} = -\frac{3\rho^{\cancel{2}}}{6a\cancel{\rho}}\overline{a_z} = -\frac{3\rho}{6a}\overline{a_z}$$

$$\overline{J_{m_2}} = -(\mu_{r_2}-1)\frac{\rho}{2a}\overline{a_z}$$

d) $H_{\phi_1}(\rho = a) \overset{?}{=} H_{\phi_2}(\rho = a)$

$$\frac{a}{4} \overset{?}{=} \frac{5}{12}a - \frac{1}{6}a$$

$$\frac{a}{4} = \frac{a}{4} \quad ! \quad \text{Q.E.D.}$$

3.8 a) 1) conductors – free charge $\overline{J} = \sigma\overline{E}$

2) electric dipoles – bound charges separated when $\overline{E}$ applied

$$\overline{J_p} = \frac{\partial\overline{P}}{\partial t}, \quad \rho_p = -\nabla\cdot\overline{P}$$

3) magnetic dipoles – magnetic dipole alligned by applied $\overline{B}$ field

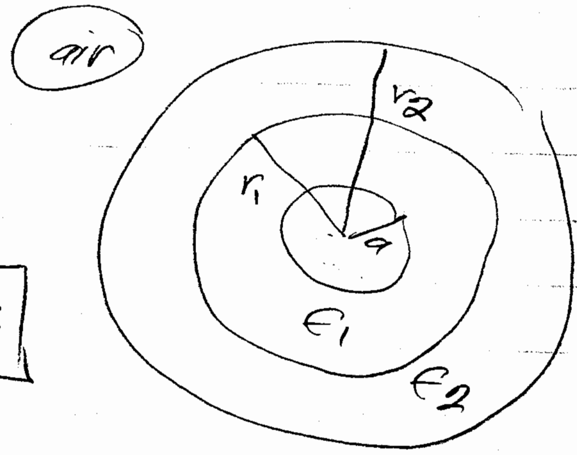
$$\overline{J_m} = \nabla\times\overline{M}$$

Must include the new ρ + $\overline{J}$ terms in Maxwell's Eq's

(881-3082 Mark)

EE 333 — Assignment 7 — (page 3)

3.8 b) charge Q on center sphere of radius "a"



$\oint \bar{D}\cdot d\bar{S} = Q$ for $r>a$

$\therefore\ 4\pi r^2 D_r = Q$ or $\boxed{D_r = \frac{Q}{4\pi r^2}}$

$\boxed{a<r\le r_1}$ $\quad E_r = \frac{Q}{4\pi\epsilon_1 r^2}$

$$\bar{P} = \bar{D} - \epsilon_0\bar{E} = \left(\frac{Q}{4\pi r^2} - \frac{Q}{4\pi\epsilon_{r_1} r^2}\right)\bar{a}_r = \frac{Q}{4\pi r^2}\left(1-\frac{1}{\epsilon_{r_1}}\right)\bar{a}_r$$

$$\rho_p = -\nabla\cdot\bar{P} = \frac{1}{r^2}\frac{\partial}{\partial r}\left[\frac{Q}{4\pi}\left(1-\frac{1}{\epsilon_{r_1}}\right)\right] = 0$$

$r_1 < r \le r_2$ $\quad E_r = \frac{Q}{4\pi\epsilon_2 r^2}$

$$\bar{P} = \frac{Q}{4\pi r^2}\left(1-\frac{1}{\epsilon_{r_2}}\right)\bar{a}_r$$

$$\rho_p = 0$$

$r_2 < r$ $\quad E_r = \frac{Q}{4\pi\epsilon_0 r}$

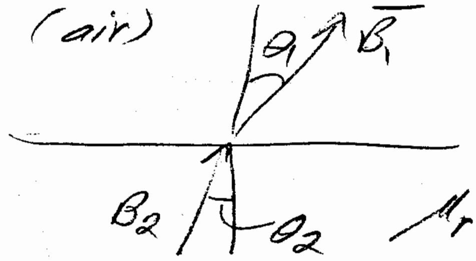
$$\bar{P} = \rho_p = 0$$

ρ_{ps} @ $r=r_1$

$$\rho_{ps} = \underbrace{-\bar{n}}_{\bar{a}_r}\cdot(\bar{P}_1-\bar{P}_2)\Big|_{r=r_1} = \frac{Q}{4\pi r_1^2}\left[\frac{\epsilon_{r_1}-1}{\epsilon_{r_1}} - \frac{\epsilon_{r_2}-1}{\epsilon_{r_2}}\right]$$

Special Problem

(air)



$\mu_r = 10^4$

$$\tan\theta_1 = \frac{B_{T1}}{B_{n1}} \;;\; \tan\theta_2 = \frac{B_{T2}}{B_{n2}}$$

but $B_{n1} = B_{n2}$ $\therefore$ $\dfrac{B_{T1}}{\tan\theta_1} = \dfrac{B_{T2}}{\tan\theta_2}$ or $\dfrac{\mu_1 \cancel{H_{T1}}}{\tan\theta_1} = \dfrac{\mu_2 \cancel{H_{T2}}}{\tan\theta_2}$

but $H_{T1} = H_{T2}$ so $\boxed{\tan\theta_2 = \dfrac{\mu_2}{\mu_1}\tan\theta_1}$

$$\theta_1 = \tan^{-1}\left[\frac{\mu_1}{\mu_2}\tan\theta_2\right] = \tan^{-1}\left[10^{-4}\tan\theta_2\right]$$

θ_2	θ_1
0°	0°
45°	$5.7 \times 10^{-3}\,^\circ$
89°	0.328°
89.9°	3.27°

EE 333 Assignment 8 3/18

3.15 a) for conductor with E_x only

$$\hat{E}_x = \hat{E}_m^+ e^{-\alpha z} e^{-j\beta z} + \hat{E}_m^- e^{+\alpha z} e^{+j\beta z}$$

$$\hat{H}_y = \frac{\hat{E}_m^+}{\hat{\eta}} e^{-\alpha z} e^{-j\beta z} - \frac{E_m^-}{\hat{\eta}} e^{+\alpha z} e^{+j\beta z}$$

1. Waves attenuate as they propagate.
2. E and H not in time phase.
3. $\bar{E} \times \bar{H}$ is in direction of propagation
4. $v_{ph} = \frac{\omega}{\beta}$ + $\lambda = \frac{2\pi}{\beta}$ for all media

$$P_{ave\,\text{free space}} = \frac{1}{2} \frac{|\hat{E}_m|^2}{\eta_0} \quad W/m^2$$

$$P_{ave\,\text{conductor}} = \frac{1}{2} \frac{|\hat{E}_m|^2}{|\hat{\eta}|} e^{-2\alpha z} \cos(\theta)$$

b) Sea water $\mu_r = 1$, $\epsilon_r = 79$, $\sigma = 3$ S/m

E_{tang} only ; $|E|_{at\ surface} = 10$ V/m

i) need $10\mu V/m$ @ receiver ; 20kHz

$$\frac{\sigma}{\omega\epsilon} = \frac{3 \times 36\pi}{2\pi \times 2\times10^4 \times 79 \times 10^{-9}} = 0.342\times10^5 >> 1$$

$$\therefore \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \sqrt{\frac{2\pi\times10^4 \times 4\pi\times10^{-7}\times 3\times2}{2}} = \sqrt{0.236} = \underline{\underline{0.486}}$$

$$10^{-6} = e^{-0.486 z} \; ; \; \boxed{z = 28.4 \text{ m}} \leftarrow$$

3.15 b) ii) $f = 20\ GHz$

$$\frac{\sigma}{\omega\epsilon} = \frac{3 \times 36\pi \times 10^9}{2\pi \times 2 \times 10^{10} \times 10^{-9} \times 79} = 0.0342 \ll 1$$

34177216

$$\text{so } \alpha = \frac{\omega\sqrt{\mu\epsilon}}{\sqrt{2}}\left[\sqrt{1+\left(\frac{\sigma}{\omega\epsilon}\right)^2}-1\right]^{1/2} = \frac{2\pi \times 2 \times 10^{10}\sqrt{79}}{\sqrt{2}\ 3 \times 10^8}\left[0.02416\right]$$

or $\alpha = 26.326 \times 10^2 \times 0.02416 = 63.6$

$$10^{-6} = e^{-\alpha z} \quad \text{or} \quad \boxed{z = 21.7\,cm}$$

c) i) $$\hat{\eta}(20\,KHz) = \sqrt{\frac{\mu}{\epsilon(1-j\frac{\sigma}{\omega\epsilon})}} \cong \sqrt{\frac{4\pi \times 10^{-7} \times 36\pi}{10^{-9}(-j\,0.34 \times 10^5)79}} = \sqrt{\frac{52.9 \times 10^{-3}}{-j}}$$

$$\boxed{\hat{\eta} = e^{j\frac{\pi}{4}} \times 0.23}$$

$$\therefore \boxed{P_{ave} = \frac{1}{2} \times 10^{-10} \times \frac{1}{0.23}\cos\frac{\pi}{4} = 1.537 \times 10^{-10}\ W/m^2}$$

ii) $$\hat{\eta} \cong \sqrt{\frac{\mu}{\epsilon}} = 120\pi\sqrt{\frac{1}{79}} = 42.41$$

$$\boxed{P_{ave} = \frac{1}{2} \times \frac{10^{-10}}{42.41} = 1.18 \times 10^{-12}\ W/m^2}$$

3.18 $$P_{ave} = \frac{1}{2}Re\left[\hat{E} \times \hat{H}^*\right] = \frac{+E_0^2 \sin^2\left(\frac{\pi x}{a}\right)}{2Z} a_z$$

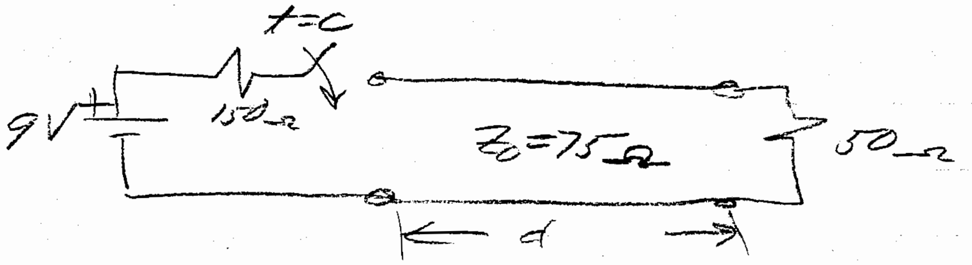
$$P_{total_z} = \int_{y=0}^{b}\int_{x=0}^{a} P_{ave}\,dx\,dy = \int_{y=0}^{b}\int_{x=0}^{a} \frac{E_0^2 \sin^2\left(\frac{\pi x}{a}\right)}{2Z}dx\,dy = \frac{E_0^2 b}{2Z}\int_0^a \sin^2\left(\frac{\pi x}{a}\right)dx$$

$$p = \frac{\pi x}{a} : dp = \frac{\pi}{a}dx \qquad \text{then } P_{total} = \frac{E_0^2 ba}{2Z\pi}\int_0^{\pi} \sin^2 p\,dp$$

$$P_{total_z} = \frac{E_0^2 ab}{2\pi Z}\left[\frac{p}{2} - \frac{\sin 2p}{4}\right]_0^{\pi} = \boxed{\frac{E_0^2 ab}{4Z}}$$

EE 333 Assignmat 9

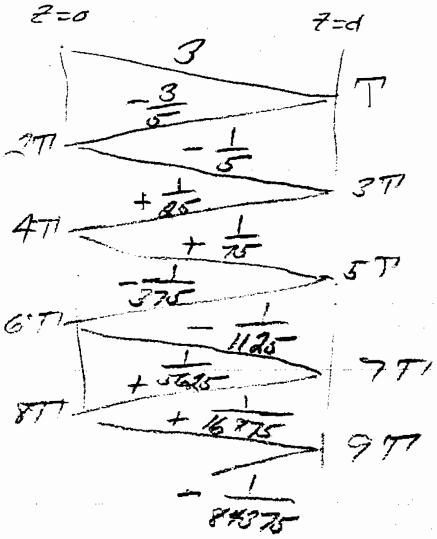
7.4

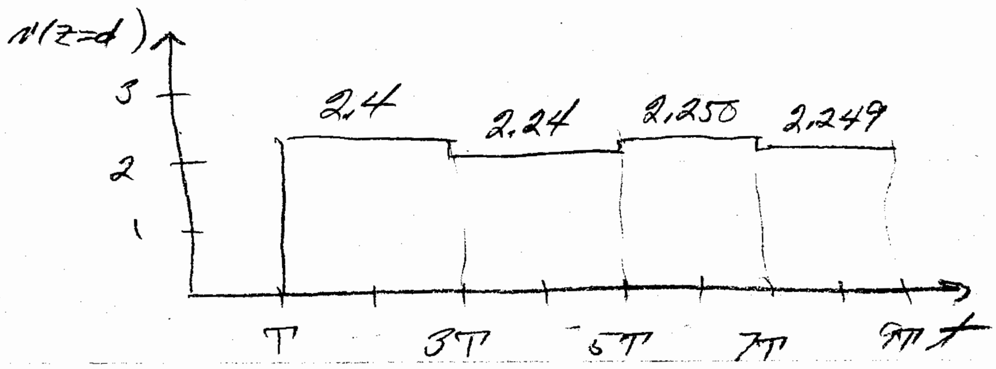


$$\boxed{T = \frac{d}{u}}$$

a) $\Gamma_g = \frac{150-75}{150+75} = \frac{75}{225} = \frac{1}{3}$; $\Gamma_T = \frac{50-75}{50+75} = \frac{-25}{125} = -\frac{1}{5}$

$v^+(z=0) = 9\,\frac{75}{225} = 3\,V$; $i^+(z=0) = \frac{3}{75} = \frac{1}{25}$

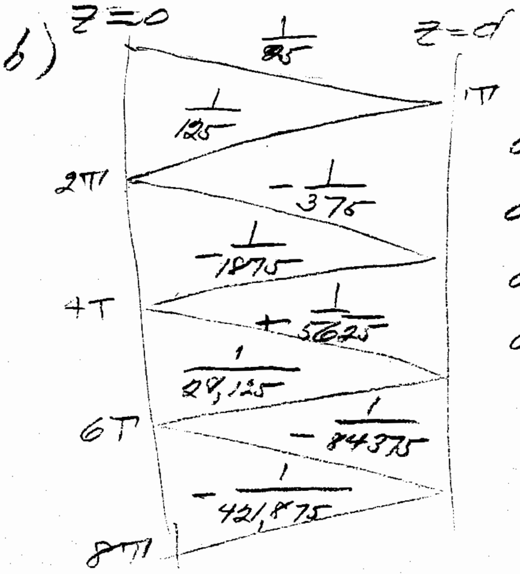


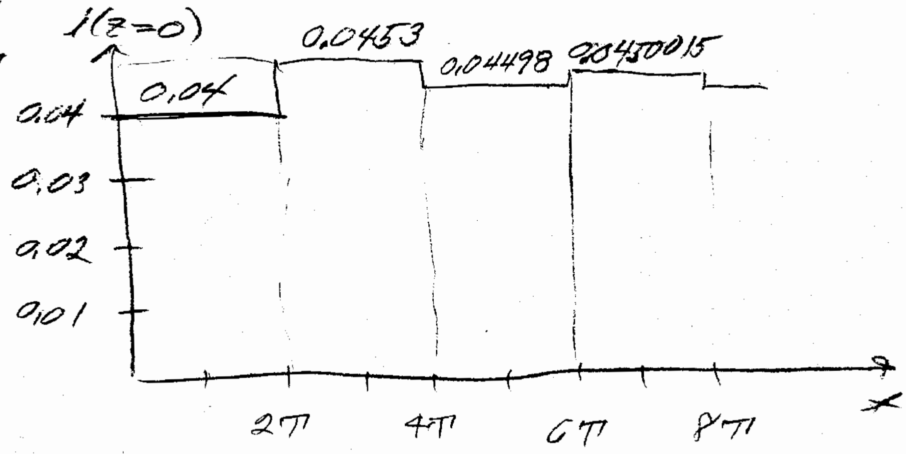


c) final value $= 9\,\frac{50}{200} = \frac{9}{4} = 2.25\,V$

$i_{final} = \frac{9}{4\times50} = \frac{9}{200} = 45\,mA$

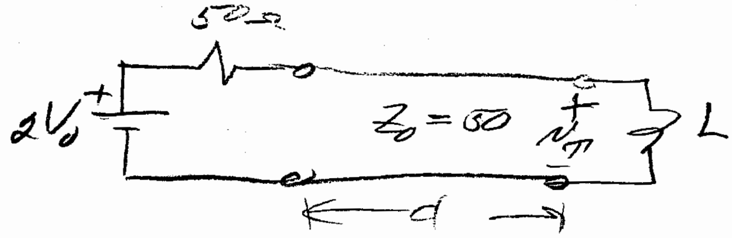
b)





EE 333 Assignment 9 (page 2)

7.5

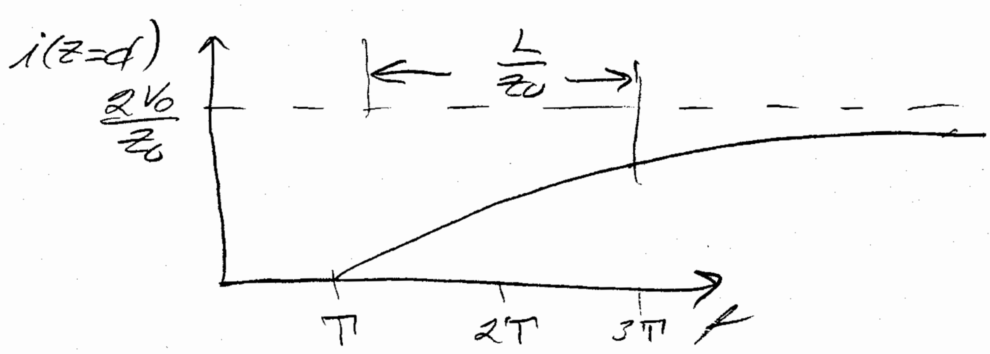


find $v_T(t)$!

$v^+ = V_0$

@ $z=d$; $2V_0 u(t-T)$

$$i(t) = \frac{2V_0}{Z_0}\left(1 - e^{-\frac{t-T}{L/Z_0}}\right)u(t-T)$$

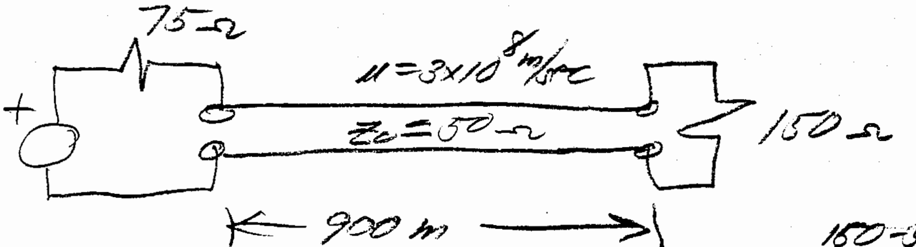


initial $i = 0$ and it can't change instantaneously

after a long time inductor looks like a short circuit and $i = \frac{2V_0}{Z_0}$.

7.12 1 volt

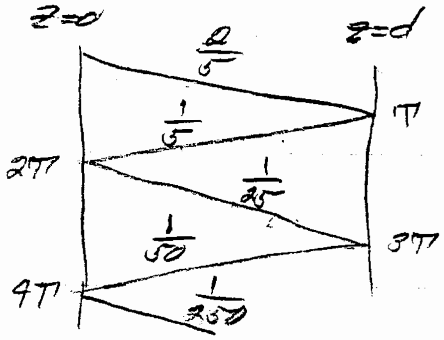
μsec

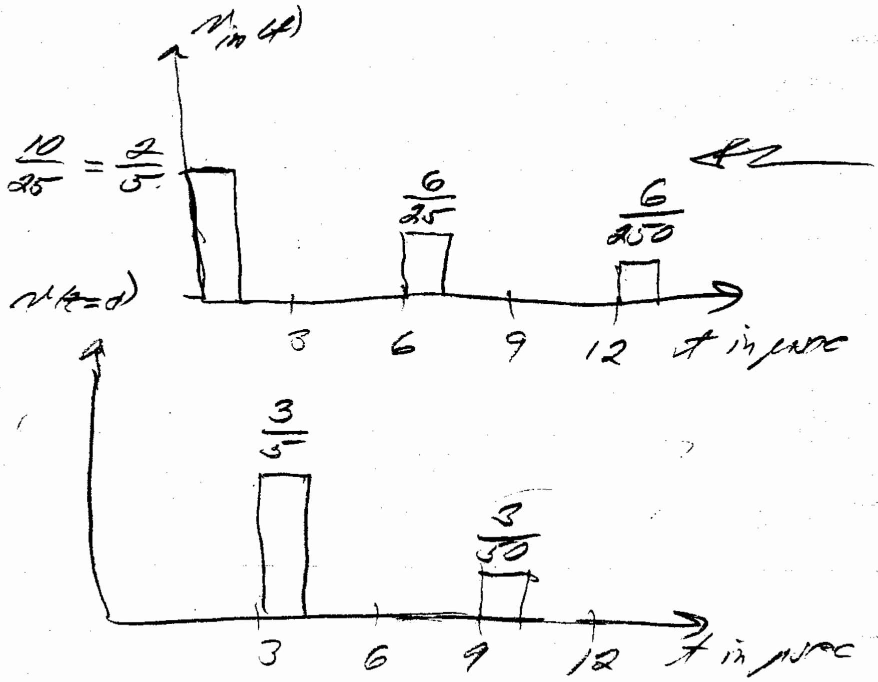


$\Gamma(0) = \frac{25}{125} = \frac{1}{5}$ $\Gamma(d) = \frac{150-50}{200} = \frac{1}{2}$

a) $\left[T = \frac{900}{3\times10^8} = 300\times10^{-8} = 3\,\mu\text{sec}\right]$ ←

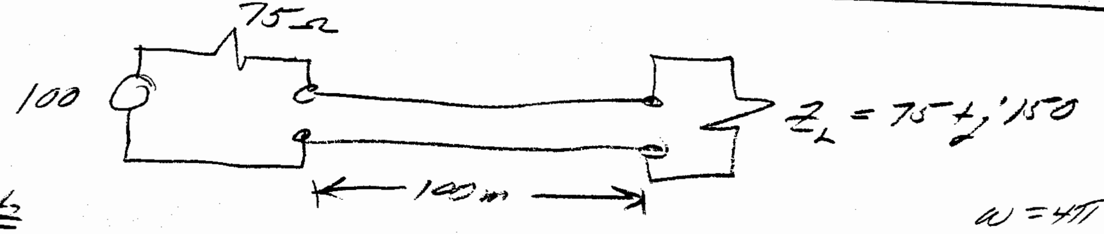
$v^+ = 1\,\frac{50}{125} = \frac{2}{5}$





b)

7.17



$f = 2$ MHz $\omega = 4\pi\times10^6$

$R = 150\ \Omega/\text{km}$, $L = 1.4$ mH/km, $C = 88$ nF/km, $G = 0.8\,\mu S/\text{km}$

a) $$Z_0 = \sqrt{\frac{R+j\omega L}{G+j\omega C}} = \sqrt{\frac{150 + j4\pi\times10^3\times1.4}{0.8\times10^{-6} + j4\pi\times88\times10^{-3}}} = \sqrt{\frac{17{,}593\,e^{j89.51^\circ}}{1.1058\,e^{j90^\circ}}} = 126\,e^{-j0.245}\ \Omega\ \ [126 - j0.54]$$ ←

$$\gamma = \sqrt{17593\times1.1058\,e^{j179.51^\circ}} = 139.48\,e^{j89.75} = \boxed{0.6 + j139.5}/\text{km}$$ ←

$\lambda = \frac{2\pi}{0.1395} = 45$ m $\boxed{\text{so } l = 2.22\ldots\lambda}$

$\therefore v_{ph} = \frac{\omega}{\beta} = \frac{4\pi\times10^6\times10^3}{1395} = 0.09\times10^9 = \underline{0.9\times10^8 \text{ m/sec}}$

7.17 b) $\Gamma_L = \dfrac{75+j150-126}{75+j150+126} = \dfrac{-51+j150}{201+j150} = \dfrac{158.4e^{j108.8}}{250.8e^{j36.73^\circ}}$

or $\Gamma_L = 0.63\,e^{j72^\circ} = 0.195 + j0.6$ ←

so $\Gamma(0) = \Gamma_L e^{-j2\beta\ell} = 0.63e^{j72^\circ} e^{-j\overbrace{2\times0.1395\times100}^{27.9\text{ radians}}}$

$\Gamma(0) = 0.63e^{j(72-1598.6)} = 0.63e^{-j1526.6^\circ}$

$\Gamma(0) = 0.63e^{-j86.55} = 0.038 - j0.629$

$\therefore Z(0) = 126\dfrac{1+\Gamma(0)}{1-\Gamma(0)} = 126\dfrac{1.038-j0.629}{0.962+j0.629} = 126\dfrac{1.214e^{-j31.21^\circ}}{1.14e^{j33.18^\circ}}$

$\boxed{Z(0) = 134e^{-j64.39^\circ} = 57.92 - j120.8}$ ←

c) $P_{ave_L} = \frac{1}{2}\text{Re}\{|\hat{I}_L|^2 Z_L\} = \frac{1}{2}|\hat{I}_L|^2 75$

$\hat{I}(0) = \dfrac{100}{75+57.92-j120.8} = \dfrac{100}{132.92-j120.8} = \dfrac{100}{179.6e^{-j42.27^\circ}}$

$\boxed{\hat{I}(0) = 0.557e^{j42.27^\circ}}$

$\dfrac{\hat{I}_L}{I(0)} = \dfrac{V^+e^{-j\beta\ell}}{V^+e^0}\cdot\dfrac{1-\Gamma_L}{1-\Gamma(0)} = e^{-j\overbrace{0.1395\times100}^{79.28^\circ}}\cdot\dfrac{1-0.195-j0.6}{1-0.038+j0.629}$

$\hat{I}_L = 0.557e^{j42.27^\circ}e^{-j79.28^\circ}\dfrac{0.805-j0.6}{0.962+j0.629} = 0.557e^{-j37^\circ}\left[\dfrac{1e^{-j36.7^\circ}}{1.14e^{j33.18^\circ}}\right]$

$\hat{I}_L = 0.488\,e^{-j106.88^\circ}$

$\therefore P_{ave_{load}} = \frac{1}{2}|0.488|^2 75 = 8.9\text{ Watts}$ ←

Iskander Problem 7-17

$R := 150 \quad L := 1.4\cdot 10^{-3} \quad C := 88\cdot 10^{-9} \quad G := 0.8\cdot 10^{-6} \quad \omega := 4\cdot\pi\cdot 10^{6} \quad j := \sqrt{-1}$

$\gamma := \sqrt{(R + j\cdot\omega\cdot L)\cdot(G + j\cdot\omega\cdot C)}$ $\qquad \gamma = 0.595 + 139.482i$

$Zo := \sqrt{\dfrac{R + j\cdot\omega\cdot L}{G + j\cdot\omega\cdot C}}$ $\qquad Zo = 126.132 - 0.538i$

$\Gamma L := \dfrac{75 + j\cdot 150 - Zo}{75 + j\cdot 150 + Zo}$ $\qquad \Gamma L = 0.195 + 0.604i$

$\Gamma o := \Gamma L\cdot e^{-2\cdot\gamma\cdot 0.1}$ $\qquad \Gamma o = 0.037 - 0.562i$

$Zin := Zo\cdot\dfrac{1 + \Gamma o}{1 - \Gamma o}$ $\qquad Zin = 68.802 - 114.39i$

$Io := \dfrac{100}{75 + Zin}$ $\qquad Io = 0.426 + 0.339i$

$IL := e^{-\gamma\cdot 0.1}\cdot Io\cdot\left(\dfrac{1 - \Gamma L}{1 - \Gamma o}\right)$ $\qquad IL = -0.142 - 0.441i$

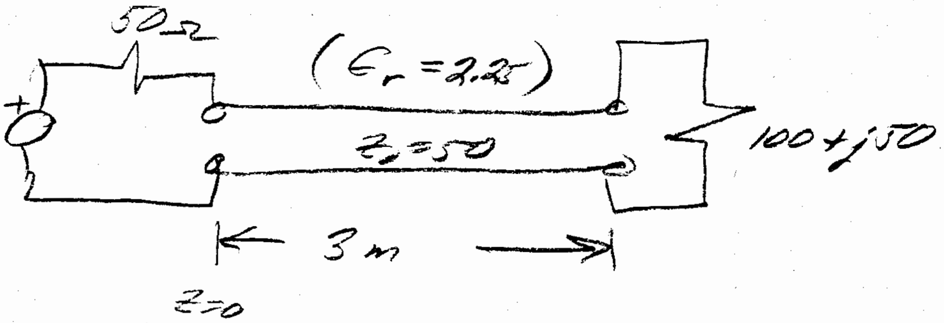
$Pave := \dfrac{1}{2}\cdot(|IL|)^{2}\cdot 75$ $\qquad Pave = 8.042$

EE 333 Assignment 11 10.3, 10.15

10.1

$\hat{V}_g = 200$

$f = 50\,MHz$



a) $j\beta = j\frac{2\pi}{\lambda} = j\frac{2\pi\sqrt{\epsilon_r}\,f}{3\times10^8} = j\frac{2\pi\times1.5\times0.5\times10^8}{3\times10^8} = j\frac{\pi}{2}$

$v_{phase} = \frac{3\times10^8}{\sqrt{\epsilon_r}} = \frac{3\times10^8}{1.5} = 2\times10^8\ m/sec = \frac{2}{3}c$

$\therefore$ line length in wavelengths $= \frac{3}{4}$

b) $\Gamma_{load} = \frac{100+j50-50}{100+j50+50} = \frac{50+j50}{150+j50} = \frac{1+j}{3+j} = \frac{\sqrt{2}\,e^{j45^\circ}}{\sqrt{10}\,e^{j18.43^\circ}} = 0.447\,e^{j26.57^\circ}$

$\Gamma(0) = 0.447\,e^{j26.57^\circ}\,e^{-j\pi 3} = -0.447\,e^{j26.57^\circ}$

(44.7% voltage reflection)

c) $Z_{in} = Z(0) = 50\,\frac{1+\Gamma(0)}{1-\Gamma(0)} = 50\,\frac{1-\frac{1+j}{3+j}}{1+\frac{1+j}{3+j}} = 50\,\frac{3+j-1-j}{3+j+1+j} = 50\,\frac{2}{4+2j}$

$$\boxed{Z_{in} = \frac{5\cancel{20}(4-2j)}{\cancel{20}} = 20-10j}$$

d) $\hat{I}_{in} = \frac{200}{50+Z_{in}} = \frac{200}{70-10j} = \frac{2\cancel{0}(7+j1)}{5\cancel{00}} = \boxed{\frac{14}{5}+j\frac{2}{5}}$ $2.8+j0.4$

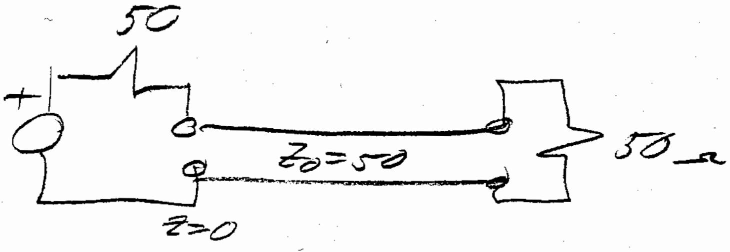
so $P_{ave_{in}} = \frac{1}{2}|I_{in}|^2\,20 = \boxed{80\ Watts}$

e) $\hat{I}_{load} = e^{-j\frac{\pi}{2}\times3}\,\hat{I}_{in}\,\frac{1-\Gamma_{load}}{1-\Gamma(0)} = \frac{2}{5}+j\frac{6}{5}$

$P_{ave_{load}} = \frac{1}{2}|I_L|^2\,100 = 80\ Watts$

$\boxed{\text{no line loss so } P_{in} = P_{load}}$

10.3



a) $\Gamma_{load} = 0 \quad \therefore \Gamma(z) = 0$

$$Z(z) = \frac{V(z)}{I(z)} = Z_0$$

b) $\hat{V}(z) = \hat{V}_m^+ e^{-j\beta z} \; ; \; \hat{I}(z) = \frac{\hat{V}_m^+}{Z_0} e^{-j\beta z}$ $\quad \boxed{\beta = \frac{\pi}{2}}$

$$\left.\begin{aligned}\hat{V}(z) &= \hat{V}_m^+ \\ \hat{I}(z) &= \hat{I}_m^+\end{aligned}\right\} @ \quad \beta z = n 2\pi \qquad n = 0, 1, 2, 3 \ldots$$

c) $\hat{I}_m = \frac{200}{50+50} = 2 = \hat{I}_m^+$

$\therefore \hat{V}_m^+ = 100$

$$\left.\begin{aligned}\hat{V}(z) &= 100 e^{-j\frac{\pi}{2}z} \\ \hat{I}(z) &= 2 e^{-j\frac{\pi}{2}z}\end{aligned}\right\}$$

d) $P_{ave_{in}} = \frac{1}{2} |I(0)|^2 50 = 100$ Watts

$P_{ave_{load}} = \frac{1}{2} |I(z=3)|^2 50 = 100$ Watts

$|\hat{I}| \neq f(z)$!

10.15 $Z_{L_{norm}} = 2 + j1 \qquad (0.75 - 0.2865)\lambda = 0.4635\lambda$

from chart $Z_{in_{norm}} = 0.4 - j0.2$!

$\Gamma_{load} = 0.45\, e^{j26^\circ}$

$\Gamma_{in} = 0.45\, e^{-j153.5^\circ}$

IMPEDANCE OR ADMITTANCE COORDINATES

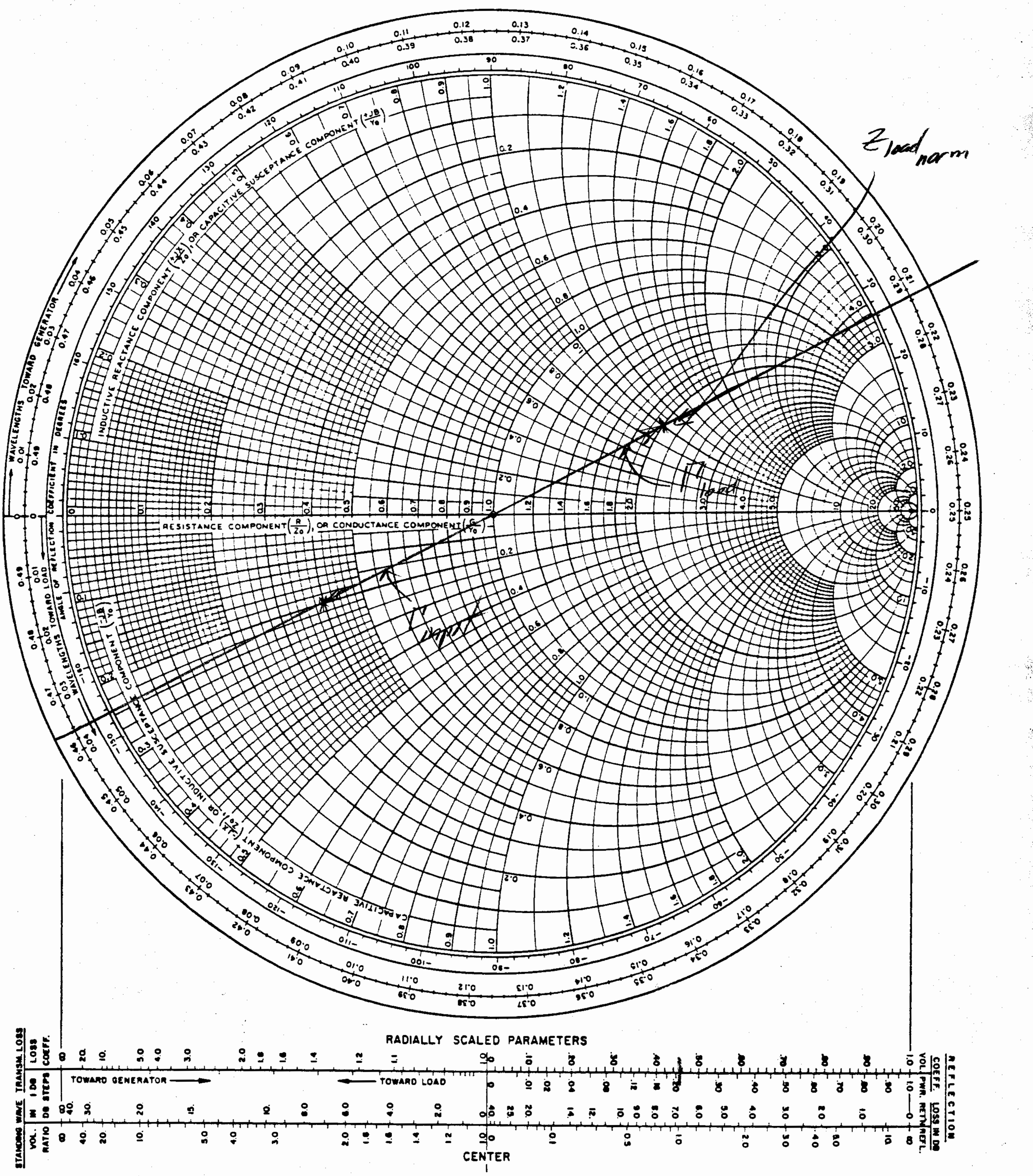
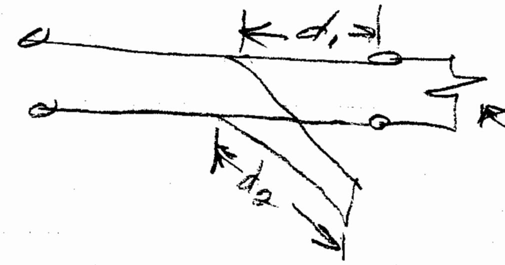


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 333 Assignment 12

7.25 $f = 1GHz$, $Z_L = \frac{Z_0}{10}$

for air line $\lambda = \frac{3\times10^8}{10^9} = 0.3m$

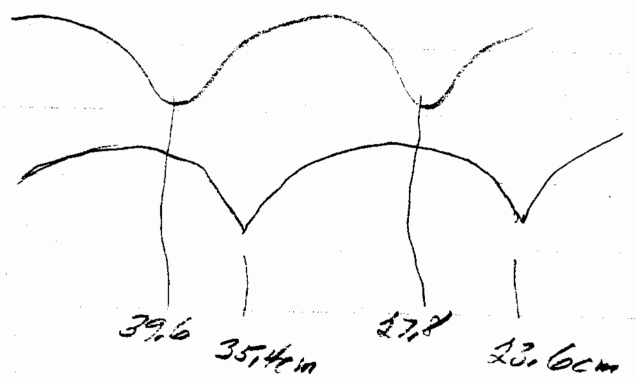


$Z_{L_n} = \frac{1}{10}$

from Smith Chart $Y_{@stub_{norm}} = 1 - j2.8$ $\boxed{\therefore d_1 = 0.05\lambda \text{ or } d_1 = 1.5\,cm}$

$Y_{stub_N} = +j2.8$ $\boxed{\therefore d_2 = 0.25 + 0.196 = 0.446\lambda = 0.134m}$

7.30 a)



VSWR = 2.6

$Z_0 = 50$

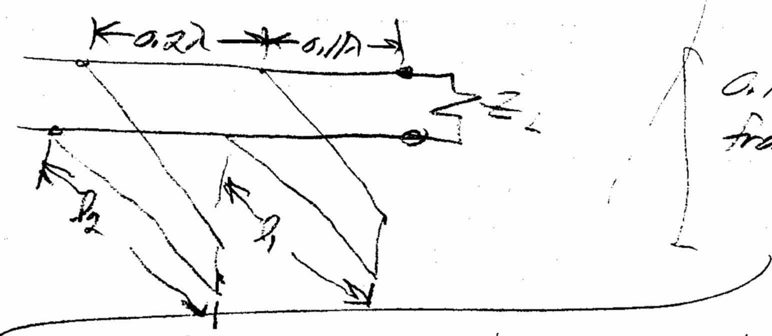
air line $v = c$

i. $\lambda = 2(39.6 - 27.8) = 23.6cm$ $\therefore$ $\boxed{f = \frac{3\times10^8}{0.236} = 1.271\,GHz}$

ii. from V_{min} move $27.8 - 23.6 = 4.2cm$ toward load (0.178λ)

$Z_{load_{norm}} = 1.23 - j1.08$ or $\boxed{Z_L = 61.5 - j54}$

b)



0.17λ toward generator from load we have $Y_n = 1.16 + j1.05$

$0.072 + .1 = 0.172$

from this point a constant G curve will not intersect the matching circle!

EE 333 Assignment 12 (page 2)

7.30 continued

added length $= 0.33 - 0.182 = 0.148\lambda$

$Y_{norm\,@1^{st}\,stub} = 1.12 - j1.04$

$Y_{stub\,1\,norm} = j(1.04 + 0.33) = j(1.37) \therefore \boxed{l_1 = 0.4\lambda}$

$Y_{norm\,gen\,side\,of\,1^{st}\,stub} = 1.12 + j0.33$

$Y_{norm\,@2^{nd}\,stub} = 1 - j0.33$

$\therefore Y_{norm\,2^{nd}\,stub} = +j0.33 \;;\; \boxed{l_2 = 0.25 + 0.051 = 0.3\lambda}$

Problem 7.25

IMPEDANCE OR ADMITTANCE COORDINATES

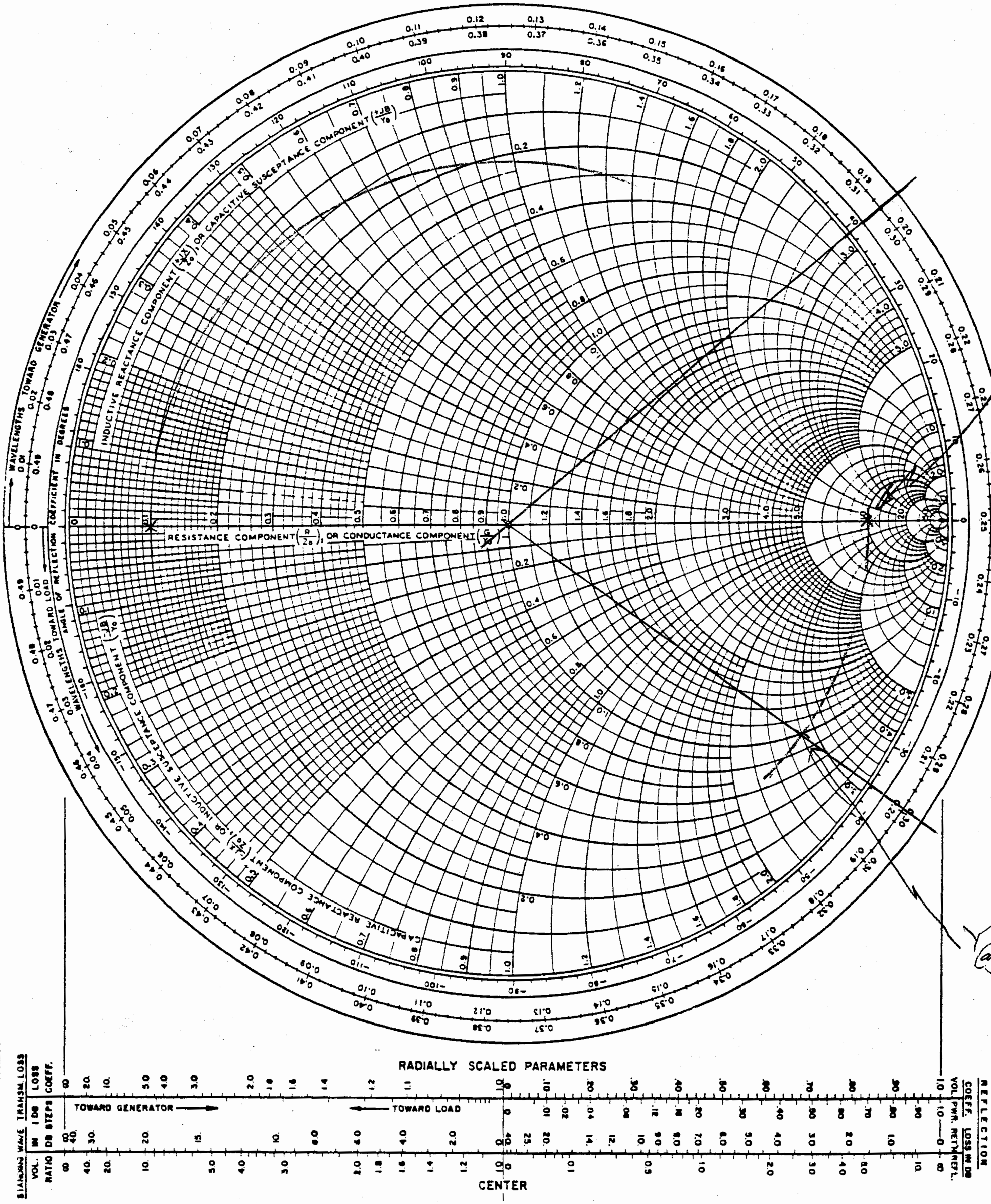


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

Problem 7.30

IMPEDANCE OR ADMITTANCE COORDINATES

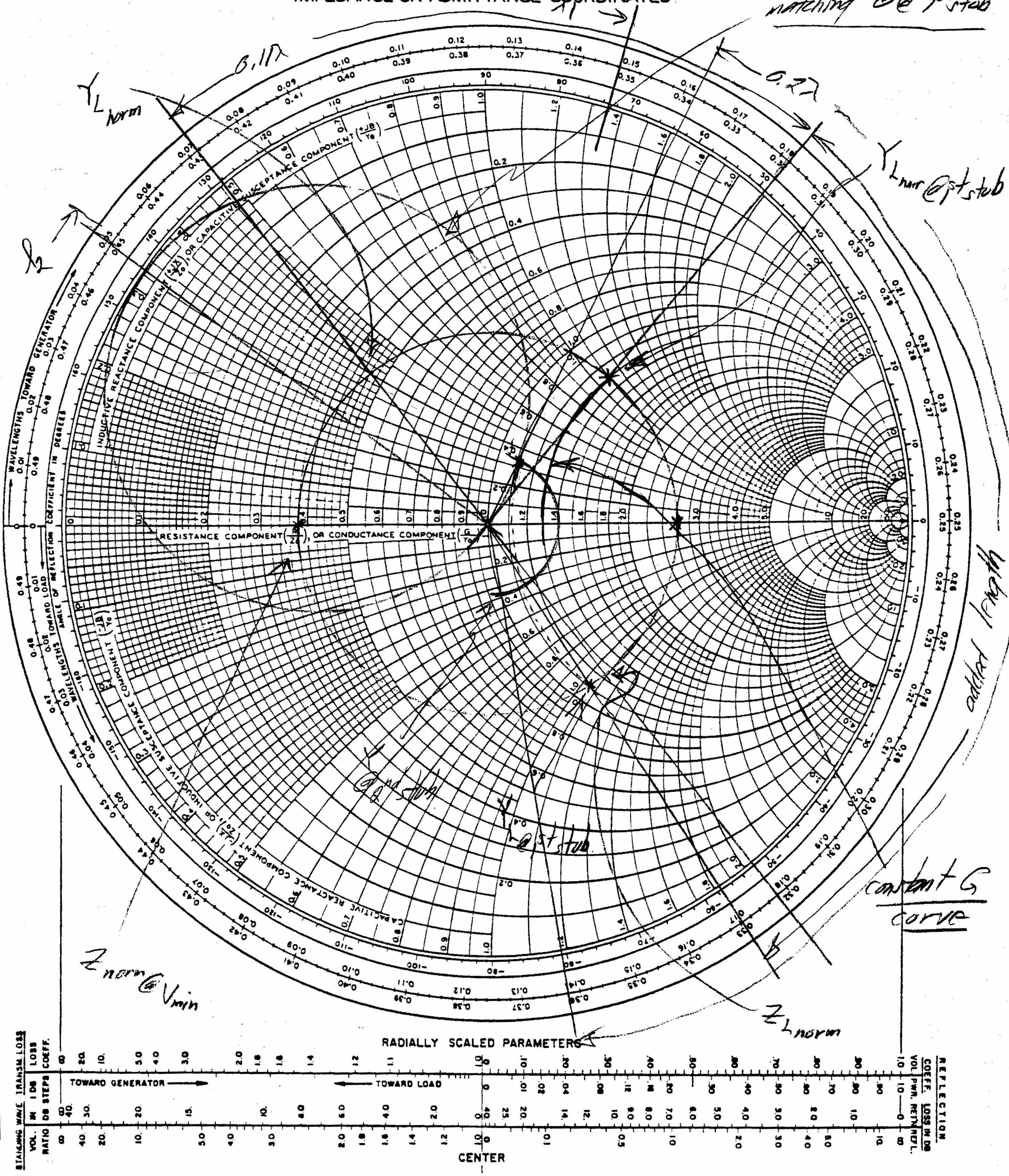
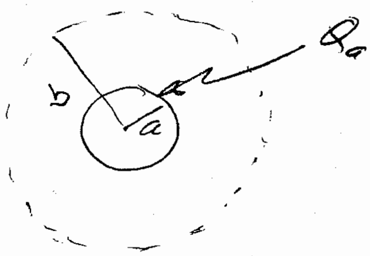


Fig. 9-3. A standard commercially available form of Smith chart graph paper. Copyrighted 1949 by Kay Electric Company, Pine Brook, N. J., and reprinted with their permission.

EE 333 Assignment 13 4.2, 4.3

4.1

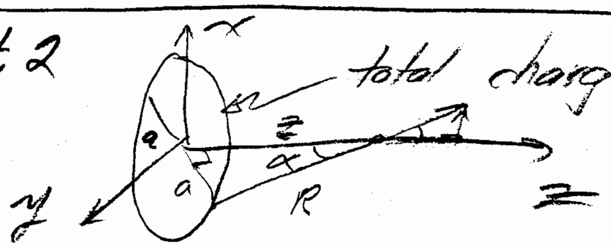


a) $\Phi_{\text{due to } Q_a} = \dfrac{Q_a}{4\pi\epsilon r}$

$$\boxed{\therefore\ \Phi(r=b) = \frac{Q_a}{4\pi\epsilon b}}$$

b) $\Phi = \Phi_{Q_A} + \Phi_{Q_B} = \dfrac{Q_a}{4\pi\epsilon b} + \dfrac{Q_b}{4\pi\epsilon b}$

4.2



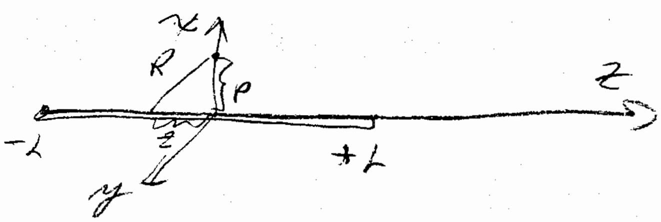
a) $\boxed{\Phi = \dfrac{Q}{4\pi\epsilon R} = \dfrac{Q}{4\pi\epsilon\sqrt{a^2+z^2}}}$

b) $\overline{E} = -\nabla\Phi = -\dfrac{\partial\Phi}{\partial\rho}\overline{a}_\rho - \dfrac{1}{\rho}\dfrac{\partial\Phi}{\partial\phi}\overline{a}_\phi - \dfrac{\partial\Phi}{\partial z}\overline{a}_z = + \dfrac{Q\,2z}{\cancel{8}\,4\pi\epsilon(a^2+z^2)^{3/2}}\overline{a}_z$

c) $dE_z = \dfrac{Q\,a\,d\phi\cos\alpha}{2\pi a\ 4\pi\epsilon R^2} = \dfrac{Q\,z\,d\phi}{8\pi^2\epsilon R^3} = \dfrac{Q\,z\,d\phi}{8\pi^2\epsilon(a^2+z^2)^{3/2}}$

$$\boxed{E_z = \int_{\phi=0}^{2\pi} dE_z = \frac{Q\,z}{4\pi\epsilon(a^2+z^2)^{3/2}}}$$ check !

4.3



a) $d\Phi = \dfrac{\rho_l\,dz}{4\pi\epsilon R} = \dfrac{\rho_l\,dz}{4\pi\epsilon\sqrt{z^2+\rho^2}}$

$\Phi(z=0)$ for length $2L$ $= \displaystyle\int_{z=-L}^{L} \frac{\rho_l\,dz}{4\pi\epsilon\sqrt{z^2+\rho^2}}$

$\Phi(z=0)$ for length $2L$ $= \dfrac{\rho_l}{4\pi\epsilon}\ln\left(z+\sqrt{z^2+\rho^2}\right)\Big|_{-L}^{L} = \dfrac{\rho_l}{4\pi\epsilon}\ln\dfrac{L+\sqrt{L^2+\rho^2}}{-L+\sqrt{L^2+\rho^2}}$

$\Phi(\rho)$ for $L\to\infty$ $= \displaystyle\lim_{L\to\infty}\frac{\rho_l}{4\pi\epsilon}\ln\frac{L+\sqrt{L^2+\rho^2}}{-L+\sqrt{L^2+\rho^2}} = \infty$

4.3 b) $\overline{E} = -\nabla\phi = -\frac{\partial \Phi}{\partial \rho}\bar{a}_\rho$

length $2L$

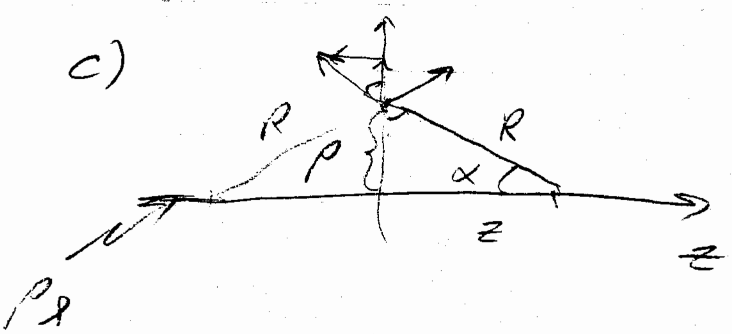
$$E_\rho = -\frac{\rho_\ell}{4\pi\epsilon}\left\{\frac{-L+\sqrt{L^2+\rho^2}}{L+\sqrt{L^2+\rho^2}}\left[(-L+\sqrt{L^2+\rho^2})^{-1}\frac{2\rho}{2\sqrt{L^2+\rho^2}} - (L+\sqrt{L^2+\rho^2})\frac{2\rho}{(-L+\sqrt{L^2+\rho^2})^{-2}\,2\sqrt{L^2+\rho^2}}\right]\right.$$

$$E_\rho = -\frac{\rho_\ell}{4\pi\epsilon}\left\{\frac{\rho}{+L+\sqrt{L^2+\rho^2}} - \frac{\rho}{-L+\sqrt{L^2+\rho^2}}\right\}\frac{1}{\sqrt{L^2+\rho^2}}$$

$$E_\rho = -\frac{\rho_\ell\rho}{4\pi\epsilon\sqrt{L^2+\rho^2}}\left\{\frac{(-L+\sqrt{L^2+\rho^2}) - (L+\sqrt{L^2+\rho^2})}{\cancel{-L^2}+\cancel{L^2}+\rho^2}\right\}$$

$$E_\rho = \frac{+2\rho_\ell\cancel{\rho}L}{\rho^{\cancel{2}}\,4\pi\epsilon\sqrt{L^2+\rho^2}};\ \lim_{L\to\infty} = \frac{\rho_\ell}{2\pi\epsilon\rho} \leftarrow$$

c)

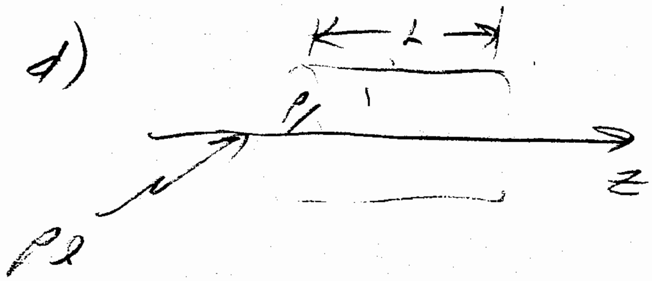


$$dE_\rho = \frac{\rho_\ell\, dz \cos\alpha}{4\pi\epsilon R^2} = \frac{\rho_\ell\, \rho\, dz}{4\pi\epsilon(\rho^2+z^2)^{3/2}}$$

$$E_\rho = \int_{z=-\infty}^{\infty} dE_\rho = \frac{\rho_\ell\rho}{4\pi\epsilon\rho^2}\cdot\left.\frac{z}{\sqrt{\rho^2+z^2}}\right|_{-\infty}^{\infty}$$

or $E_\rho = \frac{\rho_\ell}{2\pi\epsilon\rho} \leftarrow$

d)



$$\epsilon E_\rho 2\pi\rho \cancel{L} = \rho_\ell \cancel{L}$$

$$\epsilon E_\rho = \frac{\rho_\ell}{2\pi\epsilon\rho} \leftarrow$$

e) All give the same result!