[redacted]

CHAPTER 11 — SECTION 11.1

11.1 [redacted]

(c) $\mathcal{L}[(2+3t)e^{-2t}u(t)] = \mathcal{L}[2e^{-2t}u(t)] + \mathcal{L}[3te^{-2t}u(t)]$

$$= \frac{2}{s+2} + \frac{3}{(s+2)^2} = \frac{2(s+2)+3}{(s+2)^2} = \frac{2s+7}{(s+2)^2}$$

(d) $\mathcal{L}[(\cos 4t - \sin 4t)e^{-3t}u(t)]$

$$= \mathcal{L}[e^{-3t}\cos 4t\, u(t)] - \mathcal{L}[e^{-3t}\sin 4t\, u(t)]$$

$$= \frac{s+3}{(s+3)^2+4^2} - \frac{4}{(s+3)^2+4^2} = \frac{s-1}{(s+3)^2+16}$$

$$= \frac{s-1}{s^2+6s+25}$$

11.2 (a) $\mathcal{L}[u(t)] = \frac{1}{s} \Rightarrow \mathcal{L}[u(t-a)] = e^{-as}\left(\frac{1}{s}\right)$

$$\mathcal{L}[e^{-t}u(t-a)] = e^{-a(s+1)}\left(\frac{1}{s+1}\right)$$

$$\mathcal{L}[te^{-t}u(t-a)] = -\frac{d}{ds}\left[\frac{e^{-a(s+1)}}{s+1}\right] = -\left[\frac{(s+1)e^{-a(s+1)}(-a) - e^{-a(s+1)}}{(s+1)^2}\right]$$

$$= -\left[\frac{-a(s+1)-1}{(s+1)^2}\right]e^{-a(s+1)} = \frac{as+a+1}{(s+1)^2}e^{-a(s+1)}$$

(b) $\mathcal{L}[te^{-\alpha t}u(t)] = \frac{1}{(s+\alpha)^2}$

$$\mathcal{L}[(t-a)e^{-\alpha(t-a)}u(t-a)] = \frac{e^{-as}}{(s+\alpha)^2}$$

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11.3 [redacted]

(b) $\mathcal{L}[\cos(\beta t-\phi)u(t)] = \mathcal{L}[(\cos\beta t\cos\phi + \sin\beta t\sin\phi)u(t)]$

$$= \mathcal{L}[\cos\phi\cos\beta t\, u(t) + \sin\phi\sin\beta t\, u(t)]$$

$$= \frac{(\cos\phi)s}{s^2+\beta^2} + \frac{\beta\sin\phi}{s^2+\beta^2} = \frac{(\cos\phi)s + \beta\sin\phi}{s^2+\beta^2}$$

(c) From the solution of Part (a),

$$\mathcal{L}[\sin(\beta t-\phi)u(t)] = \frac{\beta\cos\phi - (\sin\phi)s}{s^2+\beta^2}$$

$$\therefore \mathcal{L}[e^{-\alpha t}\sin(\beta t-\phi)u(t)] = \frac{\beta\cos\phi - (\sin\phi)(s+\alpha)}{(s+\alpha)^2+\beta^2}$$

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(d) $\mathcal{L}\left[\sin t\, u(t) - \sin t\, u(t-\pi)\right]$

$= \mathcal{L}\left[\sin t\, u(t) + \sin(t-\pi)\, u(t-\pi)\right]$

$= \dfrac{1}{s^2+1} + e^{-s\pi}\left(\dfrac{1}{s^2+1}\right) = \dfrac{1+e^{-s\pi}}{s^2+1}$