

~~Section 11.1~~  
~~Problem 11.11~~  
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CHAPTER 11

SECTION 11.1

11.1  $\mathcal{L}\{e^{-2t}u(t-a)\} = e^{-as} \mathcal{L}\{e^{-2(t-a)}u(t-a)\}$   
 $= e^{-as} \int_a^\infty e^{-2(t-a)} e^{-st} dt = e^{-as} \int_0^\infty e^{-2\tau} e^{-s(\tau+a)} e^{-s\tau} d\tau$   
 $= e^{-as} e^{-sa} \int_0^\infty e^{-(s+2)\tau} d\tau = e^{-a(s+2)} \frac{1}{s+2}$   
 $= \frac{e^{-2a}}{s+2}$

(c)  $\mathcal{L}\{(2+3t)e^{-2t}u(t)\} = \mathcal{L}\{2e^{-2t}u(t)\} + \mathcal{L}\{3te^{-2t}u(t)\}$   
 $= \frac{2}{s+2} + \frac{3}{(s+2)^2} = \frac{2(s+2) + 3}{(s+2)^2} = \frac{2s+7}{(s+2)^2}$

(d)  $\mathcal{L}\{(\cos 4t - \sin 4t)e^{-3t}u(t)\}$   
 $= \mathcal{L}\{e^{-3t}\cos 4t u(t)\} - \mathcal{L}\{e^{-3t}\sin 4t u(t)\}$   
 $= \frac{s+3}{(s+3)^2+4^2} - \frac{4}{(s+3)^2+4^2} = \frac{s-1}{s^2+6s+25}$

11.2  $\mathcal{L}\{u(t)\} = \frac{1}{s} \Rightarrow \mathcal{L}\{u(t-a)\} = e^{-as} \left(\frac{1}{s}\right)$   
 $\mathcal{L}\{e^{-at}u(t-a)\} = e^{-as} \mathcal{L}\{e^{-a(t-a)}u(t-a)\} = e^{-as} \frac{1}{s}$   
 $\mathcal{L}\{te^{-at}u(t-a)\} = -\frac{d}{ds} \left[ \frac{e^{-a(s+1)}}{s+1} \right] = -\left[ \frac{-(s+1)e^{-a(s+1)} - e^{-a(s+1)}}{(s+1)^2} \right]$   
 $= -\left[ \frac{-a(s+1) - 1}{(s+1)^2} \right] e^{-a(s+1)} = \frac{as+a+1}{(s+1)^2} e^{-a(s+1)}$

(b)  $\mathcal{L}\{te^{-\alpha t}u(t)\} = \frac{1}{(s+\alpha)^2}$   
 $\mathcal{L}\{(t-a)e^{-\alpha(t-a)}u(t-a)\} = \frac{e^{-\alpha s}}{(s+\alpha)^2}$

11.3  $\mathcal{L}\{\cos(\beta t - \phi)u(t)\} = \mathcal{L}\{(\cos \phi \cos \beta t + \sin \phi \sin \beta t)u(t)\}$   
 $= \frac{\beta \cos \phi}{s^2 + \beta^2} + \frac{\beta \sin \phi}{s^2 + \beta^2} = \frac{\beta \cos \phi - (\sin \phi)s}{s^2 + \beta^2}$

(b)  $\mathcal{L}\{\cos(\beta t - \phi)u(t)\} = \mathcal{L}\{(\cos \phi \cos \beta t + \sin \phi \sin \beta t)u(t)\}$   
 $= \mathcal{L}\{\cos \phi \cos \beta t u(t)\} + \mathcal{L}\{\sin \phi \sin \beta t u(t)\}$   
 $= \frac{(\cos \phi)s}{s^2 + \beta^2} + \frac{\beta \sin \phi}{s^2 + \beta^2} = \frac{(\cos \phi)s + \beta \sin \phi}{s^2 + \beta^2}$

(c) From the solution of Part (a),  
 $\mathcal{L}\{\sin(\beta t - \phi)u(t)\} = \frac{\beta \cos \phi - (\sin \phi)s}{s^2 + \beta^2}$   
 $\therefore \mathcal{L}\{e^{-\alpha t} \sin(\beta t - \phi)u(t)\} = \frac{\beta \cos \phi - (\sin \phi)(s+\alpha)}{(s+\alpha)^2 + \beta^2}$

~~(c)  $\mathcal{L}\{e^{-\alpha t} \cos(\beta t - \phi)u(t)\} = \frac{\beta \sin \phi + (\cos \phi)(s+\alpha)}{(s+\alpha)^2 + \beta^2}$~~   
 ~~$\mathcal{L}\{e^{-\alpha t} \sin(\beta t - \phi)u(t)\} = \frac{\beta \cos \phi - (\sin \phi)(s+\alpha)}{(s+\alpha)^2 + \beta^2}$~~

(d) From the solution of Part (c),  
 $\mathcal{L}\{e^{-\alpha t} \cos(\beta t - \phi)u(t)\} = \frac{\beta \sin \phi + (\cos \phi)(s+\alpha)}{(s+\alpha)^2 + \beta^2}$

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$$\frac{1}{s^2+1} = \frac{1}{(s-j)(s+j)}$$

$$\frac{1}{(s-j)(s+j)} = \frac{A}{s-j} + \frac{B}{s+j}$$

$$\frac{1}{(s-j)(s+j)} = \frac{A(s+j) + B(s-j)}{(s-j)(s+j)}$$

$$1 = A(s+j) + B(s-j)$$

$$1 = As + Aj + Bs - Bj$$

$$1 = (A+B)s + (A-B)j$$

$$0 = A+B$$

$$1 = (A-B)j$$

$$A = \frac{1}{2j}$$

$$B = -\frac{1}{2j}$$

$$\frac{1}{s^2+1} = \frac{1}{2j} \left( \frac{1}{s-j} - \frac{1}{s+j} \right)$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2+1} \right\} = \frac{1}{2j} \left( \mathcal{L}^{-1} \left\{ \frac{1}{s-j} \right\} - \mathcal{L}^{-1} \left\{ \frac{1}{s+j} \right\} \right)$$

$$= \frac{1}{2j} \left( e^{jt} - e^{-jt} \right)$$

$$= \frac{1}{2j} \left( \frac{e^{jt} - e^{-jt}}{j} \right) = \frac{1}{2} \left( \frac{e^{jt} - e^{-jt}}{j^2} \right)$$

$$\mathcal{L}^{-1} \left\{ \frac{1}{s^2+1} \right\} = \frac{1}{2} \left( \frac{e^{jt} - e^{-jt}}{j^2} \right)$$

$$= \frac{1}{2} \left( \frac{e^{jt} - e^{-jt}}{-1} \right) = -\frac{1}{2} \left( e^{jt} - e^{-jt} \right)$$

$$= \frac{1}{2} \left( e^{-jt} - e^{jt} \right) = \frac{1}{2} \left( \frac{e^{-jt} - e^{jt}}{j} \right) = \frac{1}{2j} \left( e^{-jt} - e^{jt} \right)$$

$$(d) \mathcal{L} \left[ \sin t u(t) - \sin t u(t-\pi) \right]$$

$$= \mathcal{L} \left[ \sin t u(t) + \sin(t-\pi) u(t-\pi) \right]$$

$$= \frac{1}{s^2+1} + e^{-s\pi} \left( \frac{1}{s^2+1} \right) = \frac{1+e^{-s\pi}}{s^2+1}$$