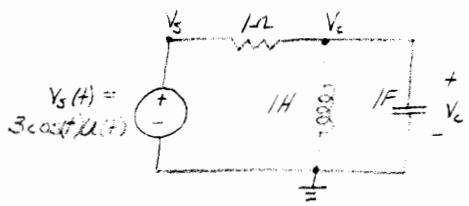
EE212 HW1 Soln

1. PR8.9



* $t \geq 0$:

@ V_c: $\frac{V_c - V_s}{1} + \frac{1}{1}\int V_c\,dt + 1\cdot\frac{dV_c}{dt} = 0$

$\Rightarrow \frac{dV_c}{dt} + V_c + \frac{d^2V_c}{dt^2} = \frac{dV_s}{dt}$

$= -3\sin(t)$

$\Rightarrow \frac{d^2V_c}{dt^2} + \frac{dV_c}{dt} + V_c = -3\sin(t)$

* $t < 0$: ZSR $\Rightarrow i_L(t) = V_c(t) = 0$

* natural response: char. eqn $= s^2 + s + 1 = 0$

$\Rightarrow \alpha = \frac{1}{2}$, $\omega_n = \sqrt{1} = 1 \Rightarrow$ UD

$\Rightarrow s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_n^2}$

$= -\frac{1}{2} \pm \sqrt{\frac{1}{4} - 1}$

$= -\frac{1}{2} \pm j\sqrt{\frac{3}{4}}$

$= -\frac{1}{2} \pm j\frac{\sqrt{3}}{2} \Rightarrow \alpha = -\frac{1}{2},\ \omega = \frac{\sqrt{3}}{2}$

$\Rightarrow V_{cn}(t) = B_1 e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) + B_2 e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)$

* forced response: $V_{cf}(t) = K_1\sin(t) + K_2\cos(t)$

V_{cf}: $K_1\sin t + K_2\cos t$

$+\ \frac{dV_{cf}}{dt}$: $K_1\cos t - K_2\sin t$

$+\ \frac{d^2V_{cf}}{dt^2}$: $-K_1\sin t - K_2\cos t$

$= -3\sin t$

$\sin t$: $K_1 - K_2 - K_1 = -3$

$\Rightarrow K_2 = 3$

$\cos t$: $K_2 + K_1 - K_2 = 0$

$\Rightarrow K_1 = 0$

$\Rightarrow V_{cf}(t) = 3\cos t$

* total response: $V_c(t) = V_{cf}(t) + V_{cn}(t) = 3\cos t + B_1 e^{-\frac{1}{2}t}\cos\frac{\sqrt{3}}{2}t + B_2 e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)$

$\rightarrow V_c(0) = 3 + B_1 + 0 = 0 \Rightarrow B_1 = -3$

$\rightarrow \frac{V_c(0) - V_s(0)}{1} + i_L(0) + \frac{dV_c(0)}{dt} = 0 \Rightarrow \frac{dV_c(0)}{dt} = -i_L(0)^{\nearrow 0} + V_s(0)^{\nearrow 3} - V_c(0)^{\nearrow 0}$ } from circuit analysis

$\Rightarrow \frac{dV_c(0)}{dt} = 3$

$\frac{dV_c(0)}{dt} = 3 = \left[-3\sin t - \frac{1}{2}e^{-\frac{1}{2}t}\left(B_1\cos\left(\frac{\sqrt{3}}{2}t\right) + B_2\sin\left(\frac{\sqrt{3}}{2}t\right)\right) + e^{-\frac{1}{2}t}\left(-\frac{1}{2}B_1\sin\left(\frac{\sqrt{3}}{2}t\right) + \frac{\sqrt{3}}{2}B_2\cos\left(\frac{\sqrt{3}}{2}t\right)\right)\right]\Big|_{t=0}$

$= -\frac{1}{2}B_1 + \frac{\sqrt{3}}{2}B_2 = +\frac{3}{2} + \frac{\sqrt{3}}{2}B_2$

$\Rightarrow B_2 = \frac{2}{\sqrt{3}}\left(3 - \frac{3}{2}\right) = \frac{2}{\sqrt{3}}\left(\frac{3}{2}\right) = \frac{3}{\sqrt{3}} = \frac{3\sqrt{3}}{3} = \sqrt{3}$

$$\Rightarrow \boxed{\begin{aligned} V_c(t) &= \left[3\cos t - 3e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t\right) + \sqrt{3}e^{-\frac{1}{2}t}\sin\left(\frac{\sqrt{3}}{2}t\right)\right]u(t)\ V \\ &= \left[3\cos t + 2\sqrt{3}e^{-\frac{1}{2}t}\cos\left(\frac{\sqrt{3}}{2}t - 150^\circ\right)\right]u(t)\ V \end{aligned}}$$

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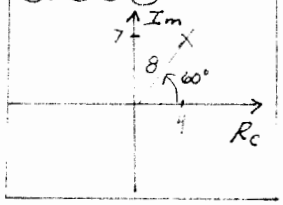
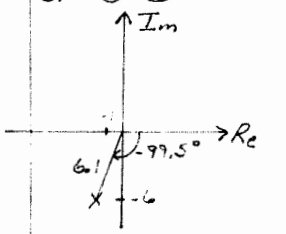
2. PR 8.11 a) $4 + j7 = \sqrt{4^2 + 7^2}\, e^{j \tan^{-1}(7/4)} = \sqrt{65}\, e^{j 1.05 \text{ rad}}$

$= 8.06\, e^{j 60.2°}$



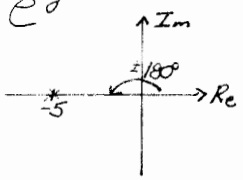
d) $-1 - j6 = \sqrt{1^2 + 6^2}\, e^{j \tan^{-1}(-6/-1)} = \sqrt{37}\, e^{-j 1.732 \text{ rad}}$

$= 6.08\, e^{-99.5°}$



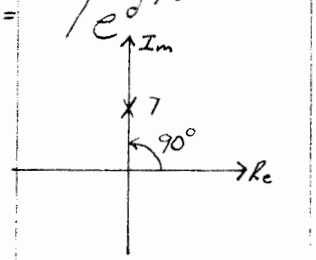
f) $-5 = \sqrt{5^2 + 0^2}\, e^{j \tan^{-1}(0/5)} = 5\, e^{\pm j\pi}$

$= 5\, e^{j \pm 180°}$



g) $j7 = \sqrt{0^2 + 7^2}\, e^{j \tan^{-1}(7/0)} = 7\, e^{j\pi/2}$

$= 7\, e^{j 90°}$



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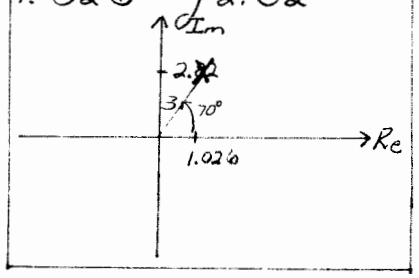
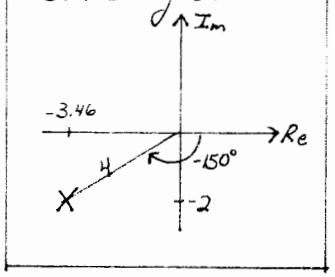
3. PR8.12 a) $3e^{j70°} = 3(\cos 70° + j\sin 70°)$

$= 1.026 + j2.82$



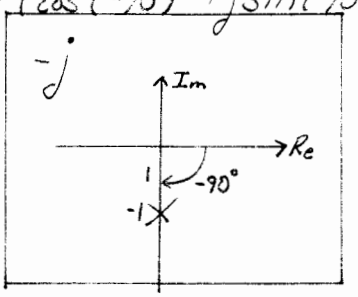
d) $4e^{-j150°} = 4(\cos(-150°) + j\sin(-150°))$

$= -3.46 - j2$



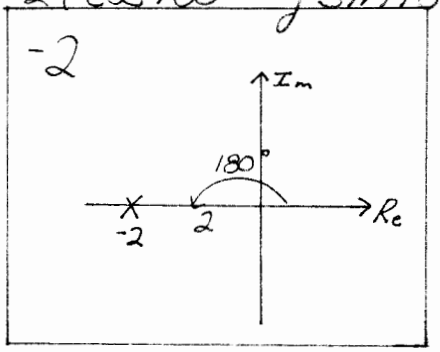
f) $e^{-j90°} = 1(\cos(-90°) + j\sin(-90°))$

$= -j$



g) $2e^{j180°} = 2(\cos 180° + j\sin 180°)$

$= -2$



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4. $4\sin(5t - 0.2\text{rad}) = 4\cos(5t - 0.2 - \pi/2)$
$= \boxed{4\cos(5t - 1.771\text{rad})}$

amp = $A = 4$
ang. freq. = $\omega = 5$ rad/sec
ord. freq = $\frac{5}{2\pi}$ Hz = 0.796 Hz = f
period = $T = \frac{1}{f} = \frac{2\pi}{5}$ sec = 1.26 sec
phase angle = $\phi = -1.771$ rad
$= -101.5°$

5. $v(t) = 3\cos(4t + 0.1\text{rad})$
$= 3\cos(4(t + 0.025))$
$i(t) = 2\cos(4t - 0.6\text{rad}))$
$= 2\cos(4(t - 0.15))$

$v(t)$ leads $i(t)$ by $0.1\text{rad} - (-0.6\text{rad}) = 0.7\text{rad}$
or $\frac{0.7\text{rad}}{4\text{rad/sec}} = 0.175$ sec

or

$i(t)$ lags $v(t)$ by 0.7rad, 0.175sec