EE 231 – Homework 3 Solutions
Due September 17, 2010

1. Find the truth table for the following functions:

   (a) $F = y'z' + y'z + xz'$

x	y	z	$y'z'$	$y'z$	xz'	$y'z' + y'z + xz'$
0	0	0	1	0	0	1
0	0	1	0	1	0	1
0	1	0	0	0	0	0
0	1	1	0	0	0	0
1	0	0	1	0	1	1
1	0	1	0	1	0	1
1	1	0	0	0	1	1
1	1	1	0	0	0	0

   (b) $F = xy + x'z'$

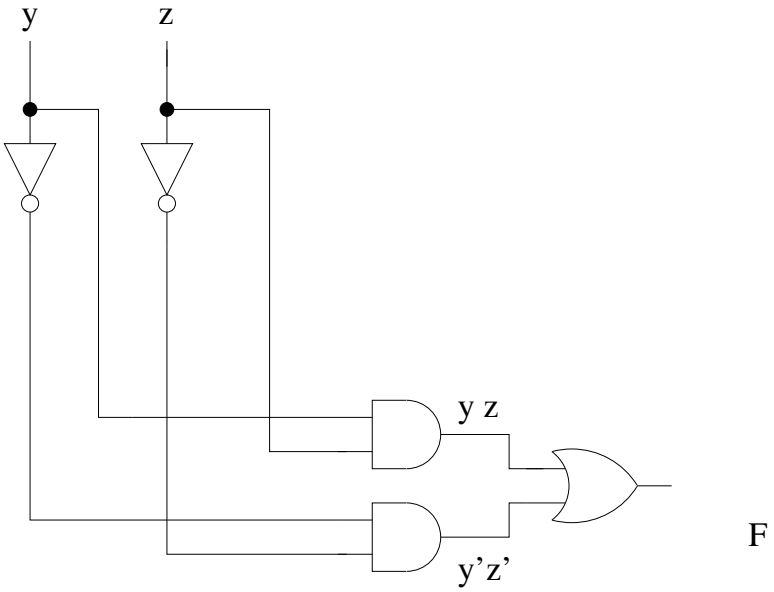
x	y	z	xy	$x'z'$	$xy + x'z'$
0	0	0	0	1	1
0	0	1	0	0	0
0	1	0	0	1	1
0	1	1	0	0	0
1	0	0	0	0	0
1	0	1	0	0	0
1	1	0	1	0	1
1	1	1	1	0	1

2. Implement the Boolean function

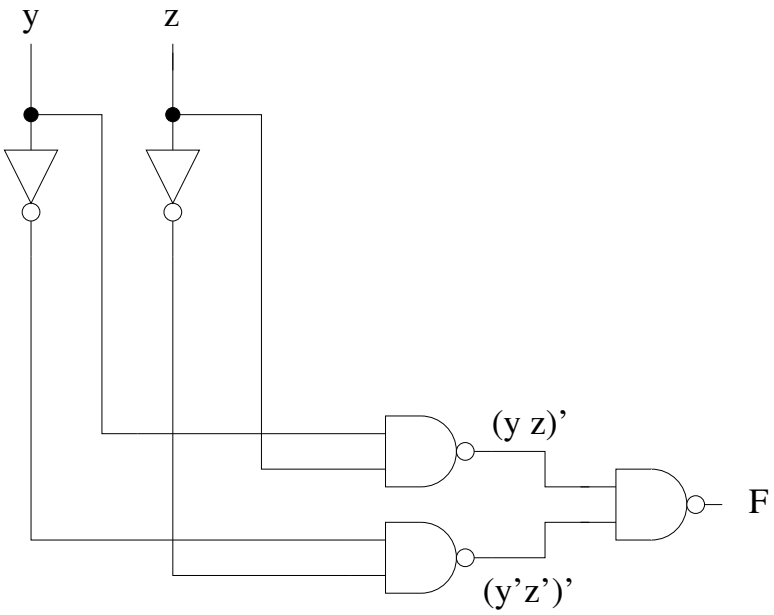
$$F = yz + y'z' + z'z$$

This can be reduced to $F = yz + y'z'$.

(a) with AND, OR and inverter gates,

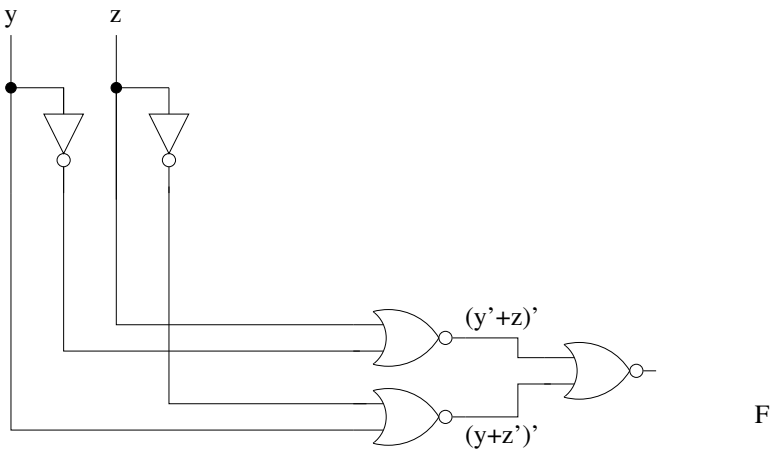


(b) with NAND and inverter gates,



(c) with NOR and inverter gates.

Convert from sum of products to product of sums: $(y+z')(y'+z) = ((y+z')'+(y'+z)')'$



3. Obtain the truth table of the following functions, and express each function as a sum-of-minterms and a product-of-maxterms:

(a) $(x + yz)(z + xz)$

	x	y	z	$(x + yz)$	$(z + xz)$	$(xyz)(z + xz)$
0	0	0	0	0	0	0
1	0	0	1	0	1	0
2	0	1	0	0	0	0
3	0	1	1	1	1	1
4	1	0	0	1	0	0
5	1	0	1	1	1	1
6	1	1	0	1	0	0
7	1	1	1	1	1	1

The minterms are the ones with 1's:

$\Sigma(3, 5, 7)$

The maxterms are the ones with the 0's

$\Pi(0, 1, 2, 4, 6)$

(b) $(xy' + yz + x'y)(x + y)$

	x	y	z	$(xy' + yz + x'y)$	$x + y$	$(xy' + yz + x'y)(x + y)$
0	0	0	0	0	0	0
1	0	0	1	0	0	0
2	0	1	0	1	1	1
3	0	1	1	1	1	1
4	1	0	0	1	1	1
5	1	0	1	1	1	1
6	1	1	0	0	1	0
7	1	1	1	1	1	1

The minterms are where there are 1's:
$F = \Sigma(2, 3, 4, 5, 7)$
The maxterms are where there are 0's:
$F = \Pi(0, 1, 6)$

4. Express the following function as a sum of minterms and as a product of maxterms:

$$F(A, B, C, D) = AC + BD' + BC' + BD$$

$$\begin{aligned} F(A, B, C, D) &= AC + BD' + BC' + BD \\ &= AC + B(D' + D) + BC' \\ &= AC + B + BC' \\ &= AC + B \end{aligned}$$

Truth table:

A	B	C	D	$AC + B$
0	0	0	0	0
0	0	0	1	0
0	0	1	0	0
0	0	1	1	0
0	1	0	0	1
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	0
1	0	0	1	0
1	0	1	0	1
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1

$F(A, B, C, D) = \Sigma(4, 5, 6, 7, 10, 11, 12, 13, 14, 15)$
$F(A, B, C, D) = \Pi(0, 1, 2, 3, 8, 9)$

5. Convert each of the following to the other canonical form:

   (a) $F(x, y, z) = \Sigma(1, 3, 6)$

   $F(x, y, z) = \Pi(0, 2, 4, 5, 7)$

   (b) $F(A, B, C, D) = \Pi(0, 2, 4, 7, 9, 13)$

   $F(A, B, C, D) = \Sigma(1, 3, 5, 6, 8, 10, 11, 12, 14, 15)$

6. Convert each of the following expressions into sum of products and products of sums:

   (a) $(BC + D)(C + AD')$

   $$\begin{aligned} F &= (BC + D)(C + AD') \\ &= BC + ABCD' + CD + ADD' \\ &= BC(1 + AD') + CD \\ &= BC + CD && \text{(Sum of Products form)} \\ &= (B + D)C && \text{(Product of Sums form)} \end{aligned}$$

   (b) $y' + y(y + z')(x' + z)$

   Note: Below we use

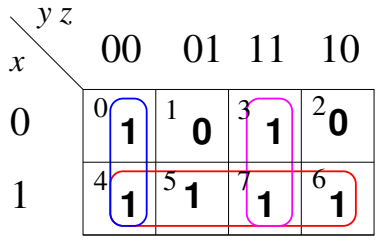
   $y'+yz = y'(z+z')+yz = y'(z+z+z')+yz = y'(z+z')+y'z+yz = y'+(y'+y)z = y'+z$

   in two places.

   $$\begin{aligned} F &= y' + y(y + z')(x' + z) \\ &= y' + x'y + yz + x'yz' + yzz' \\ &= y' + x'y(1 + z') + yz \\ &= y' + x'y + yz \\ &= y' + yx' + y' + yz \\ &= y' + x' + y' + z \\ &= y' + x' + z && \text{(SOP and POS)} \end{aligned}$$

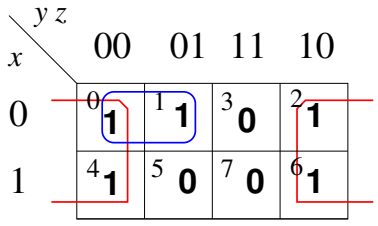
7. Simplify the following Boolean functions using three-variable maps:

   (a) $F(x, y, z) = \Sigma(0, 3, 4, 5, 6, 7)$



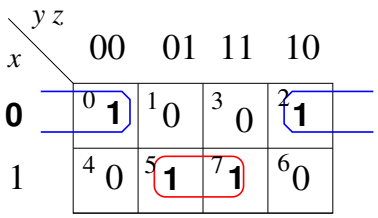
   $F = x + yz + y'z'$

   (b) $F(x, y, z) = \Pi(3, 5, 7)$



   $F = z' + x'y'$

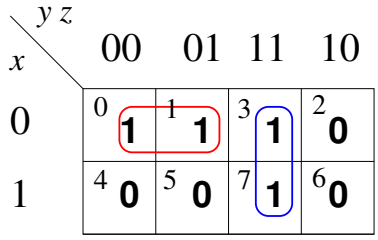
(c) $F(x, y, z) = \Sigma(0, 2, 5, 7)$



$F = x'z' + xz$

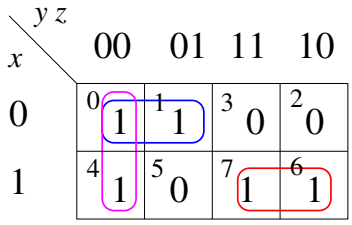
8. Simplify the followin Boolean expressions using three-variable maps:

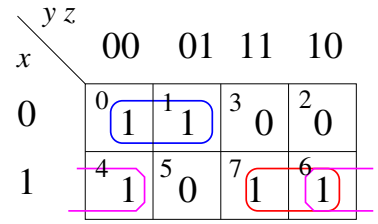
(a) $F(x, y, z) = x'y'z' + yz + x'y'z$



$F = x'y' + yz$

(b) $F(x, y, z) = xy + y'z' + x'y'z$





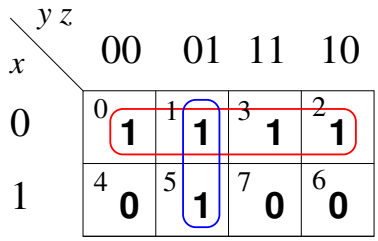
Two ways:

$F = xy + x'y' + y'z'$

or

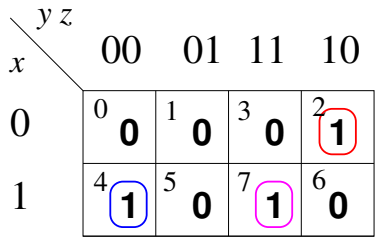
$F = xy + x'y' + xz'$

(c) $F(x, y, z) = x'y + y'z + x'z'$



$F = x' + y'z$

(d) $F(x, y, z) = xyz + xy'z' + x'yz'$

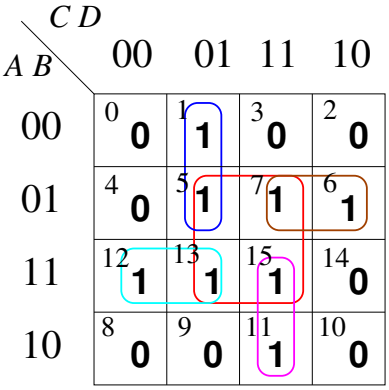


Cannot reduce

$F(x, y, z) = xyz + xy'z' + x'yz'$

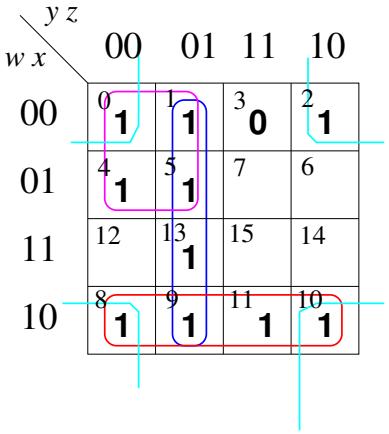
9. Simplify the following Boolean functions, using Karnaugh maps:

(a) $F(A,B,C,D) = \Sigma(1,5,6,7,11,12,13,15)$



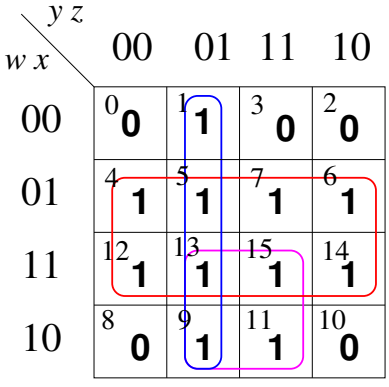
$F = BD + A'C'D + ACD + ABC' + A'BC$

(b) $F(w,x,y,z) = \Sigma(0,1,2,4,5,8,9,10,11,13)$



$F = wx' + y'z + w'y' + x'z'$

(c) $F(w,x,y,z) = \Pi(0,2,3,8,10)$



$F = B + C'D + AD$