EE 231

Homework 11

Due November 15, 2010

1. The memory units that follow are specified by the number of words times the number of bits per word. How many address lines and input-output data lines are needed in each case?

   (a) 32 x 8
   $32 = 2^5$, so 32 x 8 takes 5 address lines and 8 data lines, for a total of $5 + 8 = 13$ I/O lines.

   (b) 4M x 16
   $4\text{M} = 4 \times 2^{20} = 2^{22}$, so 4M x 16 takes 22 address lines and 16 data lines, for a total of $22 + 16 = 38$ I/O lines.

   (c) 2G x 32
   $2\text{G} = 2 \times 2^{30} = 2^1 \times 2^{30} = 2^{31}$, so 2G x 32 takes 31 address lines and 32 data lines, for a total of 63 I/O lines.

2. Give the number of bytes stored in the memories listed in Problem 1.

   (a) 32 x 8: A byte is 8 bits, so 32 x 8 has 32 bytes.

   (b) 4M x 16: $4\text{M} = 2^{22} = 4{,}194{,}304$ words. Each word is 2 bytes, so the total number of words is 8,388,608.

   (c) 2G x 32: $2\text{G} = 2^{21} = 2{,}147{,}483{,}648$. Each word is 4 bytes, so the total number of words is 8,589,934,592.

3. A DRAM chip uses two-dimensional address demultiplexing. It has 16 common address pins, with the row address having four more bits than the column address (the row address has 16 bits, the column address has 12 bits). What is the capacity of the memory?

   The DRAM chip has 16 rows and 12 columns, for a total of 28 address lines. $2^{28} = 268,435,456$ memory locations. Also, $2^{28} = 2^8 \times 2^{20} = 256 \times 2^{20} = 256$ M.

4. This problem deals with the Hamming code for error detection and correction. To be able to detect and correct one-bit errors for eight-bit words, five parity bits are required. The numbers are of the form P_0 P_1 P_2 D_3 P_4 D_5 D_6 D_7 P_8 D_9 D_{10} D_{11} D_{12}, where P_0 is the overall parity bit (called P_{13} in the text), and D_3 D_5 D_6 D_7 D_9 D_{10} D_{11} D_{12} is the original 8-bit data word.

   (a) You read the number 0001 0101 1000 0 from a memory which uses error detection and correction. What was the original 8-bit data word that was written?

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
0	0	0	1	0	1	0	1	1	0	0	0	0

   C_0 = XOR of all bits (P_0 through D_{13}). There are four ones, which is an even number, indicating no errors or a two-bit uncorrectable error.

   $C_1 = XOR(P_1, D_3, D_5, D_7, D_9, D_{11}) = XOR(0, 1, 1, 1, 0, 0) = 1$
   $C_2 = XOR(P_2, D_3, D_6, D_7, D_{10}, D_{11}) = XOR(0, 1, 0, 1, 0, 0) = 0$
   $C_4 = XOR(P_4, D_5, D_6, D_7, D_{12}) = XOR(0, 1, 0, 1, 0) = 0$
   $C_8 = XOR(P_8, D_9, D_{10}, D_{11}, D_{12}) = XOR(1, 0, 0, 0.0) = 1$
   $C_{8,4,2,1} = 1001_2 = 9_{10} \neq 0$ so there is a two-bit uncorrectable error.

   (b) Repeat (a) for the number 1010 0001 0110 0.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
1	0	1	0	0	0	0	1	0	1	1	0	0

   C_0 = XOR of all bits (P_0 through D_{13}). There are five ones, which is an odd number, indicating a one-bit correctable error.

   $C_1 = XOR(P_1, D_3, D_5, D_7, D_9, D_{11}) = XOR(0, 0, 0, 1, 1, 0) = 0$
   $C_2 = XOR(P_2, D_3, D_6, D_7, D_{10}, D_{11}) = XOR(1, 0, 0, 1, 1, 0) = 1$
   $C_4 = XOR(P_4, D_5, D_6, D_7, D_{12}) = XOR(0, 0, 0, 1, 0) = 1$
   $C_8 = XOR(P_8, D_9, D_{10}, D_{11}, D_{12}) = XOR(0, 1, 1, 0.0) = 0$
   $C_{8,4,2,1} = 0110_2 = 6_{10}$ so bit D_6 was flipped.
   Original number is $D_3 D_5 D_6' D_7 D_9 D_{10} D_{11} D_{12} = (00111100)_2 = 3C_{16}$

   (c) Repeat (a) for the number 1110 0111 1100 1.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
1	1	1	0	0	1	1	1	1	1	0	0	1

   C_0 = XOR of all bits (P_0 through D_{13}). There are nine ones, which is an odd number, indicating a one-bit correctable error.

   $C_1 = XOR(P_1, D_3, D_5, D_7, D_9, D_{11}) = XOR(1, 0, 1, 1, 1, 0) = 0$
   $C_2 = XOR(P_2, D_3, D_6, D_7, D_{10}, D_{11}) = XOR(1, 0, 1, 1, 0, 0) = 1$
   $C_4 = XOR(P_4, D_5, D_6, D_7, D_{12}) = XOR(0, 1, 1, 1, 1) = 0$
   $C_8 = XOR(P_8, D_9, D_{10}, D_{11}, D_{12}) = XOR(1, 1, 0, 0, 1) = 1$
   $C_{8,4,2,1} = 1010_2 = 10_{10}$ so bit D_{10} was flipped.
   Original number is $D_3 D_5 D_6 D_7 D_9 D_{10}' D_{11} D_{12} = (01111101)_2 = 7D_{16}$

5. This problem deals with the Hamming code for error detection and correction. To be able to detect and correct one-bit errors for eight-bit words, five parity bits are required.

   (a) What is the code to store the number 1001 0110

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
?	?	?	1	?	0	0	1	?	0	1	1	0

$P_1 = XOR(D_3, D_5, D_7, D_9, D_{11}) = XOR(1, 0, 1, 0, 1) = 1$
$P_2 = XOR(D_3, D_6, D_7, D_{10}, D_{11}) = XOR(1, 0, 1, 1, 1) = 0$
$P_4 = XOR(D_5, D_6, D_7, D_{12}) = XOR(0, 0, 1, 0) = 1$
$P_8 = XOR(D_9, D_{10}, D_{11}, D_{12}) = XOR(0, 1, 1, 0) = 0$

This gives:

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
?	1	0	1	1	0	0	1	0	0	1	1	0

P_0 = XOR of all bits (P_1 through D_{13}). There are six ones, which is an even number, so P_0 will be 0.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
0	1	0	1	1	0	0	1	0	0	1	1	0

The 13-bit encoded number is 0101 1001 0011 0.

   (b) Repeat (a) for the number 1011 1101.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
?	?	?	1	?	0	1	1	?	1	1	0	1

$P_1 = XOR(D_3, D_5, D_7, D_9, D_{11}) = XOR(1, 0, 1, 1, 0) = 1$
$P_2 = XOR(D_3, D_6, D_7, D_{10}, D_{11}) = XOR(1, 1, 1, 1, 0) = 0$
$P_4 = XOR(D_5, D_6, D_7, D_{12}) = XOR(0, 1, 1, 1) = 1$
$P_8 = XOR(D_9, D_{10}, D_{11}, D_{12}) = XOR(1, 1, 0, 1) = 1$

This gives:

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
?	1	0	1	1	0	1	1	1	1	1	0	1

P_0 = XOR of all bits (P_1 through D_{13}). There are nine ones, which is an odd number, so P_0 will be 1.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
1	1	0	1	1	0	1	1	1	1	1	0	1

The 13-bit encoded number is 1101 1011 1110 1.

(c) Repeat (a) for the number 0100 1011.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
?	?	?	0	?	1	0	0	?	1	0	1	1

$P_1 = XOR(D_3, D_5, D_7, D_9, D_{11}) = XOR(0, 1, 0.1, 1) = 1$
$P_2 = XOR(D_3, D_6, D_7, D_{10}, D_{11}) = XOR(0, 0, 0, 0, 1) = 1$
$P_4 = XOR(D_5, D_6, D_7, D_{12}) = XOR(1, 0, 0, 1) = 0$
$P_8 = XOR(D_9, D_{10}, D_{11}, D_{12}) = XOR(1.0.1, 1) = 1$

This gives:

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
?	1	1	0	0	1	0	0	1	1	0	1	1

P_0 = XOR of all bits (P_1 through D_{13}). There are seven ones, which is an odd number, so P_0 will be 1.

P_0	P_1	P_2	D_3	P_4	D_5	D_6	D_7	P_8	D_9	D_{10}	D_{11}	D_{12}
1	1	1	0	0	1	0	0	1	1	0	1	1

The 13-bit encoded number is 1110 0100 1101 1.