EE 231

Exam 1

September 24, 2008

Name: ____

No calculators allowed. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work.

1. You are tasked with designing a digital thermometer, which will display temperatures from -60^o F to $+140^o$F. (The system displays the temperature to the nearest degree, so you will not need to display fractions of a degree.) The numbers will be stored in a register in signed 2's complement form. What is the minimum number of bits your register will need to hold numbers from -60 to +140? Explain your reasoning.

   In 2's compliment form, numbers range from -2^{N-1} to $2^{N-1} - 1$, where N is the number of bits. For $N = 8$, the range is -128 to +127. For $N = 9$ the range is -256 to +255. Thus, you need a 9 bit register.

2. Convert the following decimal number to hexadecimal. You only need to keep two digits to the right of the decimal point:

   $(73.28)_{10}$

				Quot		Rem					Int		Frac
73	/	16	=	4	R	9	0.28	x	16	=	4	+	0.48
4	/	16	=	0	R	4	0.48	x	16	=	7	+	0.68

   To get the integer part, read the remainder column going up: $(73)_{10} = (49)_{16}$.

   To get the fractional part, read the integer column going down: $(0.28)_{10} = (0.47)_{16}$. (Actually, the last fractional digit should be rounded up, since 0.68 is greater than 1/2.)

   $(73.28)_{10} = (49.48)_{16}$.

3. For this problem, assume all numbers are held in 8-bit registers in a digital system, and the numbers are represented in 2's complement form.

   (a) Convert $(+87)_{10}$ to a 2's complement 8-bit hex number.

				Quot		Rem
87	/	16	=	5	R	7
5	/	16	=	0	R	5

   $(+87)_{10} = (57)_{16}$

   (b) Convert $(-95)_{10}$ to a 2's complement 8-bit hex number. First, find the hex representation for +95:

				Quot		Rem
95	/	16	=	5	R	15
5	/	16	=	0	R	5

   $(+95)_{10} = (5F)_{16}$
   Next, take the 2's complement of 5F to make it negative:
   Ones' complement of 5F is A0, add 1 to ones' complement to get two's complement: A0 + 1 = A1.
   $(-95)_{10} = (A1)_{16}$

   (c) Use the results from (a) and (b) to perform the following operation using 2's complements: $(+87)_{10} - (-95)_{10}$

   $(57)_{16} - (A1)_{16} = (57)_{16} + (5F)_{16} = (B6)_{16}$

   (d) Convert the answer to of Part (c) to its decimal equivalent. Because $(B6)_{16}$ is negative, take its 2's complement, convert that number to decimal, and put a minus sign in front:
   Ones' complement of B6 is 49, add 1 to ones' complement to get two's complement: 49 + 1 = 4A. Convert 4A to decimal: 4 x 16 + 10 x 1 = 74.
   $(B6)_{16} = (-74)_{10}$

4. Use Boolean algebra to simplify the following expressions to a minimum number of literals:

   (a) $(xy + yz' + x'z)(x + z)$

$$(xy + yz' + x'z)(x + z) = xy + xyz' + x'z + xyz + yz'z + x'z$$
$$(xy + yz' + x'z)(x + z) = xy + xyz' + x'z + xyz + x'z$$
$$(xy + yz' + x'z)(x + z) = xy + xyz' + xyz + x'z$$
$$(xy + yz' + x'z)(x + z) = xy(1 + z' + z) + x'z$$
$$(xy + yz' + x'z)(x + z) = xy + x'z$$
$$(xy + yz' + x'z)(x + z) = xy + x'z$$

(b) $(x'y' + z)' + z + xy + wz$

$$(x'y' + z)' + z + xy + wz = ((x'y')'z') + z + xy + wz$$
$$(x'y' + z)' + z + xy + wz = ((x + y)z') + z + wz + xy$$
$$(x'y' + z)' + z + xy + wz = xz' + yz' + z(1 + w) + xy$$
$$(x'y' + z)' + z + xy + wz = xz' + yz' + z + xy$$
$$(x'y' + z)' + z + xy + wz = (x + y)z' + z + xy$$
$$(x'y' + z)' + z + xy + wz = x + y + z + xy$$
$$(x'y' + z)' + z + xy + wz = x + y + z$$

5. Consider the following truth table:

x	y	z	F
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

- Write the function F as a sum of minterms.

  $$F = \sum(0, 1, 4, 6, 7) = m_0 + m_1 + m_4 + m_6 + m_7 = x'y'z' + x'y'z + xy'z' + xyz' + xyz$$

- Use Boolean algebra to reduce the expression to a minimum sum of products.

  $$F = x'y'z' + x'y'z + xy'z' + xyz' + xyz$$
  $$F = x'y'z' + x'y'z + xy'z' + xyz' + xyz' + xyz$$
  $$F = x'y'(z' + z) + xz'(y' + y) + xy(z' + z)$$
  $$F = x'y' + xz' + xy$$

  or

  $$F = x'y'z' + x'y'z + xy'z' + xyz' + xyz$$
  $$F = x'y'z' + x'y'z + x'y'z' + xy'z' + xyz' + xyz$$
  $$F = x'y'(z' + z) + y'z'(x' + x) + xy(z' + z)$$
  $$F = x'y' + y'z' + xy$$

- Write the function F as a product of maxterms.

  $$F = \prod(2, 3, 5) = M_2M_3M_5 = (x + y' + z)(x + y' + z')(x' + y + z')$$