EE 289 – Homework 6
Due October 7, 2010

1. Write a MATLAB function which will calculate the path of a projectile which has an initial velocity v_0 (in meters/second) and an initial angle θ_0 (in degrees). The relevant equations are:

$$x = v_0 \cos(\theta_0) t$$

$$y = v_0 \sin(\theta_0) t - \frac{1}{2} g t^2$$

$$x_{\text{max}} = \frac{2 v_0^2 \cos(\theta_0) \sin(\theta_0)}{g}$$

$$y_{\text{max}} = \frac{v_0^2 \sin^2(\theta_0)}{2g}$$

$$t_{\text{max}} = \frac{2 v_0 \sin(\theta_0)}{g}$$

   If given two arguments (v_0 and θ_0), the function should return x_{max}, y_{max} and t_{max}. If given three arguments (v_0, θ_0 and color), the function should return x_{max}, y_{max} and t_{max}, and should plot the path y vs x in the color given in the third argument.

   For example, to find the maximum distance, height and time:

```
[xmax, ymax, t_max] = projectile(100, 30);
```

   For example, to find the maximum distance, height and time, and to plot the path in red:

```
[xmax, ymax, t_max] = projectile(100, 30,'r');
```

2. Use the above function to plot the paths for an initial velocity of 100 m/s, and the following initial angles: 15°, 30°, 45°, 60°, 75°, and 90°. The paths should be on the same plot, and should all have different colors. (Use `hold on` after plotting the first path.) Use the `legend` command to put a legend on the plot which identifies the different paths by their initial angles. Label the x-axis and y-axis, and give the plot an appropriate title.

3. Use the function from Problem 1 to make plots of the maximum heights, the maximum distances and the maximum times as a function of the initial angle. Plot y_{max}, x_{max} and t_{max} as a function of θ_0. Use enough values of θ_0 to make the curves look smooth.

4. Write an m-file to use the function of Problem 1 to interactively plot the path. Use the `input` command to ask the user for the initial velocity and initial angle.

5. Write a function to find the length of a parametric curve. (For a parametric curve, x and y are given as functions of an independent variable, then plotted against each other. For example, $x = \cos(t)$ and $y = \sin(t)$ define a parametric curve for a circle. Also, the path y vs x of a projectile is a parametric equation with time.) The function should be given handles to the parametric equations for x and y, and the starting and ending values of the independent variable. The length of the curve can be calculated approximately as

$$L = \sum \sqrt{(\Delta x)^2 + \Delta y)^2}.$$

If you have had calculus, an exact solution is given by the following:

$$L = \int_a^b \sqrt{(dx/dt)^2 + (dy/dt)^2}\ dt.$$

MATLAB has a function called `quad` which can be used to calculate an integral. You will get more exact result if you use `quad`. For example, for a circle:

$$x = \cos(t),\ dx/dt = -\sin(t)$$
$$y = \sin(t),\ dx/dt = \cos(t)$$
$$L = \int_a^b \sqrt{(-\sin(t))^2 + (\cos(t))^2}\ dt.$$

```
f1 = @(t) sqrt((-sin(t)).^2 + (cos(t).^2));
L = quad(f1,0,2*pi);
```

This give `L` as exactly 2π, while the summation approximation using 1,000 points gives `L` correct to two parts in a million.

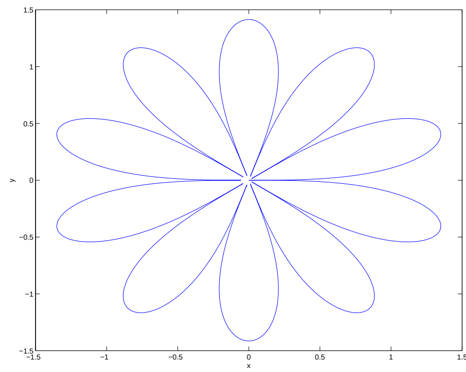
(a) To show that the function works correctly, find the circumference of a circle of radius 1, and show that it is 2π:

```
L = find_length(@cos, @sin, 0, 2*pi);
```

(b) Find the length of the curve defined by:

$$r = \sqrt{|2\sin(5\theta)|}$$
$$x(\theta) = r\cos(\theta)$$
$$y(\theta) = r\sin(\theta)$$

for $0 \le \theta \le 2\pi$.



(c) Find the length of the arc for a projectile with an initial velocity of 100 m/s and an initial angle of 45$^{\text{o}}$. Use the equations for x and y given in Problem 1, with $v_0 = 100$ and $\theta_0 = 45$. The equations can be either a function file, or a function handle. (For example, to make a function handle for the equation $x(t) = t\cos(t)$, do this: `fx = @(t) t.*cos(t);`). Find the length of the arc for $0 \leq t \leq t_{\text{max}}$.