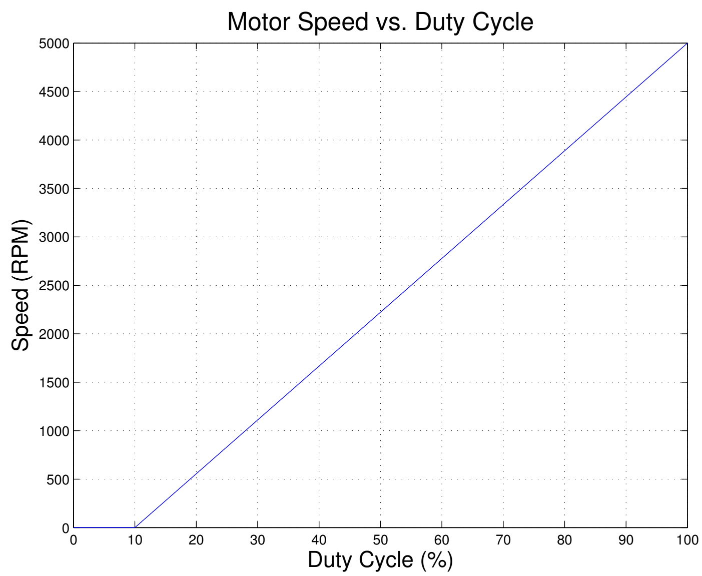
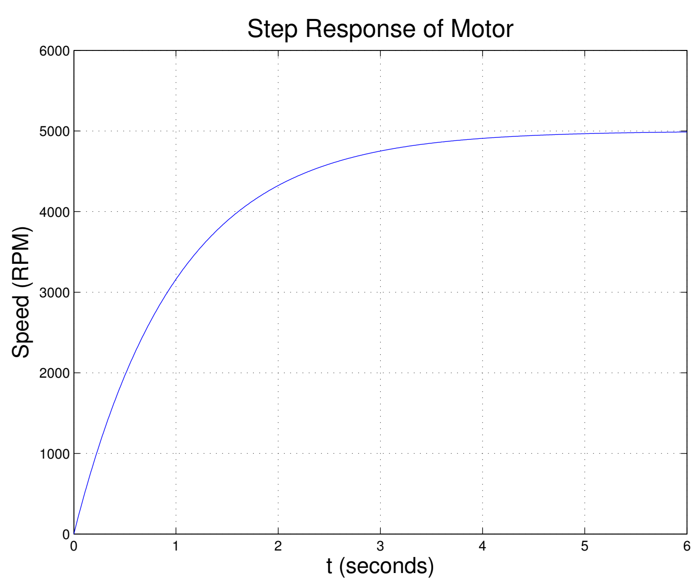
Motor Control

Consider a motor which has a maximum speed of 5000 RPM. The speed vs. duty cycle may look something like this:



The motor doesn't start rotating until it is driven with a 10% duty cycle, after which it will increase speed linearly with the increase in duty cycle.

If the motor is initially stopped, and is then turned on (with 100% duty cycle), the speed vs. time might look something like this (the step response of the motor):



We will control the motor by adjusting the duty cycle with the MC9S12. We will do this by measuring the speed and updating the duty cycle on a regular basis. Let's do the adjustments once every 8 ms. This means that we will adjust the duty cycle, wait for 8 ms to find the new speed, then adjust the duty cycle again. How much change in speed will there be in 8 ms? The motor behaves like a single time constant system, so the equation for the speed as a function of time is:

$$S(t) = S_f + e^{-t/\tau}(S_i - S_f)$$

where S_i is the speed at time 0, S_f is the speed at time ∞, and τ is the time constant of the system. With a duty cycle of D, the final speed will be:

$$S_f = \alpha D + S_0$$

where S_0 is the speed the motor would turn with a 0% duty cycle if the speed continued linearly for duty cyclces less than 10%, and α is the slope of the speed vs. duty cycle line (5000/0.9 in this example).

Here I assume that the time constant of the small motors we are using is about 1 second — i.e., it takes about 5 seconds (5 time constants) for the motor to go from a dead stop to full speed. If $T = 8$ ms, the motor will have changed its speed from S_i to

$$S(T) = S_f + e^{-T/\tau}(S_i - S_f)$$
$$S(T) = (\alpha D + S_0)(1 - e^{-T/\tau}) + e^{-T/\tau} S_i$$
$$S[n] = (\alpha D + S_0)(1 - e^{-T/\tau}) + e^{-T/\tau} S[n-1]$$

where $S[n]$ is the speed at the n^{th} cycle.

Consider an integral controller where the duty cycle is adjusted according to:

$$D[n] = D[n-1] + k(S_d - S_m[n])$$

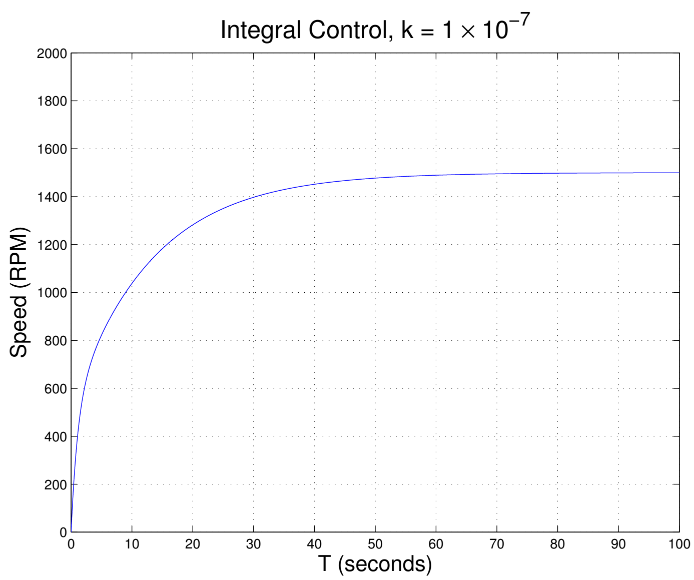
We can simulate the motor response by iterating through these equations. Start with $S_m[1] = 0$, $D[1] = 0$, and $S_d = 1500$. Then we calculate:

$$S_m[n] = (\alpha D[n-1] + S_0)(1 - e^{-T/\tau}) + e^{-T/\tau} S_m[n-1]$$
$$D[n] = D[n-1] + k(S_d - S_m[n])$$

In MATLAB we can simulate this as:

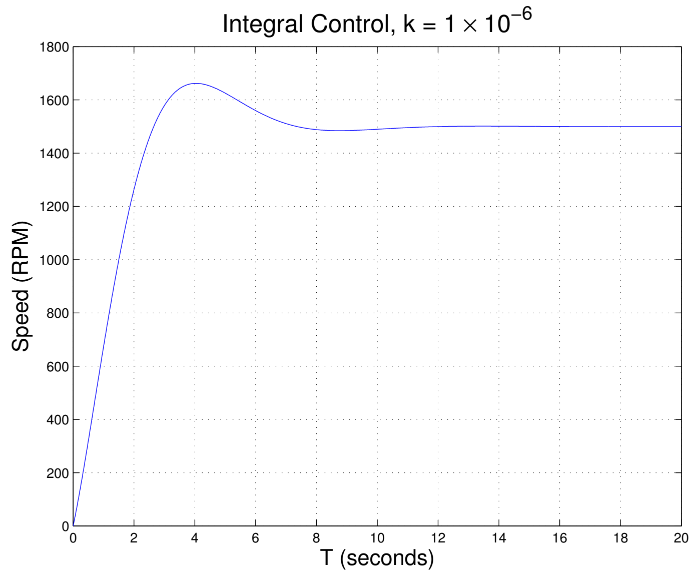
```
alpha = 5000/0.9;  % Max speed 5,000 RPM; turns on at 10% duty cycle
Sd = 1500;         % Desired Speed
S0 = -alpha*0.1;   % Speed motor would turn at 0% duty cycle if linear
tau = 1;           % One second time constant
T = 8e-3;          % Update rate is 8 ms
k = 1e-7;          % Constant for integral control

Sm = 0;            % Measured speed starts at 0
D = 0.1;           % Duty cycle starts at 10%
t = 0;
ee = exp(-T/tau);  % Precalculate this commonly used value

for n=2:1000       % Make end value bigger if needed
 Sm(n)=(alpha*D(n-1) + S0)*(1-ee) + ee*Sm(n-1);
 D(n) = k*(Sd - Sm(n)) + D(n-1);
 t(n) = t(n-1)+T;
end
plot(t,Sm);
```

By changing the value of k we can see how this parameter affects the response. Here is the curve for $k = 1.0 \times 10^{-7}$:



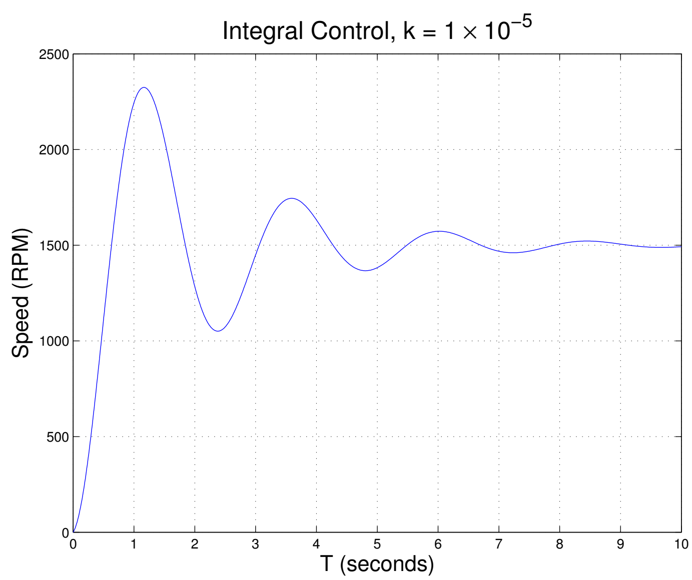
With this value of k, it will take about 1 minute for the motor to get to the desired speed.

Here is the curve for $k = 1.0 \times 10^{-6}$:

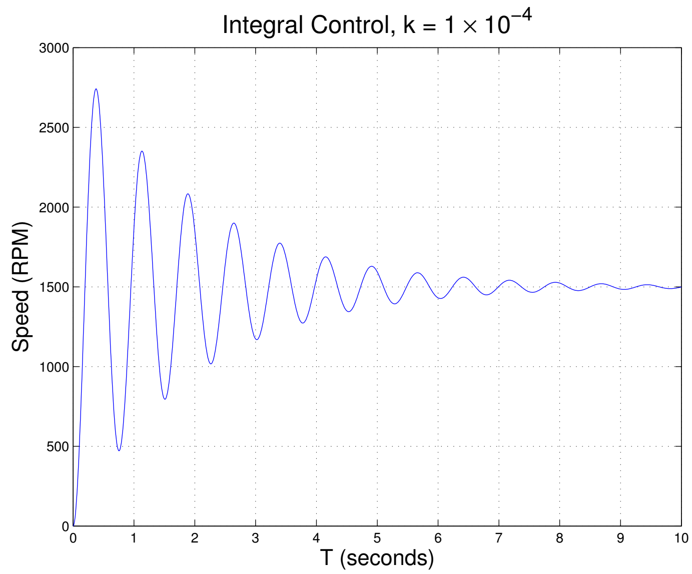


Now it takes about 10 seconds to get to the desired speed, with a little bit of overshoot.

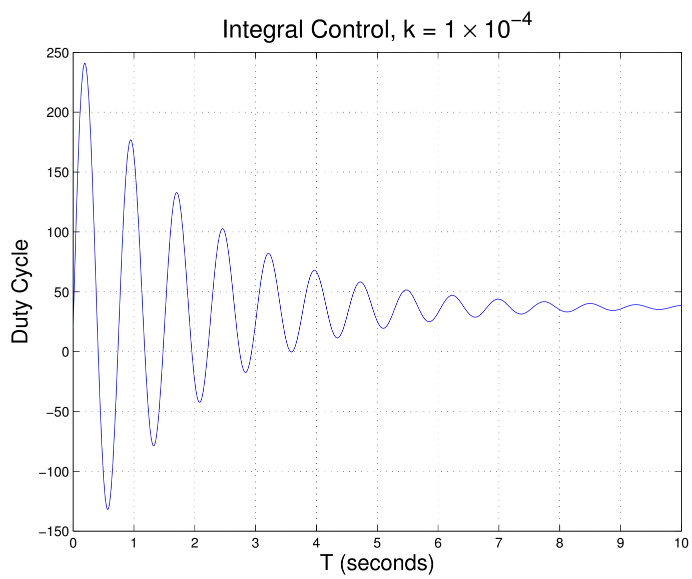
Let's try $k = 1.0 \times 10^{-5}$:



This gets to the desired value more quickly, but with a lot of oscillation. Let's increase k to 10^{-4}.



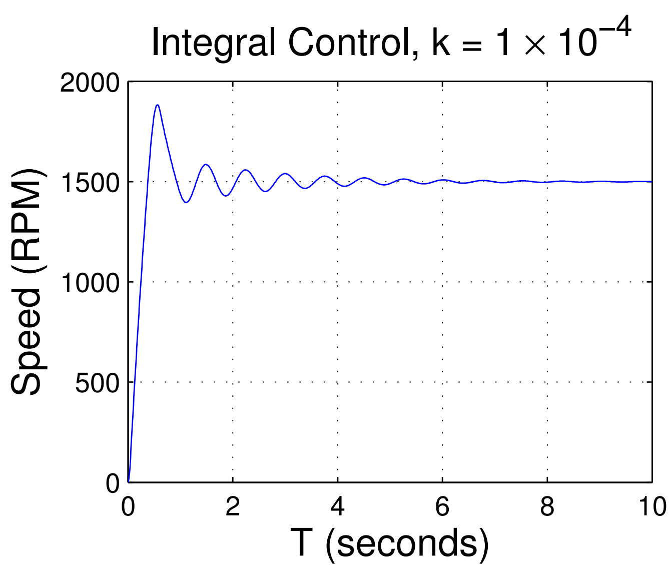
For this value of k there is a significant oscillation. However, a real motor will not act like this. If we look at the duty cycle vs time, we see:

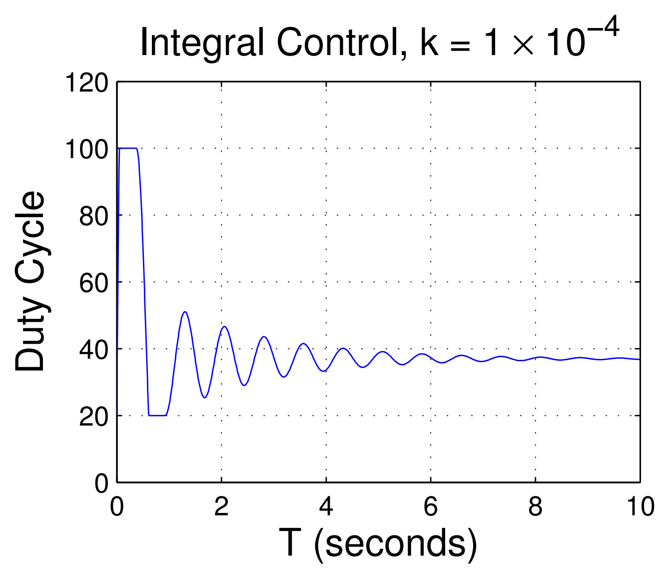


To get this oscillating response, the duty cycle must go to over 100%, and below 0%, which is clearly impossible. To get the response we expect in the lab, we need to limit the duty cycle to remain between 20% and 100%. Thus, we change our simulation to be:

```
alpha = 5000/0.9;   % Max speed 5,000 RPM; turns on at 10% duty cycle
Sd = 1500;          % Desired Speed
S0 = -alpha*0.1;    % Speed motor would turn at 0% duty cycle if linear
tau = 1;            % One second time constant
T = 8e-3;           % Update rate is 8 ms
k = 1e-7;           % Constant for integral control

Sm = 0;             % Measured speed starts at 0
D = 0.1;            % Duty cycle starts at 10%
t = 0;
ee = exp(-T/tau);   % Precalculate this commonly used value

for n=2:1000        % Make end value bigger if needed
 Sm(n)=(alpha*D(n-1) + S0)*(1-ee) + ee*Sm(n-1);
 if (Sm(n) < 0) Sm(n) = 0; end; % Motor speed cannot be less than 0
 D(n) = k*(Sd - Sm(n)) + D(n-1);
 if (D(n) > 1.0) D(n) = 1.0; end;  % Keep DC between 20% and 100%
 if (D(n) < 0.2) D(n) = 0.2; end;
 t(n) = t(n-1)+T;
end
plot(t,Sm);
```

When we use this to simulate the motor response, we get:





In your program for Lab 5, you will use a Real Time Interrupt with an 8 ms period. In the RTI interrupt service routine, you will measure the speed, and set the duty cycle based on the measured speed. Your ISR will look something like this:

```
void INTERRUPT rti_isr(void)
{
    Code to read potentiometer voltage and convert into RPM

    Code to measure speed Sm in RPM

    Code which sets duty cycle to

        DC = DC + k*(Sd-Sm)
        if (DC > 1.0) DC = 1.0;
        if (DC < 0.2) DC = 0.2;

    Code which writes the PWM Duty Cycle Register
    to generate duty cycle DC.

    Code which clears RTI flag
}
```

In the main program, you will display the measured speed, desired speed, and duty cycle on the LCD display.

Your values of k will probably be different than the values in these notes because speed vs. duty cycle, time constant, and maximum speed will most likely be different than the values I used.