1. Consider the following circuit:

The MOSFET’s are matched, and $k_n'(W/L) = 2 \text{ mA/V}^2$, and $V_t = -2$ V. Ignore the output resistance $r_o$, and assume all capacitors are large (all AC signals are passed through, all DC signals are blocked).

(a) Find $I_{D1}$, the drain current through $Q_1$.
(b) What are the bias voltages $V_A$, $V_B$, $V_C$, $V_D$, $V_E$ and $V_F$?
(c) Draw the small-signal equivalent circuit for the amplifier.
(d) What is $g_{m2}$, the transconductance of $Q_2$?
(e) What is the input resistance?
(f) What is the voltage gain $v_o/v_i$?
(a) \[ T_{D_1} = \frac{1}{2} h'_n\left(\frac{w}{L}\right)(V_{G_S_1} - V_t)^2 \] (assuming sat.)

and \[ T_{D_1} = \frac{(V_A - (-15V))}{1k} \]

\[ V_{G_S_1} = V_G - V_S_1 = -15V - V_A \]

\[ \frac{1}{2} h'_n\left(\frac{w}{L}\right)(-15V - V_A - (-3V))^2 = \frac{(V_A - (-15V))}{1k} \]

\[ \frac{1}{2} (2mA/V^2)(-13V - V_A)^2 = \frac{(V_A + 15V)}{1k} \]

Multiply through by 1k

\[ 1V^{-1}(-13V - V_A)^2 = (V_A + 15V) \]

\[ 169 + 26V_A + V_A^2 = V_A + 15 \]

\[ V_A^2 + 25V_A + 144 = 0 \]

\[ V_A = \frac{-25 \pm \sqrt{(25)^2 - 4(144)}}{2} = \frac{-25 \pm 5i}{2} = -11 - 14i \]

If \[ V_A = -11V \]

Then \[ V_{G_S_1} = V_G - V_A = -15 - (-11) = -4V \]

\[ V_{G_S_1} < V_{T_1} \]

so Q is off - cannot be

If \[ V_A = -14V \]

Then \[ V_{G_S_1} = V_G - V_A = -15 - (-14) = -1V \]

\[ V_{G_S_1} > V_{T_1} \]

so Q is on - correct

\[ V_A = -14V \]

\[ T_{D_1} = \frac{V_A - (-15V)}{1k} = \frac{-14V + 15V}{1k} = 1mA \]

\[ T_{D_1} = 1mA \]
(b) $V_A = -14\,V$  (from (a))

$V_D = 0\,V$  (can open for DC)

$V_E = 0\,V$  ("")

$I_{D_2} = I_0, \Rightarrow V_{G_{S_2}} = V_{G_{S_1}} = -1\,V$

$V_{G_{S_2}} = V_D - V_C \Rightarrow V_C = V_D - V_{G_{S_2}} = 0 - (-1\,V) = 1\,V$

$V_C = +1\,V$

$\frac{V_C - V_B}{100} = I_0 = 1\,mA$

$V_B = V_C - I_D (100) = 1\,V - (1\,mA)(100\,\Omega) = 0.9\,V$

$\frac{15\,V - V_E}{10\,k} = I_D$

$V_E = 15\,V - I_D (10\,k) = 15\,V - (1\,mA)(10\,k\,\Omega) = 5\,V$

$V_A = -14\,V$

$V_B = 0.9\,V$

$V_C = 1\,V$

$V_D = 0\,V$

$V_E = 5\,V$

$V_F = 0\,V$
(d) \[ g_m = \frac{I_D}{(V_{S2} - V_{S1})/2} = \frac{1 \text{ mA}}{(-1V - (-2V))/2} = 2 \text{ mS/V} \]

(e) No current flows into gate, so \( R_{in} = \infty \)

(f) From small signal model

\[ V_o = -g_m V_{in} (10k \parallel 10k) \]

\[ \frac{V_o}{V_{in}} = -g_m (5k) = -(2 \text{ mS/V}) (5k) = -10 \]
2. For the following problem it is known that \( V_t = 2 \) V for the NMOS FET, but \( k'_n(W/l) \) is not known.

(a) Your lab partner builds the following circuit using the MOSFET, but the circuit fails to function. What mistake has been made, and how would you correct it?

(b) After making the correction, you find that \( V_o = 6 \) V when \( V_{in} = 4 \) V. Determine \( k'_n(W/L) \) for the MOSFET.

(c) The MOSFET is next used as a switch by changing \( R_D \) from 1 k\( \Omega \) to 100 k\( \Omega \). Find \( V_D \) when \( v_{in} = 0 \) V, and and when \( v_{in} = 4 \) V.
(a) New to connect body to source, not drain

\[ V_{1n} \]

(b) \[ V_{in} = 4V \Rightarrow V_{GS} = 4V \]
\[ V_b = 6V \Rightarrow I_D = \frac{10V - 6V}{1k} = 4mA \]
\[ V_{DS} = V_b = 6V \]
\[ V_{DS} > V_{GS} - V_t \Rightarrow Q \text{ in saturation} \]
\[ I_D = \frac{1}{2} h_m' \left( \frac{W}{L} \right) (V_{GS} - V_t)^2 \]
\[ 4mA = \frac{1}{2} h_m' \left( \frac{W}{L} \right) (4 - 2)^2 \]
\[ h_m' \left( \frac{W}{L} \right) = 2 \text{ mA V}^{-2} \]

(c) \[ V_{in} = 0 \Rightarrow V_{GS} = 0 \Rightarrow Q \text{ off} \Rightarrow I_D = 0 \Rightarrow V_b = 10V \]
\[ V_{in} = 4V \Rightarrow V_{GS} > V_t \Rightarrow Q \text{ will be in triode} \]
\[ Q \text{ will act like a resistor} \]
\[ V_b = \frac{10V}{10k + r_{DS}} \]
\[ r_{DS} = \frac{1}{h_m' \left( \frac{W}{L} \right) (V_{GS} - V_t)} = \frac{1}{2 \text{ mA V}^{-2}} \frac{1}{(4V - 2V)} \]
\[ r_{DS} = \frac{250\Omega}{10k + 250} \]
\[ V_b = \frac{250\Omega}{10k + 250} 10V = 0.24V \]
3. For the MOSFETs in the circuit below \( k'_n(W/L) = 5 \, \mu A/V^2 \), and \( V_t = 2 \, V \). Select \( R \) to obtain \( V_c = 6 \, V \). Neglect the effect of \( r_o \).

\[
\begin{align*}
Q_1 \text{ and } Q_2 \text{ identical} \\
I_{D_1} = I_{D_2} \Rightarrow V_{G_3} = V_{G_2} \Rightarrow 8 - V_A = V_{A} \Rightarrow V_A = 4 \, V \\
I_{D_1} = \frac{1}{2} k'_n \left( \frac{W}{L} \right) (V_{G_3} - V_t) = \frac{1}{2} (5 \, \mu A/V^2) (4 - 2 \, V)^2 \\
= 20 \, \mu A
\end{align*}
\]

\( Q_3 \) and \( Q_2 \) identical, and \( V_{G_3} = V_{G_2} \), so

\[
I_{D_3} = I_{D_2} = I_{D_1} = 20 \, \mu A
\]

\[
I_{D_3} = \frac{qV - V_0}{R} \Rightarrow R = \frac{qV - V_0}{I_{D_3}} = \frac{8V - 6V}{20 \, \mu A} = 100 \, k\Omega
\]