EE 321 - Exam 4

December 8, 2002

Name: Solutions

Closed book. One page of notes and a calculator are allowed. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work.

For all the circuits below, assume the capacitors are large so they block DC and pass all AC signals of interest.

1. A lab technician working for you has measured $k'(W/L)$, V_t, v_{GS}, v_{DS} and i_D for a number of different MOSFETs, some n-channel and some p-channel. He forgot to indicate the type of MOSFET on the table. You need to check to see if the data is consisent. (An example of inconsistent data is if the measured current is different from the current calculated from the other data in the table.) If the data is consistent, complete the table — determine the type of MOSFET (n-channel or p-channel) and the mode of operation (triode or saturation).

	$k'(W/L)$ (mA/V^2)	V_t (V)	v_{GS} (V)	v_{DS} (V)	i_D (mA)	Type	Mode of Operation
(a)	4	1	3	4	8	N	saturation
(b)	1	2	1	2	4	—	—
(c)	2	-1	-2	-2	1	P	saturation
(d)	2	-2	1	2	8	N	triode
(e)	2	1	-1	-3	4	P	saturation

(a) Is the data consistent? If so, fill in the table. Explain your reasoning.

$v_{GS} > V_t \Rightarrow$ N-Channel $\quad V_t > 0 \Rightarrow$ enhancement $\quad v_{DS} > v_{GS} - V_t \Rightarrow$ saturation

$i_D = \frac{1}{2} k_n' \frac{W}{L} (v_{GS} - V_t)^2 = \frac{1}{2}(4\,mA/V^2)(3V - 1V)^2 = 8\,mA$ ✓

(b) Is the data consistent? If so, fill in the table. Explain your reasoning.

$v_{GS} < V_t \Rightarrow$ P-Channel $\quad V_t > 0 \Rightarrow$ depletion $\quad v_{DS} > v_{GS} - V_t \Rightarrow$ triode

$i_D = k_p' \frac{W}{L} [(v_{GS} - V_t) v_{DS} - \frac{1}{2} v_{DS}^2] = (1\,mA/V^2)[(1V - 2V)2V - \frac{1}{2}(2V)^2] = -4\,mA$ ✗

(c) Is the data consistent? If so, fill in the table. Explain your reasoning.

$v_{GS} < V_t \Rightarrow$ P-channel $\quad V_t < 0 \Rightarrow$ enhancement $\quad v_{DS} < v_{GS} - V_t \Rightarrow$ saturation

$i_D = \frac{1}{2} k_p' \frac{W}{L} (v_{GS} - V_t)^2 = \frac{1}{2}(2\,mA/V^2)(-2V - (-1V))^2 = 1\,mA$ ✓

(d) Is the data consistent? If so, fill in the table. Explain your reasoning.

$v_{GS} > V_t \Rightarrow$ N-channel $\quad V_t < 0 \Rightarrow$ depletion $\quad v_{DS} < v_{GS} - V_t \Rightarrow$ triode

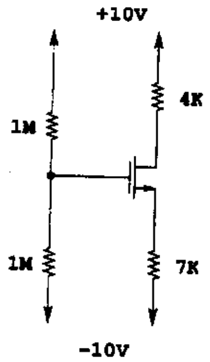
$i_D = k_n' \frac{W}{L} [(v_{GS} - V_t) v_{DS} - \frac{1}{2} v_{DS}^2] = (2\,mA/V^2)[(1V - (-2V))2V - \frac{1}{2}(2V)^2] = 8\,mA$ ✓

(e) Is the data consistent? If so, fill in the table. Explain your reasoning.

$v_{GS} < V_t \Rightarrow$ P-channel $\quad V_t > 0 \Rightarrow$ depletion $\quad v_{DS} < v_{GS} - V_t \Rightarrow$ saturation

$i_D = \frac{1}{2} k_p' (\frac{W}{L})(v_{GS} - V_t)^2 = \frac{1}{2}(2\,mA/V^2)(-1V - 1V)^2 = 4\,mA$ ✓

2. The MOSFET in the circuit below has $V_t = 2$ V and $k_n'(W/L) = 2$ mA/V^2.



(a) Find the drain current I_D.
(b) Find the gate-source voltage V_{GS}
(c) Find the drain-source voltage V_{DS}
(d) What mode is the MOSFET operating in?

(a) Assume saturation

$I_D = \frac{1}{2} k_n'\left(\frac{W}{L}\right)(V_{GS} - V_t)^2$

$V_G = \frac{1m}{1m+1m} 20V - 10V = 0V \quad V_{GS} = V_G - V_S = -V_S$

Also, $I_D = \frac{V_S - (-10V)}{7k} \Rightarrow V_S = I_D \cdot 7k - 10V$

$I_D = \frac{1}{2}(2mA/V^2)(-(7I_D - 10) - 2)^2$

$I_D = 1\, mA/V^2(-7I_D + 8)^2$

$I_D = 49I_D^2 - 112I_D + 64$

$49I_D^2 - 113I_D + 64 = 0$

$I_D = \frac{113 \pm \sqrt{(113)^2 - 4(49)(64)}}{2 \cdot 49} = \frac{113 \pm 15}{98} = 1.3mA$ or $1.0mA$

For $I_D = 1.3mA$, $V_S = I_D \cdot 7k - 10V = -0.9V \Rightarrow V_{GS} = 0.9V \Rightarrow V_{GS} < V_t \Rightarrow$ off

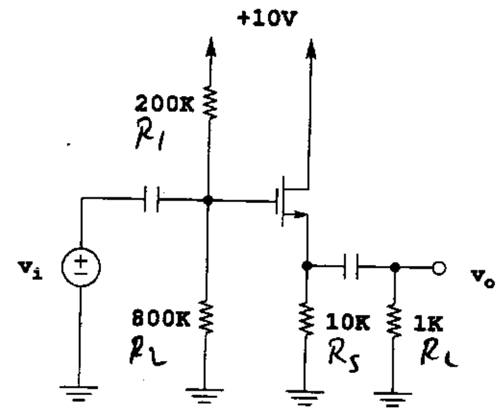
$\therefore I_D = 1mA$

(b) $V_{GS} = V_G - V_S = 0 - (7k \cdot I_D - 10V) = 3V$

(c) $V_{DS} = V_D - V_S = (10V - 4k I_D) - (-3V) = 6V + 3V = 9V$

(d) $V_{GS} > V_t$, $V_{DS} > V_{GS} - V_t \Rightarrow$ saturation

3. The MOSFET below has $V_t = 2$ V, $k_n'(W/L) = 1$ mA/V^2 and $\lambda = 0.02$ V^{-1}. The drain current has been found to be $I_D = 0.5$ mA.



(a) Draw the small signal model for the circuit.

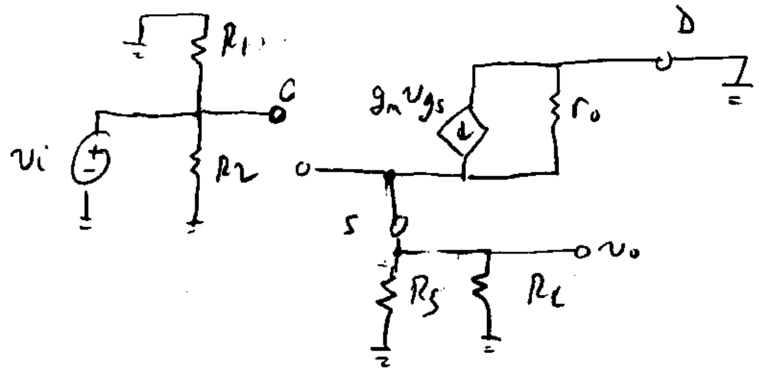
(b) Find the transconductance g_m.

(c) Find the drain-source resistance of the MOSFET, r_o.

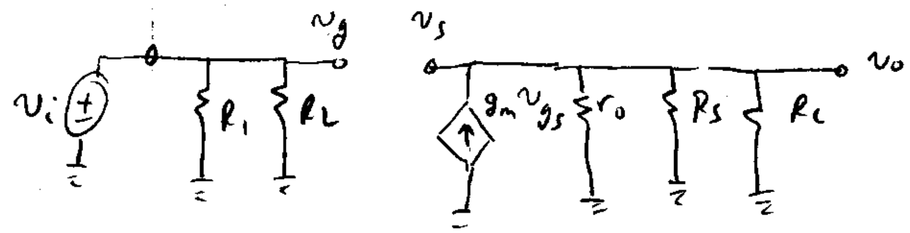
(d) Find the output gain of the circuit, v_o/v_i.

(e) Find the input resistance R_i.

(a)



Simplifies



note: $v_{gs} = v_i - v_o$

(b) $g_m = \sqrt{2 k_n' (W/L)}\sqrt{I_D} = \sqrt{2(1 \text{mA/V}^2)(0.5 \text{mA})} = 1$ mA/V

(c) $r_o = \frac{1}{\lambda I_D} = \frac{1}{(0.02 V^{-1})(0.5 mA)} = 100$ kΩ

(d) $v_o = g_m v_{gs} (r_o \parallel R_S \parallel R_L)$

$= g_m (v_i - v_o)(r_o \parallel R_S \parallel R_L)$

$$v_o = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)} v_i$$

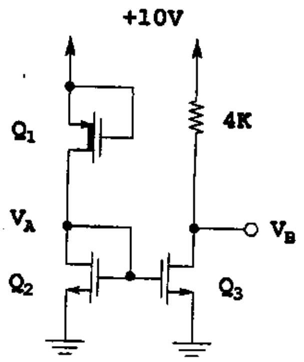
$$\frac{v_o}{v_i} = \frac{g_m (r_o \parallel R_S \parallel R_L)}{1 + g_m (r_o \parallel R_S \parallel R_L)}$$

$$= \frac{(1\,mA/V)(100k \parallel 10k \parallel 1k)}{1 + (1\,mA/V)(100k \parallel 10k \parallel 1k)}$$

$= 0.47$ V/V

(e) $R_I = R_1 \parallel R_2 = 200k \parallel 800k = 160k$

4. For all the MOSFETs below, $|V_t| = 2$ V. For Q_1, $k'_p = 0.4$ mA/V^2, and $W/L = 5$. For Q_2, $k'_n = 1.0$ mA/V^2, and $W/L = 2$. For Q_3, $k'_n = 1.0$ mA/V^2, and $W/L = 1$.



(a) Find the voltage V_A.

(b) Find the voltage V_B.

(a) For Q_1, $V_t = 2V$ (depletion p-mos)

$V_{GS} = 0 < V_t \Rightarrow$ on

$I_{D_1} = \frac{1}{2} k_{p_1}' \left(\frac{W}{L}\right)_1 (V_{GS_1} - V_t)^2 = \frac{1}{2}(0.4)(5)(0-2V)^2 = 4mA$

$I_{D_2} = I_{D_1} = 4mA$

$I_{D_2} = \frac{1}{2} k_{n_2}' \left(\frac{W}{L}\right)_2 (V_{GS_2} - V_t)^2 = \frac{1}{2}(1)(2)((V_A - 0) - V_t)^2$

$4mA = (V_A - 2)^2 \Rightarrow V_A - 2 = \pm 2 \Rightarrow V_A = 4V$ or $V_A = 0V$

$V_A = 0V$ does not make sense so $V_A = 4V$

$V_{DS_2} = 4V > V_{GS_2} - V_{t_2}$ so Q_2 in saturation as assumed

(b) $V_{GS_3} = V_A - 0 = 4V$

$I_{D_3} = \frac{1}{2} k_{n_3}' \left(\frac{W}{L}\right)_3 (V_{GS_3} - V_t)^2 = \frac{1}{2}(1)(1)(4V - 2V)^2$

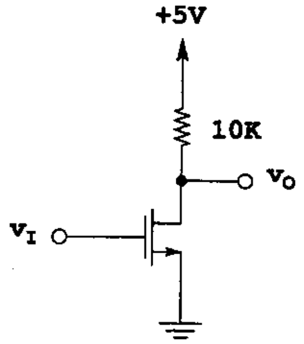
$= 2mA$

$V_B = 10V - 4K \, I_{D_3} = 10V - 8V = 2V$

$V_{DS} = V_B - 0V = 2V$ $\quad$ $V_{GS} - V_t = (V_A - 0V) - 2V = 2V$

$V_{DS} = V_{GS} - V_{t}$ so Q_3 on edge of saturation, which is ok

5. For the MOSFET in the circuit below, $k_n'(W/L) = 5\ \text{mA/V}^2$, and $V_t = 1$ V.



(a) Find the output voltage v_O when $v_I = 0$ V.

(b) Find the output voltage v_O when $v_I = 5$ V.

(a) $v_I = 0 \Rightarrow v_{GS} = 0 \Rightarrow Q$ off $\Rightarrow I_D = 0 \Rightarrow V_o = 5V$

(b) $v_I = 5V \Rightarrow v_{GS} = 5V \Rightarrow Q$ on

Because R_D is so large, Q will probably be in triode.

Assume triode:

$$r_{DS} = \frac{1}{k_n' \frac{W}{L}(v_{GS} - V_t)} = \frac{1}{(5mA/V^2)(5V - 1V)} = 50\,\Omega$$

$$v_O = \frac{r_{DS}}{r_{DS} + R_D} 5V = \frac{50}{50 + 10k} 5V \approx 25\,mV$$

Note: $v_{DS} = v_O$

$v_{DS} < v_{GS} - V_t$ so Q in triode as assumed