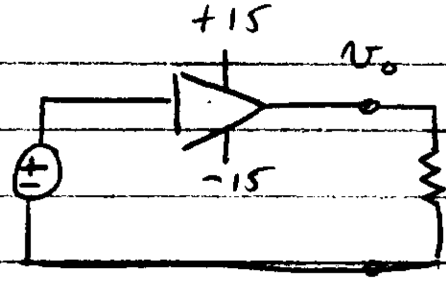
EE 321

Fall 2002

Homework #2

Solutions

1.13



$v_o = 12\sin(\omega t)$

Power from +15V supply is $VI = 15^V \cdot 2ma = 30mW$

Power from -15V supply is also 30mW

Total power used is 60mW

Power dissipated by load resistor is

$$P_L = (V_{o\,rms})^2/R_L = \left(\frac{12V}{\sqrt{2}}\right)^2/2K\Omega = 36mW$$

Power dissipated by amp is $60mW - 36mW = 24mW$

D1.15 $v_o = 10 - 5(v_I - 2)^2 \qquad 2 \le v_I \le v_o + 2$

When $v_I = v_o + 2$, we have $v_o = 10 - 5(v_o + 2 - 2)^2$

$5v_o^2 + v_o - 10 = 0 \qquad v_o = -1.518,\ 1.318$

$v_o + 2 = 0.482,\ 3.318$

Thus $2 \le v_I \le 3.318$

(a) When $v_I = 2$, $v_o = 10$, so $L_+ = 10V$

When $v_I = 3.318$, $v_o = 1.318$, so $L_- = 1.318V$

(b) ~~$A_v = \frac{dv_o}{dv_I}\Big|_{v_I}$~~ $\quad v_o = 5V \Rightarrow 5 = 10 - 5(v_I - 2)^2$

$v_I = 1V,\ 3V$

1V is outside range, so $V_I = 3V$ gives $V_o = 5V$

(c) $A_v = \left.\frac{dv_o}{dv_I}\right|_{v_I = 3V} = \left.-10(v_I - 2)\right|_{v_I = 3} = -10$

(d) $v_I = V_I + V_i \cos\omega t = 3 + V_i \cos\omega t$

$$v_O = 10 - 5(v_I - 2)^2 = 10 - 5(1 + V_i \cos\omega t)^2$$

$$\cancel{= 10 - 5(1 + V_i\ \cos}$$

$$= 10 - 5(1 + 2V_i \cos\omega t + V_i^2 \cos^2\omega t)$$

$$= 5 - 10 V_i \cos\omega t - 5 V_i^2 \cos^2\omega t$$

$$= V_O - 10 V_i \cos\omega t - 5 V_i^2 \cos^2\omega t$$

$$v_o = -10 V_i \cos\omega t - 5 V_i^2 \cos^2\omega t$$

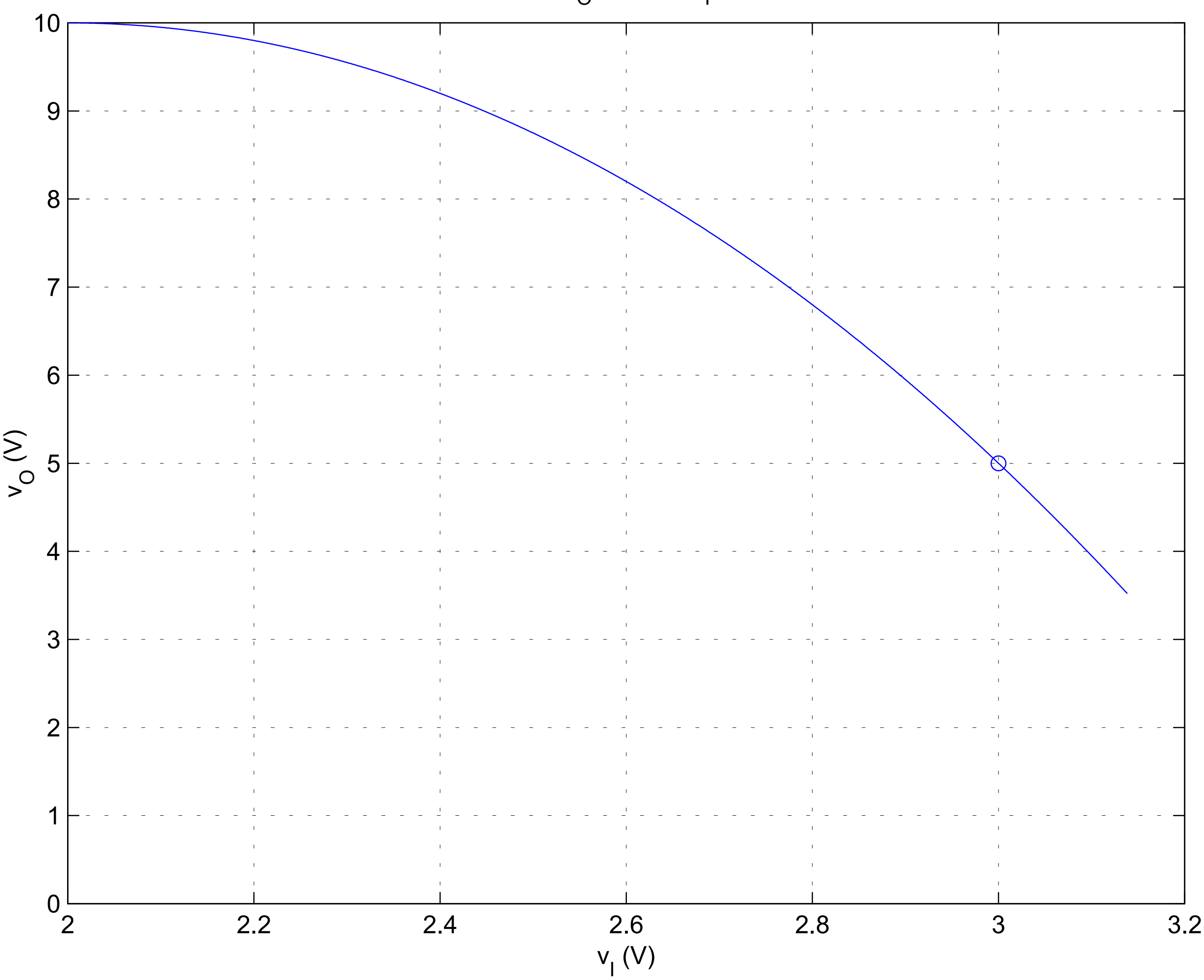
$$= -10 V_i \cos\omega t - 5\left(\frac{V_i^2}{2} + \frac{V_i^2}{2}\cos 2\omega t\right)$$

$$= -\frac{5V_i^2}{2} \underbrace{- 10 V_i \cos\omega t}_{\text{fundamental}} \underbrace{- \frac{5}{2} V_i^2 \cos 2\omega t}_{2^{\text{nd}} \text{ harmonic}}$$

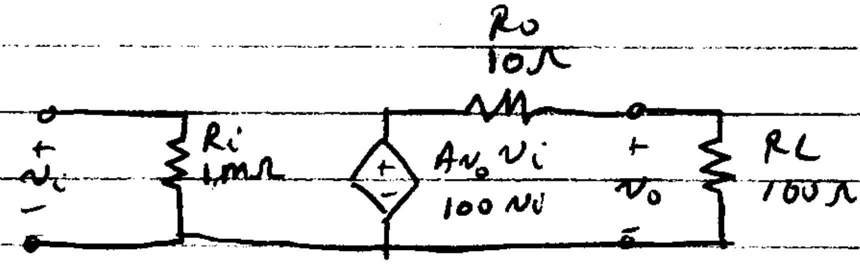
Want $\dfrac{\frac{5}{2} V_i^2}{10 V_i} \leq 0.01 \Rightarrow V_i \leq 0.04\,\text{V} = 40\,\text{mV}$

```
% PROBELM 1.15
%
% Using algebra, found the following values:
vI_min = 2;  vI_max = 3.138;
% Make vector for input voltage with 1 mV spacing
vI = vI_min:0.001:vI_max;
% Calculate output voltage
vO = 10 - 5*(vI - 2).^2;
% Plot vO vs. vI
plot(vI,vO);
axis([2 3.2 0 10])
grid
title('Plot of v_O = 10 - 5(v_I - 2)^2')
xlabel('v_I (V)')
ylabel('v_O (V)')
% From plot or algebra, find vO = 5V when vI = 3V
VI = 3; VO = 5;
% Plot Q point on vO vs. vI plot
hold on
plot(VI,VO,'o')
print
```

Plot of $v_O = 10 - 5(v_I - 2)^2$



1.18



$$v_o = \frac{R_L}{R_o + R_L} A_{v_o} v_i = \frac{100}{10+100} 100\,v_i = 90.9\,v_i$$

$$A_v = \frac{v_o}{v_i} = 90.9 \qquad A_v(dB) = 20\log_{10} A_v = 39\ dB$$

$$P_o = \frac{v_o^2}{R_L} = \frac{(90.9\,v_i)^2}{100}$$

$$P_i = \frac{v_i^2}{R_i} = \frac{v_i^2}{10^6}$$

$$A_p = \frac{P_o}{P_i} = \frac{90.9^2/100}{1/10^6} = 8.26 \times 10^7$$

$$A_p(dB) = 10\log_{10} A_p = 79\ dB$$

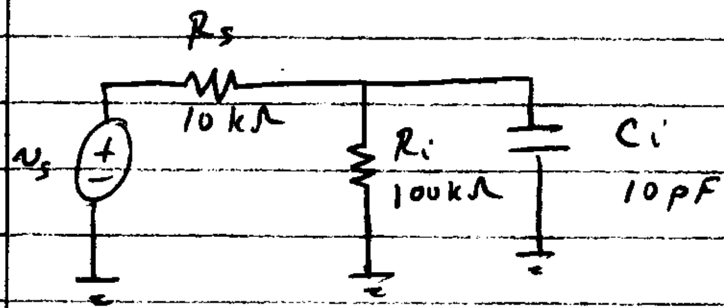
$$v_{o_{max}} = \cancel{\ \ } \ i_{o_{max}} R_L = \cancel{90.9\,v_i\ 100}\ 90.9\,v_{i_{max}}$$

$$v_{i_{max}} = \frac{i_{o\,max} R_L}{90.9} = \frac{0.1 \times 100}{90.9} = 0.11\,V$$

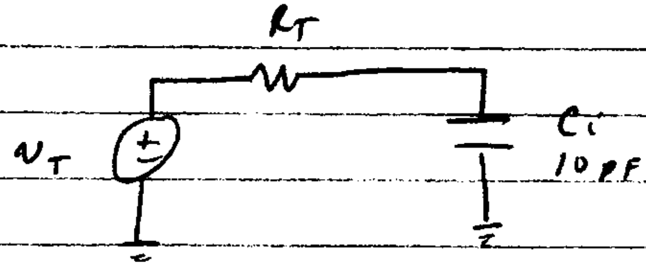
$$v_{i_{Rms}} = \frac{0.11V}{\sqrt{2}} = 0.079 = 79\,mV$$

$$P_o = (i_{Rms})^2 R_L = \left(\frac{0.1}{\sqrt{2}}\right)^2 100 = 0.5W$$

1.34



Thevenin Equiv:



$R_T = R_s \parallel R_i = 9.1\,k\Omega \quad v_T = \dfrac{R_i}{R_i + R_s} v_s = 0.91\, v_s$

$$T(s) = \frac{V_i(s)}{V_s(s)} = \frac{\frac{1}{C_i s}}{R_T + \frac{1}{C_i s}} \frac{V_T(s)}{V_s(s)}$$

$$= \frac{1}{1 + R_T C_i s} \frac{R_i}{R_i + R_s} \frac{V_s(s)}{V_s(s)}$$

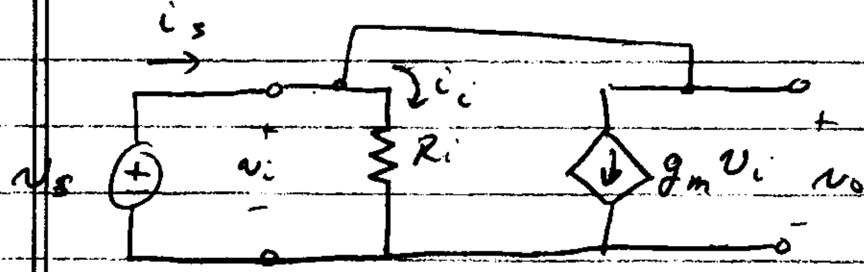
$$= \frac{1}{1 + R_T C_i s} \frac{R_i}{R_i + R_s} \quad \text{STC eqn}$$

$$= \cancel{\frac{1}{1 + (9.1k)(10p)s}}$$

$$\omega_{3dB} = \frac{1}{R_T C_i} = \frac{1}{(9.1k)(10pF)} = 1.1 \times 10^7 \text{ rad/sec}$$

$$f_{3dB} = \frac{\omega_{3dB}}{2\pi} = 1.8\text{ MHz}$$

1.27

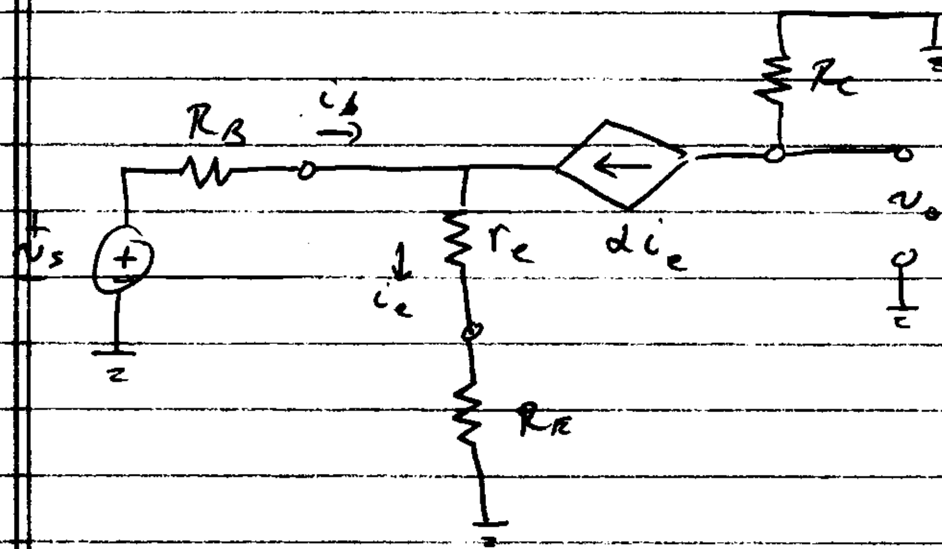


$$i_s = i_i + g_m v_i \quad (KCL)$$

$$= \frac{v_s}{R_i} + g_m v_s = v_s\left(\frac{1}{R_i} + g_m\right)$$

$$R_{in} = \frac{v_s}{i_s} = \frac{v_s}{v_s\left(\frac{1}{R_i} + g_m\right)} = \frac{R_i}{1 + g_m R_i}$$

1.29



$v_o = -\alpha i_e R_C$

$i_b + \alpha i_e = i_e \Rightarrow i_b = (1-\alpha) i_e$

Need to relate i_e to v_s: use KVL

$$v_s = i_b R_B + i_e r_e + i_e R_E$$

$$= (1-\alpha) i_e R_B + i_e r_e + i_e R_E$$

$$= \left((1-\alpha) R_B + r_e + R_E\right) i_e$$

$$v_o = -\alpha i_e R_c = \frac{-\alpha R_C v_s}{(1-\alpha) R_B + r_e + R_E}$$

$$\frac{v_o}{v_s} = \frac{-\alpha R_c}{(1-\alpha) R_B + r_e + R_E}$$

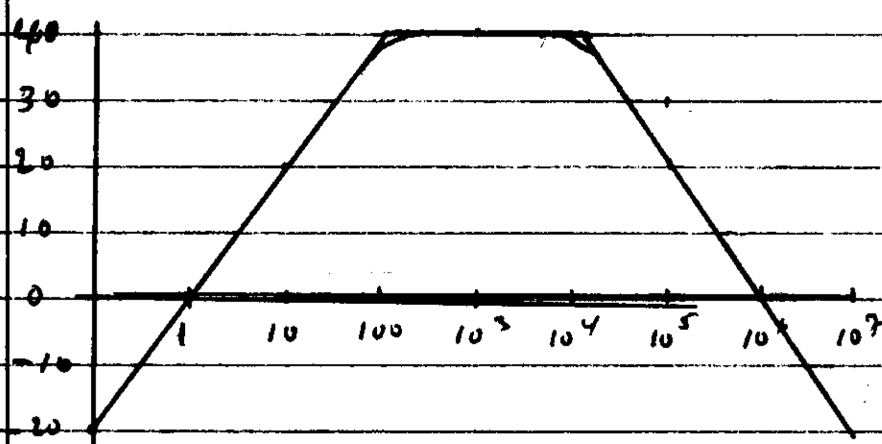
1.42 $A_v = \dfrac{100}{\left(1 + j\frac{f}{10^4}\right)\left(1 + 10^2/jf\right)}$

For $f = 10^3$ gain is highest,

$A_v \simeq \dfrac{100}{(1)(1)} = 100 = 40\ dB$

Lower 3dB freq is 10^2 Hz
Upper 3dB freq is 10^4 Hz

Below 10^2 Hz and above 10^4 Hz signal falls off at 20 dB/decade

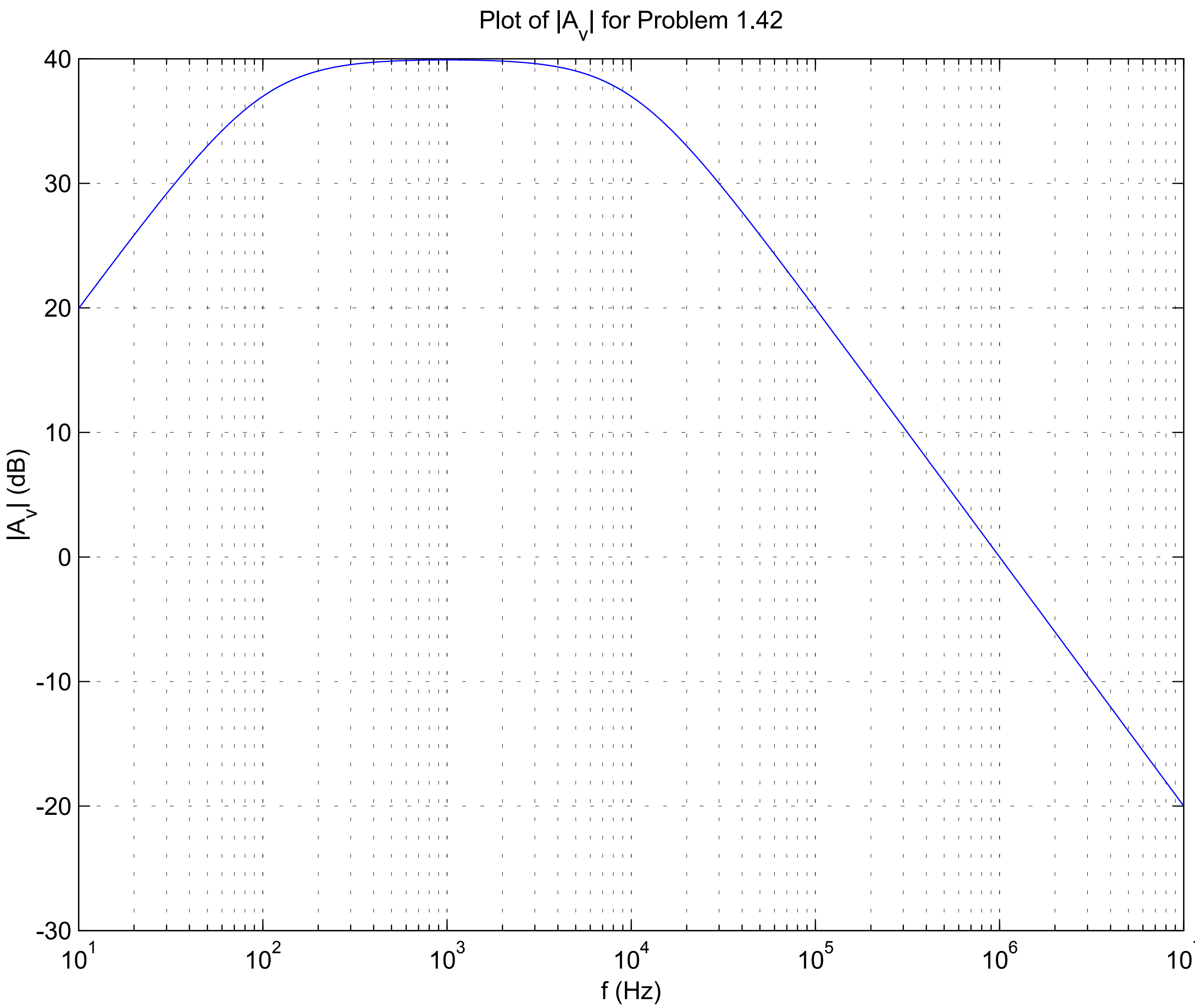


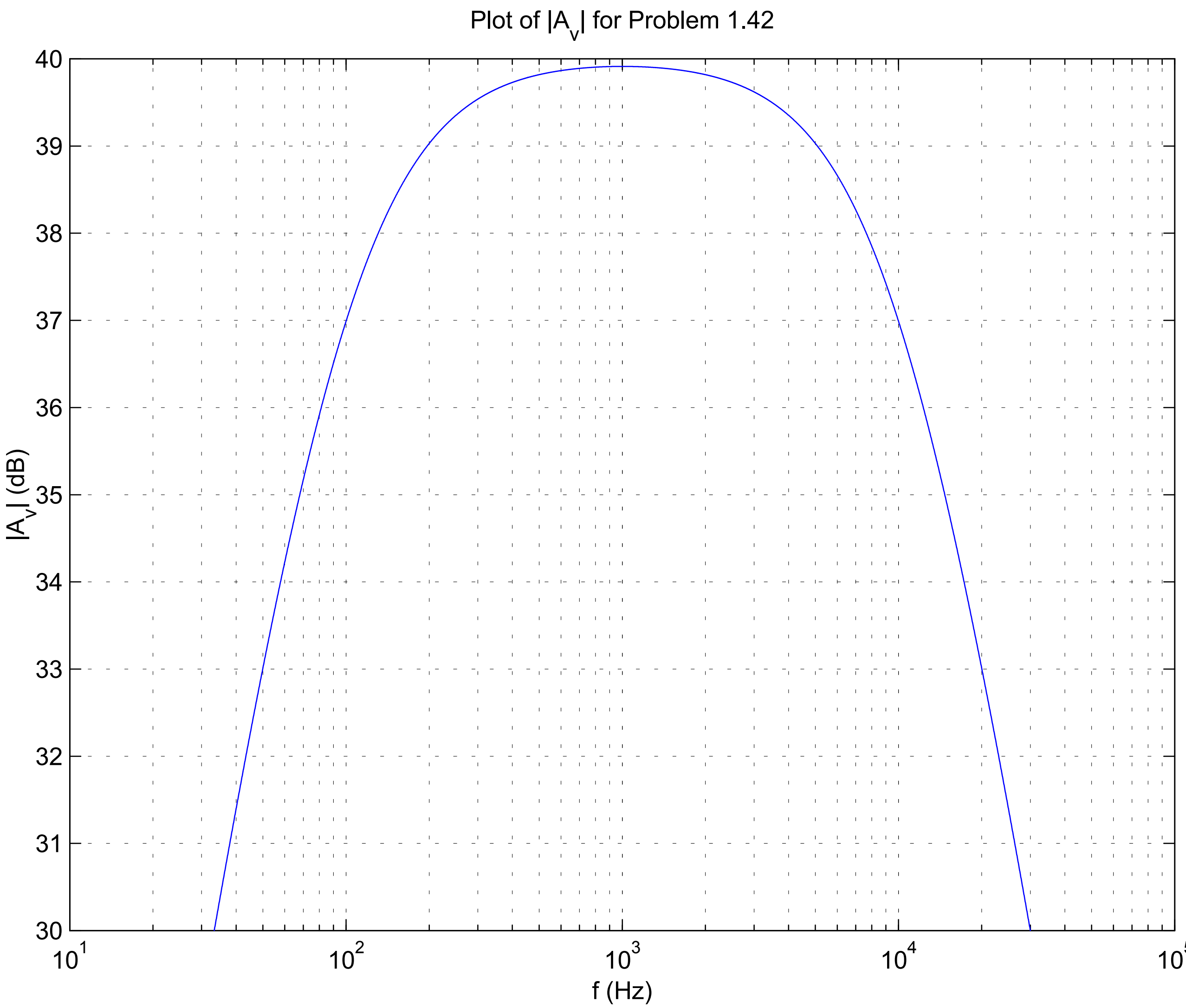
f (Hz)	10	10^2	10^3	10^4	10^5	10^6	10^7
$\lvert A_v \rvert$ (dB)	20	37	40	37	20	0	-20

BW: $10^4 - 10^2 = 9{,}900$ Hz

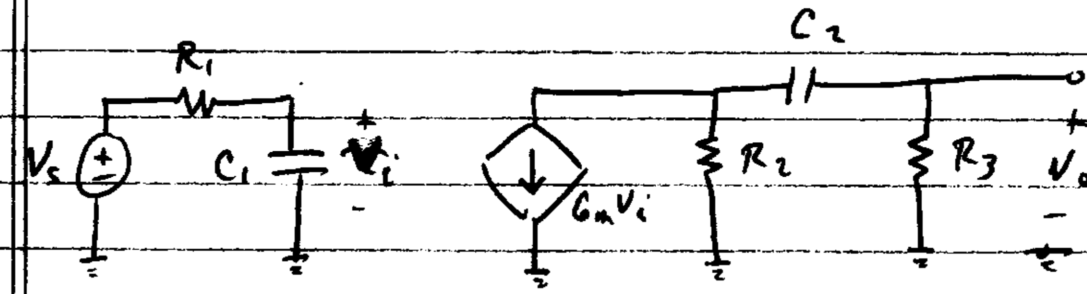
```
% PROBLEM 1.42
%
% Want f to go from 10^1 to 10^7 on log scale
% Make vector for this
f = logspace(1,7,10000);
% Calculate Av
Av = 100./((1+j*f/10^4).*(1+10^2./(j*f)));
Av_dB = 20*log10(abs(Av));
semilogx(f,Av_dB);
grid
title('Plot of |A_v| for Problem 1.42')
xlabel('f (Hz)');
ylabel('|A_v| (dB)')
print
% From plot can easily see gain at 10, 10^3, 10^5, 10^6, 10^7 Hz
% gain at 10,000 Hz is 37 dB

% Zoom in on region between 30 dB and 40 dB to find gain at 100 Hz and
% 10000 Hz
axis([10 10^5 30 40])
print
% From plot can easily see gain at 100 Hz is 37 dB,
% gain at 10,000 Hz is 37 dB
```

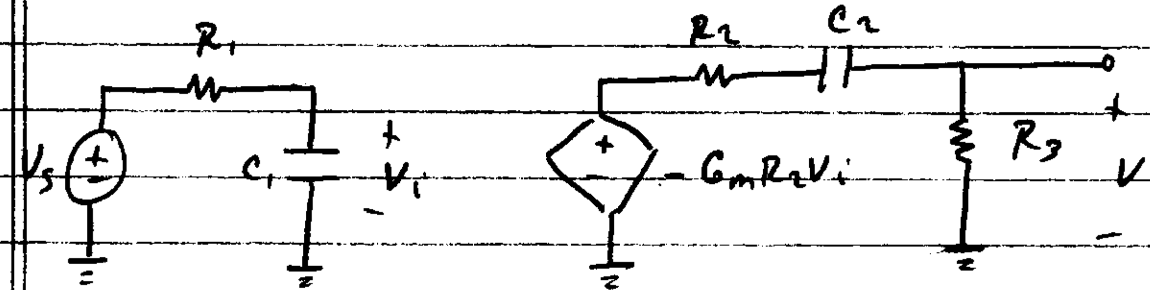

Plot of $|A_v|$ for Problem 1.42
$|A_v|$ (dB)
40
30
20
10
0
-10
-20
-30
10^1
10^2
10^3
10^4
10^5
10^6
10^7
f (Hz)


Plot of $|A_v|$ for Problem 1.42
$|A_v|$ (dB)
40
39
38
37
36
35
34
33
32
31
30
10^1
10^2
10^3
10^4
10^5
f (Hz)

1.43



Redraw output in Thevenin equiv form!



$$V_i = \frac{\frac{1}{C_1 s}}{R_1 + \frac{1}{C_1 s}} V_s = \frac{1}{1 + R_1 C_1 s} V_s$$

$$T_i(s) = \frac{1}{1 + R_1 C_1 s} \qquad \text{STC Low Pass}$$

$$V_o(s) = \frac{R_3}{R_2 + \frac{1}{C_2 s} + R_3} (-G_m R_2 V_i)$$

$$T_o(s) = \frac{V_o}{V_i} = \frac{-G_m R_2 R_3}{R_2 + R_3 + \frac{1}{C_2 s}} = \frac{-G_m R_2 R_3 C_2 s}{1 + (R_2 + R_3) C_2 s}$$

$$T_o(s) = \frac{-G_m \frac{R_2 R_3}{R_2 + R_3} s}{s + \frac{1}{(R_2 + R_3) C_2}} \qquad \text{STC High Pass}$$

For $T_i(s)$ $\quad f_{3dB} = \frac{1}{2\pi R_1 C_1} = 15.9\ kHz$

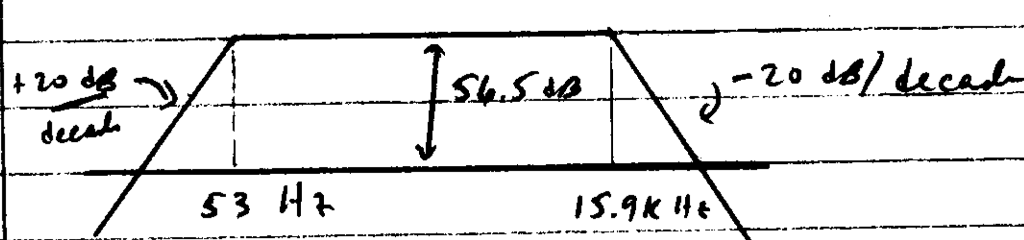
For $T_o(s)$ $\quad f_{3dB} = \frac{1}{2\pi (R_2 + R_3) C_2} = 53\ Hz$

$$T(s) = T_i(s)\, T_o(s)$$

$$= \frac{-G_m \, R_2 \| R_3 \, s}{(1 + R_1 C_1 s)\left(s + \frac{1}{(R_2 + R_3) C_2}\right)}$$

Gain at center freq $= -G_m \, R_2 \| R_3 = 667$

$|K|_{dB} = 20 \log_{10} 667 = 56.5 \text{ dB}$



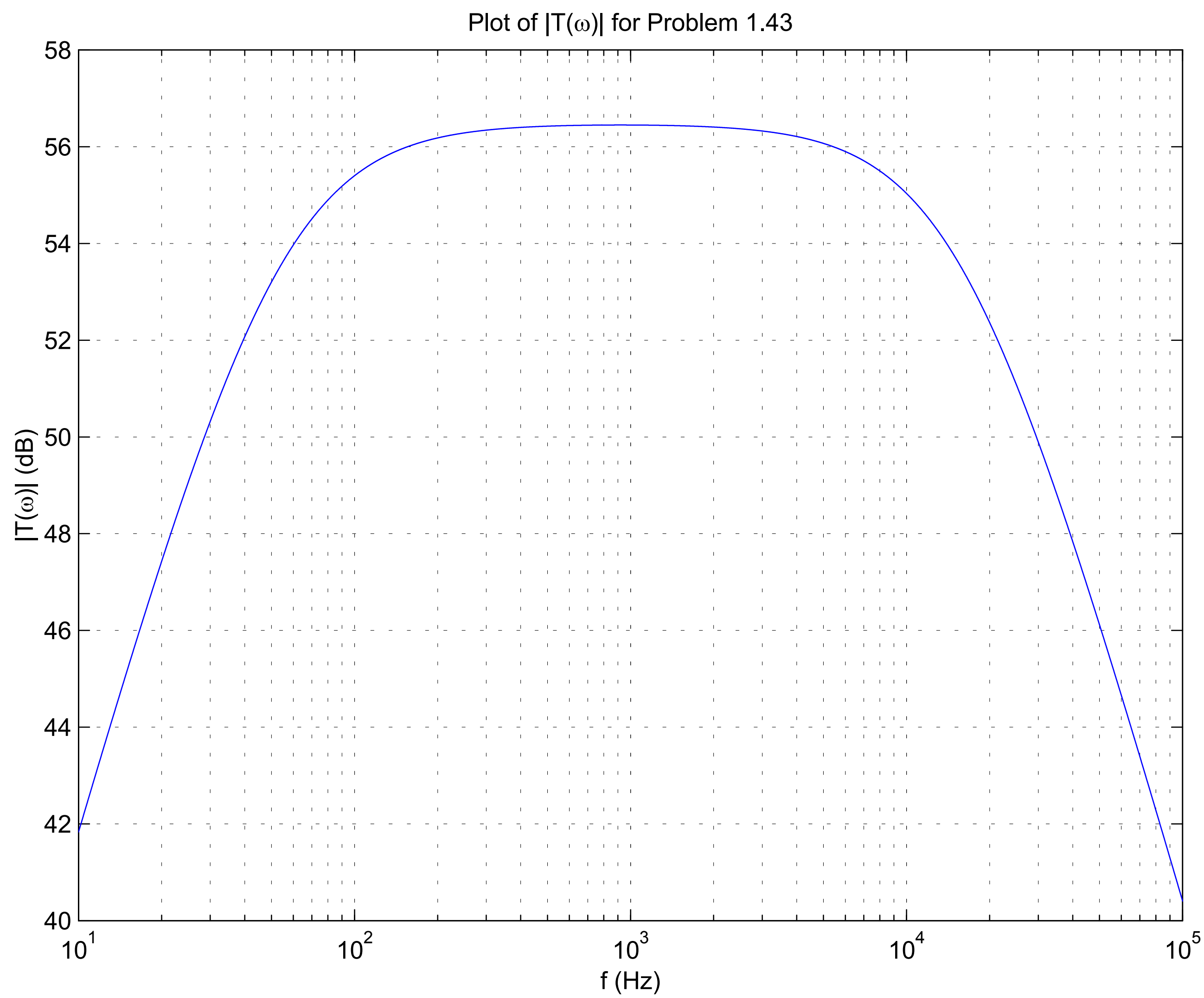
```
% PROBLEM 1.43
%
% Values from book
R1 = 1e6; R2 = 10e3; R3 = 20e3; C1 = 10e-12; C2 = 100e-9; Gm = 0.1;

% From work, find corner frequencies are 53 Hz and 16 kHz
% Make vector for frequencies covering a little more than this range
f = logspace(1,5,10000);

% Transfer function is in terms of s = j w, where w = 2 pi f
w = 2*pi*f;

s = j*w;

T = (1./(1 + R1*C1*s)).*(s*Gm*(R2*R3/(R2+R3))./(s+1/((R2+R3)*C2)));
T_dB = 20*log10(abs(T));
semilogx(f,T_dB);
grid
title('Plot of |T(\omega)| for Problem 1.43')
xlabel('f (Hz)');
ylabel('|T(\omega)| (dB)')
print
```


Plot of |T(ω)| for Problem 1.43
58
56
54
52
50
48
46
44
42
40
|T(ω)| (dB)
10^1
10^2
10^3
10^4
10^5
f (Hz)