EE 321

Fall 2002

Homework #3

Solutions

2.8 (a) Standard inverting op-amp circuit

$$\frac{v_o}{v_i} = -\frac{R_2}{R_1} = -\frac{100k}{10k} = -10$$

$R_{in} = R_1 = 10 K\Omega$

(b) The extra $10 k\Omega$ resistor on the output has no effect on v_o – op amp will adust v_o until $v_- = v_+$

$\frac{v_o}{v_i} = -10 \qquad R_{in} = 10 k\Omega$

(c) Extra $10 k\Omega$ from v_- to ground has no effect – v_- is a virtual ground so no current flows through extra resistor

$\frac{v_o}{v_i} = -10 \qquad R_{in} = -10 k\Omega$

(d) Extra $10 k\Omega$ from v_+ to ground has no effect – No current flows into + input, so no voltage drop across resistor, so $v_+ = 0$ as before

$\frac{v_o}{v_i} = -10 \qquad R_{in} = 10 k\Omega$

(e) $v_- = v_+ = 0$. v_o connected directly to v_-, so $v_o = 0$

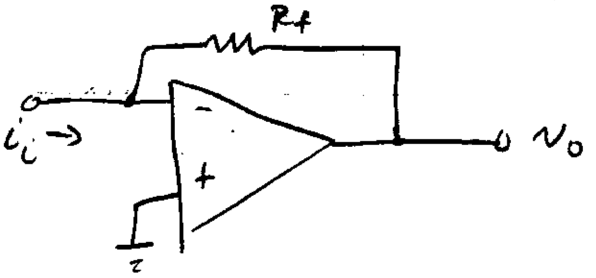
$\frac{v_o}{v_i} = 0 \qquad R_{in} = 10 k\Omega$

(f) $v_- = v_i \qquad v_o = A(v_+ - v_-) = -A v_- = -A v_i$

For ideal op amp, $A = \infty$, so $v_o = -\infty v_i$

$\frac{v_o}{v_i} = -\infty \qquad R_{in} = 0$

2.16 (a)



i_i goes through R_f and $v_- = v_+ = 0$, so

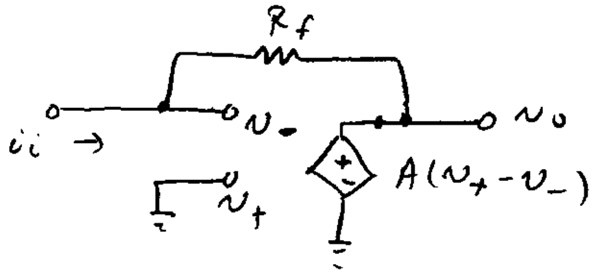
$$i_i = \frac{v_- - v_o}{R_f} = \frac{-v_o}{R_f} \Rightarrow v_o = -R_f\, i_i$$

For a transresistance amplifier (Table 1.1)

$$\boxed{R_m = \frac{v_o}{i_i} = \frac{-R_f\, i_i}{i_i} = -R_f}\,.$$

$$R_{in} = \frac{v_i}{i_i} \quad v_i = v_- = 0, \text{ so } \boxed{R_{in} = \frac{0}{i_i} = 0}$$

(b)



$$v_o = A(v_+ - v_-) = -A v_- \Rightarrow v_- = -v_o/A$$

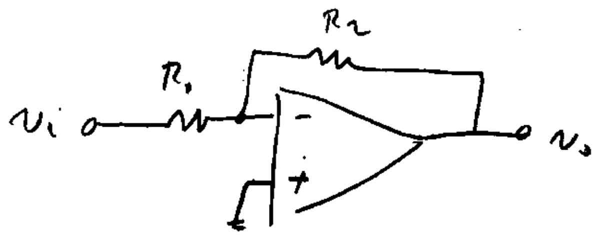
$$i_i = \frac{v_- - v_o}{R_f} = \frac{-v_o/A - v_o}{R_f} = \frac{-(1+A)}{A R_f} v_o$$

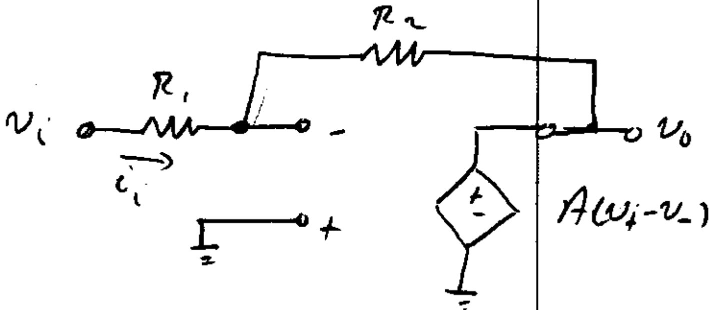
$$v_o = \frac{-A R_f}{1+A} i_i \qquad \boxed{R_m = \frac{v_o}{i_i} = \frac{-R_f}{(1+1/A)}}$$

$$v_- = -v_o/A = \frac{R_f}{1+A} i_i$$

$$\boxed{R_{in} = \frac{v_i}{i_i} = \frac{v_-}{i_i} = \frac{R_f}{1+A}}$$

2.17





① $v_o = A(v_+ - v_-) = -Av_-$ (because $v_+ = 0$)

② $$i_i = \frac{v_i - v_-}{R_1} = \frac{v_- - v_o}{R_2} \Rightarrow v_- = \frac{v_o R_1 + v_i R_2}{R_1 + R_2}$$ ③

$$v_o = -Av_- = -A\,\frac{v_o R_1 + v_i R_2}{R_1 + R_2}$$

Solve for v_o:

④ $$v_o = \frac{-A v_i R_2}{R_1(1+A) + R_2}$$

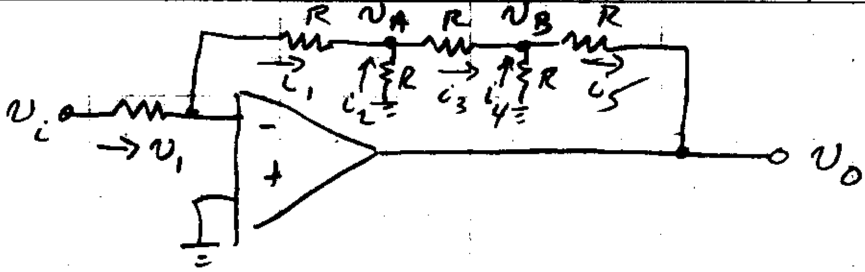
Note that $$i_i = \frac{v_i - v_o}{R_1 + R_2}$$

$$\therefore\ i_i = \frac{v_i - \left(\frac{-A v_i R_2}{R_1(1+A) + R_2}\right)}{R_1 + R_2} = \frac{v_i(1+A)}{R_1(1+A) + R_2}$$

$$v_i = i_i\left[\frac{R_1(1+A) + R_2}{1+A}\right]$$

$$\boxed{R_{in} = \frac{v_i}{i_i} = \frac{R_1(1+A) + R_2}{1+A}}$$

2.22



$$v_- = v_+ = 0 \Rightarrow i_1 = \frac{v_i}{R}$$

$$i_1 = \frac{v_- - v_A}{R} = \frac{-v_A}{R} \Rightarrow v_A = -v_i$$

$$i_2 = \frac{0 - v_A}{R} = \frac{v_i}{R} = i_1 = \frac{v_i}{R}$$

$$i_3 = i_1 + i_2 = 2i_1 = \frac{2v_i}{R}$$

$$i_3 = \frac{v_A - v_B}{R} \Rightarrow v_B = v_A - i_3 R = -v_i - 2i_1 R = -3v_i$$

$$i_4 = \frac{0 - v_B}{R} = \frac{3v_i}{R} = 3i_1 = \frac{3v_i}{R}$$

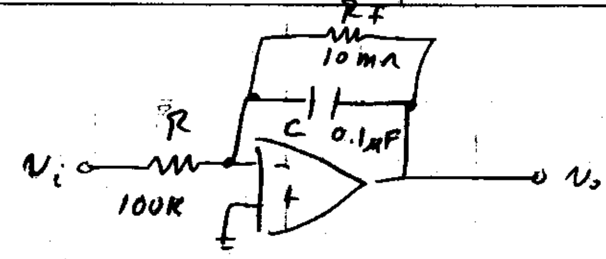
$$i_5 = i_3 + i_4 = 5i_1 = \frac{5v_i}{R}$$

$$i_5 = \frac{v_B - v_o}{R} = \frac{-3v_i - v_o}{R} = \frac{5v_i}{R}$$

$$-3v_i - v_o = 5v_i \Rightarrow v_o = -8v_i$$

$$\boxed{\frac{v_o}{v_i} = -8}$$

2.31



$$T(s) = -\frac{Z_2}{Z_1} \qquad Z_2 = R_f \parallel C = \frac{R_f \frac{1}{Cs}}{R_f + \frac{1}{Cs}} = \frac{R_f}{1 + R_f C s}$$

$$Z_1 = R$$

① $$T(s) = -\frac{R_f/R}{1 + R_f C s} = -\frac{10m/100k}{1 + (10 m\Omega)(0.1 \mu F) s} = \frac{-100}{1+s}$$

For ideal integrator, $R_f = \infty$ so $T_{ideal}(s) = -\frac{1}{RCs}$

② $$T_{ideal}(s) = -\frac{1}{RCs} = \frac{-1}{(100 k\Omega)(0.1 \mu F) s} = \frac{-100}{s}$$

Bode plot for ①: DC gain $= 100 = 40_{dB}$

$\omega_3 = 1 \qquad f_b = \frac{\omega_b}{2\pi} = 0.16\ Hz$

Bode plot for ② - $T_{ideal}(\omega) = \frac{-100}{j\omega} \qquad |T_{ideal}(\omega)| = \frac{100}{\omega}$

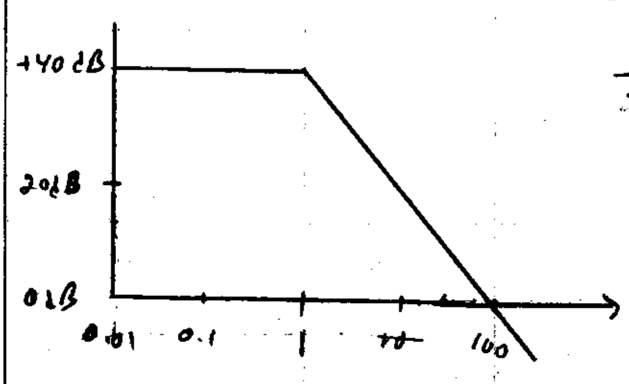
$$|T_{ideal}(\omega)|_{dB} = 20 \log_{10}(100) - 20 \log_{10}(\omega)$$

$$= \underbrace{40 - 20 \log_{10}(\omega)}$$

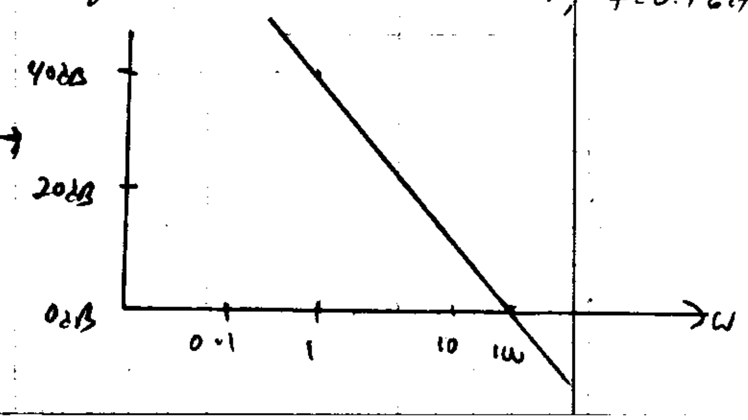
straight line, slope = -20 dB/decade

Passes through 40 dB when $\omega = 1$, $f = 0.16$ Hz

Actual



Ideal →



$f > 0.16\ Hz$

Circuit acts as amplifier for $\omega < \omega_2$ $\quad \omega < 1\ rad/s$

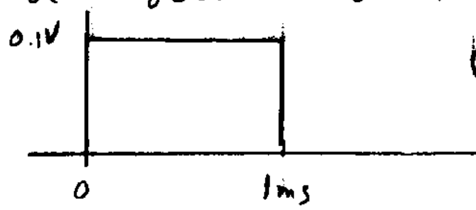
$f < 0.16\ Hz$

(b) High freq ⇒ short duration.

For short-duration pulses ($t < \frac{1}{\omega} = 1\,sec$) circuit acts as integrator

For single-time constant circuits

$$v_o(t) = v_o(\infty) - [v_o(\infty) - v_o(0+)]\, e^{-t/\tau} \quad \text{(eqn F.9)}$$



For $v_i = 0.1V$, $v_o = 100 v_i = -10V$ if v_i left for a long time

For $t < 0$, $v_o = 0$

For $0 < t < 1ms$ $\quad v_o(t) = -10 - [-10 - 0]\, e^{-t/\tau}$

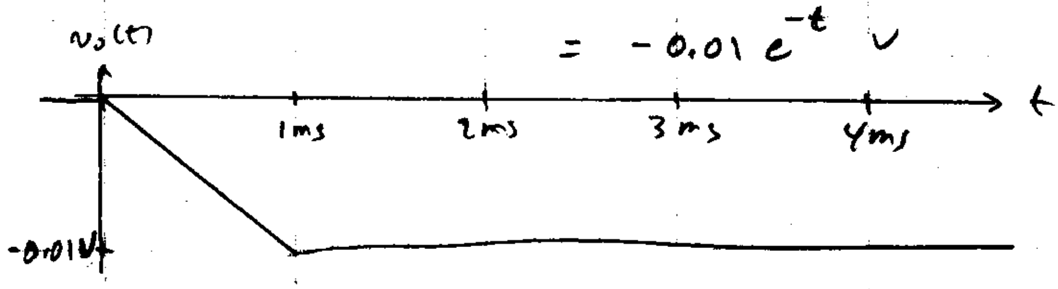
$$v_o(t) = -10\,(1 - e^{-t/\tau})$$

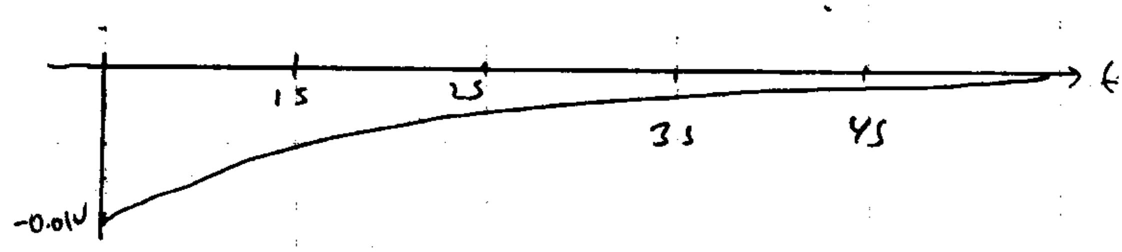
Because $t \ll \tau = 1 sec$, $e^{-t/\tau} \cong 1 - t/\tau$ and $v_o(t) \cong -10\, t/\tau = -10t$

At $t_o = 1ms$, $v_o(t_o) = -10(0.001)V = -0.01V$

For $t > 1ms$, $v_o(t) = 0V - [0V - (-0.01V)]\, e^{-t/\tau}$

$$= -0.01\, e^{-t}\ V$$





```
% Define component values
R = 100e3; C = 0.1e-6; Rf = 10e6;

% Find w0 and tau
tau = Rf*C;   w0 = 1/tau;

% Create vector of frequencies
w = logspace(-2,3,1000);
s = j*w;

% Find T(w) = (-Rf/R)/(1+Rf C s)
T = (-Rf/R)./(1+Rf*C*s);
T_dB = 20*log10(abs(T));

% Find ideal integrator response, Tid = -1/RCs
Tid = -1./(R*C*s);
Tid_dB = 20*log10(abs(Tid));

figure(1)
semilogx(w,Tid_dB,'r')
hold on
semilogx(w,T_dB)
grid
xlabel('\omega (rad/sec)');
ylabel('|T(\omega)| (dB)');
title('Bode plot for Problem 2.31');
legend('Ideal Integrator','Actual Circuit')

% Plot time response
t = -1:(1/10000):5;
t0 = 0.001;       % time pulse is turned off
v = zeros(size(t));     % Vector of all zeros same size as t
ii = find((t>0)&(t<t0));      % Time pulse is on
v(ii) = -10*(1-exp(-t(ii)/tau));
ii = find(t>t0);              % Time pulse is off
v(ii) = -0.01*exp(-(t(ii)-t0)/tau);
figure(2)
subplot(211)
plot(t*1000,v)      % Plot in milliseconds
axis([-t0*1000 5*t0*1000 -0.012 0.012])
grid
xlabel('t (ms)');
ylabel('v (volts)');
title('Pulse response of Problem 2.31')

subplot(212)
plot(t,v)
axis([-1 5 -0.012 0.012])
grid
xlabel('t (sec)');
ylabel('v (volts)');
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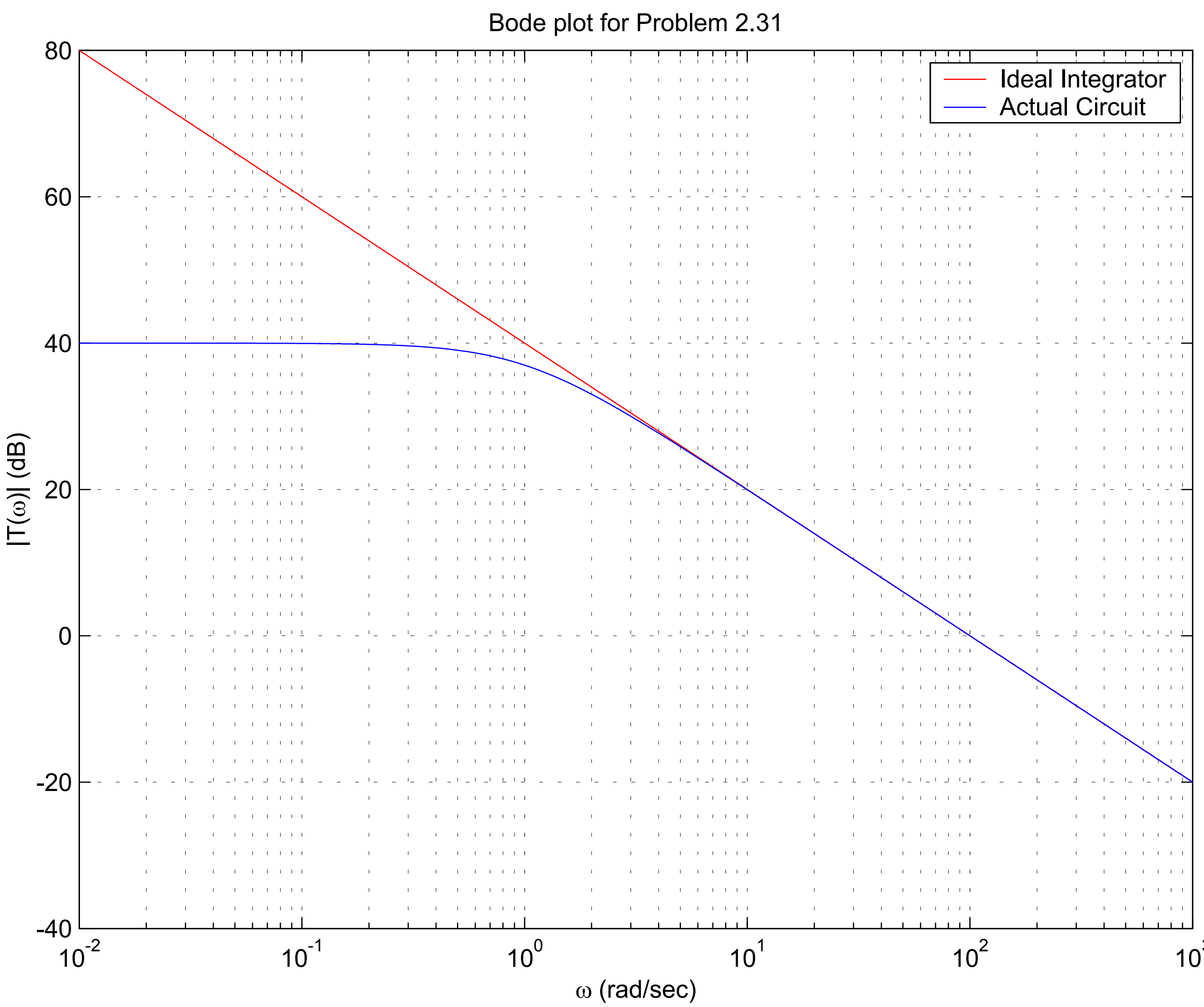
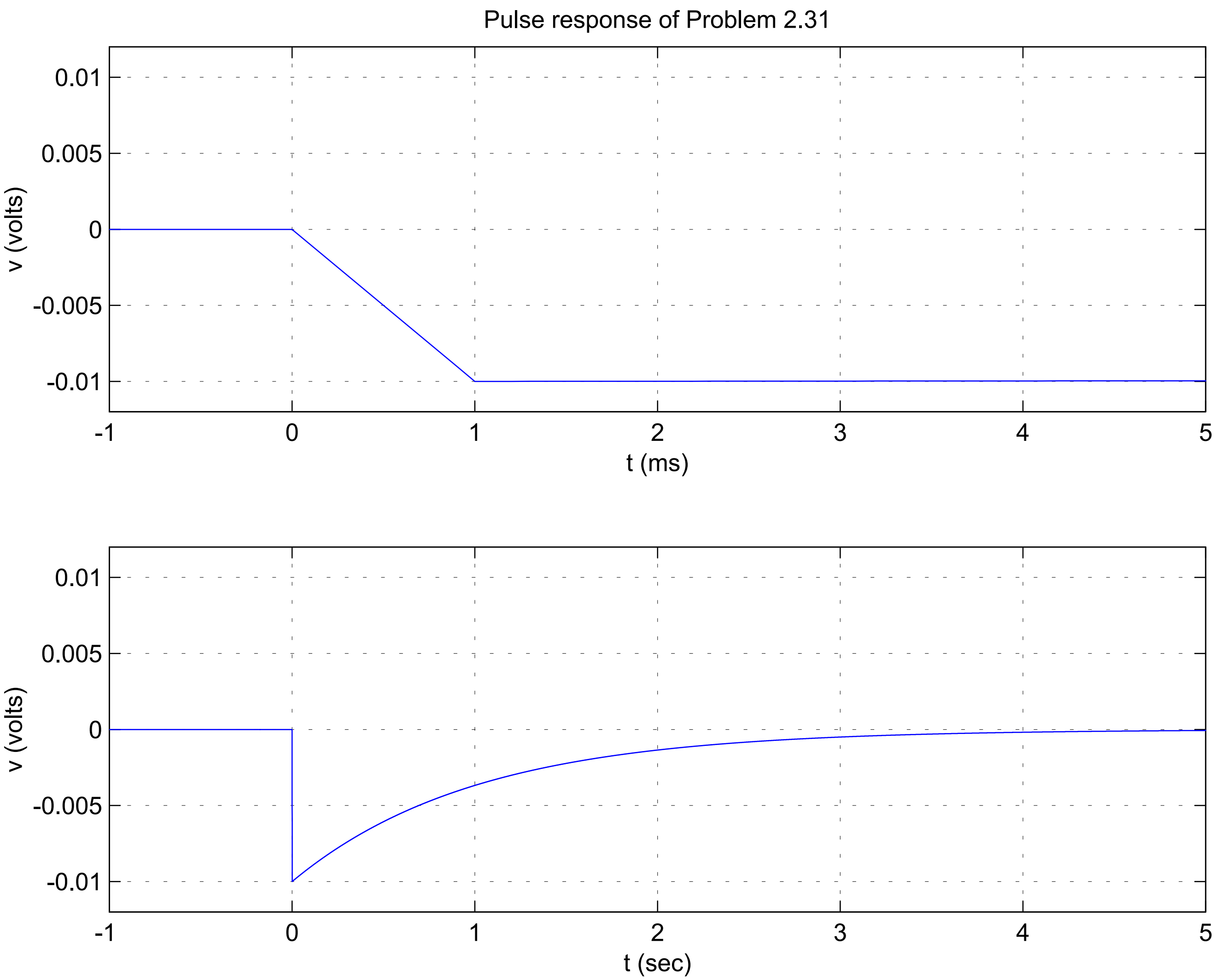
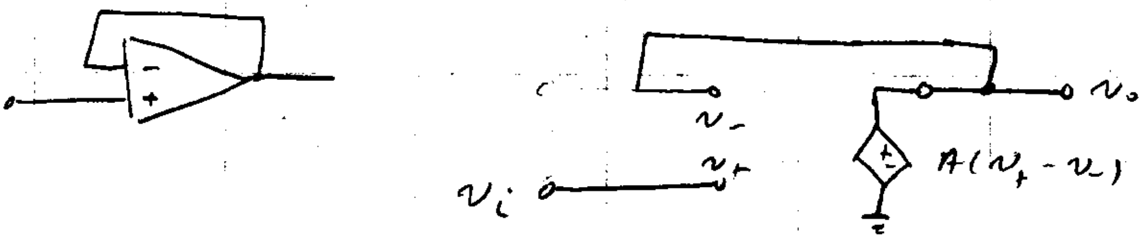


Bode plot for Problem 2.31
Ideal Integrator
Actual Circuit
|T(ω)| (dB)
ω (rad/sec)
80
60
40
20
0
-20
-40
10^{-2}
10^{-1}
10^{0}
10^{1}
10^{2}
10^{3}


Pulse response of Problem 2.31
v (volts)
0.01
0.005
0
-0.005
-0.01
-1
0
1
2
3
4
5
t (ms)
v (volts)
0.01
0.005
0
-0.005
-0.01
-1
0
1
2
3
4
5
t (sec)

2.50



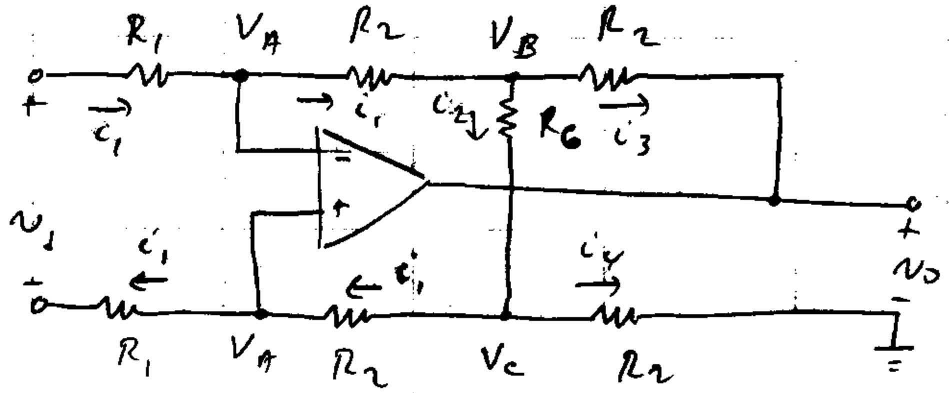
$$v_o = A(v_+ - v_-) \qquad v_+ = v_i \qquad v_- = v_o$$

$$\therefore\ v_o = A(v_i - v_o) \Rightarrow v_o = v_i\left(\frac{A}{1+A}\right)$$

$$\frac{v_o}{v_i} = \frac{A}{1+A}$$

A	v_o/v_i	$\left(1-\frac{v_o}{v_i}\right)\times 100\%$
1000	0.999	0.1 %
100	0.99	1 %
10	0.91	9.1 %

2.61



$$v_+ = v_- \Rightarrow \quad i_1 = \frac{v_d}{2R_1}$$

$$i_1 = \frac{V_A - V_B}{R_2} = \frac{v_d}{2R_1} \Rightarrow V_B = \frac{2V_A R_1 - v_d R_2}{2R_1}$$

$$i_1 = \frac{V_C - V_A}{R_2} = \frac{v_d}{2R_1} \Rightarrow V_C = \frac{2V_A R_1 + v_d R_2}{2R_1}$$

$$i_2 = \frac{V_B - V_C}{R_G} = -v_d \frac{R_2}{R_1 R_G}$$

$$i_1 = i_2 + i_3 \Rightarrow i_3 = i_1 - i_2 = \frac{v_d}{2R_1} + \frac{v_d R_2}{R_1 R_G}$$

$$i_3 = \frac{R_G + 2R_2}{2R_1 R_G} v_d$$

$$i_2 = i_1 + i_4 \Rightarrow i_4 = i_2 - i_1 = -i_3 = -\frac{R_G + 2R_2}{2R_1 R_G} v_d$$

$$v_o = -i_3 R_2 + i_2 R_G + i_4 R_2$$

$$= -\left(\frac{R_G + 2R_2}{2R_1 R_G}\right) v_d R_2 + \left(-\frac{R_2 v_d}{R_1 R_G}\right) R_G - \left(\frac{R_G + 2R_2}{2R_1 R_G}\right) v_d R_2$$

$$= \frac{-2R_2 R_2 - 2R_G R_2}{R_1 R_6} v_d = -2\frac{R_2}{R_1}\left[1 + \frac{R_2}{R_G}\right] v_d$$

$$\boxed{\frac{v_o}{v_d} = -2\frac{R_2}{R_1}\left[1 + \frac{R_2}{R_G}\right]}$$