# EE 321

# Fall 2002

# Homework #10

# Solutions

4.66 See attached MATLAB plot

For $i_B = 10\mu A$, $i_C = 1\,mA$ and $V_{CE} = 4V$

For $i_B = 40\mu A$, $i_C = 4\,mA$ and $V_{CE} = 1V$

Because $V_E = 0V$, $V_C = V_{CE}$, so $V_C$ will swing from 4V to 1V, or 3V total.

When $V_{CE} = \frac{1}{2} V_{CC} = 2.5V$, $I_C = 2.5\,mA$

$I_B = I_C/\beta = 25\mu A$

$V_{BB} = I_B R_B + V_{BE} = (25\mu A)(100\,k\Omega) + 0.7V = 3.2V$

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% Problem 4.66

% Find i_C for i_B = 1, 10, 20, 30, 40
% i_C = beta * i_B for all V_CE
beta = 100;
iB=[1 10 20 30 40]*1e-6;
VCE = [0:1:8];
for ii=1:5
    iC(ii,:) = beta*iB(ii)*ones(size(VCE));
end
plot(VCE,iC*1000)
legend('i_B = 1 uA','i_B = 10 uA','i_B = 20 uA','i_B = 30 uA','i_B = 40 uA')
hold on
xlabel('V_{CE} (V)','FontSize',14)
ylabel('i_{C} (mA)','FontSize',14)

% Calculate load line
RC = 1e3;
VL = [0 5];
IL = [5/RC 0];
plot(VL,IL*1000,'k')

% Plot line showing V_CE = 2.5 V
plot([2.5 2.5],[0 beta*50e-6]*1000)
grid
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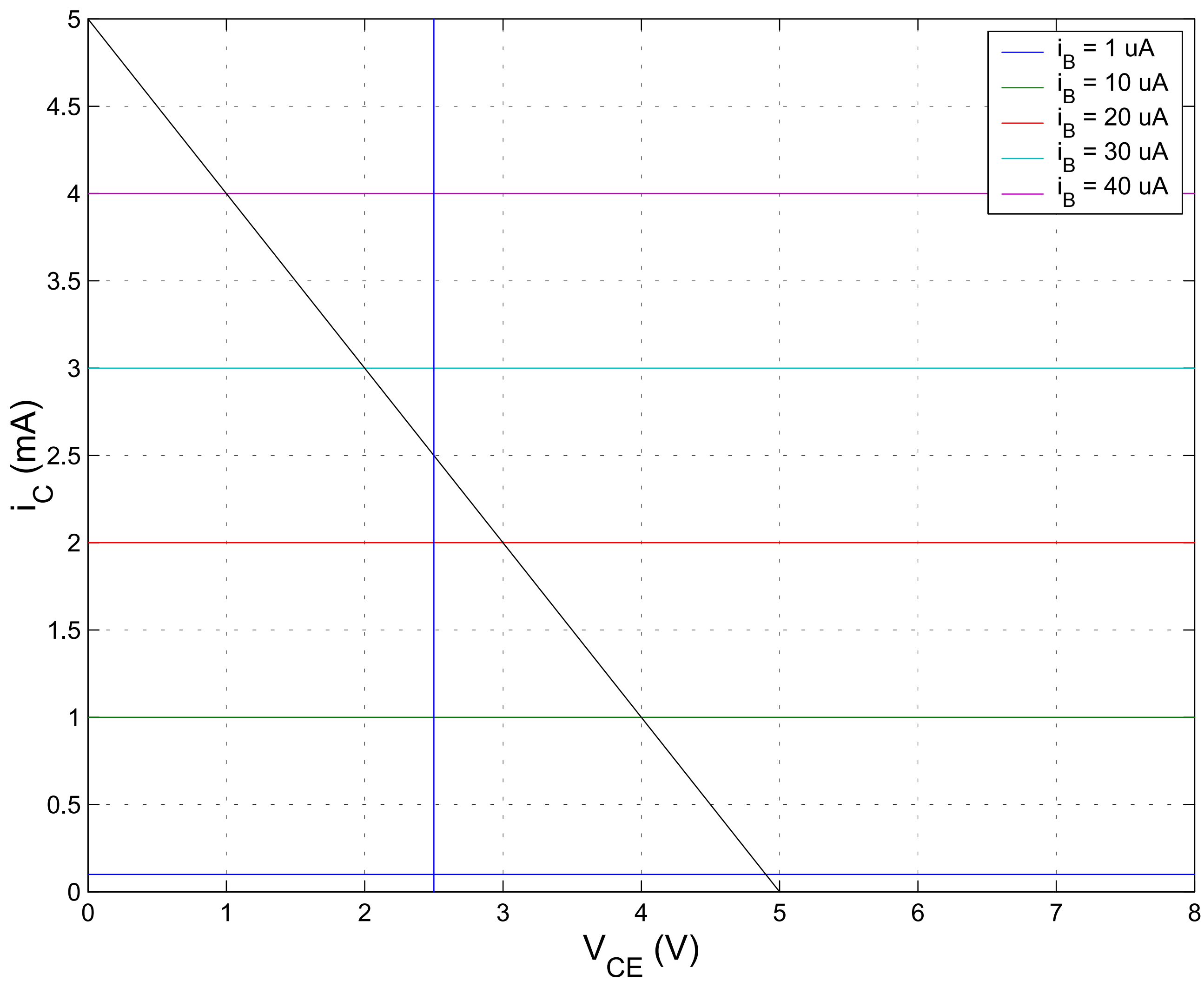
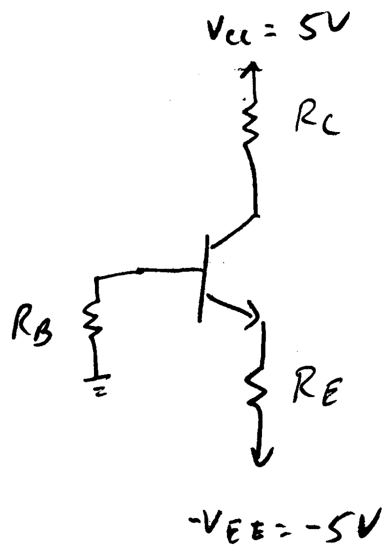


5
4.5
4
3.5
3
2.5
2
1.5
1
0.5
0
$i_C$ (mA)
0
1
2
3
4
5
6
7
8
$V_{CE}$ (V)
$i_B$ = 1 uA
$i_B$ = 10 uA
$i_B$ = 20 uA
$i_B$ = 30 uA
$i_B$ = 40 uA

4.70




For $\beta = \infty$: $\quad I_B = 0 \Rightarrow V_B = 0 \Rightarrow V_E = -0.7V$

Want $V_C$ 40% between $V_E$ and $V_{cc}$:

$$V_c = 0.4(V_{cc} - V_E) + V_E = 0.4(5V - (-0.7V)) + (-0.7V) = \underline{\underline{1.58V}}$$

$$V_c = V_{cc} - I_C R_C \Rightarrow R_c = \frac{V_{cc} - V_c}{I_c} = \frac{5V - 1.58V}{0.1mA} = 34.2\,k\Omega$$

The nearest 5% resistor is $\boxed{R_C = 33\,k\Omega}$

$$I_B R_B + V_{BE} + I_E R_E + V_{EE} = 0$$

$$R_E = \frac{-V_{EE} - V_{BE} - I_B R_B}{I_E} \qquad \text{For } \beta = 0,\ I_B = 0 \text{ and } I_E = I_C$$

$$R_E = \frac{-(-5V) - 0.7V + 0}{0.1mA} = 43\,k\Omega$$

43 kΩ is a standard 5% resistor, so $\boxed{R_E = 43\,k\Omega}$

For $\beta \neq 0$:

$$I_B R_B + V_{BE} + (\beta+1) I_B R_E + V_{EE} = 0$$

$$I_E = \frac{I_B}{\beta+1} = \frac{-V_{EE} - V_{BE}}{R_E + \frac{R_B}{\beta+1}} = \frac{4.3V}{43k + \frac{R_B}{\beta+1}}$$

$$I_E = \frac{V_E - V_{EE}}{R_E} \Rightarrow V_E = V_{EE} + I_E R_E = V_{EE} - \frac{V_{EE} + V_{BE}}{R_E + \frac{R_B}{\beta+1}}$$

$$V_E = -5 + \frac{4.3V \times R_E}{R_E + R_B/\beta+1}$$

$$V_C = V_{CC} - I_C R_C = V_{CC} - \alpha I_E R_C = 5 - \frac{\alpha\, 4.3\,V \cdot R_C}{R_E + R_B/(\beta+1)}$$

$$V_{CE} = V_C - V_E = 5 - \frac{\alpha (4.3v)(R_C)}{R_E + R_B/_{\beta+1}} + 5 - \frac{(4.3v)\, R_E}{R_E + R_B/(\beta+1)}$$

$$= 10V - \frac{(4.3v)(\alpha R_C + R_E)}{R_E + R_B/(\beta+1)}$$

Solve for $R_B$

$$R_B = (\beta+1)\left[\frac{4.3v\ (\alpha R_C + R_E)}{10V - V_{CE}}\right]$$

For $\beta = \infty$, $V_C = V_{CC} - I_C R_C = 4.7v$, $V_E = 0.7v \Rightarrow V_{CE} = 2.4\,V$

For $\beta = 70$, want $V_{CE}$ to increase by no more than 20 %

$$V_{CE} < 1.2 \times 2.4\,v = 2.88\,v$$

$$R_B < (\beta+1)\left[\frac{4.3V(\alpha R_C + R_E)}{10 - 2.736} - R_E\right] = 186\,k\Omega$$

Any $R_B$ less than $186\,k\Omega$ will work

The largest 5% resistor less than $186 k\Omega$ is $180\,k\Omega$

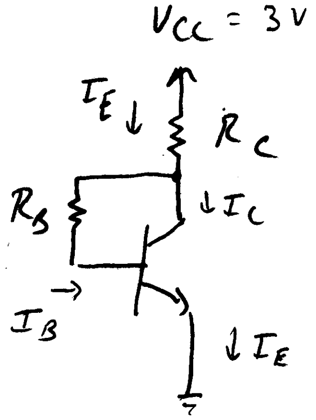
Let $\boxed{R_B = 180\,k\Omega}$

$$I_E = \frac{4.3v}{43k + R_B/_{\beta+1}} \Rightarrow I_E = 0.0944\,mA \qquad \boxed{I_C = \alpha I_E = 0.093\,mA}$$

For $\beta = 70$

$$V_{CE} = 10V - \frac{4.3v(\alpha R_C + R_E)}{R_E + R_B/_{\beta+1}} \Rightarrow \boxed{V_{CE} = 2.87\,V}$$

4.73




Want $V_{CE} = 1.4V \Rightarrow V_C = 1.4V$

$$I_E = \frac{I_C}{\alpha} = \frac{\beta+1}{\beta} I_C = \frac{101}{100}(0.1mA) = 0.101\ mA$$

$$V_{CC} - V_C = I_E R_C \Rightarrow R_C = \frac{V_{CC} - V_C}{I_E} \approx \frac{3V - 1.4V}{0.101mA} = 15.84k\Omega$$

Nearest 5% value is $\boxed{R_C = 16\ k\Omega}$

$I_B = I_C/\beta = 0.001\ mA$

$V_C = 1.4V, \quad V_B = 0.7V$

$$I_B = \frac{V_C - V_B}{R_B} \Rightarrow R_B = \frac{V_C - V_B}{I_B} = \frac{1.4V - 0.7V}{0.001mA} = 700k\Omega$$

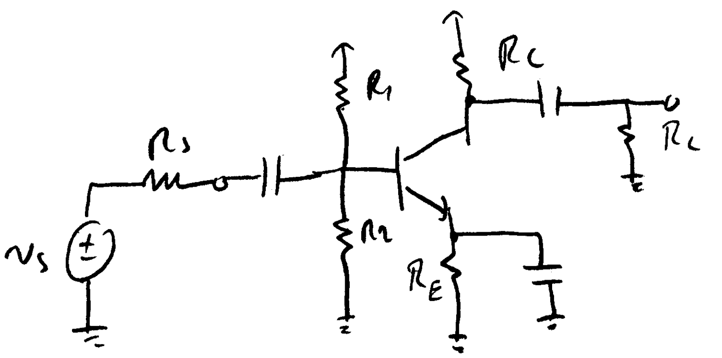
The nearest 5% resistor is $\boxed{R_B = 680k\Omega}$

$$V_{CC} = I_E R_C + I_B R_B + V_{BE} = ((\beta+1) I_B R_C + I_B R_B + V_{BE}$$

$$I_B = \frac{V_{CC} - V_{BE}}{(\beta+1)R_C + R_B} \qquad V_C = V_{CC} - I_E R_C = V_{CC} - (\beta+1) I_B R_C$$

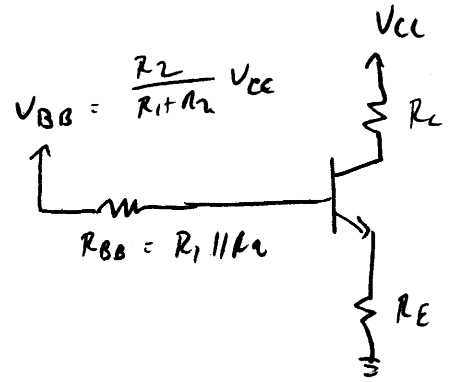
| $\beta$ | $I_B$ | $I_C$ | $V_{CE} = V_C$ |
|---|---|---|---|
| 50 | 0.0015mA | 0.077mA | 1.75V |
| 200 | 0.00059mA | 0.118mA | 1.10V |

4.78




$\beta = 100$

For DC



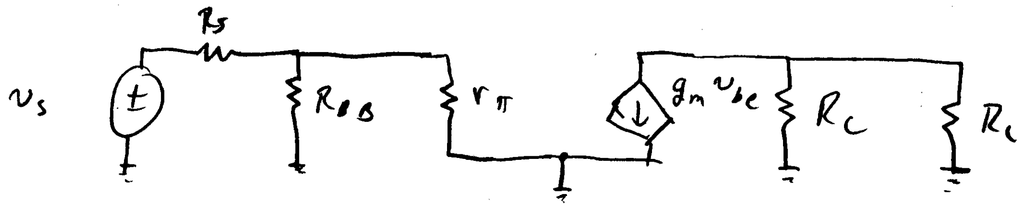

$$V_{BB} = \frac{15k}{27k + 15k} 9V = 3.21V$$

$$R_{BB} = R_1 \| R_2 = 9.64k$$

$$V_{BB} = I_B R_{BB} + V_{BE} + (\beta + 1) I_B R_E$$

$$I_B = \frac{V_{BB} - V_{BE}}{R_{BB} + (\beta + 1) R_E} = \frac{9V - 0.7V}{9.64k + (101)(1.2k)} = 0.0634mA$$

$$I_E = (\beta + 1) I_B = 6.41mA$$

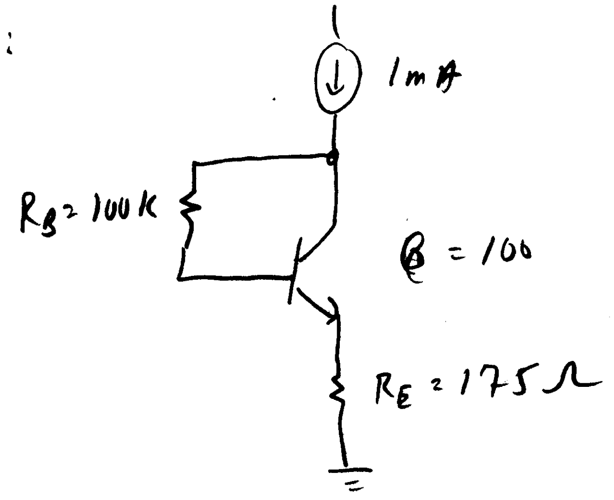



$$g_m = \frac{I_C}{V_T} = \frac{\beta I_B}{V_T} = 254 \text{ mA/V}$$

$$r_\pi = \frac{V_T}{I_B} = 394\ \Omega$$

4.85

(a)
DC:



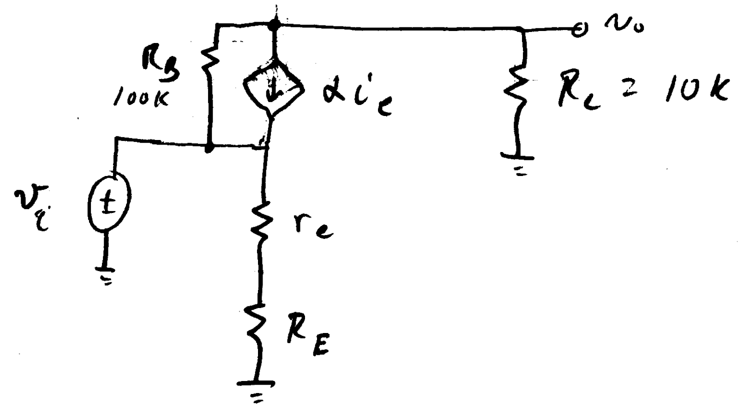

$$I_E = 1\,mA \Rightarrow I_C = \alpha I_E = \frac{\beta}{\beta+1} I_E = \boxed{0.99\,mA = I_C}$$

$$V_C = I_B R_B + V_{BE} + I_E R_E = \frac{I_E R_B}{\beta+1} + V_{BE} + I_E R_E$$

$$\boxed{V_C = 1.865\,V}$$

$$r_e = \frac{V_T}{I_E} = 25\Omega$$

(b) Small signal




$$i_e = \frac{v_i}{R_E + r_e}$$

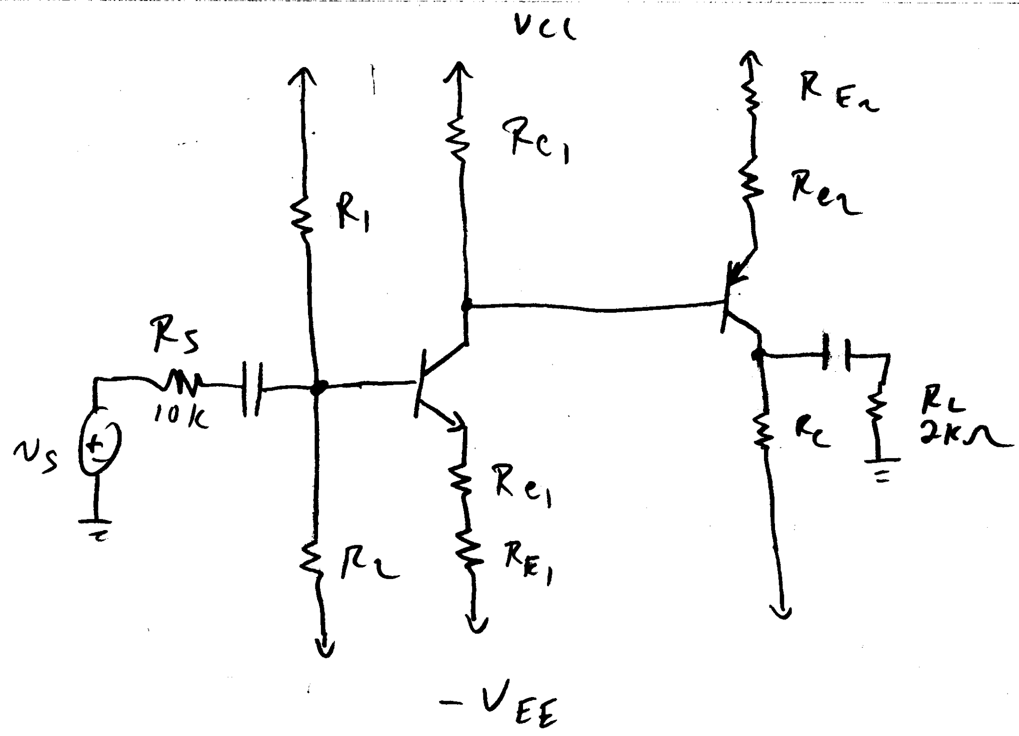
$$\frac{v_i}{R_E + r_e}$$

KCL at $v_o$: $$\frac{v_o - v_i}{R_B} + \alpha i_e + \frac{v_o}{R_L} = 0$$

Solve for $v_o$:

$$\frac{v_o}{v_i} = (R_B \| R_L)\left(\frac{1}{R_B} - \frac{\alpha}{R_E + r_e}\right) = -44.9\ V/V$$

6.

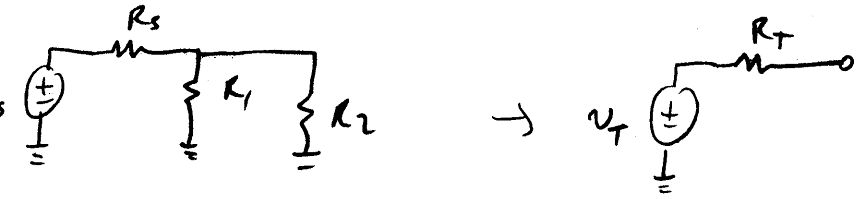



In the small signal analysis Stage 1 sees $R_{C1}$ in parallel with $R_{i2}$ (the input resistance of Stage 2), so

$$A_{v1} = \frac{-\beta (R_{C1} \parallel R_{i2})}{R_{S1} + (\beta+1)(r_{e1} + R_{e1})} \quad \text{(eq. 4.75)}$$

where $R_{i2} = (\beta+1)(r_{e2} + R_{e2})$ (eq 4.70)

The input to stage 1 can be redrawn:




where $V_T = \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_S$ and $R_T = R_S \parallel R_1 \parallel R_2$

The output voltage of Stage 1 is

$$v_{o1} = A_{v1} v_T = A_{v1} \frac{R_1 \parallel R_2}{R_S + R_1 \parallel R_2} v_S$$

and $R_{S1} = R_T$

We will make $R_{BB} = R_1 \| R_2 >> R_S$ so

$$R_T = R_S \| R_B \simeq R_S, \quad \text{and} \quad v_T = \frac{R_1 \| R_2}{R_1 \| R_2 + R_S} \simeq v_S$$

Then $v_{o_1} = Av_1 v_S$ and $R_{S_2} = R_S$

Stage 2 sees $R_L$ is parallel with $R_{C_2}$, so

$$Av_2 = \frac{-\beta(R_{C_2} \| R_L)}{R_{S2} + (\beta+1)(r_{e_2} + R_{e2})}$$

$R_{S_2}$ is the output resistance from Stage 1

$R_{S_2} = R_{o_1} = R_{c_1}$ (eq on Page 488 of text)

The input voltage to Stage 2 is the output voltage from Stage 1, so

$$v_o = Av_2 v_{i_2} = Av_2 v_{o_1} = Av_2 Av_1 v_S$$

$$\frac{v_o}{v_s} = \frac{-\beta(R_{c_1} \| [(\beta+1)(r_{e_2} + R_{e_2})])}{R_S + (\beta+1)(r_{e_1} + R_{e_1})} \cdot \frac{\beta(R_{c_2} \| R_L)}{R_{c_1} + (\beta+1)(r_{e_2} + R_{e_2})}$$

To make gain independent of $\beta$, want $R_{e_1}$ and $R_{e_2}$ fairly large, but this reduces the gain.

Also, want $R_{C2}$ fairly large so $R_{C2} \| R_L$ is large. However, if $R_{C2}$ is too large, the emitter current of stage 2 will not be able to supply the 2 ma needed by the load

Let $R_{C_2} = 2.2\,k\Omega$, $R_{E_2} = 1k$, $I_{E_2} = 7\,mA$

Then $V_{C_2} = I_{C_2} R_{C_2} - 15V \simeq I_{E_2} R_{C_2} - 15V = 0.4V$

$V_{E_2} = 15V - I_{E_2} R_{E_2} = 8V$

This gives plenty of voltage swing for $v_o$ (need $\pm 4V$)

$V_{E_2} = 8V \Rightarrow V_{C_1} = V_{E_2} - V_{EB_2} = 7.3V$

Let $I_{E_1} = 1\,mA$ (don't need much current in stage 1, because it just drives the base current for stage 2)

$$R_{C_1} = \frac{15V - V_{C_1}}{I_{C_1}} \simeq \frac{15V - V_{C_1}}{I_{E_1}} = 7.7\,k\Omega \rightarrow 7.5\,k\Omega$$

(nearest 5%)

Let $V_{B_1} = -5V$ ($\frac{1}{3}$ of way between $V_{EE}$ and $V_{CC}$)

We can do this with $R_2 = 50k\Omega \rightarrow 51k\Omega$, $R_1 = 100k\Omega$

Then $V_{E_1} = V_{B_1} - V_{BE_1} = -5.7V$

$$R_{E_1} = \frac{V_{E_1} - (-15V)}{I_{E_1}} = \frac{-5.7V + 15V}{1\,mA} = 9.3k \rightarrow 9.1k$$

Note that $r_{e_1} = \frac{V_T}{I_{E_1}} = 25\,\Omega$

$$r_{e_2} = \frac{V_T}{I_{E_2}} = 3.6\,\Omega$$

To find gains assume $\beta = 150$

Want $A_{v_2} = 10$:

$$A_{v_2} = \frac{\beta\,(R_{C_2} \| R_L)}{R_{C_1} + (\beta+1)(r_{e_2} + R_{E_2})}$$

$$R_{e_2} = \frac{\left[\dfrac{\beta\,(R_{C_2} \| R_L)}{A_{v_2}} - R_{C_1}\right]}{\beta+1} - r_{e_2} = 52\,\Omega \rightarrow 51\,\Omega$$

$$A_{v_1} = \frac{\beta\,(R_{C_1} \| R_{i_2})}{R_S + (\beta+1)(r_{e_1} + R_{e_1})}$$

$R_{i_2} = (\beta+1)(r_{e_2} + R_{e_2}) \doteq 8.2\,k$

$$R_{e_1} = \frac{\left[\dfrac{\beta\,(R_{C_1} \| R_{i_2})}{A_{v_1}} - R_S\right]}{\beta+1} - r_{e_1} = 105\,\Omega \rightarrow 100\,\Omega$$

$R_1 = 100k$ $\quad R_2 = 51k$

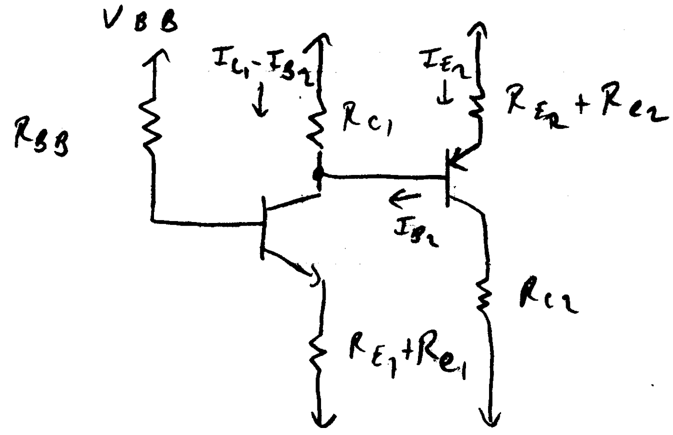
$R_{C_1} = 7.5k$ $\quad R_{C_2} = 2.2k$

$R_{E_1} = 9.1k$ $\quad R_{E_2} = 1k$

$R_{e_1} = 100\,\Omega$ $\quad R_{e_2} = 51\,\Omega$

Solve for $\beta = 100$. Find actual bias voltages, currents, $r_{e_1}$, $r_{e_2}$, gain.

Look at DC only for bias:




$$V_{BB} = \frac{3V}{R_1 + R_2} \times R_2 - 15V = -5V$$

$$R_{BB} = R_1 \parallel R_2 = 34k\Omega$$

KVL from $V_{BB}$ through $R_{BB}$, $V_{BE_1}$, $R_{E_1}$ to $V_{EE}$

$$10 = I_{B_1} R_{BB} + 0.7V + (\beta+1) I_{B_1} (R_{E_1} + R_{e_1})$$

$$I_{B_1} = \frac{10V - 0.7V}{R_{BB} + (\beta+1)(R_{E_1} + R_{e_1})} = 0.0097 \text{ mA}$$

$$I_{E_1} = (\beta+1) I_{B_1} = 0.98 \text{ mA}$$

$$V_{E_1} = I_{E_1}(R_{E_1} + R_{e_1}) - 15V = -6.0 \text{ V}$$

$$I_{C_1} = \beta I_{B_1} = 0.97 \text{ mA}$$

KVL from $V_{CC}$ through $R_{E_2}$, $V_{EB_2}$, $R_{C_1}$ to $V_{CC}$

$$0V = (\beta+1) I_{B_2} R_{E_2} + 0.7V - (I_{C_1} - I_{B_2}) R_{C_1}$$

$$I_{B_2} = \frac{I_{C_1} R_{C_1} - 0.7V}{R_{C_1} + (\beta+1)(R_{E_2} + R_{e_2})} = 0.058 \text{ mA}$$

$$I_{E_2} = (\beta+1) I_{B_2} = 5.8 \text{ mA} \qquad I_{C_2} = \beta I_{B_2} = 5.8 \text{ mA}$$

$$V_{E_2} = V_{CC} - I_{E_2}(R_{E_2} + R_{e_2}) = -8.9V$$

$$V_{C_2} = I_{C_2} R_{C_2} - 15V = -2.8 \text{ V}$$

$$r_{e_1} = \frac{V_T}{I_{E_1}} = 26\,\Omega \qquad r_{e_2} = \frac{V_T}{I_{E_2}} = 4.3\,\Omega$$

$$R_{i_2} = (\beta+1)(r_{e_2} + R_{e_2}) = 5.6\ k\Omega \qquad (11.k)$$

$$\frac{v_o}{v_i} = \frac{\beta(R_{c_1} \| R_{i_2})}{R_s + (\beta+1)(r_{e_1} + R_{e_1})} \quad \frac{\beta(R_{c_2} \| R_L)}{R_{c_1} + (\beta+1)(r_{e_2} + R_{e_2})} = 113 \ V/V$$

25

Do the same for $\beta = 200$

| | $\beta=100$ | $\beta=200$ |
|---|---|---|
| $I_{B_1}$ | 0.0097 mA | 0.0049 mA |
| $I_{E_1}$ | 0.98 mA | 0.99 mA |
| $I_{C_1}$ | 0.97 mA | 0.99 mA |
| $V_{E_1}$ | −6.0 V | −5.9 V |
| $V_{C_1}$ | 8.2 V | 7.8 V |
| $I_{B_2}$ | 0.058 mA | 0.031 mA |
| $I_{E_2}$ | 5.8 mA | 6.2 mA |
| $I_{C_2}$ | 5.8 mA | 6.1 mA |
| $V_{E_2}$ | 8.9 V | 8.5 V |
| $V_{C_2}$ | −2.2 V | −1.5 V |
| $r_{e_1}$ | 26 Ω | 25 Ω |
| $r_{e_2}$ | 4.3 Ω | 4.0 Ω |
| $\frac{v_o}{v_i}$ | 113 | 287 |

Will work, but gain is fairly dependent on $\beta$. Could improve this by isolating the stages with emitter followers