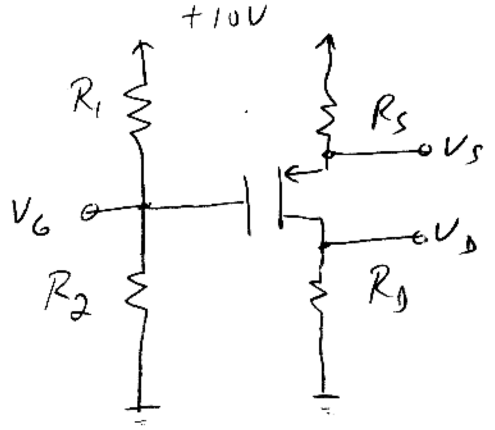
EE 321

Fall 2002

Homework #13

Solutions

5.63 (a)



Want 10μA through $R_1 + R_2$, so

$$R_1 + R_2 = \frac{10V}{10\mu A} = 1M\Omega$$

$$k_p' \frac{W}{L} = 0.5 mA/V^2$$

$$V_t = -1V$$

Want $I_D = 1mA$

$V_D = 3V$

$$V_D = I_D R_D \quad R_D = \frac{V_D}{I_D} = \frac{3V}{1ma} = 3k\Omega$$

$$I_D = \frac{1}{2} k_p' \frac{W}{L} (V_{GS} - V_t)^2$$

$$k_p' \frac{W}{L} = 0.5 mA/V^2 \qquad V_t = -1V$$

$$1mA = \frac{1}{2}(0.5mA/V^2)(V_{GS} - (-1V))^2$$

$$4V^2 = (V_{GS} + 1V)^2$$

$$V_{GS} = -3V \text{ or } +1V$$

For PMOS need $V_{GS} < V_t$, so $V_{GS} = -3V$

At triode, $V_{DS} = V_{GS} - V_t = -3V - (-1V) = -2V$

To be 1V above triode, need $V_{DS} = -3V$

$$V_S = V_D - V_{DS} = 3V - (-3V) = 6V$$

$$V_G = V_{GS} + V_S = -3V + 6V = 3V$$

$$V_G = \frac{R_2}{R_1 + R_2} V_{DD} \Rightarrow R_2 = \frac{V_G}{V_{DD}}(R_1 + R_2) = \frac{3V}{10V} 1M\Omega = 300k\Omega$$

$$V_S = V_{DD} - I_D R_S \Rightarrow R_S = \frac{V_{DD} - V_S}{I_D} = \frac{10V - 6V}{1mA} = 4k\Omega$$

$\therefore$ $R_1 = 300K \qquad R_S = 4k\Omega$

$R_2 = 700k \qquad R_D = 3k\Omega$

(b) As in (a) $R_1 + R_2 = 10\,M\Omega$ $\quad R_D = 3\,k\Omega$

$$I_D = \frac{1}{2} k_p' \frac{W}{L} (V_{GS} - V_t)^2$$

$|V_t| = 2V \Rightarrow V_t = +2V$ (Depletion P-MOS)

$k_p' \frac{W}{L} = 0.5\,mA/V^2$

$$1mA = \frac{1}{2}(0.5\,mA/V^2)(V_{GS} - 2V)^2$$

$$4V^2 = (V_{GS} - 2V)^2$$

$V_{GS} = 0V, 4V$ $\quad$ Nee $V_{GS} < V_t$, so $V_{GS} = 0V$

At triode, $V_{DS} = V_{GS} - V_t = 0 - 2V = -2V$

To be 1V above triode need $V_{DS} = -3V$

$V_S = V_D - V_{DS} = 3V - (-3V) = 6V$

$V_S = I_{DD} - I_D R_S \Rightarrow R_S = \frac{V_{DD} - V_S}{I_D} = 4\,k\Omega$

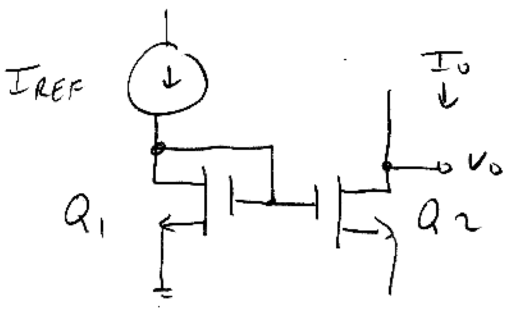
$V_G = V_{GS} + V_S = 0 + 6V = 6V$

$V_G = \frac{R_2}{R_1 + R_2} V_{DD} \Rightarrow R_2 = \frac{V_G}{V_{DD}}(R_1 + R_2) = 600\,k\Omega$

$\therefore$ $R_1 = 400\,k\Omega$ $\quad R_S = 4\,k\Omega$

$R_2 = 600\,k\Omega$ $\quad R_D = 3\,k\Omega$

5.71



$k_n' \frac{W}{L} = 40 \mu A / V^2$

$V_t = 0.8 V$

$V_A = 20 V$

$I_{REF} = 10 \mu A$

$I_{D1} = \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_t)^2 = I_{REF}$

$10 \mu A = \frac{1}{2} (40 \mu A / V^2) (V_{GS} - 0.8 V)^2$

$\frac{1}{2} V^2 = (V_{GS} - 0.8 V)^2$

$V_{GS} = 0.8 V \pm \sqrt{\frac{1}{2} V^2} = 1.51 V, \; -0.93 V$

Need $V_{GS} > V_t$, so $V_{GS} = 1.51 V$

For $I_o = I_{REF}$ need $V_{DS2} = V_{DS1}$

$V_{D2} = V_{G2}$ so $V_{DS_2} = 1.51 V \Rightarrow V_{DS_2} = 1.51 V \Rightarrow V_{D_2} = 1.51 V$

$V_o = V_{D_2} = 1.51 V$

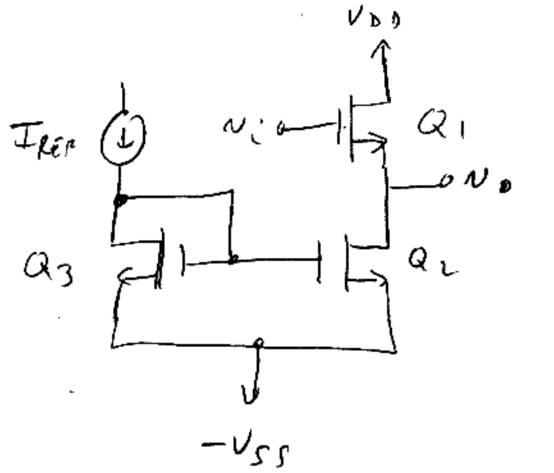
$r_{o2} = \frac{V_A}{I_D} = \frac{20 V}{10 \mu A} = 2 M\Omega$

$\Delta v_{DS} = r_o \Delta i_D$

$\Delta i_D = \frac{\Delta v_{DS}}{r_o} = \frac{2 V}{2 M\Omega} = 1 \mu A$

I_o is $11 \mu A$ when v_{DS} increases by $2V$

5.80



$k_n' = 20\mu A/V^2$

$V_t = 1V$

$V_A = 100V$

$\frac{W}{L} = 10$

$I_{REF} = 200\mu A$

$\chi = 0.1$

Because the transistors are identical,

$I_{D2} = I_{D3} = 200\mu A$

$I_{D1} = I_{D2} = 200\mu A$

$g_{m1} = \sqrt{2k_n'}\sqrt{\frac{W}{L}}\sqrt{I_{D1}} = \sqrt{2(20\mu A/V^2)(10)(200\mu A)}$

$= 0.28\,mA/V$

$r_{o1} = r_{o2} = \frac{V_A}{I_D} = \frac{100V}{200\mu A} = 500\,k\Omega$

eqn 5.70

$$A_v = \frac{g_{m1}}{g_{m1} + g_{mb1} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} = \frac{g_{m1}}{g_{m1}(1+\chi) + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

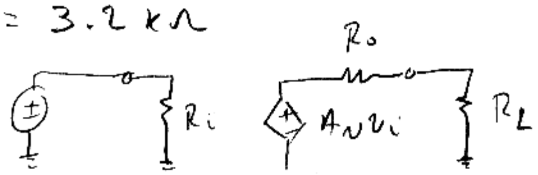
$$= \frac{0.28\,mA/V}{0.28m/V(1+0.1) + \frac{1}{500k} + \frac{1}{500K}}$$

$= 0.90\ V/V$

eqn 5.73

$$R_o = \left(\frac{1}{g_{m1}}\right) \| \left(\frac{1}{g_{mb1}}\right) \| r_{o1} \| r_{o2} = \frac{1}{g_{m1} + g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}}$$

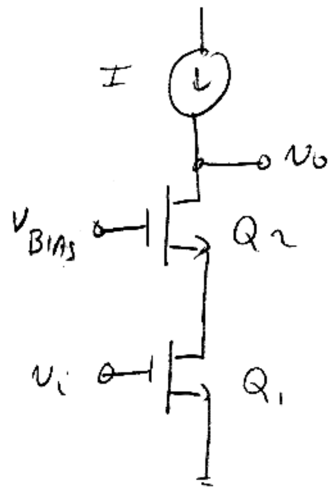
$= 3.2\,k\Omega$



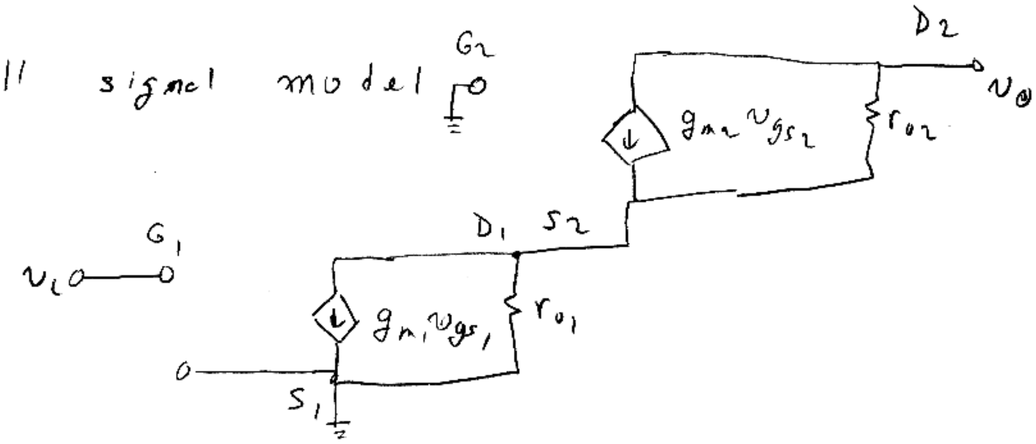
$$A_{v_{new}} = \frac{R_L}{R_L + R_o} A_v = \frac{10k}{10k + 3.2k}\, 0.9\,V/V$$

$= 0.68\,V/V$

5.81

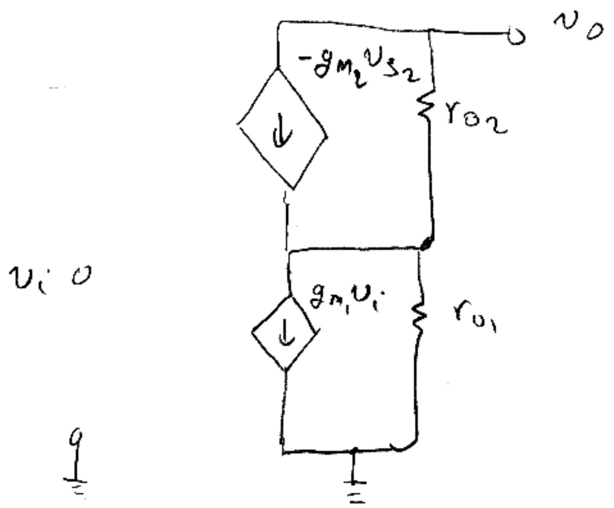


Small signal model

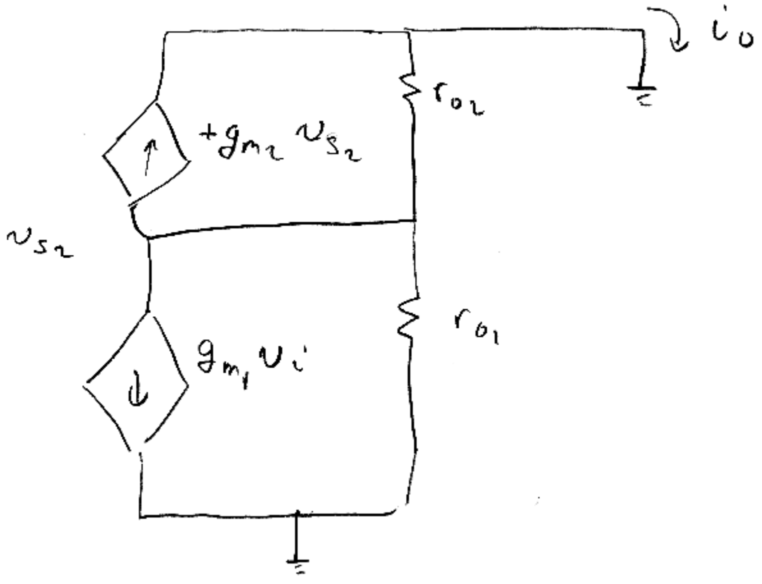


$v_{gs_1} = v_i \qquad v_{gs_2} = -v_{s_2}$

Redraw



(a) Find short-circuit output current



Node eqn at S_2:

$$g_{m1} v_i + g_{m2} v_{s2} + \frac{v_{s2}}{r_{o1}} + \frac{v_{s2}}{r_{o2}} = 0$$

$$v_{s2} = \frac{-g_{m1}}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} v_i$$

$$i_o = g_{m2} v_{s2} + \frac{v_{s2}}{r_{o2}}$$

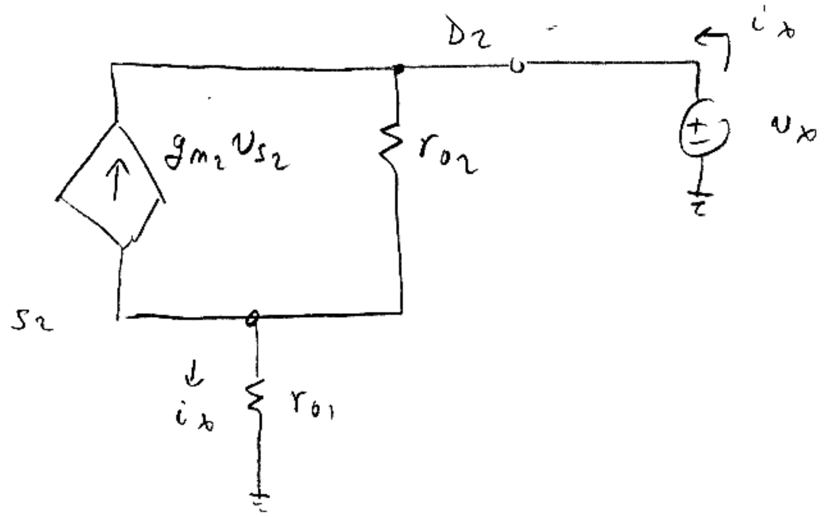
$$= \frac{-g_{m1}\left(g_{m2} + \frac{1}{r_{o2}}\right)}{g_{m2} + \frac{1}{r_{o1}} + \frac{1}{r_{o2}}} v_i$$

$$\cong -g_{m1} v_i \qquad (\text{for large } r_{o1}, r_{o2})$$

$$\frac{i_o}{v_i} \cong -g_{m1}$$

(b) With v_i shorted, $v_i = 0 \Rightarrow g_{m1} v_i = 0$

Model becomes



Node eqn at D_2:

$$i_x + g_{m2} v_{s2} + \frac{v_{s2} - v_x}{r_{o2}} = 0$$

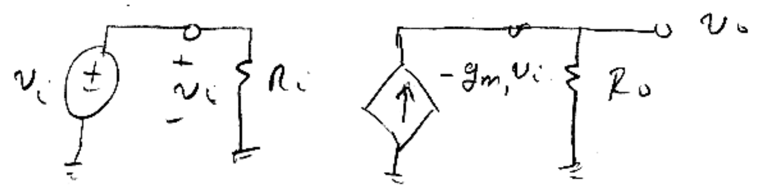
Also, $v_{s2} = i_x r_{o1}$, so

$$i_x + g_{m2} i_x r_{o1} + \frac{i_x r_{o1} - v_x}{r_{o2}} = 0$$

$$i_x \left(1 + g_{m2} r_{o1} + \frac{r_{o1}}{r_{o2}} \right) = \frac{v_x}{r_{o2}}$$

$$R_o = \frac{v_x}{i_x} = r_{o1} + r_{o2} + g_{m2} r_{o1} r_{o2} \simeq g_{m2} r_{o1} r_{o2}$$

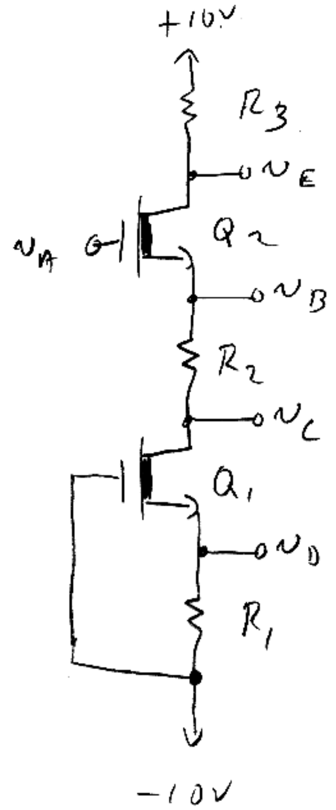
(c)



$$v_o = -g_{m1} R_o v_i \simeq -g_{m1} g_{m2} r_{o1} r_{o2} v_i$$

$$A_{vo} \cong -g_{m1} g_{m2} r_{o1} r_{o2}$$

5.84



$I_{DSS} = 4mA$

$|V_t| = 2V \Rightarrow V_t = -2V$ (Depletion NMOS)

$I_{DSS} = \frac{1}{2} k_n' \frac{W}{L} V_t^2$

$\therefore k_n' \frac{W}{L} = \frac{2 I_{DSS}}{V_t^2} = \frac{2(4mA)}{(-2V)^2} = 2mA/V^2$

$I_{D_1} = \frac{1}{2} k_n' \left(\frac{W}{L}\right) (V_{GS_1} - V_t)^2$

$1mA = \frac{1}{2}(2mA/V^2)(V_{GS_1} - (-2V))^2$

$1V^2 = (V_{GS_1} + 2V)^2$

$V_{GS_1} = -1V$ or $-3V$

Need $V_{GS_1} > V_t$, so $V_{GS_1} = -1V$

$\therefore$ 1V drop across R_1

$R_1 = \frac{V_1}{I_D} = \frac{1V}{1mA} = 1k\Omega$

$R_2 = R_1 = 1k\Omega$

$V_E = V_{DD} - R_3 I_D \Rightarrow R_3 = \frac{10V - 6V}{1mA} = 4k\Omega$

Because the transistors are equal, and have equal I_{D_1}

$V_{GS_2} = V_{GS_1} = -1V$

$V_{GS_2} = V_{G_2} - v_{S_2} = v_A - v_{S_2} \Rightarrow v_{S_2} = v_A - V_{GS_2} = 0 - (-1V) = 1V$

$v_B = v_{S_2} = 1V$

$v_C = v_B - R_2 I_D = 1V - (1k)(1mA) = 0V$

Note that $v_{S2} = v_A - V_{GS2} = v_A + 1V$, $v_B = v_{S2} = v_A + 1V$

$v_C = v_B - I_D R_1 = v_A + 1V - (1mA)(1k\Omega) = v_A$

$v_C = v_A$ as long as both transistors remain in saturation

Q_2 enters triode when

$$v_{DS_2} = v_{GS_2} - V_t = -1V - (-2V) = 1V$$

$$v_{D_2} = v_E = 6V$$

$$v_A = v_{S2} + v_{GS_2} = v_{D2} - v_{DS_2} + v_{GS2}$$
$$= 6V - 1V - 1V = 4V$$

Q_1 enters triode when

$$v_{DS_1} = v_{GS_1} - V_t = -1V - (-2V) = 1V$$

$$v_A = v_{Dc} = v_{D_1} = v_{DS_1} + v_{S_1} = v_{DS_1} + I_D R_1 - 10V$$
$$= 1V + (1mA)(1k\Omega) - 10V$$
$$= -8V$$

Circuit works as a follower with unity gain and no offset for $-8V < v_A < 4V$