EE 321

Fall 2002

Homework #14

Solutions

5.34 Exercise. V_{th} is the voltage where both the transistors are in saturation:

$$i_{DN} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n (V_{th} - V_{tn})^2 \qquad \text{Eqn 5.88}$$

$$i_{DP} = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_p (V_{DD} - V_{th} - |V_{tp}|)^2 \qquad \text{Eqn 5.90}$$

Because $i_{DP} = i_{DN}$ we have

$$\frac{1}{2} k_n' \left(\frac{W}{L}\right)_n (V_{th} - V_{tn})^2 = \frac{1}{2} k_p' \left(\frac{W}{L}\right)_p (V_{DD} - V_{th} - |V_{tp}|)^2$$

$$\therefore (V_{th} - V_{tn})^2 = r^2 (V_{DD} - V_{th} - |V_{tp}|)^2$$

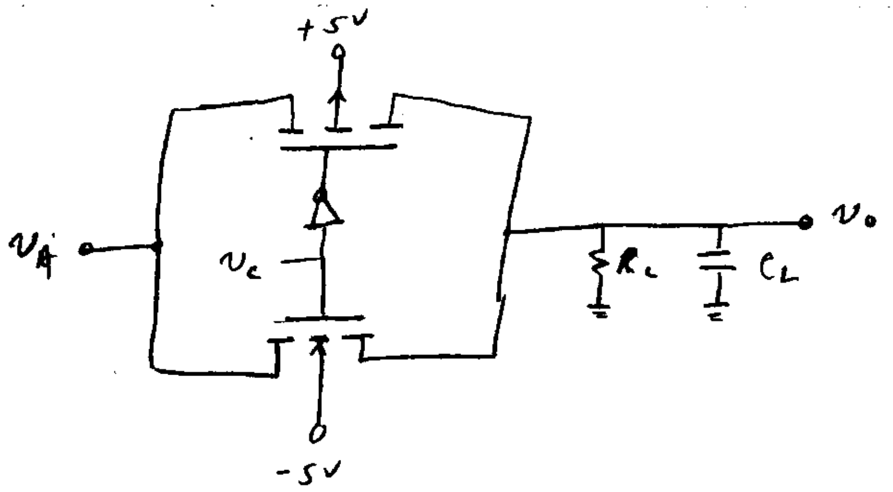
where $r = \sqrt{\dfrac{k_p' \left(\frac{W}{L}\right)_p}{k_n' \left(\frac{W}{L}\right)_n}}$

$$\therefore (V_{th} - V_{tn}) = r(V_{DD} - V_{th} - |V_{tp}|)$$

$$V_{th} + r V_{th} = r(V_{DD} - |V_{tp}|) + V_{tn}$$

$$V_{th} = \frac{r(V_{DD} - |V_{tp}|) + V_{tn}}{1 + r}$$

5.40 Exercise



$|V_t| = 2V$

$k'\frac{W}{L} = 0.1 mA/V^2$

$R_L = 50k$

$v_G = 5V$ (Nmos)

$v_G = -5V$ (Pmos)

(a) For $v_A = -5V$

Pmos: $v_G = -5V$, $v_S = v_A = -5V$ $\quad v_{GS} = 0V > V_{tp}$

Pmos off $\quad r_{DS_P} = \infty$

Nmos: $v_G = 5V$ $\quad v_S = v_A = -5V$ $\quad v_{GS} = 5-(-5) = 10V > V_{tn}$

Nmos on

$r_{DSN} = \left[k'\frac{W}{L}|v_{GS} - V_t|\right]^{-1} = \left[(0.1mA/V^2)|10V - 2V|\right]^{-1}$

$= 1.25\ k\Omega$

$R = r_{DS_N} = 1.25k\Omega$

$$v_o = \frac{R_L}{R+R_L}v_A = \frac{50k}{1.25k+50k}(-5V) = -4.878V$$

(b) For $v_A = -2V$

Pmos: $v_S = v_A = -2V$ $\quad v_{GS} = -3V < V_{tp}$

Pmos on

Nmos: $v_S = v_A = -2V$, $\quad v_{GS} = 5-(-2) = 7V > V_{tn}$

Nmos on

$r_{DS_P} = \left[(0.1mA/V^2)|-3V-(-2V)|\right]^{-1} = 10\ k\Omega$

$r_{DS_N} = \left[(0.1mA/V^2)|7V-2V|\right]^{-1} = 2k\Omega$

$$R = r_{DS_N} \parallel r_{DS_P} = 2k\Omega \parallel 10k\Omega = 1.67k\Omega$$

$$v_O = \frac{R_L}{R + R_L} v_A = \frac{50k}{1.67k + 50k}(-2V) = -1.94V$$

(c) $v_A = 0V$

Pmos: $v_S = v_A = 0V$ $\quad v_{GS} = -5V - 0V = -5V < V_{tP}$

Pmos on

Nmos: $v_S = v_A = 0V$ $\quad v_{GS} = 5V - 0V = 5V > V_{tn}$

Nmos on

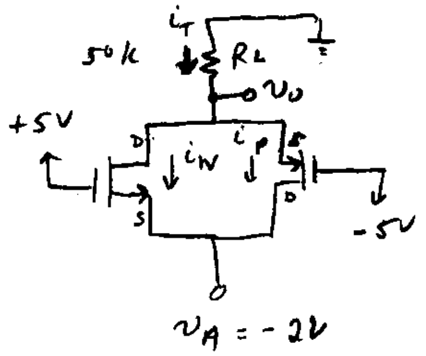
$$r_{DS_P} = \left[(0.1\,mA/V^2)\,|-5V - (-2V)|\right]^{-1} = 3.33\,k\Omega$$

$$r_{DS_N} = \left[(0.1\,mA/V^2)(5V - 2V)\right]^{-1} = 3.33\,k\Omega$$

$$R = r_{DS_P} \parallel r_{DS_N} = 3.33k \parallel 3.33k = 1.67k\Omega$$

$$v_O = \frac{R_L}{R + R_L} v_A = \frac{50k}{1.67k + 50k}\, 0V = 0V$$

Answer in the book to part (b) is slightly different. To get answer in book:



$$v_O = -i_T R_L = -(i_N + i_P) R_L \qquad (1)$$

$$i_N = k_n'\left(\frac{W}{L}\right)_n \left[(v_{GS_n} - V_{tn}) v_{DS_n} - \tfrac{1}{2} v_{DS_n}^2\right]$$

$$\cong k_n'\left(\frac{W}{L}\right)_n (v_{GS_n} - V_{tn}) v_{DS_n}$$

$$i_N = 100\mu A/V^2 \,(5 - (-2)) - 2)\,(v_O - (-2)) \qquad (2)$$

$$i_P \cong k_p'\left(\frac{W}{L}\right)_p (v_{GS_p} - V_{tp}) v_{DS_p}$$

$$i_P = 100\mu A/V^2 (-5 - v_O - (-2))(-2 - v_O) \qquad (3)$$

Put (2) + (3) into (1), and get an equation for v_O

$$v_o = -\left[(500\,\mu A/V)(v_o + 2V) + (100\,\mu A/V^2)(-3 - v_o)(-2 - v_o)\right] R_L$$

$$v_o = -25(v_o + 2) - 5(6 + 5v_o + v_o^2)$$

$$5v_o^2 + 51v_o + 80 = 0$$

$$v_o = -1.936V,\ -8.236V$$

$-8.236V$ doesn't make sense, so

$$v_o = -1.936V$$

This gives

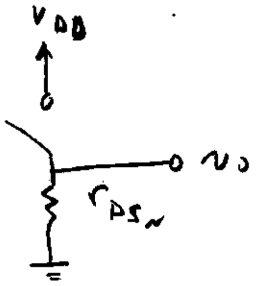
$$i_N = 31.9\,\mu A$$

$$i_P = 6.8\,\mu A$$

$$i_T = 38.7\,\mu A$$

$$R = \frac{v_o - v_A}{i_T} = \frac{-1.936V - (-2V)}{38.7\,\mu A} = 1.649\,k$$

5.92 (a) When $v_O = V_{OL}$, output of inverter looks like



$$R_O = r_{DS_N} = \left[k_n'\left(\frac{W}{L}\right)_n (V_{DD} - V_{tn})\right]^{-1}$$

$$= \left[(81\mu A/V^2)\left(\frac{1.8\mu m}{1.2\mu m}\right)(5V - 0.75V)\right]^{-1}$$

$$= 1.94\ k\Omega$$

(b) When $v_O = V_{OH}$, output of inverter looks like

$$R_O = r_{DS_P} = \left[k_p'\left(\frac{W}{L}\right)_p (V_{DD} - |V_{tp}|)\right]^{-1}$$

$$= \left[(27\mu A/V^2)\left(\frac{5.4\mu m}{1.2\mu m}\right)(5V - 0.75V)\right]^{-1}$$

$$= 1.94\ k\Omega$$

5.93 Two ways:

(1) When output is low, voltage drop across r_{DS_N} is

$$0.2V = r_{DS_N} i_D \Rightarrow i_D = \frac{0.2V}{r_{DS_N}} = 0.10\ mA$$

When output is high, voltage drop across r_{DS_P} is

$$0.2V = r_{DS_P} i_D \Rightarrow i_D = \frac{0.2V}{1.94k\Omega} = 0.10\ mA$$

(2) When output low, NMOS in triode; $v_I = V_{DD}$

$$i_{D_N} = k_n'\left(\frac{W}{L}\right)_n \left[(V_{DD} - V_{tn}) v_O - \frac{1}{2} v_O^2\right]$$

$$= 81\mu A/V^2 \left(\frac{1.8\mu m}{1.2\mu m}\right)\left[(5 - 0.75)(0.2V) - \frac{1}{2}(0.2V)^2\right]$$

$$= 0.11\ mA$$

Same for i_{D_P} when output high

5.100 Both transistors in saturation ; $v_I = V_{th} = 2.5V$

$$i_D = i_{Dn} = \frac{1}{2} k_n' \left(\frac{W}{L}\right)_n (v_I - V_{tn})^2$$

$$= \frac{1}{2}\, 81\mu A/V^2 \left(\frac{1.8\mu m}{1.2\mu m}\right)(2.5V - 0.75V)^2$$

$$= 0.18\ mA$$

5.103 $t_{PHL} = \dfrac{1.6C}{k_n'\left(\frac{W}{L}\right)_n V_{DD}}$ eqn 5.102

$$W_n = \frac{1.6C}{t_{PHL}\, k_n' \frac{1}{L_n} V_{DD}}$$

$$= \frac{1.6(0.05pF)}{(60ps)(81\mu A/V^2)\left(\frac{1}{1.2\mu m}\right)5V}$$

$$= 6.3\mu m$$

$$W_p = \frac{1.6C}{t_{PLH}\, k_p' \frac{1}{L_p} V_{DD}}$$

$$= 12\mu m$$

5.107 $r_{DS} = \dfrac{1}{k_n'\left(\frac{W}{L}\right)(V_{GS}-V_t)}$

When on, $v_G = V_C = +10\,V$

$v_{GS} = v_G - v_S = v_G - v_{in} = 10\,V - v_{in}$

For $v_{in} = +5\,V$:

$$r_{DS} = \frac{1}{(100\,\mu A/V^2)(10-5-1)} = 2.5\,k\Omega$$

For $v_{in} = -5\,V$:

$$r_{DS} = \frac{1}{(100\,\mu A/V^2)(10-(-5)-1)} = 0.71\,k\Omega$$

Want $\dfrac{r_{DS}}{R+r_{DS}} < 0.01$

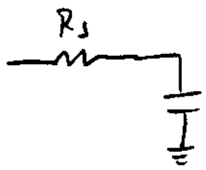
$$R > \frac{r_{DS}}{0.01} - r_{DS} = 99\,r_{DS}$$

For $r_{DS} = 2.5\,k\Omega$, need $R > 248\,k\Omega$

5.109 Input is about 0 V, so $r_{DS_N} = r_{DS_P}$, and $v_{GS_N} = 5V - 0V = 5V$

$$R = r_{DS_N} \parallel r_{DS_P} = \tfrac{1}{2} r_{DS_N} = \tfrac{1}{2}\,\frac{1}{k_n'\left(\frac{W}{L}\right)_n (V_{GS}-V_{tn})}$$

$$= \tfrac{1}{2}\,\frac{1}{(100\,\mu A/V^2)(5V - 1V)} = 1.25\,k\Omega$$

When switch is on, v_i 

$C_L = 1000\,pF$

$$f_0 = \frac{1}{2\pi R_s C_L} = 127\,kHz$$