

EE 341 - Exam 2

October 26, 2005

Name: Solutions

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes and a calculator.

1. Find the Fourier series representation of the continuous-time periodic signal

$$x(t) = 2 \cos\left(\frac{2\pi}{3}t\right) + 4 \sin\left(\frac{2\pi}{3}t\right) + \cos\left(\frac{\pi}{7}t\right)$$

$$\omega_1 = \frac{2\pi}{3} \quad T_1 = \frac{2\pi}{\omega_1} = 3$$

$$\omega_2 = \omega_1 \quad T_2 = T_1 = 3$$

$$\omega_3 = \frac{\pi}{7} \quad T_3 = \frac{2\pi}{\omega_3} = 14$$

$$\frac{T_1}{T_3} = \frac{3}{14}$$

$$T = 8T_1 = 14 \cdot 3 = 42$$

$$\omega_0 = \frac{2\pi}{T} = \frac{2\pi}{42} = \frac{\pi}{21}$$

$$\omega_1 = \frac{2\pi}{3} = 14\omega_0$$

$$\omega_2 = 14\omega_0$$

$$\omega_3 = \frac{\pi}{7} = 3\omega_0$$

$$X(\omega) = 2 \cos(14\omega_0 t) + 4 \sin(14\omega_0 t) + \cos(3\omega_0 t)$$

$$= 2 \frac{e^{j14\omega_0 t} + e^{-j14\omega_0 t}}{2} + 4 \frac{e^{j14\omega_0 t} - e^{-j14\omega_0 t}}{2j} + \frac{e^{j3\omega_0 t} + e^{-j3\omega_0 t}}{2}$$

$$= e^{j14\omega_0 t} (1 - j2) + e^{-j14\omega_0 t} (1 + j2) + e^{j3\omega_0 t} \left(\frac{1}{2}\right) + e^{-j3\omega_0 t} \left(\frac{1}{2}\right)$$

$$C_{14} = 1 - j2 \quad C_{-14} = 1 + j2 \quad C_3 = \frac{1}{2} \quad C_{-3} = \frac{1}{2} \quad C_{ke} = 0 \text{ for all other } k$$

2. Determine the following:

(a) Find the Fourier transform of $x(t) = e^{-2t} \cos(2\pi t) u(t)$

$$e^{-2t} u(t) \leftrightarrow \frac{1}{2+j\omega}$$

$$e^{-2t} \cos(2\pi t) u(t) \leftrightarrow \frac{1}{2} \left[\frac{1}{2+j(\omega+2\pi)} + \frac{1}{2+j(\omega-2\pi)} \right]$$

(b) Find the Fourier transform of $x(t) = \text{sinc}(t) * \text{sinc}(2t)$ (the convolution of $\text{sinc}(t)$ and $\text{sinc}(2t)$).

$$\text{sinc}(t) = \frac{1}{2\pi} \left[2\pi \text{sinc} \left(\frac{2\pi t}{2\pi} \right) \right] \Leftrightarrow \frac{1}{2\pi} [2\pi P_{2\pi}(\omega)] = P_{2\pi}(\omega)$$

$$\text{sinc}(2t) = \frac{1}{4\pi} \left[4\pi \text{sinc} \left(\frac{4\pi t}{2\pi} \right) \right] \Leftrightarrow \frac{1}{4\pi} [2\pi P_{4\pi}(\omega)] = \frac{1}{2} P_{4\pi}(\omega)$$

$$\text{sinc}(t) * \text{sinc}(2t) \Leftrightarrow \frac{1}{2} P_{2\pi}(\omega) P_{4\pi}(\omega) = \frac{1}{2} P_{2\pi}(\omega)$$

(c) Find the inverse Fourier transform $x(t)$ when $X(\omega) = \cos(10\omega)$.

Use duality

$$\cos(10t) \Leftrightarrow \pi [\delta(\omega+10) + \delta(\omega-10)]$$

$$2\pi \pi [\delta(\omega+10) + \delta(\omega-10)] \Leftrightarrow \cos(10t)$$

$$X(t) \Leftrightarrow 2\pi X(-\omega)$$

$$\pi [\delta(t+10) + \delta(t-10)] \Leftrightarrow 2\pi \cos(-10\omega) = 2\pi \cos(10\omega)$$

$$\frac{1}{2} [\delta(t+10) + \delta(t-10)] \Leftrightarrow \cos(10\omega)$$

OR: $\cos(10\omega) = \frac{e^{j10\omega} + e^{-j10\omega}}{2} \Leftrightarrow \frac{\delta(\omega+10) + \delta(\omega-10)}{2}$ using $e^{j\omega c} \Leftrightarrow \delta(\omega+c)$

3. The frequency response of a system is:

$$H(\omega) = \frac{j\omega}{j\omega + 100}$$

(a) What type of system is this: low pass, high pass, band pass or all pass? Explain.

$$H(0) = \frac{0}{100} = 0 \quad H(\infty) = \frac{j\infty}{j\infty} = 1$$

High pass - blocks DC, passes high frequencies

(b) The input to the system is $x(t) = 2 + 2 \cos(50t + \pi/2)$. Find the output $y(t)$ from the system.

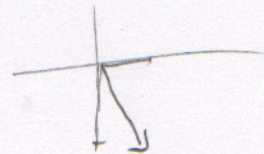
System blocks DC signal of two

Signal with $\omega = 50$:

$$H(50) = \frac{j50}{j50 + 100} = \frac{50}{50 - j100} = \frac{1}{1 - j2}$$

$$|H(50)| = \frac{1}{\sqrt{1^2 + 2^2}} = \frac{1}{\sqrt{5}}$$

$$\angle H(50) = \angle 1 - \angle (1 - j2) = 0 - \tan^{-1}\left(\frac{-2}{1}\right) = 63^\circ$$



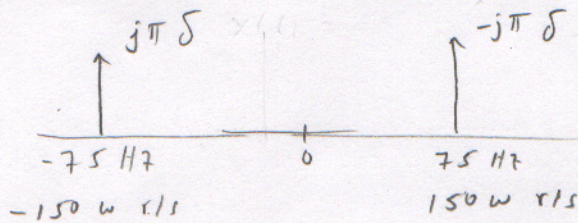
$$y(t) = \frac{1}{\sqrt{5}} 2 \cos(50t + \pi/2 + 63^\circ)$$

$$= \frac{2}{\sqrt{5}} \cos(50t + 153^\circ)$$

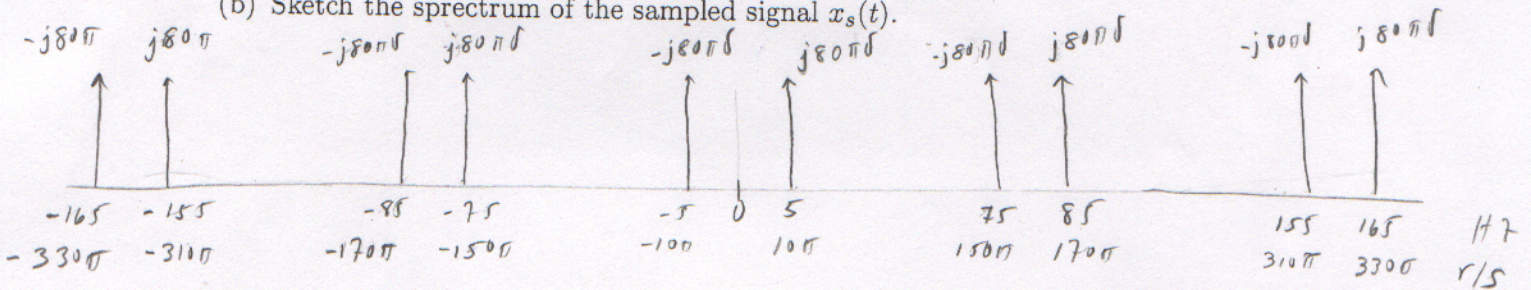
4. The signal $x(t) = \sin(150\pi t)$ is sampled with $T = 0.0125$ sec ($F_s = 80$ Hz).

(a) Sketch the spectrum of the signal $x(t)$.

$$X(\omega) = j\pi (\delta(\omega + 150\pi) - \delta(\omega - 150\pi))$$

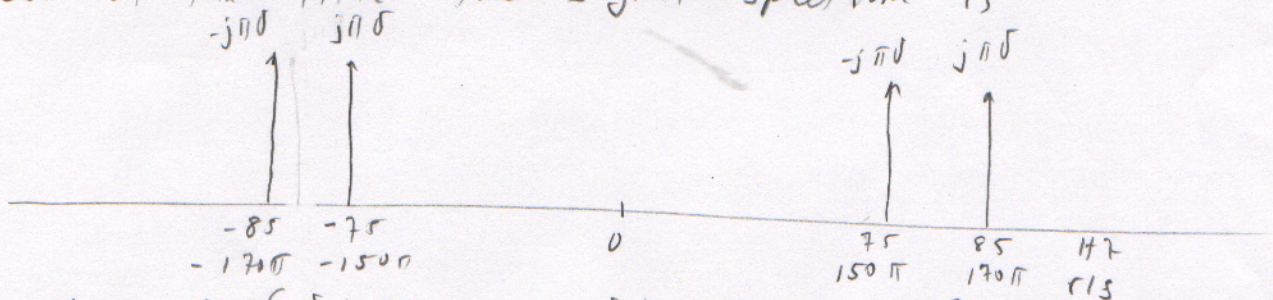


(b) Sketch the spectrum of the sampled signal $x_s(t)$.



(c) $x_s(t)$ is passed through an ideal bandpass filter with pass frequencies between 60 Hz and 100 Hz (ω between 120π and 200π rad/sec). Find the output $y(t)$ from the bandpass filter. $H(\omega) = \frac{1}{80}$

Out of the filter the signal spectrum is



$$Y(\omega) = j\pi (\delta(\omega + 150\pi) - \delta(\omega - 150\pi)) - j\pi (\delta(\omega + 170\pi) - \delta(\omega - 170\pi))$$

$$y(t) = \sin(150\pi t) - \sin(170\pi t)$$