EE 341 - Final Exam

December 14, 2004

Name: ______________________________

Closed book. Show all work. Partial credit will be given. No credit will be given if an answer appears with no supporting work. You may use one page of notes and a calculator.

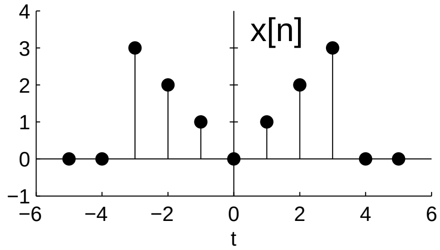
1. Determine whether or not each of the following signals is periodic. If a signal is periodic, determine its fundamental period. If a signal is not periodic, explain why it isn't.

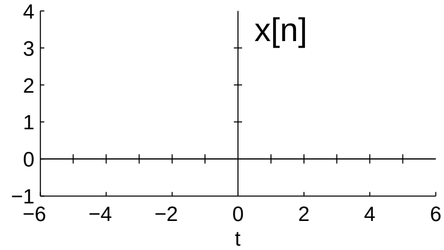
   (a) $x(t) = \cos\left(t + \frac{\pi}{4}\right) + \sin\left(2\,t + \frac{1}{4}\right)$

   (b) $x(t) = \cos^2\left(\frac{\pi}{8}\,t\right)$

   (c) $x[n] = \cos\left(\frac{\pi}{3}\,n\right) + \cos\left(\frac{\pi}{4}\,n\right)$

2. A discrete-time signal $x[n]$ is shown below. Sketch and label the signal $x[n-2]\{u(n+2)-u(n)\}$.





3. A system may or may not be (i) Memoryless, (ii) Causal, (iii) Linear, (iv) Time Invariant. Determine which of these properties hold and which do not hold for each of the following systems. Justify your answer. In each example, $y(t)$ or $y[n]$ denotes the system output and $x(t)$ or $x[n]$ is the system input.

(a) $y(t) = \begin{cases} 0 & \text{if } t < 0 \\ x(t) + x(t-2) & \text{if } t \geq 0 \end{cases}$

(i)______ (ii)______ (iii)______ (iv)______

(b) $y[n] - 2y[n-1] = x[n] + 2nx[n-2]$

(i)______ (ii)______ (iii)______ (iv)______

4. Consider the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n],$$

with the input

$$x[n] = \left(\frac{1}{3}\right)^n u[n].$$

The initial condition is $y[-1] = -2$.

Find a closed form expression for the output $y[n]$, solving the system in the time domain.

5. Determine the following convolutions:

   (a) $x[n] = \delta(n) + 2\delta(n-1) + 3\delta(n-2)$, and $v[n] = \delta(n) - 2\delta(n-2)$. Find the linear convolution of $x[n]$ and $v[n]$.

   (b) $x[n] = \delta(n) + 2\delta(n-1) + 3\delta(n-2)$, and $v[n] = \delta(n) - 2\delta(n-2)$. Find the 3-point circular convolution of $x[n]$ and $v[n]$.

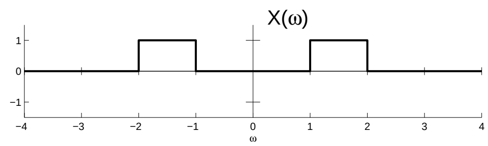
   (c) $x[n] = (\frac{1}{2})^n u[n]$ and $h[n] = u[n+2]$. Find the linear convolution of $x[n]$ and $h[n]$.

6. Find the Fourier series representation of the continuous-time periodic signal

$$x(t) = 2 + \cos\left(\frac{2\pi}{3}t\right) + 4\sin\left(\frac{5\pi}{3}t\right)$$

7. Below are plots of the Fourier transforms $X(\omega)$ of some continuous-time signals $x(t)$. Answer the questions about the signals $x(t)$. Be sure to explain your answers.

   (a)

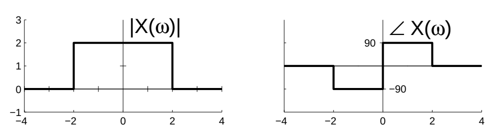


   i. Is $x(t)$ real, imaginary, or complex?

   ii. Is $x(t)$ even or odd?

   iii. Find $x(0)$.

   (b)

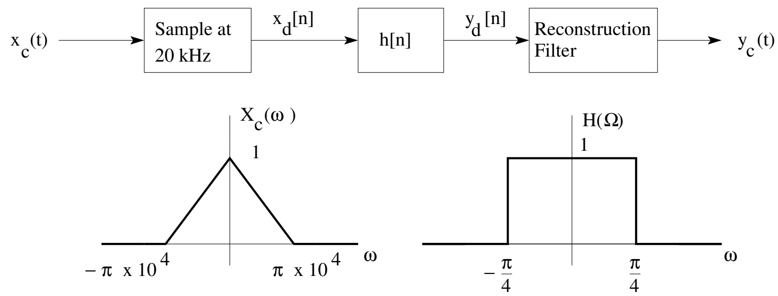


   i. Is $x(t)$ real, imaginary, or complex?

   ii. Is $x(t)$ even or odd?

   iii. Find $x(0)$.

8. The figure below shows a system for filtering a continuous-time signal using a discrete-time filter. Also shown are the Fourier transform $X_c(\omega)$ of the input signal $x_c(t)$ and the frequency response $H(\Omega)$ of the discrete-time filter $h[n]$.



(a) Sketch $X_d(\Omega)$, the Fourier transform of $x_d[n]$.

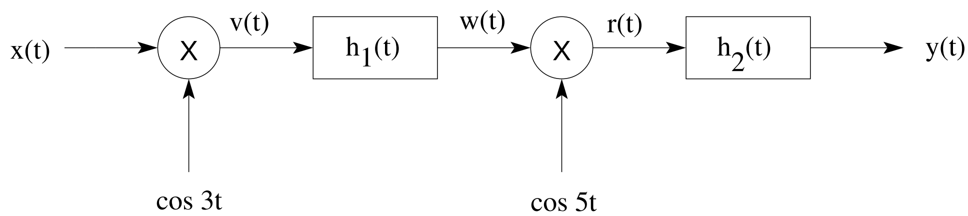
(b) Sketch $Y_d(\Omega)$, the Fourier transform of $y_d[n]$.

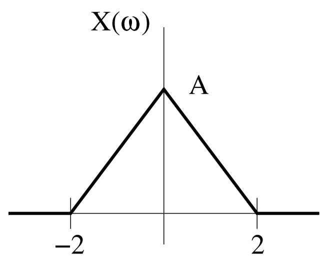
(c) Sketch $Y_c(\omega)$, the Fourier transform of $y_c(t)$.

9. Consider the system below. The frequency responses of the filters are given by

$$H_1(\omega) = \begin{cases} 0, & |\omega| < 3 \\ 1, & |\omega| \geq 3 \end{cases} \qquad H_2(\omega) = \begin{cases} 1, & |\omega| < 3 \\ 0, & |\omega| \geq 3 \end{cases}$$

The input $x(t)$ has the spectrum $X(\omega)$.





(a) Sketch $V(\omega)$

(b) Sketch $W(\omega)$

(c) Sketch $R(\omega)$

(d) Sketch $Y(\omega)$

10. Find the following Fourier transforms:

    (a) $x[n] = n\left(\frac{1}{4}\right)^n u[n]$

    (b) $x[n] = \left(-\frac{1}{2}\right)^n \sin\left(\frac{\pi}{4}n\right) u[n]$