EE 341 - Homework 2

Due September 7, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. In each of the following systems, $x(t)$ is the input and $y(t)$ is the output. Classify each system in terms of linearity, time invariance, memory, and causality. Justify your answers

   (a) $y(t) = x(t)\cos(2\pi\, f_o\, t)$ (Amplitude modulation)

   (b) $y(t) = \cos\left[2\pi\, f_0\, t\, x(t)\right]$ (Frequency modulation)

   (c) $y(t) = x(4t)$

   (d) $y(t) = x^2(t) + 2\,x(t+1)$

   (e) $\dfrac{d^2y(t)}{dt^2} + 3\dfrac{dy(t)}{dt} = 2\dfrac{dx(t)}{dt} + x(t)$

   (f) $\dfrac{d^2y(t)}{dt^2} + \cos(st)\dfrac{dy(t)}{dt} = 2\,x(t)$

2. In each of the following systems, $x[n]$ is the input and $y[n]$ is the output. Classify each system in terms of linearity, time invariance, memory, and causality. Justify your answers

   (a) $y[n] - y[n-1] = 2\,x[n] - x[n-1]$

   (b) $y[n] - y[n-1] = 2\,x[n+2] - x[n+1]$

   (c) $y[n] = 3^n\, x[n]$

   (d) $y[n] = x[n] + \cos[0.5\pi(n+1)]$

3. Problem 1.33.

4. Problem 2.1 (b) (e).

5. Problem 2.9.

EE 341 HW#2

1.(a) $y(t) = x(t)\cos(2\pi f_0 t)$

Memoryless – $y(t)$ depends only on current time

Causal – memoryless systems are causal

Time-varying – coefficient of $x(t)$ depends on time

Linear – coefficient of $x(t)$ does not depend on $x(t)$ or $y(t)$

(b) $y(t) = \cos(2\pi f_0 t\, x(t))$

Memoryless – see (a)

Causal – see (b)

Time-varying: Let $\tilde{x}(t) = x(t-t_0)$

$\tilde{y}(t) = \cos(2\pi f_0 t\, x(t-t_0))$

But $y(t-t_0) = \cos(2\pi f_0 (t-t_0)\, x(t-t_0)) \neq \tilde{y}(t)$

Non-linear – If $x(t)=0$, $y(t)=1$

Zero input does not give zero output

(c) $y(t) = x(4t)$

Has memory: $y(-2) = x(-8)$

Non-causal: $y(2) = x(8)$

Linear – coefficient of x is constant

Time-varying: Let $\tilde{x}(t) = x(t-t_0)$

$\tilde{y}(t) = \tilde{x}(4t) = x(4t-t_0)$

But $y(t-t_0) = x(4(t-t_0)) = x(4t-4t_0) \neq \tilde{y}(t)$

(d) $y(t) = x^2(t) + x(t+1)$

Has memory: $y(0)$ depends on $x(1)$

Non-causal – $y(0)$ depends on $x(1)$

Non-linear – Let $\tilde{x}(t) = 2x(t)$ $\tilde{y}(t) = 4x^2(t) + 2x(t+1)$

$= 2x^2(t) + 2x^2(t) + 2x(t+1)$

$= 2x^2(t) + 2y(t) \neq 2y(t)$

Time-invariant. Let $\tilde{x}(t) \equiv x(t-t_0)$

$$\tilde{y}(t) = x^2(t-t_0) + x(t-t_0) = y(t-t_0)$$

(e) $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} = 2\frac{dx}{dt} + x(t)$

This is Causal - derivatives can be found with current and past values of input and output.

Has memory - need to know about current and past values to calculate derivatives.

Linear - equations of form 1.57 (p. 45 of text) are linear

Time-invariant - equations of form 1.57 with constant coefficients are time invarient

(f) $\frac{d^2y}{dt^2} + \cos(5t)\frac{dy}{dt} = 2x(t)$

Causal, has memory, linear - same reasons as (e)

Time-varying - equations of the form 1.57 with coefficients which depend on time

2. (a) $y[n] - y[n-1] = 2x[n] - x[n-1]$

$y[n] = y[n-1] + 2x[n] - x[n-1]$

Causal - $y[n]$ depends on current and past times

Has memory - depends on past times

Linear and time-invariant - of form of eq 1.59 with constant coefficients

(b) $y[n] = y[n-1] + x[n+2] - x[n+1]$

Non-causal - depends on future inputs

Has memory - depends on other times

Linear and time invariant - same reason as (a)

(c) $y(n) = 3^n x(n)$

Causal - does not depend on future times

Memoryless - depends only on current time

Linear - $y_1(n) = 3^n x_1(n)$ $y_2(n) = 3^n x_2(n)$

Let $\tilde{x}(n) = a_1 x_1(n) + a_2 x_2(n)$

$$\tilde{y}(n) = 3^n \tilde{x}(n) = 3^n(a_1 x_1(n) + a_2 x_2(n))$$
$$= a_1 3^n x_1(n) + a_2 3^n x_2(n)$$
$$= a_1 y_1(n) + a_2 y_2(n)$$

Time-varying - let $\tilde{x}(n) = x(n-n_0)$

$$\tilde{y}(n) = 3^n \tilde{x}(n) = 3^n x(n-n_0)$$

But $y(n-n_0) = 3^{n-n_0} x(n-n_0) \neq \tilde{y}(n)$

(d) $y(n) = x(n) + \cos(0.5\pi(n+1))$

Causal - Does not depend on future inputs

Memoryless - Depends on current input only

Non-linear - Zero input gives non-zero output

Time-varying - let $\tilde{x}(n) = x(n-n_0)$

$$\tilde{y}(n) = \tilde{x}(n) + \cos(0.5\pi(n+1)) = x(n-n_0) + \cos(0.5\pi(n+1))$$

But $y(n-n_0) = x(n-n_0) + \cos(0.5\pi(n-n_0+1)) \neq \tilde{y}(n)$

3. Problem 1.33

(a) See MATLAB

(b) See MATLAB $x_2(n) = x_1(n-1)$ and $y_2(n) = y_1(n-1)$

(c) See MATLAB

(d) See MATLAB $x_4(n) = x_3(n-4)$ and $y_4(n) = y_3(n-4)$

(e) See MATLAB $y_2(n) \neq y_1(n-1)$ $y_4(n) \neq y_3(n-4)$

Time-varying - the output from a delayed input is different than the original output delayed

4. Problem 2.1 (b) (e)

2.1 Use eqn 2.13

$$y(t) = e^{-at}[y(0^-) - b_1 x(0^-)] + \int_{0^-}^{t} e^{-a(t-\lambda)}(b_0 - ab_1) x(\lambda)\, d\lambda + b_1 x(t)$$

(b) $\frac{dy}{dt} + 2y = x(t) \quad x(t) = u(t) \quad y(0^-) = 4$

$a = 2 \quad b_0 = 1 \quad b_1 = 0 \quad x(0^-) = 0$

$$y(t) = e^{-2t}[4 - 0 \cdot 0] + \int_{0^-}^{t} e^{-2(t-\lambda)}(1 - 2\cdot 0)\, u(\lambda)\, d\lambda + 0 \cdot x(t)$$

$$= 4e^{-2t} + e^{-2t}\int_{0^-}^{t} e^{2\lambda}\, d\lambda = 4e^{-2t} + e^{-2t}\left(\frac{e^{2\lambda}}{2}\right)_0^t$$

$$= 4e^{-2t} + e^{-2t}\left(\frac{e^{2t}-1}{2}\right) = 4e^{-2t} + \frac{1}{2} - \frac{1}{2}e^{-2t}$$

$$= \frac{1}{2} + \frac{7}{2}e^{-2t} \qquad t \geq 0$$

(e) $\frac{dy}{dt} - 2y = \frac{dx}{dt} - 2x \quad x(t) = u(t) \quad y(0^-) = 1$

$a = -2 \quad b_0 = 1 \quad b_1 = -2 \quad x(0^-) = 0$

$$y(t) = e^{2t}[1 + (-2)\cdot 0] + \int_{0^-}^{t} e^{2(t-\lambda)}(1 - (-2)(-2))\, u(\lambda)\, d\lambda + (-2)u(t)$$

$$= e^{2t} + e^{2t}\int_{0^-}^{t} e^{-2\lambda}(1+4)\, d\lambda - 2u(t)$$

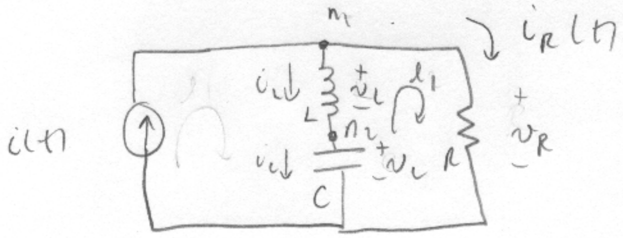
$$= e^{2t} + 5e^{2t}\left(\frac{e^{-2\lambda}}{-2}\right)_0^t - 2u(t)$$

$$= e^{2t} - \frac{5}{2}e^{2t}(e^{-2t} - 1) - 2u(t)$$

$$= e^{2t} - \frac{5}{2} + \frac{5}{2}e^{2t} - 2u(t)$$

$$= \frac{7}{2}e^{2t} - \frac{9}{2} \qquad t \geq 0$$

5. Problem 2.9



① Node n_1: $i = i_L + i_R$

② Node n_2: $i_L = i_C$

③ Loop l_1: $v_R = v_L + v_C$

Element equations: $v_R = R\,i_R$ $\quad v_L = L\dfrac{di_L}{dt}$ $\quad i_C = C\dfrac{dv_C}{dt}$

Take derivative of ③

$$\frac{dv_R}{dt} = \frac{dv_L}{dt} + C\frac{dv_C}{dt}$$

Use element equations:

$$R\frac{di_R}{dt} = L\frac{d^2 i_L}{dt^2} + \frac{i_C}{C}$$

From ①, $i_L = i - i_R$ From ②, $i_C = i_L = i - i_R$

$$R\frac{di_R}{dt} = L\frac{d^2(i - i_R)}{dt^2} + \frac{i - i_R}{C}$$

Rearrange

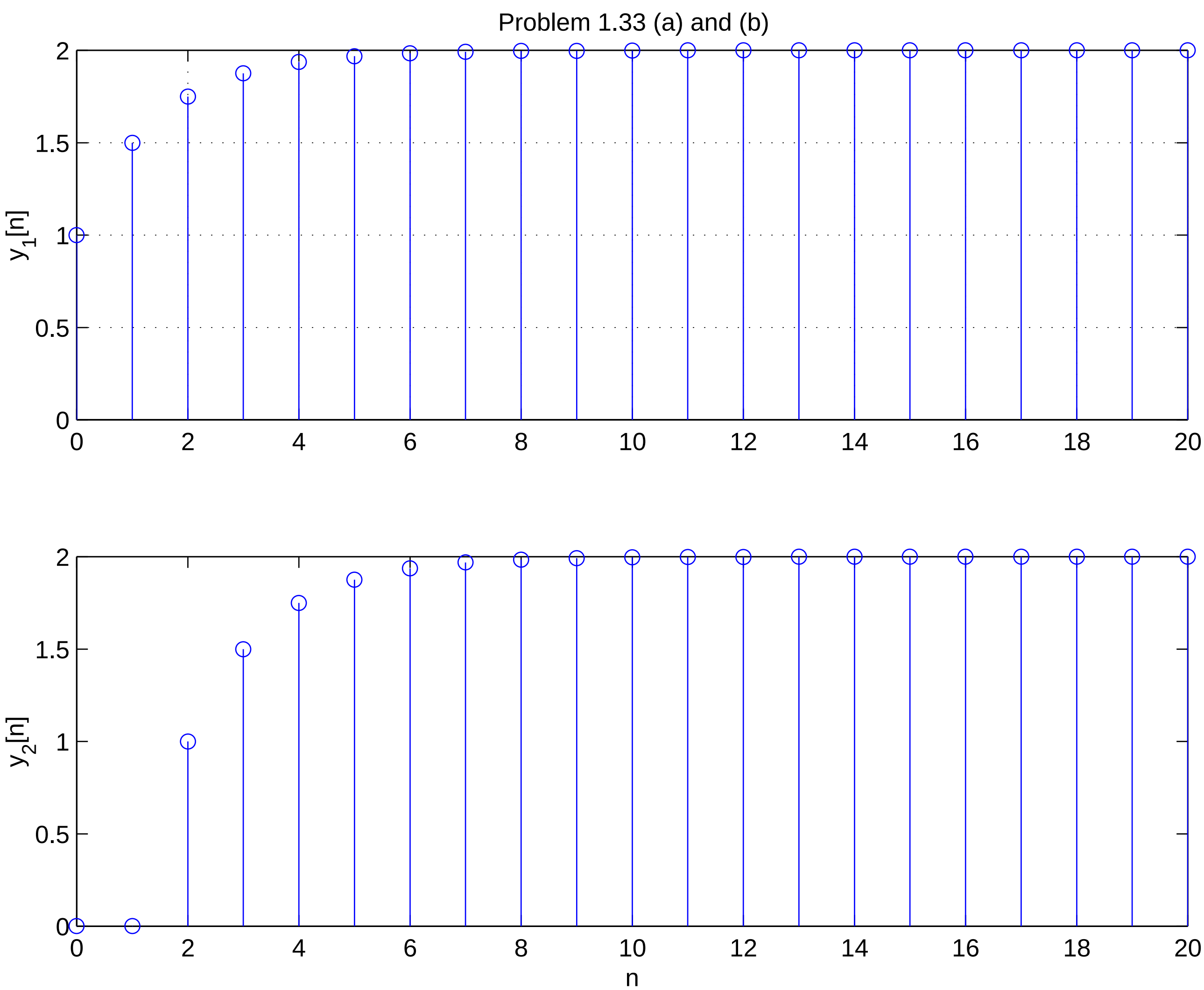
$$L\frac{d^2 i_R}{dt^2} + R\frac{di_R}{dt} + \frac{i_R}{C} = L\frac{d^2 i}{dt} + \frac{1}{C}i$$

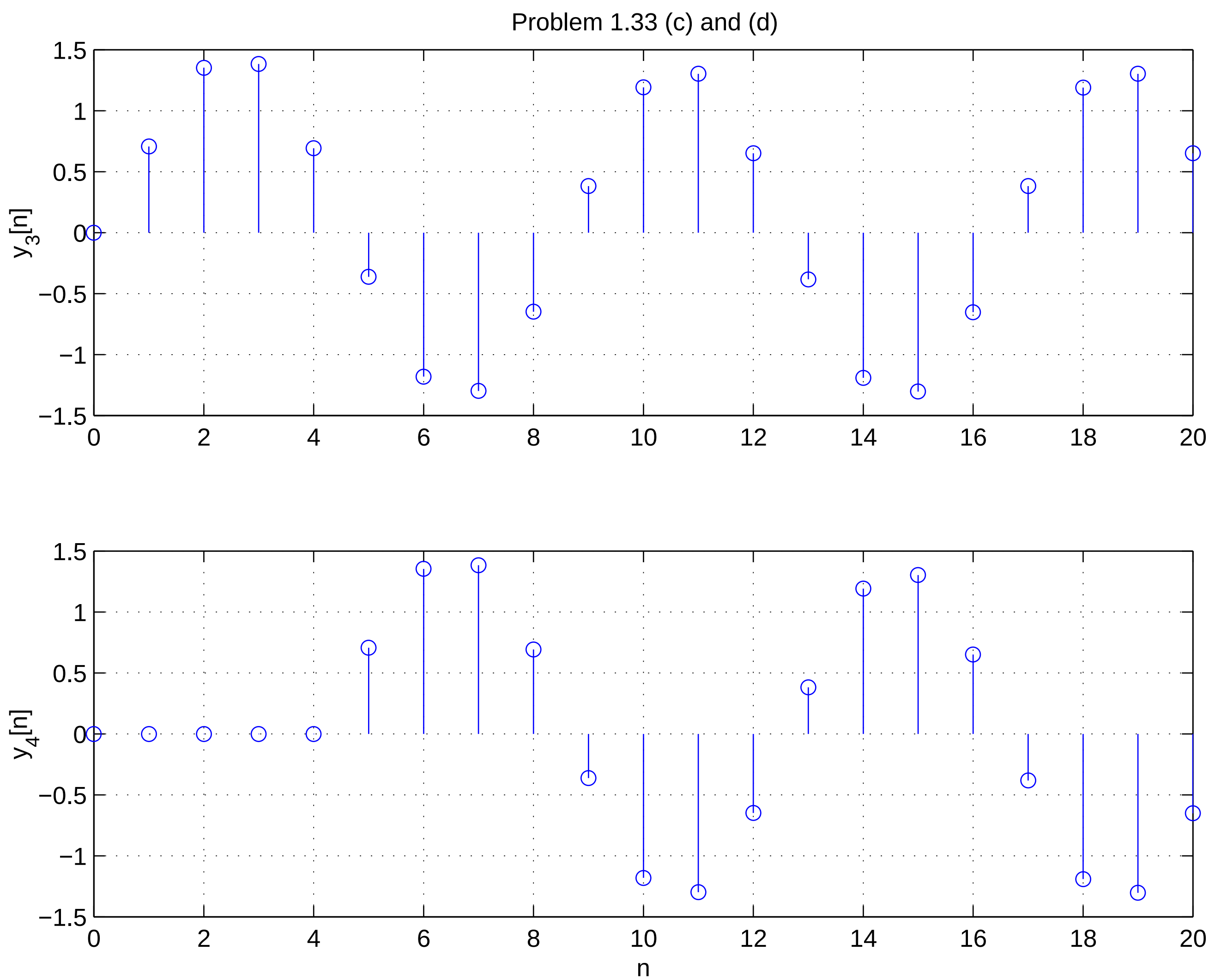
$$\frac{d^2 i_R}{dt^2} + \frac{R}{L}\frac{di_R}{dt} + \frac{1}{LC}i = \frac{d^2 i}{dt^2} + \frac{1}{LC}i$$

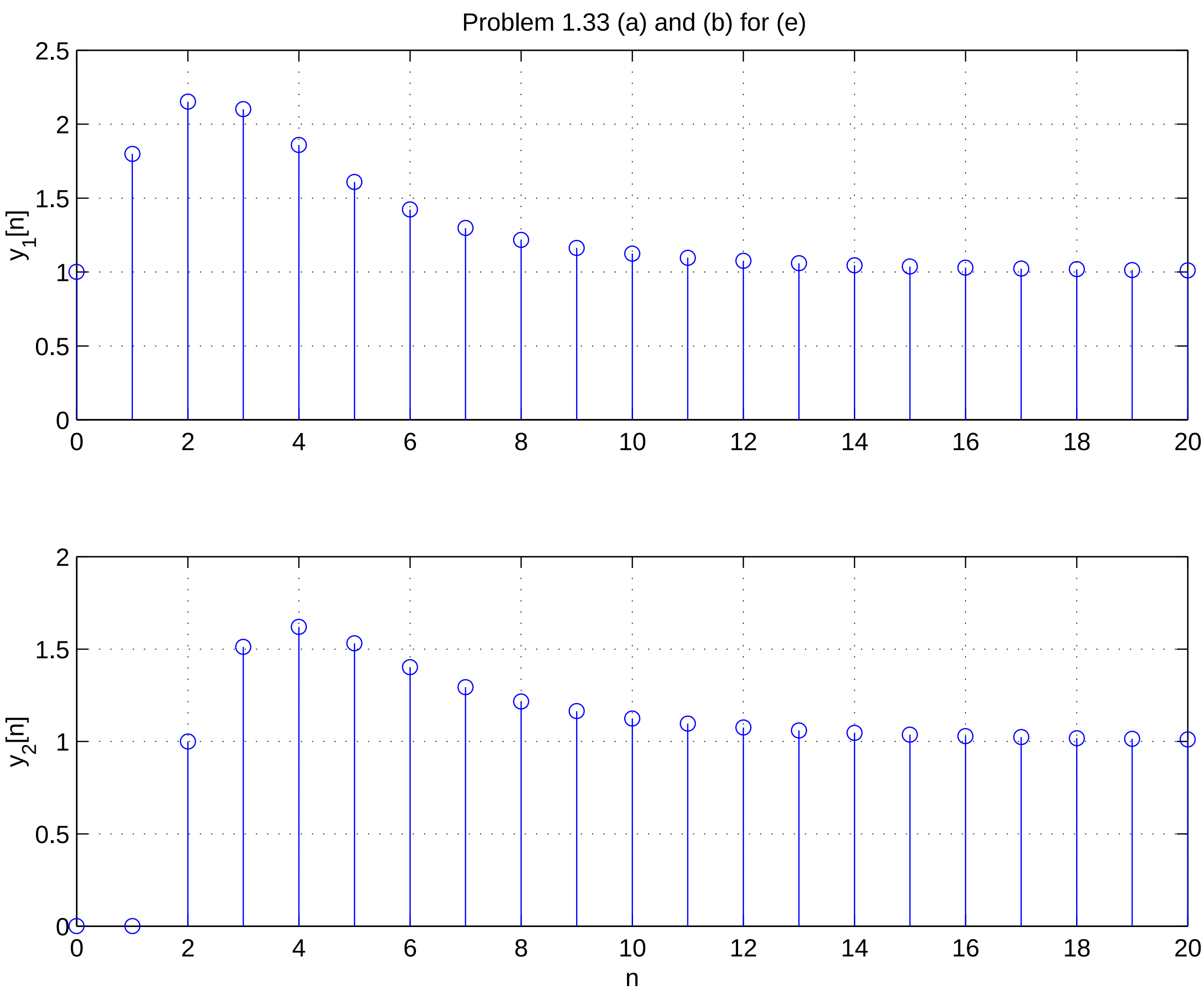
```
% Problem 1.33

% (a-d) y[n] = 0.5 y[n-1] + x[n], y[-1] = 0

% (a)   x1[n] = u[n]
figure(1)
clf

n = 0:20;
ym1 = 0;
y0 = 0.5*ym1 + 1;              % Input at time 0 is 1
y(1) = 0.5*y0 + 1;             % Input at time 1 is 1
for k = 2:20
    y(k) = 0.5*y(k-1) + 1;     % Input at times 2-20 is 1
end;
y1 = [y0 y];
subplot(211)
stem(n,y1)
grid
title('Problem 1.33 (a) and (b)')
ylabel('y_1[n]')

% (b) x2[n] = u[n-2]
ym1 = 0;
y0 = 0.5*ym1 + 0;              % Input at time 0 is 0
y(1) = 0.5*y0 + 0;             % Input at time 1 is 0
for k = 2:20
    y(k) = 0.5*y(k-1) + 1;     % Input at times 2-20 is 1
end;
y2 = [y0 y];
subplot(212)
stem(n,y2)
grid
ylabel('y_2[n]')
xlabel('n')

% (c)   x3[n] = sin(pi n/4)u[n]
figure(2)
clf

n = 0:20;
ym1 = 0;
y0 = 0.5*ym1 + sin(pi*0/4);              % Input at time 0 is sin(pi 0/4)
y(1) = 0.5*y0 + sin(pi*1/4);             % Input at time 1 is sin(pi 1/4)
for k = 2:20
    y(k) = 0.5*y(k-1) + sin(pi*k/4);     % Input at times 2-20 is sin(pi k/4)
end;
y3 = [y0 y];
subplot(211)
stem(n,y3)
grid
title('Problem 1.33 (c) and (d)')
ylabel('y_3[n]')

% (b) x2[n] = sin(pi (n-4)/4) u[n-4]
ym1 = 0;
y0 = 0.5*ym1 + 0;              % Input at time 0 is 0
y(1) = 0.5*y0 + 0;             % Input at time 1 is 0
y(2) = 0.5*y0 + 0;             % Input at time 2 is 0
y(3) = 0.5*y0 + 0;             % Input at time 3 is 0
for k = 4:20
    y(k) = 0.5*y(k-1) + sin(pi*(k-4)/4); % Input at times 4-20 is sin(pi (k-4)/4)
end;
y4 = [y0 y];
```

```
subplot(212)
stem(n,y4)
grid
ylabel('y_4[n]')
xlabel('n')

% (e) y[n] = 0.8^n y[n-1] + x[n], y[-1] = 0

% (a)   x1[n] = u[n]
figure(3)
clf

n = 0:20;
ym1 = 0;
y0 = (0.8)^0*ym1 + 1;              % Input at time 0 is 1
y(1) = (0.8)^1*y0 + 1;             % Input at time 1 is 1
for k = 2:20
    y(k) = (0.8)^k*y(k-1) + 1;    % Input at times 2-20 is 1
end;
y1 = [y0 y];
subplot(211)
stem(n,y1)
grid
title('Problem 1.33 (a) and (b) for (e)')
ylabel('y_1[n]')

% (b) x2[n] = u[n-2]
ym1 = 0;
y0 = (0.8)^0*ym1 + 0;              % Input at time 0 is 0
y(1) = (0.8)^1*y0 + 0;             % Input at time 1 is 0
for k = 2:20
    y(k) = (0.8)^k*y(k-1) + 1;    % Input at times 2-20 is 1
end;
y2 = [y0 y];
subplot(212)
stem(n,y2)
grid
ylabel('y_2[n]')
xlabel('n')

% (c)   x3[n] = sin(pi n/4)u[n]
figure(4)
clf

n = 0:20;
ym1 = 0;
y0 = (0.8)^0*ym1 + sin(pi*0/4);              % Input at time 0 is sin(pi 0/4)
y(1) = (0.8)^1*y0 + sin(pi*1/4);             % Input at time 1 is sin(pi 1/4)
for k = 2:20
    y(k) = (0.8)^k*y(k-1) + sin(pi*k/4);    % Input at times 2-20 is sin(pi k/4)
end;
y3 = [y0 y];
subplot(211)
stem(n,y3)
grid
title('Problem 1.33 (c) and (d) for (e)')
ylabel('y_3[n]')

% (b) x2[n] = sin(pi (n-4)/4) u[n-4]
ym1 = 0;
y0 = (0.8)^0*ym1 + 0;              % Input at time 0 is 0
y(1) = (0.8)^1*y0 + 0;             % Input at time 1 is 0
y(2) = (0.8)^2*y0 + 0;             % Input at time 2 is 0
y(3) = (0.8)^3*y0 + 0;             % Input at time 3 is 0
```

```
for k = 4:20
    y(k) = (0.8)^k*y(k-1) + sin(pi*(k-4)/4); % Input at times 4-20 is sin(pi (k-4)/4)
end;
y4 = [y0 y];
subplot(212)
stem(n,y4)
grid
ylabel('y_4[n]')
xlabel('n')
```


Problem 1.33 (a) and (b)
y1[n]
y2[n]
n
0
0.5
1
1.5
2
0
2
4
6
8
10
12
14
16
18
20


Problem 1.33 (c) and (d)
$y_3[n]$
$y_4[n]$
n
1.5
1
0.5
0
−0.5
−1
−1.5
0
2
4
6
8
10
12
14
16
18
20


Problem 1.33 (a) and (b) for (e)
y_1[n]
y_2[n]
n
0
0.5
1
1.5
2
2.5
0
2
4
6
8
10
12
14
16
18
20

Problem 1.33 (c) and (d) for (e)