EE 341 - Homework 3

Due September 14, 2005

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. For the difference equations given below, find the $y[n]$ for $n = 0$ to 4. Do this by hand.
   (a) $y[n] = 0.5y[n-1] + x[n-1]$; $x[n] = u[n]$; $y[-1] = 0$.
   (b) $y[n] = 2y[n-1]$; $y[-1] = 1$.
   (c) $y[n] = 0.25y[n-1]+0.125y[n-2]+x[n]+0.25x[n-1]$; $x[n] = u[n]$; $y[-1] = 1$, $y[-2] = -1$.
   (d) $y[n] = y[n-1] - 0.25y[n-2] + x[n] - x[n-2]$; $x[n] = \delta[n]$; $y[-1] = 0$, $y[-2] = 0$
   (e) $y[n] = -0.25y[n-2] + x[n]$; $x[n] = 0$; $y[-1] = 1$, $y[-2] = 1$

2. For the difference equations from Problem 1, use the *recur* function (on page 69 of the text, or download it from the textbook website) to find $y[n]$ for $n = 0$ to 10. Plot $y[n]$ using the `stem` function of MATLAB. Verify that the results for the first few values match those you found in Problem 1.

3. Using the techniques in the class handout, find a closed-form equation for $y[n]$ for the difference equations in Problem 1. Use MATLAB to find and plot $y[n]$ for $n = 0$ to 10. Verify that the results match those of Problem 2.

4. Consider the difference equation

$$y[n] = y[n-1] + y[n-2] + x[n-1]$$

   with $x[n] = \delta[n]$ and $y[-1] = 0$, $y[-2] = 0$.
   (a) Show that this difference equation gives the Fibonacci sequence 0, 1, 1, 2, 3, 5, 8, 13, $\cdots$, where a term is equal to the sum of the previous two terms.
   (b) Using the techniques in the class handout, find a closed-form equation for $y[n]$.
   (c) Use the equation from (b) to verify the $y[50] = 12586269025$.

EE 341 HW #3

1.(a) $y(n) = 0.5y(n-1) + x(n-1) \quad x(n) = u(n) \quad y(-1) = 0$

$y(0) = 0.5y(-1) + u(-1) = 0.5(0) + 0 = 0$

$y(1) = 0.5y(0) + u(0) = 0.5(0) + 1 = 1$

$y(2) = 0.5y(1) + u(1) = 0.5(1) + 1 = 1.5$

$y(3) = 0.5y(2) + u(2) = 0.5(1.5) + 1 = 1.75$

$y(4) = 0.5y(3) + u(3) = 0.5(1.75) + 1 = 1.875$

(b) $y(n) = 2y(n-1) \quad y(-1) = 1$

$y(0) = 2y(-1) = 2(1) = 2$

$y(1) = 2y(0) = 2 \cdot 2 = 4$

$y(2) = 2y(1) = 2 \cdot 4 = 8$

$y(3) = 2y(2) = 2 \cdot 8 = 16$

$y(4) = 2y(3) = 2 \cdot 16 = 32$

(c) $y(n) = 0.25y(n-1) + 0.125y(n-2) + x(n) + 0.25x(n-1)$

$x(n) = u(n) \quad y(-1) = 1 \quad y(-2) = -1$

$y(0) = 0.25y(-1) + 0.125y(-2) + u(0) + 0.25u(-1)$

$= 0.25(1) + 0.125(-1) + 1 + 0.25(0) = 1.125$

$y(1) = 0.25y(0) + 0.125y(-1) + u(1) + 0.25u(0)$

$= 0.25(1.125) + 0.125(1) + 1 + 0.25(1) = 1.6563$

$y(2) = 0.25y(1) + 0.125y(0) + u(2) + 0.25u(1)$

$= 0.25(1.6563) + 0.125(1.125) + 1 + 0.25(1) = 1.8047$

$y(3) = 0.25y(2) + 0.125y(1) + u(3) + 0.25u(2)$

$= 0.25(1.8047) + 0.125(1.6563) + 1 + 0.25(1) = 1.9082$

$y(4) = 0.25y(3) + 0.125y(2) + u(4) + 0.25u(3)$

$= 0.25(1.9082) + 0.125(1.8047) + 1 + 0.25(1) = 1.9526$

(d) $y(n) = y(n-1) - 0.25y(n-2) + x(n) - x(n-2)$

$x(n) = \delta(n) \qquad y(-1) = 0 \quad y(-2) = 0$

$y(0) = y(-1) - 0.25y(-2) + \delta(0) - \delta(-2)$

$= 0 - 0.25(0) + 1 - 0 = 1$

$y(1) = y(0) - 0.25y(-1) + \delta(1) - \delta(-1) =$

$= 1 - 0.25(0) + 0 - 0 = 1$

$y(2) = y(1) - 0.25y(0) + \delta(2) - \delta(0)$

$= 1 - 0.25(1) + 0 - 1 = -0.25$

$y(3) = y(2) - 0.25y(1) + \delta(3) - \delta(1)$

$= -0.25 - 0.25(1) + 0 - 0 = -0.5$

$y(4) = y(3) - 0.25y(2) + \delta(4) - \delta(2)$

$= -0.5 - 0.25(-0.25) + 0 - 0 = -0.4375$

(e) $y(n) = -0.25y(n-2) + x(n) \qquad x(n) = 0 \quad y(-1) = 1 \quad y(-2) = 1$

$y(0) = -\frac{1}{4}y(-2) + 0 = -\frac{1}{4}(1) = -\frac{1}{4}$

$y(1) = -\frac{1}{4}y(-1) + 0 = -\frac{1}{4}(1) = -\frac{1}{4}$

$y(2) = -\frac{1}{4}y(0) + 0 = -\frac{1}{4}(-\frac{1}{4}) = \frac{1}{16}$

$y(3) = -\frac{1}{4}y(1) + 0 = -\frac{1}{4}(-\frac{1}{4}) = \frac{1}{16}$

$y(4) = -\frac{1}{4}y(2) + 0 = -\frac{1}{4}(\frac{1}{16}) = -\frac{1}{64}$

2 See MATLAB. The resuts agee

3. (a) $y(n) - 0.5y(n-1) = x(n-1)$ $\quad x(n) = u(n)$ $\quad y(-1) = 0$

$y_h(n) - 0.5y_h(n-1) = 0$

$\lambda - 0.5 = 0$ $\quad \lambda = 0.5$ $\quad y_h(n) = A\,0.5^n$

$y_p(n) = k$

$y(n) = y_h(n) + y_p(n) = A\,0.5^n + k$

$y(0) = A + k = 0$

$y(1) = 0.5A + k = 1$

$A = -2, \ k = 2$

$y(n) = -2(0.5)^n + 2$ $\quad n \geq 0$

See MATLAB. Results match

(b) $y(n) - 2y(n-1) = 0$ $\quad y(-1) = 0$

$\lambda - 2 = 0$ $\quad \lambda = 2$ $\quad y_h(n) = A\,2^n$ $\quad y_p(n) = 0$

$y(n) = A\,2^n$

$y(0) = A = 2$

$y(n) = 2 \cdot 2^n = 2^{n+1}$ $\quad n \geq 0$

c. $y(n) - 0.25y(n-1) - 0.125y(n-2) = x(n) + 0.25x(n-1)$

$y_h(n) - 0.25y_h(n-1) - 0.125y_h(n-2) = 0$

$\lambda^2 - \frac{1}{4}\lambda - \frac{1}{8} = 0$

$(\lambda + \frac{1}{4})(\lambda - \frac{1}{2}) = 0 \qquad \lambda = -\frac{1}{4}, \frac{1}{2}$

$y_h(n) = A_1(-\frac{1}{4})^n + A_2(\frac{1}{2})^n$

$x(n) = u(n) \Rightarrow y_p(n) = K$

$y(n) = A_1(-\frac{1}{4})^n + A_2(\frac{1}{2})^n + K$

$y(0) = A_1 + A_2 + K = 1.125$

$y(1) = -\frac{1}{4}A_1 + \frac{1}{2}A_2 + K = 1.6563$

$y(2) = \frac{1}{16}A_1 + \frac{1}{4}A_2 + K = 1.8047$

$$\text{rref}\begin{bmatrix} 1 & 1 & 1 & 1.125 \\ -\frac{1}{4} & \frac{1}{2} & 1 & 1.6563 \\ \frac{1}{16} & \frac{1}{4} & 1 & 1.8047 \end{bmatrix} \Rightarrow \begin{array}{l} A_1 = -\frac{1}{8} \\ A_2 = -\frac{3}{4} \\ K = 2 \end{array}$$

5

(d) $y(n) - y(n-1) + 0.25y(n-2) = x(n) - x(n-2)$

$y_h(n) - y_h(n-1) + 0.25 y_h(n-2) = 0$

$\lambda^2 - \lambda + \frac{1}{4} = 0 \qquad \lambda = \frac{1}{2}, \frac{1}{2}$

$y_h(n) = A_1 (\frac{1}{2})^n + A_2 n (\frac{1}{2})^n$

$x(n) = \delta(n) \Rightarrow \qquad y_p(n) = k\,\delta(n)$

$y(n) = A_1 (\frac{1}{2})^n + A_2 n (\frac{1}{2})^n + k\,\delta(n)$

$y(0) = A_1 + 0A_2 + k = 1$
$y(1) = \frac{1}{2}A_1 + \frac{1}{2}A_2 + 0 \cdot k = 1$
$y(2) = \frac{1}{4}A_1 + \frac{1}{2}A_2 + 0 \cdot k = -\frac{1}{4}$

$$\text{rref}\left(\begin{bmatrix} 1 & 0 & 1 & 1 \\ \frac{1}{2} & \frac{1}{2} & 0 & 1 \\ \frac{1}{4} & \frac{1}{2} & 0 & -\frac{1}{4} \end{bmatrix}\right) \Rightarrow \begin{aligned} A_1 &= 5 \\ A_2 &= -3 \\ k &= -4 \end{aligned}$$

(e) $y(n) + 0.25\, y(n-2) = x(n)$

$y_h(n) + 0.25\, y_h(n-2) = 0$

$\lambda^2 + \frac{1}{4} = 0 \qquad \lambda = \frac{j}{2}, -\frac{j}{2}$

$y_h(n) = A_1 \left(\frac{j}{2}\right)^n + A_2 \left(-\frac{j}{2}\right)^n$

$x(n) = 0 \Rightarrow y_p(n) = 0$

$y(n) = A_1 \left(\frac{j}{2}\right)^n + A_2 \left(-\frac{j}{2}\right)^n$

$y(0) = A_1 + A_2 = -1/4$

$y(1) = \frac{j}{2} A_1 - \frac{j}{2} A_1 = -1/4$

$\text{rref} \begin{bmatrix} 1 & 1 & -1/4 \\ \frac{j}{2} & -\frac{j}{2} & -1/4 \end{bmatrix} \Rightarrow \begin{aligned} A_1 &= -\frac{1}{8} + \frac{j}{4} \\ A_2 &= -\frac{1}{8} - j/4 \end{aligned}$

$y(n) = \left(-\frac{1}{8} - \frac{j}{4}\right)\left(\frac{j}{2}\right)^n + \left(-\frac{1}{8} + \frac{j}{4}\right)\left(\frac{-j}{2}\right)^n$

4. (a) $y[0] = y(-1) + y(-2) + \delta(-1) = 0 + 0 + 0 = 0$

$y(1) = y(0) + y(-1) + \delta(0) = 0 + 0 + 1 = 0$

$x(n-1) = \delta(n-1) = 0$ for $n > 1$, so for $n > 1$, we have

$y(n) = y(n-1) + y(n-2)$

Each term is the sum of the previous two

(b) $y(n) - y(n-1) - y(n-2) = x(n-1)$

$y_h(n) - y_h(n-1) - y_h(n-2) = 0$

$\lambda^2 - \lambda - 1 = 0 \Rightarrow \lambda_1 = \frac{1-\sqrt{5}}{2}, \lambda_2 = \frac{1+\sqrt{5}}{2}$

$x(n) = \delta(n) \Rightarrow \quad y_p(n) = K\,\delta(n)$

$y_t(n) = A_1 \lambda_1^n + A_2 \lambda_2^n + k\,\delta(n)$

$y(0) = A_1 + A_2 + k = 0$

$y_1 = \frac{1-\sqrt{5}}{2} A_1 + \frac{1+\sqrt{5}}{2} A_2 + 0 = 1$

$y_2 = \left(\frac{1-\sqrt{5}}{2}\right)^2 A_1 + \left(\frac{1+\sqrt{5}}{2}\right)^2 A_2 + 0 = 1$

$$\text{rref} \begin{bmatrix} 1 & 1 & 1 & 0 \\ \lambda_1 & \lambda_2 & 0 & 1 \\ \lambda_1^2 & \lambda_2^2 & 0 & 1 \end{bmatrix} \Rightarrow \quad \begin{aligned} A_1 &= -\frac{1}{\sqrt{5}} \\ A_2 &= \frac{1}{\sqrt{5}} \\ k &= 0 \end{aligned}$$

$$y(n) = -\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n + \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$$

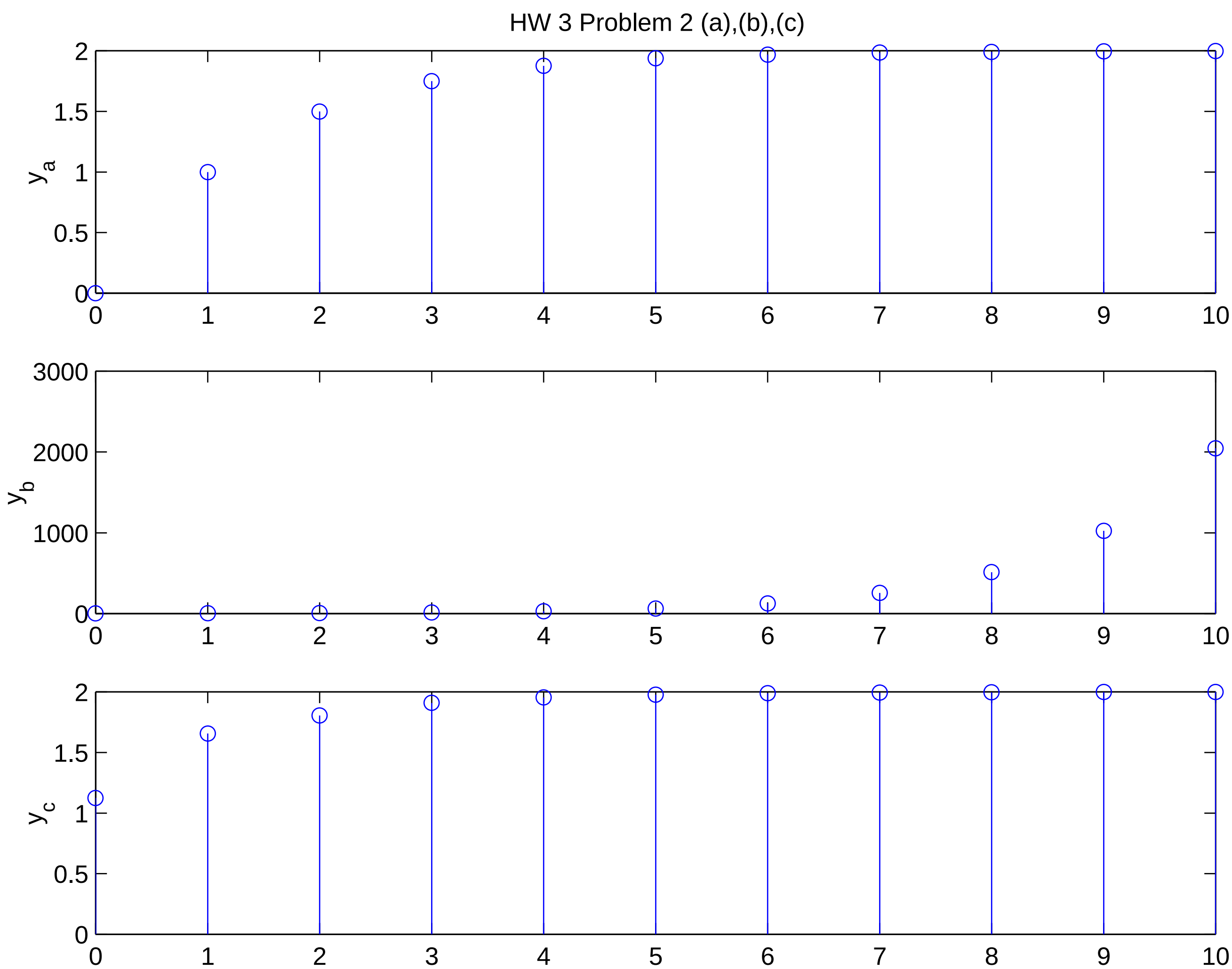
(c) $$y[50] = -\frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^{50} + \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^{50}$$

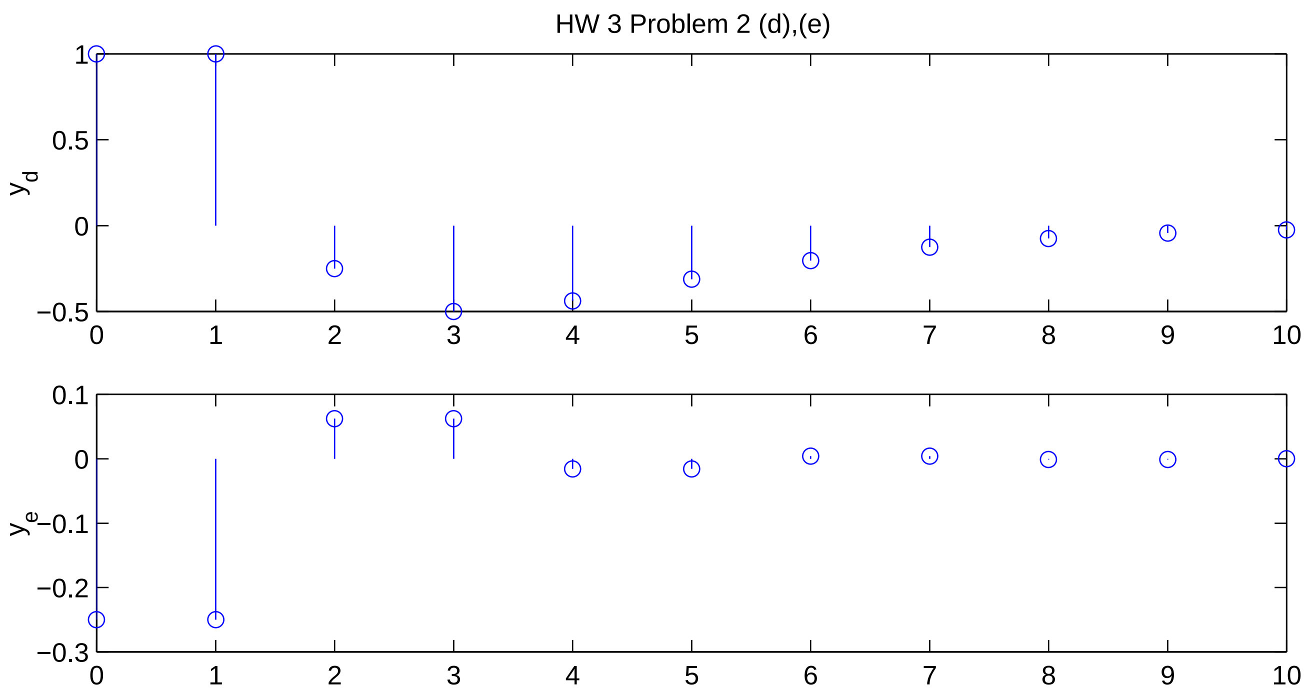
$$= 12{,}586{,}269{,}025$$

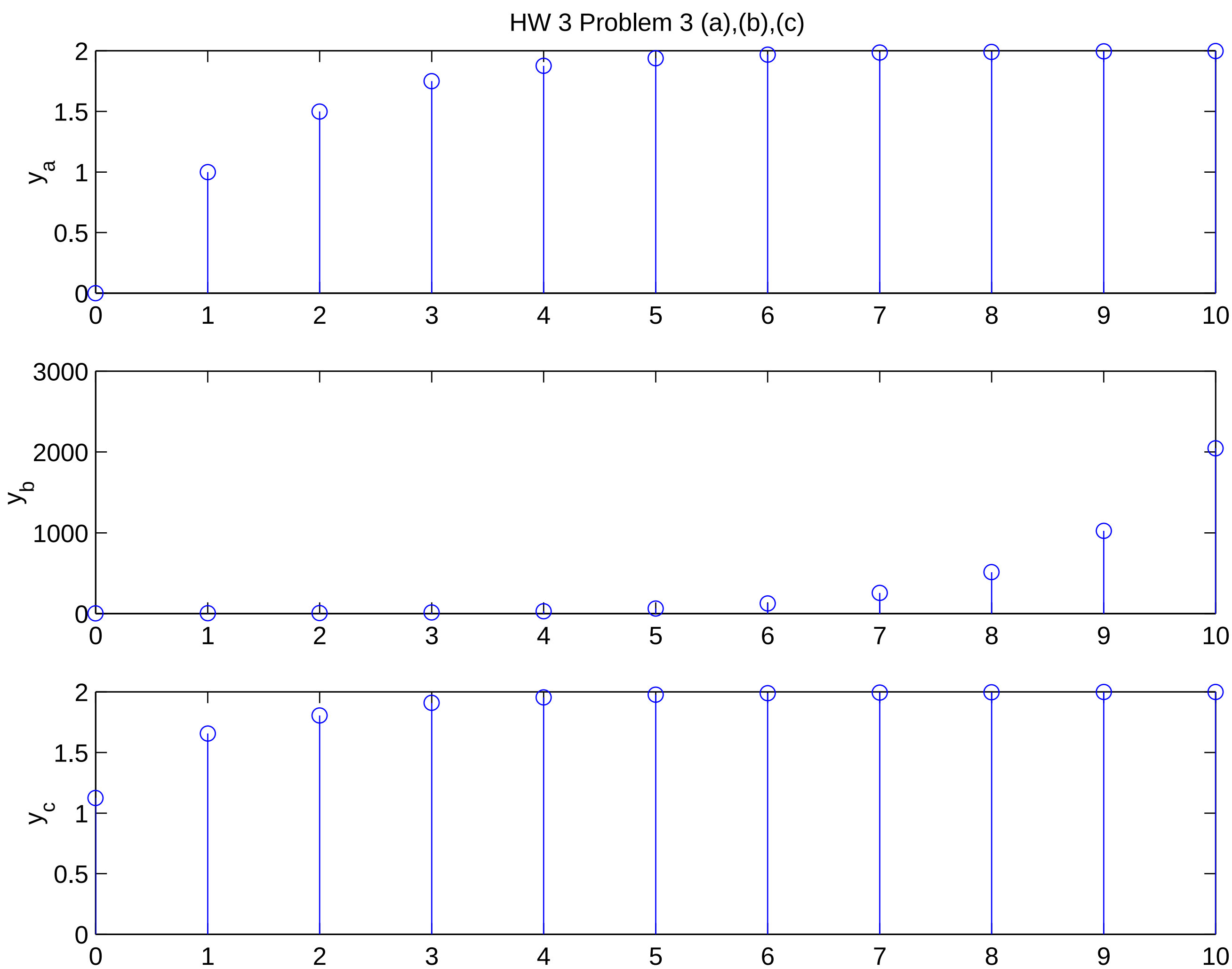
```
% Problem 2
figure(1)
clf

% (a) y[n] - 0.5 y[n-1] = x[n-1]

a = -0.5;       % -0.5 y[n-1]
b = [0 1];      % 0 x[n] + 1 x[n-1]
n = 0:10;
x = (n >= 0);   % x[n] = u[n]
y0 = 0;         % y[-1]
x0 = 0;         % x[-1]
ya = recur(a,b,n,x,x0,y0);
subplot(311)
stem(n,ya)
ylabel('y_a')
title('HW 3 Problem 2 (a),(b),(c)')

% (b) y[n] - 2 y[n-1] = 0

a = -2;       % -2 y[n-1]
b = [0 0];        % 0 x[n]
n = 0:10;
x = zeros(size(n));        % no input
y0 = 1;       % y[-1]
x0 = 0;
yb = recur(a,b,n,x,x0,y0);
subplot(312)
stem(n,yb)
ylabel('y_b')

% (c) y[n] - 0.25 y[n-1] - 0.125 y[n-2] = x[n] + 0.25 x[n-1]

a = [-0.25 -0.125];
b = [1 0.25];
n = 0:10;
x = (n >= 0);      % x[n] = u[n]
y0 = [-1 1];       % y[-2] y[-1]
x0 = 0;            % x[-1]
yc = recur(a,b,n,x,x0,y0);
subplot(313)
stem(n,yc)
ylabel('y_c')
print -dpsc2 'p2_abc.ps'

% (d) y[n] - y[n-1] + 0.25 y[n-2] = x[n] - x[n-2]
figure(2)
clf


a = [-1 0.25];
b = [1 0 -1];
n = 0:10;
x = (n == 0);     % x[n] = delta[n]
y0 = [0 0];       % y[-2] y[-1]
x0 = [0 0];       % x[-2] x[-1]
yd = recur(a,b,n,x,x0,y0);
subplot(311)
stem(n,yd)
ylabel('y_d')
title('HW 3 Problem 2 (d),(e)')

% (e) y[n] + 0.25 y[n-2] = x[n]
a = [0 0.25];
```

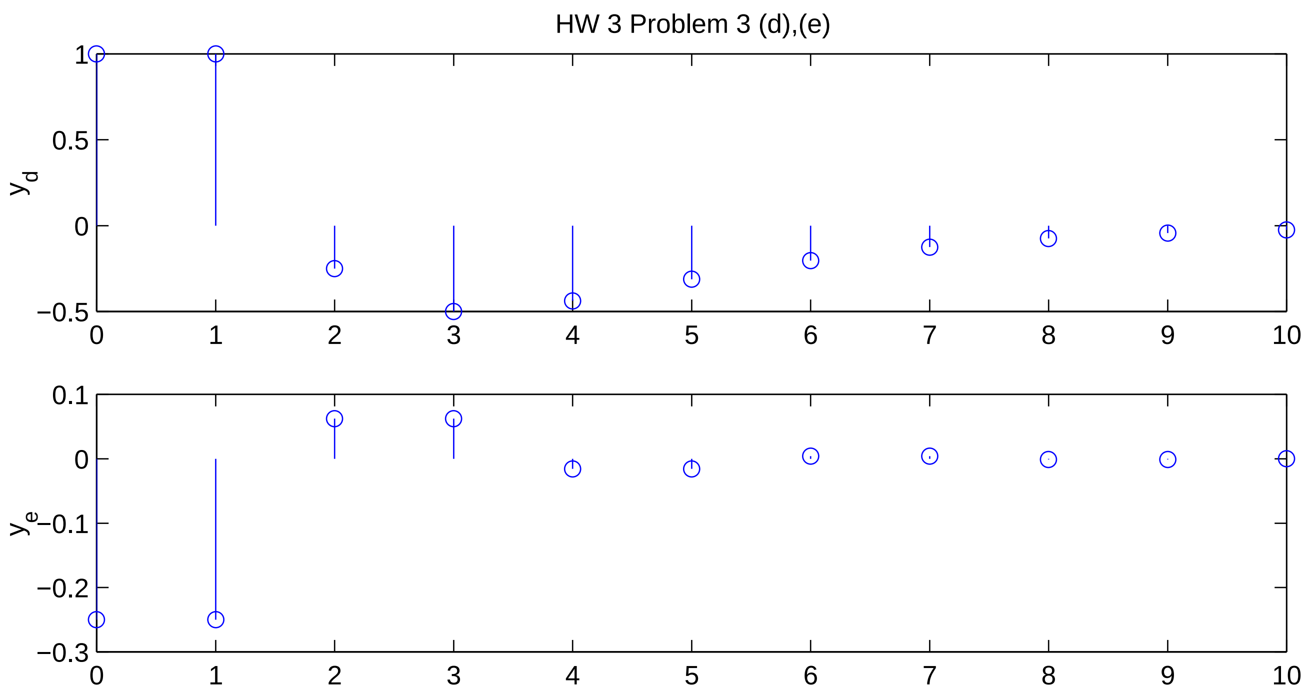
```
b = [1 0];
n = 0:10;
x = zeros(size(n));     % x[n] = 0
y0 = [1 1];       % y[-2] y[-1]
x0 = 0;       % x[-2] x[-1]
ye = recur(a,b,n,x,x0,y0);
subplot(312)
stem(n,ye)
ylabel('y_e')
print -dpsc2 'p2_de.ps'

% Problem 3
figure(3)
clf

% (a)
l = 1/2;       % lambda
Aa = -2;
Ka = 2;
n = 0:10;
ya3 = Aa*l.^n + Ka;
subplot(311)
stem(n,ya3)
ylabel('y_a')
title('HW 3 Problem 3 (a),(b),(c)')

% (b)
l = 2;
Ab = 2;
n = 0:10;
yb3 = Ab*l.^n;
subplot(312)
stem(n,yb3)
ylabel('y_b')

l1 = -1/4;     % lambda_1
l2 =  1/2;     % lambda_1
A = rref([1 1 1 yc(1); l1 l2 1 yc(2); l1^2 l2^2 1 yc(3)]);
A1c = A(1,4);
A2c = A(2,4);
Kc  = A(3,4);
n = 0:10;
yc3 = A1c*l1.^n + A2c*l2.^n + Kc;
subplot(313)
stem(n,yc3)
ylabel('y_c')
print -dpsc2 'p3_abc.ps'

% (d)
figure(4)
clf

l1 = 1/2;     % lambda_1
l2 = 1/2;     % lambda_1
A = rref([1 0 1 yd(1); l1 1*l2 0 yd(2); l1^2 2*l2^2 0 yd(3)]);
A1d = A(1,4);
A2d = A(2,4);
Kd  = A(3,4);
n = 0:10;
yd3 = A1d*l1.^n + A2d*n.*l2.^n + Kd*(n==0);
subplot(311)
stem(n,yd3)
ylabel('y_d')
title('HW 3 Problem 3 (d),(e)')
```

```
% (e)
l1 =  j/2;    % lambda_1
l2 = -j/2;    % lambda_2
A = rref([1 1 -1/4;l1 l2 -1/4]);
A1e = A(1,3);
A2e = A(2,3);
n = 0:10;
ye3 = A1e*l1.^n + A2e*l2.^n;
subplot(312)
stem(n,ye3)
ylabel('y_e')
print -dpsc2 'p3_de.ps'
```


HW 3 Problem 2 (a),(b),(c)
y_a
y_b
y_c
0 1 2 3 4 5 6 7 8 9 10
0 0.5 1 1.5 2
0 1000 2000 3000


HW 3 Problem 2 (d),(e)
y_d
y_e
1
0.5
0
−0.5
0.1
0
−0.1
−0.2
−0.3
0 1 2 3 4 5 6 7 8 9 10


HW 3 Problem 3 (a),(b),(c)
y_a
y_b
y_c
0
0.5
1
1.5
2
1000
2000
3000
0 1 2 3 4 5 6 7 8 9 10


HW 3 Problem 3 (d),(e)
y_d
1
0.5
0
−0.5
0
1
2
3
4
5
6
7
8
9
10
y_e
0.1
0
−0.1
−0.2
−0.3
0
1
2
3
4
5
6
7
8
9
10