## EE 341 - Homework 14

**Due December 7, 2005**

For problems which require MATLAB, please include a MATLAB m-file which shows how you made your plots.

1. Problem 11.17.
2. Problem 11.19.
3. Problem 11.22.
4. Problem 11.27.
5. Problem 11.29.
6. Problem 11.31.
7. Problem 11.43. Do not use the Jury test. Use MATLAB or your calculator to find the pole and zero locations.
8. Problem 11.47. For (a), find the steady-state response only. To do this, you do not need to take an inverse z-transform. Also, plot the pole-zero diagram.
9. Problem 11.48. For (a), find the steady-state response only. To do this, you do not need to take an inverse z-transform. Also, plot the pole-zero diagram.

# EE 341
# HW # 14

1. Problem 11.17

$y(n) + 0.7\,y(n-1) = x(n) \qquad x(n) = u(n) \qquad H(z) = \frac{1}{1+0.7z^{-1}} = \frac{z}{z+0.7}$

(a) $Y(z) + 0.7\left(z^{-1}Y(z) + y(-1)\right) = \frac{z}{z-1}$

$Y(z)\left(1+0.7z^{-1}\right) = -0.7 + \frac{z}{z-1} = \frac{0.3z+0.7}{z-1}$

$Y(z) = \frac{0.3z+0.7}{(1+0.7z^{-1})(z-1)} = \frac{z(0.3z+0.7)}{(z+0.7)(z-1)}$

$\frac{Y(z)}{z} = \frac{0.3z+0.7}{(z+0.7)(z-1)} = -\frac{49}{170}\frac{1}{z+0.7} + \frac{10}{17}\frac{1}{z-1}$

$Y(z) = \frac{-49}{170}\frac{z}{z+0.7} + \frac{10}{17}\frac{z}{z-1}$

$y(n) = \frac{-49}{170}(-0.7)^n + \frac{10}{17} \qquad n \geq 0$

(b) See MATLAB

2. Problem 11.19

$y(n+2) + y(n) = 2x(n+1) - x(n)$

$y(n) + y(n-2) = 2x(n-1) - x(n-2)$

(a) $H(z) = \frac{2z^{-1} - z^{-2}}{1+z^{-2}} = \frac{2z-1}{z^2+1}$

$\frac{H(z)}{z} = \frac{2z-1}{z(z^2+1)} = \frac{2z-1}{z(z+j)(z-j)} = \frac{-1}{z} + \left(\frac{1}{2}-j\right)\frac{1}{z-j} + \left(\frac{1}{2}+j\right)\frac{1}{z+j}$

$H(z) = -1 + \frac{\sqrt{5}}{2}e^{-j1.071}\frac{z}{z-e^{j\pi/2}} + \frac{\sqrt{5}}{2}e^{j1.071}\frac{z}{z-e^{-j\pi/2}}$

$h(n) = -\delta(n) + \sqrt{5}\cos\left(\frac{\pi}{2}n - 1.071\right)u(n)$

(b) $G(z) = H(z)\,U(z) = \dfrac{2z-1}{z^2+1}\,\dfrac{z}{z-1}$

$$\frac{G(z)}{z} = \frac{2z-1}{(z^2+1)(z-1)} = \frac{1}{2}\frac{1}{z-1} + \left(\frac{-1-j3}{4}\right)\frac{1}{z-j} + \frac{-1+j3}{4}\,\frac{1}{z+j}$$

$$G(z) = \frac{1}{2}\frac{z}{z-1} + \frac{\sqrt{10}}{4}e^{-j1.8925}\frac{z}{z-e^{j\pi/2}} + \frac{\sqrt{10}}{4}e^{j1.8925}\frac{z}{z-e^{-j\pi/2}}$$

$$g(n) = \frac{1}{2}u(n) + \frac{\sqrt{10}}{2}\cos\left(\frac{\pi}{2}n - 1.8925\right)u(n)$$

(c) $x(n) = 2^n u(n) \qquad y(-1) = 3 \qquad y(-2) = 2$

$$Y(z) + z^{-2}Y(z) + y(-2) + z^{-1}y(-1) = (2z^{-1} - z^{-2})X(z) = (2z^{-1} - z^{-2})\frac{z}{z-2}$$

$$z^2 Y(z) + Y(z) + 2z^2 + 3z = (2z-1)\frac{z}{z-2}$$

$$(z^2+1)Y(z) = -2z^2 - 3z + \frac{2z^2 - z}{z-2} = \frac{-2z^3 + 3z^2 + 5z}{z-2}$$

$$Y(z) = \frac{-2z^3 + 3z^2 + 5z}{(z^2+1)(z-2)}$$

$$\frac{Y(z)}{z} = \frac{-2z^2 + 3z + 5}{(z^2+1)(z-2)} = \frac{0.6}{z-2} + (-1.3 + j1.1)\frac{1}{z-j} + (-1.3 - j1.1)\frac{1}{z+j}$$

$$Y(z) = 0.6\frac{z}{z-2} + 1.7029\,e^{j2.4393}\frac{z}{z-e^{j\pi/2}} + 1.7029\,e^{-j2.4393}\frac{z}{z-e^{-j\pi/2}}$$

$$y(n) = 0.6(2^n) + 3.4059\cos\left(\frac{\pi}{2}n + 2.4393\right) \qquad n \geq 0$$

(d) $Y(z) = H(z)X(z) \qquad X(z) = \dfrac{Y(z)}{H(z)} \qquad y(n) = (\sin n\pi)u(n)$

$$Y(z) = \frac{(\sin\pi)z}{z^2 - 2(\cos\pi)z + 1} = \frac{0}{z^2 + 2z + 1} \qquad H(z) = \frac{2z-1}{z^2+1}$$

$$X(z) = \frac{\frac{0}{z^2+2z+1}}{\frac{2z-1}{z^2+1}} = 0 \qquad x(n) = 0$$

(e) $X(z) = \frac{Y(z)}{H(z)}$ $\quad y(n) = \delta(n-1)$ $\quad Y(z) = z^{-1} = \frac{1}{z}$

$$X(z) = \frac{\frac{1}{z}}{\frac{2z-1}{z^2+1}} = \frac{z^2+1}{z(2z-1)} = \frac{1}{2}\,\frac{z^2+1}{z(z-\frac{1}{2})}$$

$$\frac{X(z)}{z} = \frac{1}{2}\,\frac{z^2+1}{z^2(z-\frac{1}{2})} = \frac{-2}{z} + \frac{-1}{z^2} + \frac{5/2}{z-\frac{1}{2}}$$

$$x(n) = -2\delta(n) - \delta(n-1) + \frac{5}{2}\left(\frac{1}{2}\right)^n u(n)$$

(f) See MATLAB

3. Problem 11.22

$$H(z) = \frac{-0.4z^{-1} - 0.5z^{-2}}{(1-0.5z^{-1})(1-0.8z^{-1})} = \frac{-0.4z - 0.5}{(z-0.5)(z-0.8)}$$

(a) $G(z) = H(z)\,U(z) = \frac{-0.4z-0.5}{(z-0.5)(z-0.8)}\,\frac{z}{z-1}$

$$\frac{G(z)}{z} = \frac{-0.4z-0.5}{(z-0.5)(z-0.8)(z-1)} = \frac{-14/3}{z-0.5} + \frac{41/3}{z-0.8} - \frac{9}{z-1}$$

$$g(n) = -\frac{14}{3}(0.5)^n u(n) + \frac{41}{3}(0.8)^n u(n) - 9\,u(n)$$

(b) See MATLAB

4. Problem 11.27

$x(n) = 0.5\,u(n)$ $\quad X(z) = \frac{z}{z-0.5}$

$y(n) = 4(0.5)^n u(n) - n(0.5)^n u(n) - (-0.5)^n u(n)$

$$Y(z) = \frac{4z}{z-0.5} - \frac{0.5z}{(z-0.5)^2} - \frac{z}{z+0.5}$$

$$= \frac{4z^3 - 1.5z^2 - 0.25z - 0.25}{(z^2 - z + 0.25)(z+0.5)}$$

$$Y(z) = \frac{-(a_1 y(-1) + a_2 y(-2))z^2 - a_2 y(-1) z}{z^2 + a_1 z + a_2} + \frac{b_0 z^3 + b_1 z^2 + b_2 z + b_3}{z(z^2 + a_1 z + a_2)}$$

$$= \frac{-(8a_1 + 4a_2)z^2 - 8a_2 z}{z^2 + a_1 z + a_2} + \frac{b_0 z^3 + b_1 z^2 + b_2 z + b_3}{z(z^2 + a_1 z + a_2)} \frac{z}{z+0.5}$$

By equating denominators, we see $a_1 = -1$, $a_2 = 0.25$

$$Y(z) = \frac{7z^2 - 2z}{z^2 - z + 0.25} + \frac{b_0 z^3 + b_1 z^2 + b_2 z + b_3}{(z^2 - z + 0.25)(z+0.5)}$$

$$= \frac{(b_0+7)z^3 + (b_1 + 1.5)z^2 + (b_2 - 1)z + b_3}{(z^2 - z + 0.25)(z+0.5)}$$

By equating numerators, we see

$b_0 + 7 = 4 \Rightarrow b_0 = -3$

$b_1 + 1.5 = -1.5 \Rightarrow b_1 = -3$

$b_2 - 1 = -0.25 \Rightarrow b_2 = 0.75$

$b_3 = -0.25$

$$H(z) = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + b_3 z^{-3}}{1 + a_1 z^{-1} + a_2 z^{-2}} = \frac{-3 - 3z^{-1} + 0.75 z^{-2} - 0.25 z^{-3}}{1 - z^{-1} + 0.25 z^{-2}}$$

5. Problem 11.29

$h(0) = 0$

$h(1) = 1$

$h(n) = h(n-2) + h(n-1)$ for $n \geq 2$

Can write as

$h(n) = h(n-2) + h(n-1) + \delta(n-1)$ for $n \geq 0$

$h(n) - h(n-1) - h(n-2) = \delta(n-1)$

$H(z) - z^{-1}H(z) - z^{-2}H(z) = z^{-1}$

$$H(z) = \frac{z^{-1}}{1 - z^{-1} - z^{-2}} = \frac{z}{z^2 - z - 1}$$

6 Problem 11.31

(i) (a) $$H(z) = \frac{z^2 - 0.75z}{\left(z - \frac{3}{4} - j\frac{\sqrt{27}}{4}\right)\left(z - \frac{3}{4} + j\frac{\sqrt{27}}{4}\right)} = \frac{z^2 - 0.75z}{(z - 1.5e^{j\pi/3})(z - 1.5e^{-j\pi/3})}$$

Transient response will be exponentially increasing sinusoid

(b) $g(n) = C_1 |1.5|^n \cos\left(\frac{\pi}{3}n + \theta\right) u(n) + C_2 u(n)$

(c) See MATLAB - $g(n)$ grows as $1.5^n$

(ii) $$H(z) = \frac{z^2 - 0.5z}{z^2 - z + 1} = \frac{z^2 - 0.5z}{(z - e^{j\pi/3})(z - e^{-j\pi/3})}$$

(a) Transient response: undamped sinusoid, frequency $\frac{\pi}{3}$

(b) $g(n) = C_1 \cos\left(\frac{\pi}{3}n + \theta\right) + C_2 u(n)$

(c) See MATLAB - signal oscillates with frequency $\frac{\pi}{3}$

(iii) (a) $$H(z) = \frac{z^2 - 0.25z}{z^2 - 0.5z + 0.25} = \frac{z^2 - 0.25z}{(z - 0.5e^{j\pi/3})(z - 0.5e^{-j\pi/3})}$$

Transient response: damped sinusoid, frequency $\frac{\pi}{3}$

(b) $g(n) = C_1 (0.5)^n \cos\left(\frac{\pi}{3}n + \theta\right) u(n) + C_2 u(n)$

(c) See MATLAB - decaying exponential

7. Problem 11.43

(a) $H(z) = \dfrac{z-4}{z^2+1.5z+0.5} = \dfrac{z-4}{(z+1)(z+0.5)}$

Marginally stable, pole at $-1$

(b) $H(z) = \dfrac{z^3-3z+1}{z^3+z^2-0.5z+0.5} = \dfrac{z^3-3z+1}{(z+1.537)(z-0.2685+j0.5032)(z-0.2685-j0.503)}$

Unstable - pole at $-1.537$

(c) $H(z) = \dfrac{1}{z^3+0.5z+0.1} = \dfrac{1}{(z+0.1869)(z-0.73e^{j1.44})(z-0.73e^{-j1.44})}$

Stable - poles inside unit circle

8. Problem 11.47

(a) $y(n)+0.3y(n-1)+0.02y(n-2) = x(n-1)+3\,x(n-2)$

$H(z) = \dfrac{z^{-1}+3z^{-2}}{1+0.3z^{-1}+0.02z^{-2}} = \dfrac{z+3}{z^2+0.3z+0.02}$

$x(n) = \cos(\pi n)\,u(n)$

$H(z)\Big|_{z=e^{j\pi}} = \dfrac{e^{j\pi}+3}{(e^{j\pi})^2+0.3e^{j\pi}+0.02} = \dfrac{-1+3}{1-0.3+0.02} = 2.78$

$y_{ss}(n) = 2.78\cos(\pi n)$

(b) See MATLAB. Steady state value correct.

9. Problem 11.48

$$H(z) = \frac{z}{(z^2+0.25)(z-0.5)^2}$$

$x(n) = 12\cos(\frac{\pi}{2}n)u(n)$

(a) $$H(z)\Big|_{z=e^{j\frac{\pi}{2}}} = \frac{j}{(j^2+0.25)(j-0.5)^2} = 1.067\, e^{j0.6435}$$

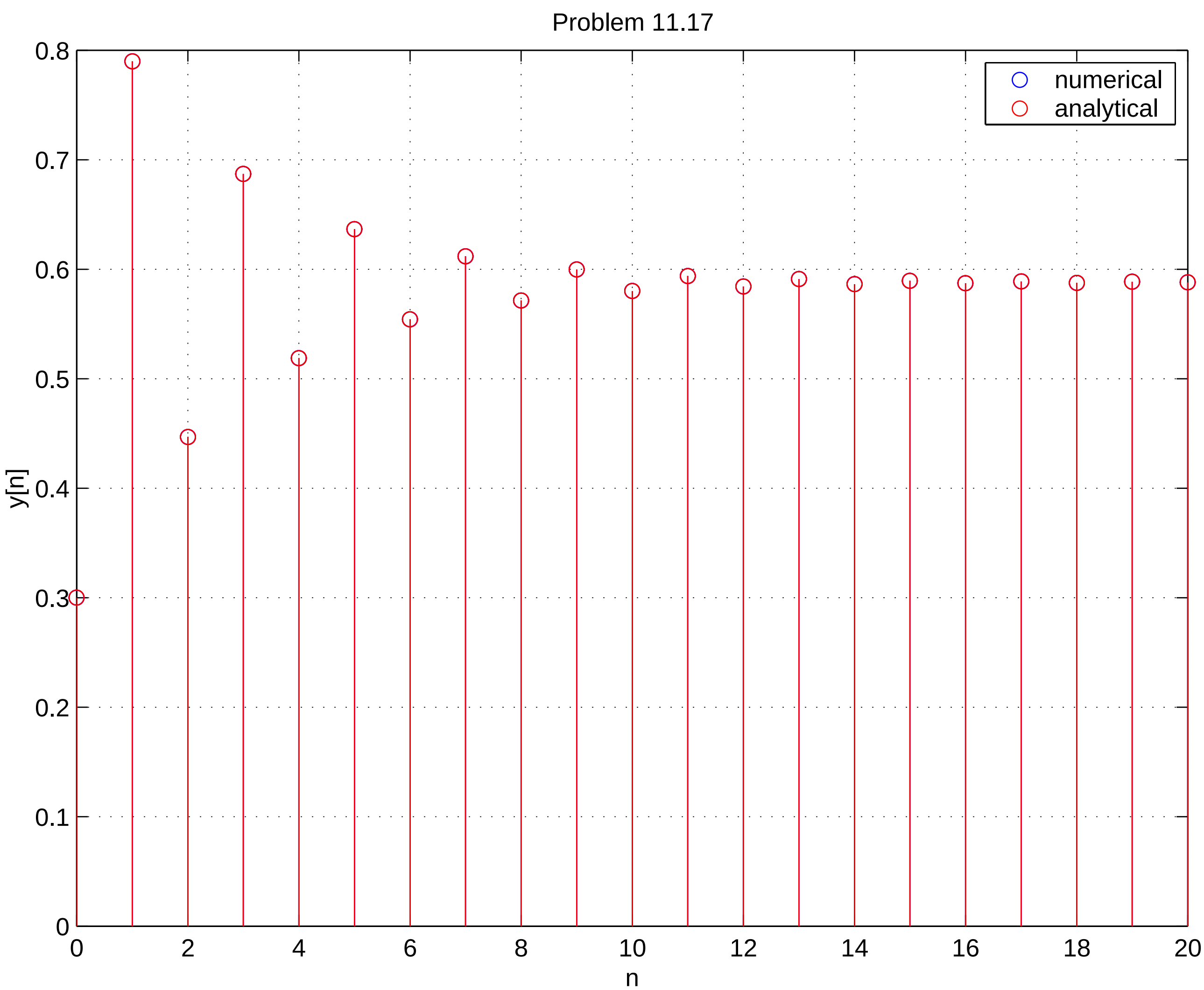
$y(n) = (1.067)(12)\cos(\frac{\pi}{2}n + 0.6435) = 7.722\cos(\frac{\pi}{2}n + 0.6435)$

(b) See MATLAB. Steady state values agree

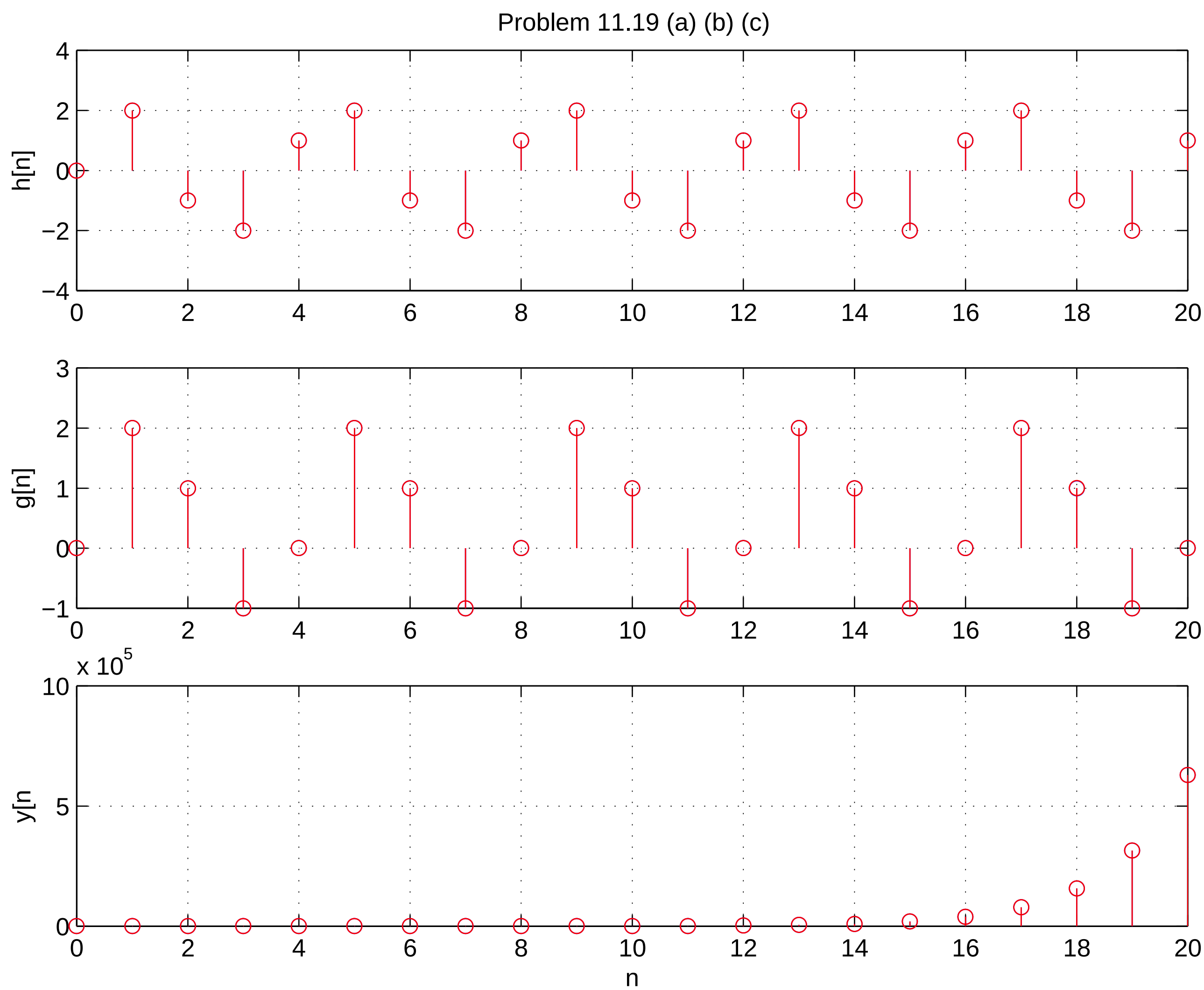
```
% EE 341 HW 13

% Problem 11.17
% Enter H(z)
figure(1)
clf
a = [1 0.7];
b=[1 0];
% Set up initial conditions, as on Page 584
a1 = 0.7;
ym1 = 1;
zi = [-a1*ym1];
% Input is step -- all ones
n = 0:20;
x = ones(1,length(n));
y = filter(b,a,x,zi);
stem(n,y);
hold on
ya = (-49/170)*(-0.7).^n + 10/17;
stem(n,ya,'r')
legend('numerical','analytical');
grid
xlabel('n');
ylabel('y[n]');
title('Problem 11.17');
print -dpsc2 p11_17.ps

% Problem 11.19
figure(2)
clf
clear
% (a)
subplot(311)
a = [1 0 1];
b = [0 2 -1];
n = 0:20;
h = dimpulse(b,a,n);
stem(n,h)
grid
hold on
ha = -(n==0)+sqrt(5)*cos(pi*n/2+angle(0.5-j));
stem(n,ha,'r');
ylabel('h[n]')
title('Problem 11.19 (a)  (b)  (c)')
% (b)
subplot(312)
g = dstep(b,a,n);
stem(n,g);
grid
hold on
ga = 0.5+(sqrt(10)/2)*cos(pi*n/2+angle(-1-j*3));
stem(n,ga,'r');
ylabel('g[n]')
% (c)
subplot(313)
a1 = 0;
a2 = 1;
ym1 = 3;
ym2 = 2;
zi = [-a1*ym1-a2*ym2 -a2*ym1];
x = 2.^n;
y = filter(b,a,x,zi);
stem(n,y)
grid
```

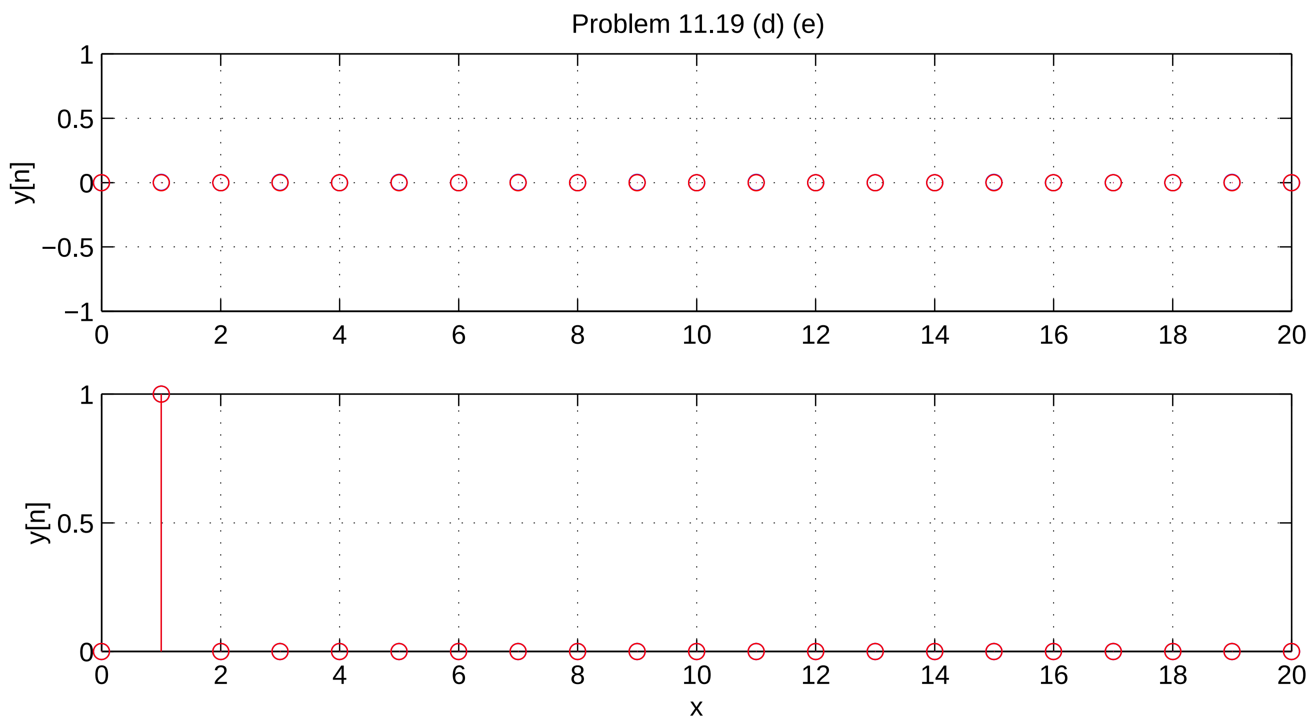
```
hold on
ya = 0.6*2.^n +2*abs(-1.3+j*1.1)*cos(pi*n/2+angle(-1.3+j*1.1));
stem(n,ya,'r')
ylabel('y[n');
xlabel('n');
print -dpsc2 p11_19_abc.ps
% (d)
figure(3)
clf
subplot(311)
y = sin(pi*n);
stem(n,y)
axis([0 20 -1 1])
grid
hold on
x = zeros(size(n));
yn = filter(b,a,x);
stem(n,yn,'r')
ylabel('y[n]')
title('Problem 11.19 (d) (e)')
% (e)
subplot(312)
y = (n==1);
stem(n,y)
grid
hold on
x = -2*(n==0) - (n==1) + (5/2)*(1/2).^n;
yn = filter(b,a,x);
stem(n,yn,'r')
ylabel('y[n]')
xlabel('x')
print -dpsc2 p11_19_de.ps

% Probelm 11.22
figure(4)
clf
b=[-0.4 -0.5];
a=conv([1 -0.5],[1 -0.8]);
n = 0:20;
g = dstep(b,a,n);
stem(n,g);
hold on
ga = (-14/3)*(0.5).^n+(41/3)*(0.8).^n-9;
stem(n,ga,'r')
grid
xlabel('n')
ylabel('g[n]')
title('Problem 11.22')
print -dpsc2 p11_22.ps

% Probelm 11.31
figure(5)
clf
n = 0:20;
% (i)
subplot(311)
b = [1 -0.75 0];
a = [1 -1.5 2.25];
gi = dstep(b,a,n);
stem(n,gi)
grid
ylabel('g_i[n]');
title('Problem 11.31')
% (ii)
```

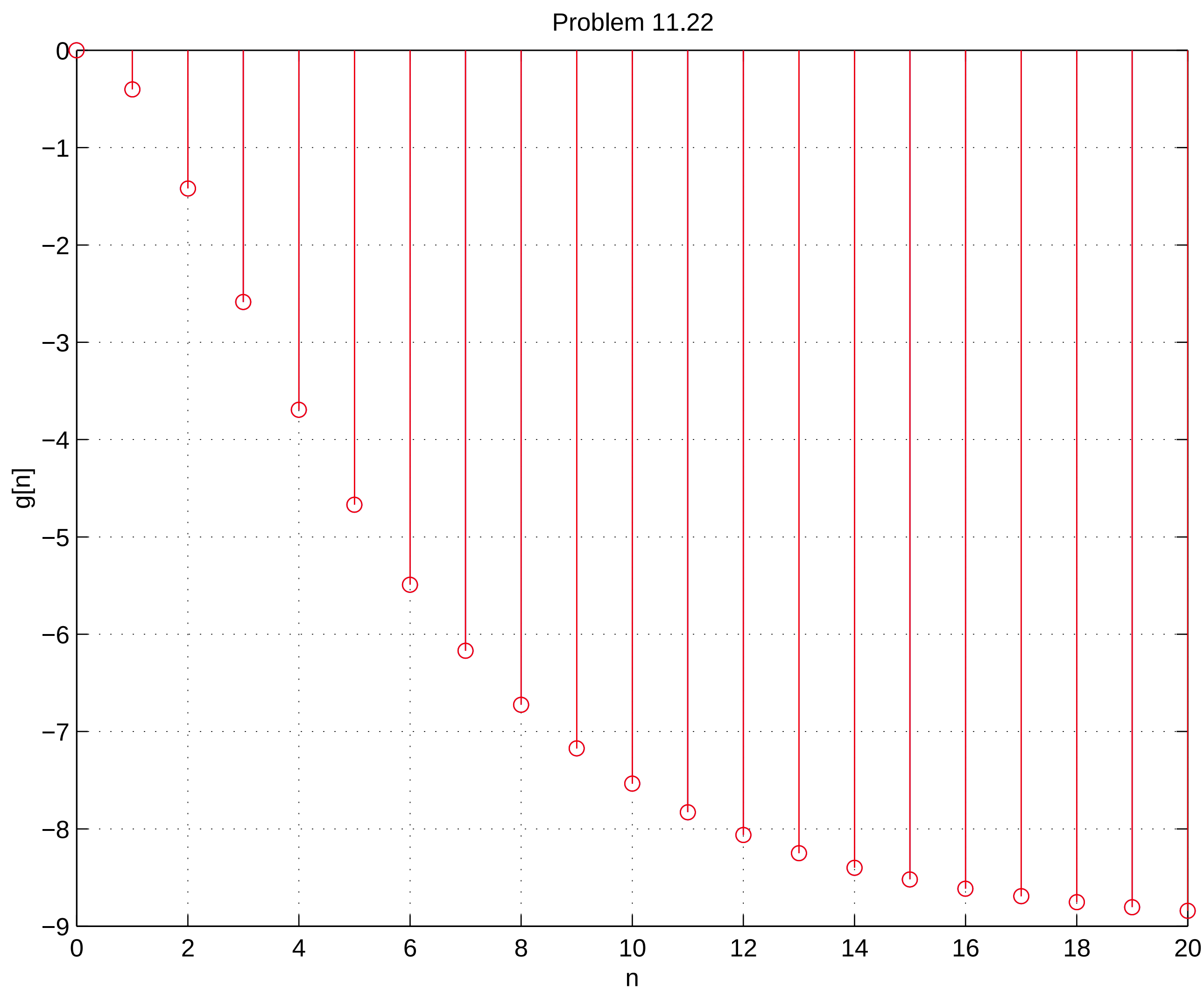
```
subplot(312)
b = [1 -0.5 0];
a = [1 -1 1];
gii = dstep(b,a,n);
stem(n,gii)
grid
ylabel('g_{ii}[n]');
% (iii)
subplot(313)
b = [1 -0.25 0];
a = [1 -0.5 0.25];
giii = dstep(b,a,n);
stem(n,giii)
grid
ylabel('g_{iii}[n]');
print -dpsc2 p11_31.ps

% Problem 11.47
figure(6);
clf
b = [0 1 3];
a = [1 0.3 0.02];
n = 0:20;
x = cos(pi*n);
y = filter(b,a,x);
stem(n,y);
hold on
yss = 2.7778*cos(pi*n);
stem(n,yss,'r');
grid
title('Problem 11.47')
xlabel('n')
ylabel('y[n], y_{ss}[n]')
print -dpsc2 p11_47.ps


% Problem 11.48
figure(7);
clf
b = [0 0 0 1 0];
a = conv([1 0 0.25],conv([1 -0.5],[1 -0.5]));
n = 0:20;
x = 12*cos(pi*n/2);
y = filter(b,a,x);
stem(n,y);
hold on
Hpiover2 = j/((j*j+0.25)*(j-0.5)^2);
yss = abs(Hpiover2)*12*cos(pi*n/2 + angle(Hpiover2));
stem(n,yss,'r');
grid
title('Problem 11.48')
xlabel('n')
ylabel('y[n], y_{ss}[n]')
print -dpsc2 p11_48.ps
```

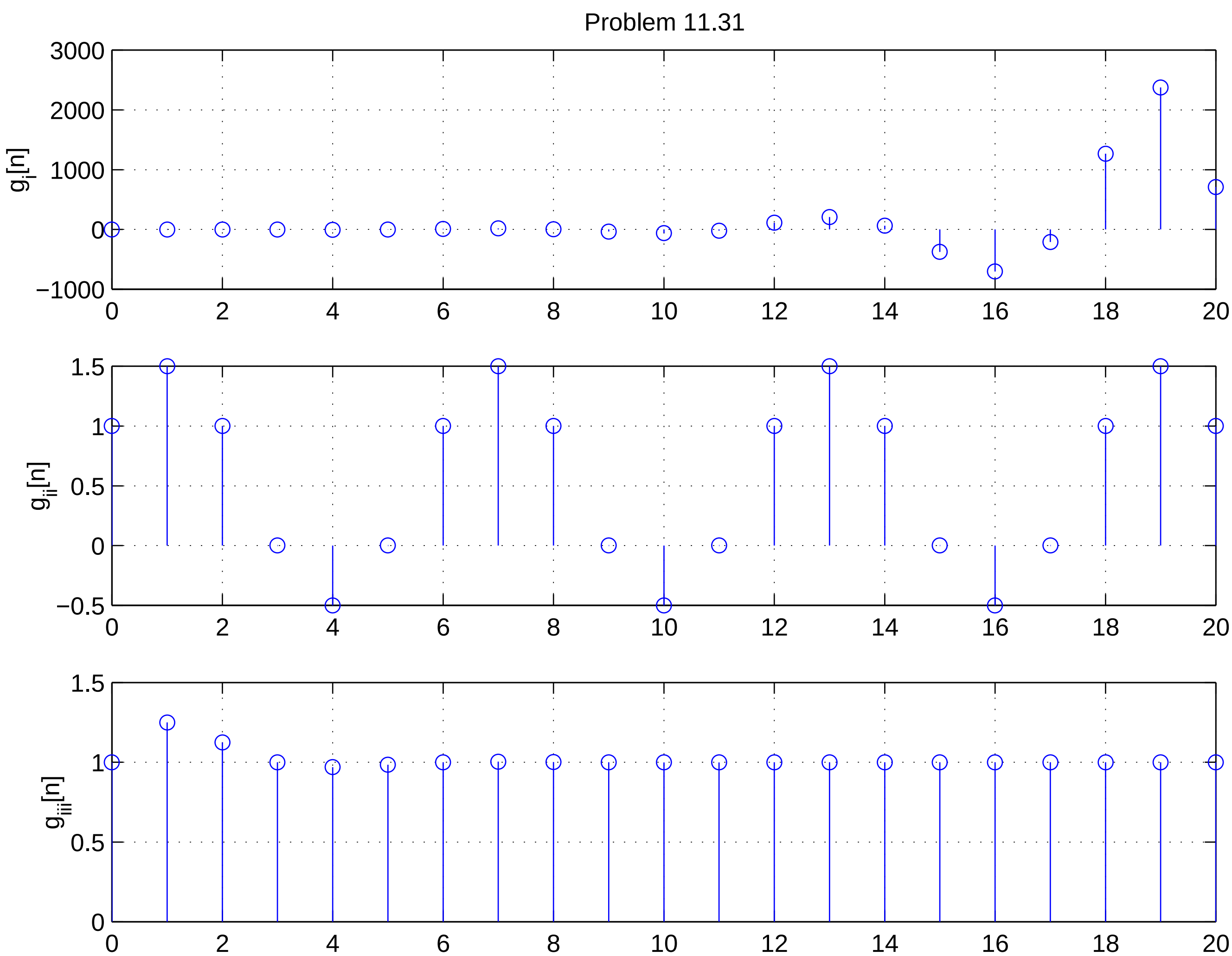


Problem 11.17
numerical
analytical
y[n]
n
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
2
4
6
8
10
12
14
16
18
20

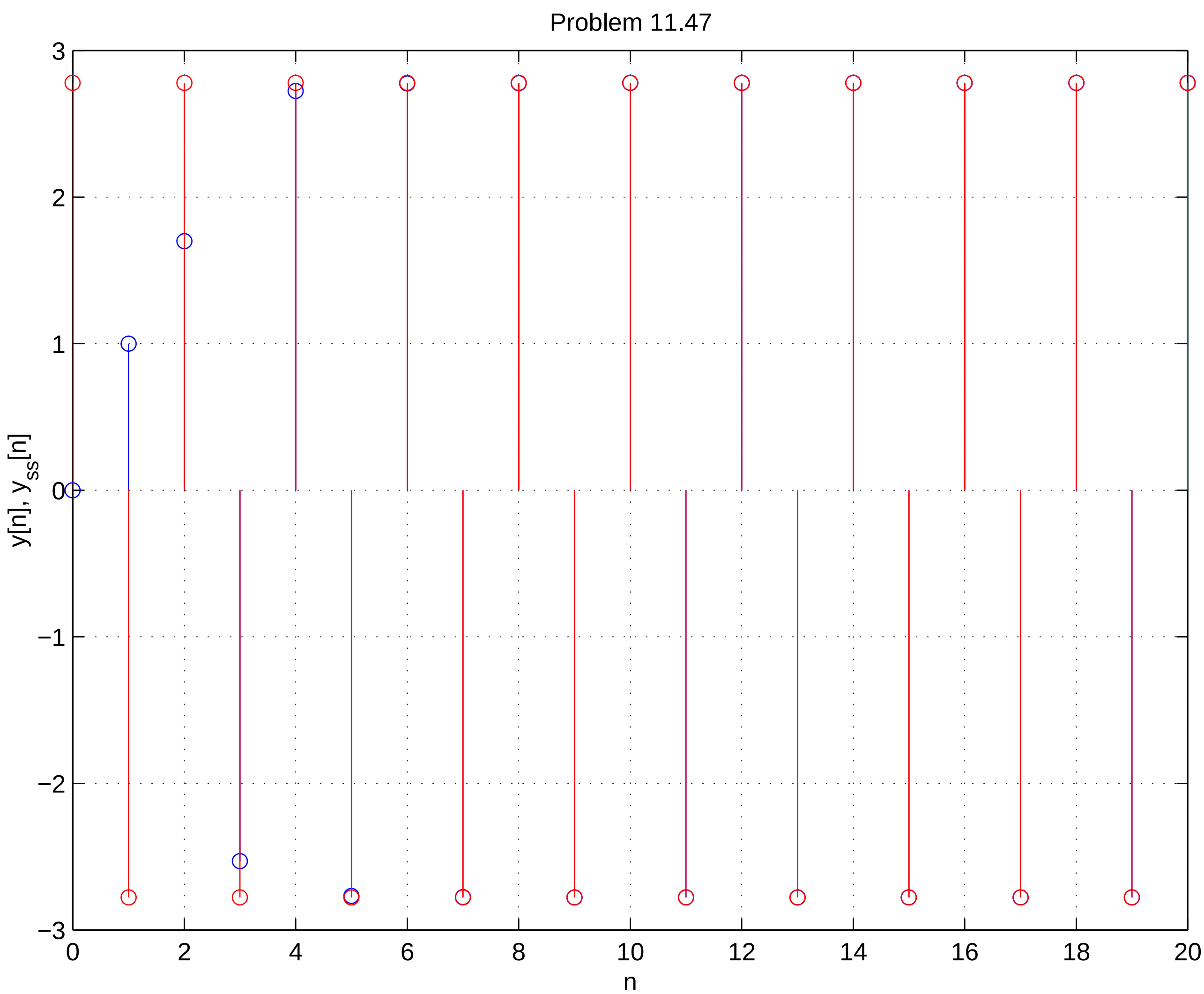

Problem 11.19 (a) (b) (c)
h[n]
4
2
0
−2
−4
0
2
4
6
8
10
12
14
16
18
20
g[n]
3
2
1
0
−1
x 10^5
10
5
0
y[n
n

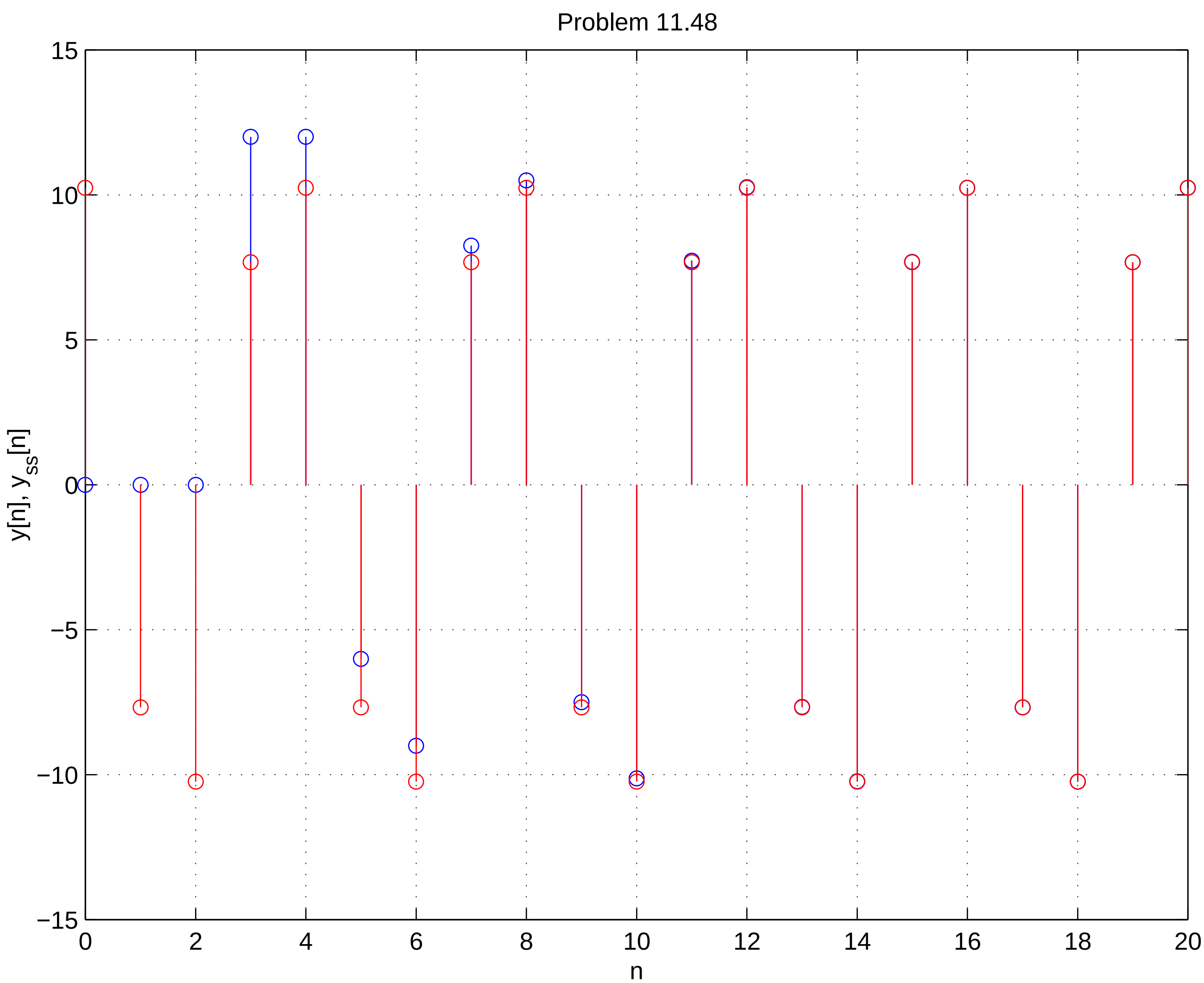

Problem 11.19 (d) (e)
1
0.5
0
−0.5
−1
y[n]
0
2
4
6
8
10
12
14
16
18
20
1
0.5
0
y[n]
0
2
4
6
8
10
12
14
16
18
20
x


Problem 11.22
g[n]
n
0
−1
−2
−3
−4
−5
−6
−7
−8
−9
0
2
4
6
8
10
12
14
16
18
20



Problem 11.31
g_i[n]
g_ii[n]
g_iii[n]
3000
2000
1000
0
−1000
1.5
1
0.5
0
−0.5
0
2
4
6
8
10
12
14
16
18
20


Problem 11.47
y[n], y_ss[n]
n
3
2
1
0
−1
−2
−3
0
2
4
6
8
10
12
14
16
18
20


Problem 11.48
y[n], y_ss[n]
n
15
10
5
0
−5
−10
−15
0
2
4
6
8
10
12
14
16
18
20